Role of magnetosheath force balance in regulating the dayside reconnection potential


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When the interplanetary magnetic field (IMF) is southward, most of the ionospheric potential is generated by merging between the IMF and the magnetospheric field. Typically, the ionospheric potential responds linearly to the magnitude of the southward IMF. However, when the IMF magnitude is large, the ionospheric potential saturates and it becomes relatively insensitive to further increases in the IMF magnitude. We present evidence from simulations that under purely southward IMF conditions, the value of the portion of the potential due to reconnection is controlled by the divergence of the magnetosheath flow, which determines the geoeffective length in the solar wind. Typically, the gradient in the plasma pressure controls the magnetosheath flow, so as the southward IMF increases in magnitude, the change in the magnetosheath force balance is negligible, the geoeffective length in the solar wind does not change, and the reconnection potential increases linearly with the magnitude of the IMF. However, when the IMF magnitude increases to the point where \( \mathbf{J} \times \mathbf{B} \) becomes the dominant force in the magnetosheath, further increases in IMF magnitude do affect the overall force balance, diverting more flow away from the merging line, decreasing the geoeffective length, and limiting the global merging rate. Thus magnetosheath force balance can be seen as a single organizing factor that regulates the geoeffective length in the solar wind for the entire range of solar wind parameters.


1. Introduction and Background

[2] As the solar wind flows around the magnetosphere, it transmits forces to the magnetosphere-ionosphere system that causes plasma to circulate. It is generally agreed that this convection of plasma is driven by magnetic reconnection and a viscous interaction, both of which were proposed at the beginning of the space age [Cowley, 1982]. The viscous interaction [Axford and Hines, 1961] is a mechanical transfer of momentum across the magnetopause that entrains magnetospheric plasma on closed field lines, which then moves antisunward. This motion sets up a return flow further inside the magnetosphere. These plasma motions exert a force on the closed field lines, which is then transmitted to the ionosphere, producing a mirror of the viscous cell convection. The flows in the ionosphere require a self-consistent electric field, with the flow velocity being the \( \mathbf{E} \times \mathbf{B} \) drift. The electric field is the negative gradient of the ionospheric potential that is generated by this process. The ionospheric potential created by the viscous interaction is generally estimated to be around 20–30 kV [Reiff et al., 1981; Cowley, 1982; Boyle et al., 1997; Sonnerup et al., 2001; Newell et al., 2008].

Also associated with this force exerted on the ionosphere there exists a self-consistent Birkeland current generated by the velocity shear in the viscous cells, which creates a magnetic shear in the same sense as the velocity shear. In fact, one can model the generation of the ionospheric potential from the viscous interaction for a uniform Pederson conductance as a Poisson equation for the potential, where the source term is the Birkeland current density consistent with the magnetic shear generated in the viscous cell divided by the conductance [e.g., Lyon et al., 2004].

[3] The second, and more important, process in generating the ionospheric potential is the merging between the solar wind magnetic field and the dayside magnetospheric field [Dungey, 1961]. This process produces up to about 200 kV of ionospheric potential, ten times the potential drop produced by the viscous interaction [Reiff and Luhmann, 1986; Boyle et al., 1997; Hairston et al., 2003; Ober et al., 2003].

In the simplest case, southward directed interplanetary magnetic field (IMF) merges with the northward field on the dayside. Magnetic flux is then transported to the nightside, where eventually it reconnects in the tail, powering phenomena such as substorms [e.g., Baker et al., 1996]. Dayside...
merging imposes a flow on the ionospheric plasma that is antisunward across the polar cap, as open field lines are dragged antisunward by the solar wind flow. There is a return flow at lower latitudes, which leads to a two-cell convection pattern in the ionosphere that for southward IMF is in the same sense as the viscous cell convection. The rate at which the convection is driven by reconnection depends on the rate at which flux crosses the merging line, and the rate at which flux crosses the merging line is just the potential drop along the merging line. That potential drop is communicated to the ionosphere along newly merged field lines. A schematic of the generation regions of the ionospheric potential and the resulting two-cell ionospheric convection is presented in Figure 1.

Figure 1. A schematic illustration of the sources of ionospheric convection.

[4] In the frame of Earth, the flowing solar wind has an electric field perpendicular to flow direction and the magnetic field. For purely southward IMF and flow along the Earth-Sun line (negative $X$ direction), this electric field has magnitude $VB_z$ and it points in the dawn-to-dusk (positive $Y$) direction. What should be the potential drop imposed by the solar wind along the merging line? The $Y$-extent of the magnetosphere from terminator to terminator is about 32 $R_E$ under nominal conditions [Sibeck et al., 1991]. For a steady southward IMF and a solar wind speed of 400 km/s, the solar wind electric field is 0.4 mV/m in the solar wind. If the electric field and potential drop under steady southward IMF conditions were imposed across the entire dayside merging region (which we take to be roughly the magnetopause sunward of termination) then we would expect a reconnection potential of 204 kV for every mV/m of electric field in the solar wind, or alternatively 81.5 kV for each nT of southward IMF. For an IMF with $B_z = -4$ nT, this would yield an ionospheric potential of 326 kV (not including the viscous contribution), and it has been known for a long time [e.g., Stern, 1973] that this value is much larger than indicated by observations. Thus not all of the solar wind magnetic flux that intersects the cross-section of the magnetosphere is actually delivered to the dayside merging line. The solar wind flow is diverted around the magnetosphere, and the forces that produce that diversion allow only a fraction of the impinging solar wind magnetic flux to actually reach the merging line.

[5] Figure 2 presents the ionospheric potential difference across the polar cap determined from DMSP F13 observations during polar cap passes during the main phase of moderate to large ($Dst < -85$ nT) magnetic storms. The DMSP spacecraft orbit at about 800 km with an orbital period of 101 min. As DMSP F13 crosses the polar ionosphere along (roughly) the dawn-dusk direction it measures the plasma flow velocity perpendicular to the spacecraft track [e.g., Hairston et al., 2003]. This allows one to integrate the

Figure 2. Ionospheric potentials derived from DMSP F13 data as a function of averaged solar wind $E \cdot (V_x B_z)$ during the DMSP F13 pass derived from 5-min OMNI data.
electric field associated with the plasma motion along the satellite track and determine the potential difference across the ionosphere along the DMSP trajectory over the 15 min that it takes the satellite to traverse the polar cap. Since DMSP F13 does not always reach the highest latitudes or sample the maximum and minimum potential regions in every pass, the potential calculated on a given pass is actually a lower limit during that period. Also if there are big variations in the potential within the 15 min it take to complete a polar cap pass, the calculated potential will not be accurate. In Figure 2 we have selected passes during the main phases of magnetic storms when the solar wind driver was fairly steady during the period so that we could compare the calculated potential with the average solar wind electric field \( \langle V \rangle \) during the period extending up to 10 min before the pass (5-min resolution OMNI data were used) and including the first 5 min after the pass. This window allows for potential timing errors and ionospheric response time to solar wind input [e.g., Lopez et al., 1999].

[6] What can be seen is that for low values of the solar wind electric field the potential does increase with the electric field. However, for solar wind electric field values greater than about 4.5 mV/m, the ionospheric potential does not increase further. This effect is the well-known saturation of the potential [e.g., Russell et al., 2000]. A line has been drawn as a rough envelope of the distribution of observations in Figure 2 for solar wind electric field values less than 4.5 mV/m. The slope of the line is roughly 46 kV/(mV/m) and the intercept is 15 kV. While this is not an actual regression line calculated from the data, it is a reasonable representation of the variation of the potential. Several things are evident from Figure 2. First, as discussed above, only a fraction of the solar wind potential across the magnetosphere is actually seen in the ionosphere. Thus some of the solar wind flow and the magnetic flux it carries must be diverted so that it does not reach the merging line, as discussed by Burke et al. [1999]. Second, when the merging potential goes to zero (when \( V_B = 0 \)), the residual potential (which we identify as the viscous potential) is about 15 kV, consistent with other estimates. Therefore if we subtract that viscous potential from the total ionospheric potential we should be left with the reconnection component of the ionospheric potential. Finally, there are two regimes for the ionospheric potential: a regime below 4.5 mV/m in which the ionospheric potential responds linearly to increases in the solar wind electric field and the saturation regime above 4.5 mV/m in which the ionospheric potential seems to be insensitive to further increases in the solar wind electric field.

[7] To examine these issues, we will utilize the Lyon-Fedder-Mobarry (LFM) global magneto-hydrodynamic simulation, which solves 3-D time-dependent, single fluid MHD equations in a modified spherical simulation domain that extends from \(+30 > X > -300 \, R_E\) and from \(+100 \) to \(-100 \, R_E\) in the \( Y \) and \( Z \) directions. The fixed computational grid places the highest resolution in regions of a priori interest to the solar wind–magnetosphere system. The magnetospheric MHD simulation takes solar wind input as the upstream boundary condition and couples to a simple 2-D height-integrated ionospheric model below the 3 \( R_E \) magnetospheric inner boundary condition. The details of the LFM code are discussed by Lyon et al. [2004]. We will demonstrate that the LFM code reproduces the essentials of the response of the ionospheric potential to changes in solar wind conditions. We will then present a comprehensive explanation of the behavior of the reconnection potential in both the linear and saturation regimes that is based on the role of magnetosheath force balance in regulating the flow into the dayside merging region. Specifically, we argue that the reconnection portion of the transpolar ionospheric potential is uniquely determined by the amount of flux that the solar wind flow carries across the merging line.

[8] The amount of flow across the merging line depends on the pattern of flow in the magnetosheath that is determined, in MHD, by the momentum equation, i.e., by force balance. Consider the flow of a time-independent solar wind around a magnetosphere that has come to equilibrium with the flow. The pattern of the flow around the magnetosphere is controlled by the magnetosheath forces, so we can write:

\[
\rho \frac{d\vec{V}}{dt} = \rho \vec{V} \cdot \nabla \vec{V} = -\nabla P + \vec{J} \times \vec{B}.
\]

The LHS is the divergence of the flow around the magnetosphere, and the RHS are the forces that cause the divergence. The resulting pattern of the flow determines the amount of solar wind magnetic flux that is transported to and across the dayside merging line. What we will demonstrate below is that magnetosheath force balance not only explains the behavior of the reconnection portion of the ionospheric potential and the relationship to solar wind parameters when the IMF has a small magnitude but also when it has a large magnitude and the saturation effect is seen. The basic argument is that when the IMF magnitude is small, the first term of the RHS of equation (1) (pressure) dominates the flow. Increasing the IMF magnitude does not change to force balance much, but it does increase the flux delivered to the merging line. But when the IMF is large and the second RHS term dominates the flow (magnetic), increasing the IMF increases the divergence of the flow, less flow reaches the merging line, and even though there is more flux per unit flow, the amount of flux per unit time delivered to the merging line does not appreciably change. While this paper focuses on simulations done with purely southward IMF, the line of argument holds for all orientations of the IMF, even though the efficiency of reconnection depends on the relative orientation of the magnetic fields. Thus the relationship between the solar wind parameters and the reconnection portion of the ionospheric potential can be explained for all solar wind values by force balance in the magnetosheath, which determines the geoeffective length, and the orientation of the solar wind field, which determines the projection of the solar wind electric field along the merging line.

[9] A general point should be made regarding our perspective on reconnection and the role of the solar wind as a boundary condition. Recent work [e.g., Borovsky et al., 2008; Cassak and Shay, 2007] has focused on the role of local plasma parameters in determining the local rate of reconnection. In particular Borovsky et al. [2008] modified the local parameters in a global MHD model and got agreement with the Cassak-Shay formula for the reconnection rate. Our view is that the local reconnection rate may be modified by local conditions, but that the global reconnection rate, and the reconnection-generated portion of the
ionospheric potential, will be determined by the magnitude of the solar wind potential drop that is imposed across the dayside merging line, and that this is controlled by the amount of solar wind flow that actually reaches the merging line. This statement is not inconsistent with the Borovsky et al. [2008] results. In that paper, the authors investigate the effect of a plasmaspheric plume reaching the magnetopause and present their results in Figure 14 of the paper. As predicted by the Cassak-Shay formula, the local reconnection rate drops in the region where the plume intersects the merging line, but in nearby sectors the reconnection rate increases. In fact, the integrated reconnection rate across the region plotted ($|Y| < 6 R_E$) does not change much when the plume hits. This would be consistent with the global reconnection rate being controlled by the total potential applied across the merging line by the solar wind (that is to say the total amount of magnetic flux crossing the merging line per unit time) while the local reconnection rate might be strongly affected by local conditions. Such local variations in the reconnection rate might produce important local variations in the ionospheric potential patterns and the distribution of the Region 1 currents, but they will not change the magnitude of the portion of the ionospheric potential due to reconnection.

2. LFM Ionospheric Potential

[10] Figure 3 presents the ionospheric potential from the LFM model as a function of the solar wind electric field. The runs were all done with a purely southward IMF, a solar wind speed of 400 km/s, a density of 5 cm$^{-3}$, a sound speed ($C_s$) of 40 km/s (about 116,000 K), zero dipole tilt, and a uniform ionosphere with a Pederson conductance of 5 mhos. To obtain the ionospheric potential, the simulation is run with a specified solar wind input until it reaches a steady state (even though there may be some variations or oscillations), then the total potential difference across the ionosphere (maximum value minus minimum value) is averaged over 2 h in the Northern Hemisphere. The error bars are the standard deviation of the averages, which are rather small. The four lowest values of the electric field correspond to $B_z$ values of $-1$ nT to $-4$ nT, and for those values the ionospheric potential responds linearly to the solar wind electric field. At high values of the solar wind electric field the potential exhibits the saturation effect. Also, the values of the potential are high compared to the observations presented in Figure 2 by a factor of about 1.5, which is a well-known feature of LFM [e.g., Lopez et al., 2009].

[11] In the linear regime the potential does not go to zero when the solar wind electric field goes to zero. We presume that the part of the potential produced by the dependence on the solar wind electric field is the reconnection component of the potential, and that the residual potential when the solar wind electric field is zero is the potential due to the viscous interaction. This gives a viscous potential of 33.9 kV, which, while larger than the commonly accepted value of about 20 kV, is on the order of upper estimates of the viscous potential value derived from data [Reiff et al., 1981] and consistent with the overall tendency of LFM to produce potentials that are high by a factor of 1.5. We can calculate the portion of the potential produced by reconnection by subtracting the viscous potential term, which according to Newell et al. [2008] is a function of solar wind density and velocity. We also expect that the viscous potential is a function of ionospheric conductivity. The viscous potential is generated by the magnetosheath flow entraining antisunward flow in the LLBL. This flow drags the field, and with the sunward return flow deeper inside the magnetosphere, a magnetic shear is produced (i.e., a current) with Region 1 polarity. This shear is propagated into the ionosphere, driving flow, for which there must be a self-consistent
electric field, which is the negative gradient of the viscous potential. But the stiffness of the ionosphere depends upon the conductivity. In the limit of infinite conductivity there would be no ionospheric flow at all. So the viscous potential should be inversely dependent on the ionospheric conductivity. Using our definition of the viscous potential, we find that this is in fact the case in LFM. Therefore the viscous potential must be calculated separately for each set of solar wind plasma parameters and ionospheric conductivity.

\[12\] Figure 4 shows the reconnection potential (total ionospheric potential minus the viscous potential) for three different sets of runs of the LFM code. The error bars on the potential values are omitted in Figure 4, but they are all on the order of a few kilovolts as seen in Figure 3. Two of the runs have the same solar wind velocities but different densities, and one has a higher ionospheric conductivity, but with the same velocity as the other runs and a density of 5 cm\(^{-3}\). All three sets of runs exhibit a region of linear behavior for low solar wind electric field, from which the viscous potential was determined and then subtracted from the total potential to yield the reconnection potential that is plotted. The viscous potentials for each run are listed in Table 1, and one can see that in fact the viscous potential does depend on the ionospheric conductivity. All three sets of runs also show evidence of the saturation of the potential at high values of the solar wind electric field. However, there are differences between the runs, such as the much lower potentials for the higher conductivity run. In the next two sections we will discuss generation of the potential in the linear regime and the saturation regime. We will show that the behavior of the potential as a function of solar wind input in both regimes can be understood with a single, comprehensive explanation, namely that the force balance in the magnetosheath determines how much solar wind flow (and magnetic flux) reaches the dayside merging line, and that determines the reconnection potential.

3. Explaining the Linear Regime

\[13\] Below about 4.5 mV/m, the potential is linearly related to the solar wind electric field. Since in these simulation runs we hold the solar wind velocity at a constant value of 400 km/s, this relationship is really a relationship between the potential and the magnitude of southward \(B_z\). As discussed above, a direct application of the solar wind potential drop across the magnetosphere would lead to ionospheric potentials that are well in excess of what is actually observed, so that only a fraction of the solar wind potential is actually imposed across the dayside merging region, even in the linear regime. All three sets of runs in Figure 4 exhibit a linear regime, which we can characterize through a linear fit to the results for the lowest magnitudes of \(B_z\). These results are summarized in Table 1 in which two sets of runs have the same uniform ionospheric conductance (5 mhos), and one set has a higher conductance (10 mhos). As described above, the intercept of the fit is what we interpret to be the viscous potential. We assume that the viscous potential is independent of the solar wind magnetic field, which may not be correct. But for the purposes of this

\[\text{Table 1. Linear Fits to the Total Ionospheric Potential as a Function of } V B_z\]

<table>
<thead>
<tr>
<th>(n) (cm(^{-3}))</th>
<th>(V) (km/s)</th>
<th>Conductivity (mhos)</th>
<th>Slope [kV/(mV/m)]</th>
<th>Intercept (kV)</th>
</tr>
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<tbody>
<tr>
<td>5</td>
<td>400</td>
<td>5</td>
<td>52.8</td>
<td>33.9</td>
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<tr>
<td>5</td>
<td>400</td>
<td>10</td>
<td>39.4</td>
<td>25.0</td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td>5</td>
<td>51.2</td>
<td>41.6</td>
</tr>
</tbody>
</table>
Figure 5. A perspective view of the equatorial plane for a simulation run with standard conditions ($V=400\text{ km/s}$, $n=5\text{ cm}^{-3}$, $C_s=40\text{ km/s}$, $\Sigma_p=5\text{ mhos}$) and a 5 nT purely southward IMF. The yellow lines are magnetic field lines showing that the merging region extends across the entire dayside magnetopause.

paper this is our operating assumption, and we have not encountered any behaviors in the runs studied so far that are inconsistent with this assumption.

[14] One possible interpretation of the slope is that it is related to the length of the merging line. However, closer examination of the simulations indicates that this is not the case, since the maximum slope indicates an effective merging line of just over $8R_E$. In Figure 5 the two field lines near the dawn and dusk terminators are clearly recently merged field lines. Thus in the simulation the merging line is much longer than $8R_E$; it extends across the entire dayside. Therefore the length given by the slope must be the extent in the $Y$-direction of the solar wind flow that actually intersects the merging line, in other words the geoeffective length in solar wind. Since the plasma flow lines are equipotentials, the potential drop in the solar wind that would be applied to the merging region would be the $Y$-extent of the solar wind flow that intersects the merging line times the solar wind electric field. Solar wind streamlines will be deflected in the magnetosheath by forces in the magnetosheath in order to divert the flow around the magnetospheric obstacle, thus limiting the actual amount of solar wind potential that is applied across the merging region. Burke et al. [1999] came to the same conclusion based on DE-2 observations.

[15] We can test this idea directly by examining the solar wind flow streamlines to see if they intersect the merging region. A flow line that intersects the merging region would be deflected out of $X-Y$ plane as the forces at the merging line deflect the plasma flow. We can see this in Figure 6, which shows flow lines initiated in the solar wind near the equatorial plane ($Z=0.2R_E$) at various $Y$ positions ($\pm Y=10$, $5.2$, $4.6$, $4$, $3.7$, $2.9$, and $2.5R_E$) for a single time step in the LFM simulation run with standard conditions ($V=400\text{ km/s}$, $n=5\text{ cm}^{-3}$, $C_s=40\text{ km/s}$, $\Sigma_p=5\text{ mhos}$) and a purely southward IMF of 5 nT. The flow lines that do not intersect the merging region simply divert around the magnetosphere in the equatorial plane, while flow lines that intersect the merging region are significantly deflected out of the equatorial plane. Figure 6 (bottom) is very similar to Figure 1 of Burke et al. [1999]. The projection into the solar wind of the flow lines that intersect the merging region is not always symmetric, and from time step to time step there are some variations in the potential, but the results are quite consistent. In this case, the $Y$-projection of the solar wind potential imposed across the merging region extends from $-4.1R_E$ to $4.5R_E$, for a total $Y$-extent of $8.6R_E$. This is twice the length determined by Burke et al. [1999], but the ionospheric potentials here are twice those reported by Burke et al. [1999]. If we multiply this length ($8.6R_E$) times the solar wind electric field (2 mV/m), we obtain a reconnection potential of $109.6\text{ kV} \pm 5\text{ kV}$, where we estimate the possible error in the determination of the $Y$-extent is about $\pm 0.4R_E$ (interpolated flow line start positions are examined in steps of $0.1R_E$). This is almost exactly equal to the reconnection component of the potential we determine from the ionospheric results by taking the total value of the ionospheric potential in that time step (144 kV) and subtracting the viscous potential (33.9 kV).

[16] Using this method of identifying flow lines that intersect the merging region we have calculated the $Y$-extent of the solar wind flow that intersects the merging line for all of the runs with the plasma and ionospheric conditions in the case above. From these lengths we have calculated the reconnection potential for each case by multiplying that $Y$-extent with the solar wind electric field. Figure 7 shows the potential calculated in the LFM ionosphere as well as the potential calculated by determining the reconnection potential from the flow lines and adding to that the viscous potential from Table 1. The curves are essentially on top of each other. This is quite remarkable. The ionospheric potential is calculated by solving current continuity, which yields a simple 2-D Poisson equation $\Sigma_p \nabla^2 \Phi = J_{||}$, where
the Birkeland current source term is simply the field-aligned current at the inner edge of the MHD boundary. The source term does not directly depend on magnetospheric electric fields. From the total potential we subtract the viscous potential (itself derived from the ionospheric solutions) to arrive at the reconnection potential. On the other hand, the streamline analysis method for determining the reconnection potential just follows the solar wind flow to see how much reaches the merging region, so it does not directly depend on either the magnetospheric currents or magnetospheric electric field. Yet both methods, through completely independent means, end up with the same reconnection potential.

Figure 6. Color coded density in the equatorial plane with plasma flow streamlines (in yellow) that begin in the solar wind near the equatorial plane \((Z = 0.2 \, R_E)\) at \(± Y = 10, 5.2, 4.6, 4, 3.7, 2.9, \) and \(2.5 \, R_E\) for a single time step in the LFM simulation run with \(V = 400 \, \text{km/s}, \, n = 5 \, \text{cm}^{-3}, \, C_s = 40 \, \text{km/s}, \, \Sigma_p = 5 \, \text{mhos}, \) and \(B_z = -5 \, \text{nT}.\) (top) The view from sunward of the magnetosphere, just above the equatorial plane, and (bottom) a view from above the magnetosphere.

While these results are from just one of the sets of runs, the other runs in Figure 4 also have the same behavior. As best as can be determined, the reconnection potential is indeed determined by the magnitude of the solar wind potential that is applied across the dayside merging region. This result allows us to understand the dependence of the potential on the solar wind electric field in the linear regime. In the linear regime, where \(B_z\) is small and negative, the \(Y\)-extent of the solar wind flow that intersects the merging line is the same \((8.6 \, R_E)\) for all values of the solar wind \(B_z,\) as long as the solar wind plasma parameters and ionospheric conductivity remain unchanged. Thus as the magnitude of the southward IMF increases, the
reconnection potential increases proportionally, which results in the linear dependence of the reconnection potential on the solar wind electric field (for fixed solar wind speed).

The discussion presented above assumes that there is no potential drop between the dayside merging line and the ionosphere. However, Merkin et al. [2003, 2005] and Lopez et al. [2009] reported significant differences (tens of kV) between magnetospheric voltages and ionospheric voltages, although the differences did not impact the results of those studies. The exact nature of these differences is still being investigated; however, some of the differences seems be due to the interpolation in the analysis software, CISM-DX [Wiltberger et al., 2005], which results in path-dependent potential differences, whereas when one looks at the actual electric field on the LFM grid and that is used to calculate potential differences, the potential differences are path-independent. In fact, it has been determined that using the LFM electric field on the grid (not interpolations) there are no substantial potential differences between the dayside merging line potential and the ionospheric potential between the foot points of the field lines that connect to the edge of the merging region [Ouellette, 2009]. And since what concerns this study is the relationship between the potential applied across the merging line and the ionospheric potential, the work of Ouellette [2009] indicates that those potentials should be the same, which is the result we have obtained here as presented in Figure 7. Finally, the remarkable correspondence between the reconnection potential determined from the streamline analysis and from the ionospheric calculation, which are completely independent calculation methods, argues in favor of our interpretation.

Next, let us consider the density dependence. Table 1 shows a slight difference between the slopes, hence the Y-extent of the solar wind flow that intersects the respective merging lines, and this difference is evident in Figure 4 where the reconnection potential in the largest electric field still in the linear regime (4 mV/m) is larger for smaller density (and the difference is significantly different from the few kV standard deviation for the averages). A larger solar wind density will result in a larger solar wind pressure and thus a smaller magnetosphere. Given that the magnetosphere shape is basically self-similar for a variety of pressures [Sibeck et al., 1991], we would expect that the size of the merging region should scale roughly like \( n^{-1/6} \) (Chapman-Ferraro scaling). If we assume that the flow patterns are essentially invariant under this scaling, then the Y-extent of the solar wind flow that intersects the merging line in the linear regime should also follow Chapman-Ferraro scaling. This assumption was used by Siscoe et al. [2002a, 2002b] in the derivation of the functional dependence of the reconnection potential at the magnetopause upon solar wind parameters (equation (2) in that paper).

The ratio of the slopes in Table 1 for the 5 cm\(^{-3}\) and 8 cm\(^{-3}\) runs (all other conditions equal) is 1.03 while the ratio of the densities to the 1/6 power is 1.08. Alternatively, if we include in the fit the reconnection potential of all of the points up to 4 mV/m, then the ratio of the slopes is 1.11. And if we divide the reconnection potential of the 5 cm\(^{-3}\) runs by the reconnection potential of the 8 cm\(^{-3}\) runs, and calculate the average of the ratios of the reconnection potentials for all of the points with solar wind electric field from 0.4 mV/m to 4 mV/m, we get an average ratio of 1.06. Thus the density dependence in the slopes in the linear regime seems to be somewhat consistent with what one would expect from the change in the actual size of the magnetosphere, and hence the length of the merging region, according to Chapman-Ferraro scaling. However, this issue deserves more study because there may be nonlinear effects.

Figure 7. LFM ionospheric potential for a set of simulation runs with \( V = 400 \) km/s, \( n = 5 \) cm\(^{-3}\), \( C_s = 40 \) km/s, \( \Sigma_p = 5 \) mhos with the average potential calculated directly in the ionosphere, as well the sum of the viscous potential plus the solar wind potential imposed on the merging line as calculated from the geoeffective length determined from the analysis of flow streamlines that intersect the merging region. The two match well across the entire range of solar wind electric field.
effects that come into play because of the dependence of the viscous potential on the density. We note that the ratio of the intercepts for the two 5 mho sets of runs in Table 1, which we interpret to be the value of the viscous potential, is about the same as the square root of the ratio of the densities, just as predicted by the formula of Newell et al. [2008]. A change in the viscous potential can produce a change in the magnitude and/or distribution of Region 1 currents. This may result in a slightly different magnetospheric shape that could influence the force balance and plasma flow in the magnetosheath. Changing the magnetosheath flow pattern would in turn change the amount of flux crossing the merging line, thus changing the reconnection potential. And so the dependence of the reconnection potential on density in the linear regime may be more complicated and include more subtleties than what one might deduce from simple arguments.

In addition to dependence of the reconnection potential on density, there is a very significant dependence of the reconnection potential on ionospheric conductivity. In the linear regime this manifests itself as a lower slope for a larger conductivity. Given our previous interpretation we would expect that the $Y$-extent of the solar wind flow that intersects the merging line to be smaller for larger ionospheric conductivity. Figure 8, which presents results from a run identical to the run in Figure 6 except that $\Sigma_p = 10$ mhos, and $B_z = -5$ nT, demonstrates that this is in fact the case. The two LFM runs are identical except for the ionospheric conductivity, and a smaller $Y$-extent

Figure 8. Color-coded density in the equatorial plane with plasma flow streamlines (in yellow) that begin in the solar wind near the equatorial plane ($Z = 0.2 R_E$) at $Y = 10, 5.2, 4.6, 4, 3.7, 2.9,$ and $2.5 R_E$ for a single time step in the LFM simulation run with $V = 400$ km/s, $n = 5$ cm$^{-3}$, $C_s = 40$ km/s, $\Sigma_p = 10$ mhos, and $B_z = -5$ nT. (top) View from sunward of the magnetosphere, just above the equatorial plane, and (bottom) view from above the magnetosphere.
of solar wind flow lines intersects the merging region in the 10 mho case compared to the 5 mho case. Thus a higher conductivity run has a smaller geoeffective length in the solar wind. The reason for this is evident if we compare the lower panels of Figures 6 and 8. The 10 mho magnetosphere is blunter, the subsolar point is closer to Earth, and the magnetosheath is thicker. This configurational difference is due to the larger Region 1 current in the 10 mho case, even though the potential is smaller. In the higher conductivity case the solar wind flow has to traverse a greater distance across a wider magnetosheath and hence experiences a greater deflection due to the forces in the magnetosheath that are diverting the flow around the wider magnetosphere. Thus a smaller Y-extent of solar wind flow is able to reach the dayside merging region, resulting in a smaller geoeffective length in the solar wind and a lower reconnection potential. However, as in the set of 5 mho runs, the linear behavior is due to a constant Y-extent of flow that reaches the dayside merging region for a given ionospheric conductivity, so that as the solar wind $B_z$ becomes more negative, the reconnection potential increases. This is essentially the same result and interpretation discussed by Merkin et al. [2005].

The potential also has a velocity dependence that is consistent with our perspective on the role of solar wind force balance in the magnetosheath. All of the simulation results presented above vary the (purely) southward IMF to get the dependence of the reconnection potential on the solar wind electric field. And in the rest of this paper, we will focus primarily on southward IMF and its variation, with some discussion of the dependence on ionospheric conductivity. However, we briefly consider here the fact that for a given a solar wind magnetic field, we can increase the electric field by increasing the velocity. Under such circumstances, how would we expect the potential to react?

As the solar wind velocity increases, so does the dynamic pressure, which results in a smaller magnetosphere. The magnetosphere size will scale like $V^{-1/3}$ (Chapman-Ferraro scaling due to pressure balance with the dipole) so the geoeffective length in the solar wind will scale that way as well. But that is not the whole story. A larger solar wind speed will produce a larger plasma pressure in the magnetosheath. In the linear regime where the IMF magnitude is small, the solar wind has a large magnetosonic Mach number and the compression ratio across the bow shock is near its maximum value of 4 so that there can be little additional compression across the shock when the solar wind speed increases. The additional kinetic energy of the flow must go into plasma heating, creating a larger plasma pressure. The larger pressure forces will produce a greater divergence of the magnetosheath flow, leading to a smaller geoeffective length in the solar wind and a smaller reconnection potential for a given solar wind electric field.

[24] For the $V = 400$ km/s runs, the geoeffective length in the linear regime (as seen in Figure 3) is 8.3 $R_E$. We have conducted the same analysis for the linear regime for runs with the same density and ionospheric conductivity but with $V = 600$ km/s and $V = 800$ km/s. The results to the linear fits of the potential as a function of $V B_z$ for low values of southward $B_z$ are given in Table 2. Two things are evident. First, the viscous potential increases dramatically with solar wind speed. This is consistent with the Newell et al. [2008] result. Second, the geoeffective length decreases with solar wind velocity. Since the reconnection potential should be that length times $V B_z$, we find that the reconnection potential should be rather insensitive solar wind speed. Moreover, the reduction in the geoeffective length with increasing $V$ is beyond what one would expect from the Chapman-Ferraro scaling. The geoeffective length scales roughly as the inverse of the solar wind speed, so it would appear that the reconnection potential is fairly insensitive to increases in solar wind speed in the linear regime. This dependence of the geoeffective length on the solar wind speed is easily explained by considering force balance in the magnetosheath. A faster solar wind will have more energy/mass and hence the magnetosheath will have a larger pressure/mass compared to a slower solar wind case. If the pressure gradient force dominates the evolution of the magnetosheath flow, a faster solar wind with the larger magnetosheath pressure and pressure gradient will produce a more rapid diversion of the flow around the magnetosphere compared to a slower solar wind, and thus a smaller amount of flow will reach the merging line. Thus the geoeffective length in the solar wind of fast solar wind will be smaller than for the slow solar wind (all other things equal).

[25] We note that the total ionospheric potential does increase with $V$. For $B_z = -5$ nT, and with $V = 400$, 600, 800 km/s, the potential is 142, 177, and 224 kV, respectively. This increase in the total potential is driven by the increase in the value of the viscous potential with increasing velocity. We note that our result is very similar to the result obtained by Boyle et al. [1997] from observations, where it was determined that the best fit to the potential in the linear regime contained two terms: a term proportional to the IMF magnitude times a clock angle-dependent term (with no $V$ dependence) that must represent the portion of the potential produced by reconnection, and a term that depends on $V^2$ (but no $B_z$ dependence), the same dependence that Newell et al. [2008] found for the viscous potential. Thus it is possible that during periods of high-speed, small IMF magnitude solar wind the viscous potential is larger than the reconnection potential. This limiting of the geoeffective length does not produce the saturation effect because if you increase the magnitude of the southward IMF when the IMF is small, you still get an increase in the ionospheric potential. However, by further considering the nature of force balance in the magnetosheath, but now with large magnetic fields, we can arrive at an explanation for the saturation effect that is based on the same physics discussed above.

### 4. Behavior of the Potential in the Nonlinear Regime

[26] Initial studies about solar wind-magnetosheath-ionosphere (SW-M-I) coupling showed that the transpolar...
potential was linearly related to the interplanetary electric field (IEF) [Reiff and Luhmann, 1986; Boyle et al., 1997], except for large values of IEF (>4.0 mV/m), when the linear model would over predict the transpolar potential. Evidence for the saturation of the ionospheric potential has been provided by assimilated mapping of ionospheric electrodynamics (AMIE) reconstructions of ionospheric potentials [Russell et al., 2000, 2001; Liemohn et al., 2002], high-latitude radar reconstruction of synoptic-scale ionospheric flows [Shepherd et al., 2002], and DMSP drift meter measurements [Hirston et al., 2003; Ober et al., 2003]. The transpolar potential rarely reaches 200 kV and almost never much beyond that value, as illustrated in Figure 2. Moreover, ionospheric potential saturation has been identified in various global MHD models [Raeder et al., 2001; Siscoe et al., 2002a, 2002b], including the LFM simulation code as discussed here and elsewhere [Merkin et al., 2003, 2005].

As is evident in Figure 2, the saturation effect becomes noticeable when the solar wind electric field exceeds 4.5 mV/m, consistent with the result of Nagatsuma [2002]. For the typical solar wind speed of 400 km/s, this translates to \( B_z = -11.25 \) nT. Above that value, the ionospheric potential becomes less sensitive to continued increases in the solar wind electric field (that is to say \( B_z < -11.25 \) nT) in what we refer to as the nonlinear regime. In the section above, we focused on the linear regime and showed that the reconnection potential determined in the ionosphere was equal to the amount of solar wind potential applied to the dayside merging line. Figure 7 shows that this is essentially the case in the nonlinear regime as well. Above 4.5 mV/m, the \( Y \)-extent of the solar wind flow that intersects the dayside merging region (which is constant below 4.5 mV/m) shrinks as the solar wind magnetic field magnitude increases. This can readily be seen in Figure 9, which is the same as Figure 6, except that the run was done with \( B_z = -20 \) nT instead of \(-5 \) nT. Comparing Figure 6 and Figure 9, it is obvious that some of the flow lines that intersect the merging region in the \(-5 \) nT case do not do so in the \(-20 \) nT case. Thus the geoeffective length in the solar wind in the \(-20 \) nT case has shrunk relative to the \(-5 \) nT, so that the increase in the magnitude of the southward magnetic field is offset by a shrinking of the length of solar wind that applies potential to the dayside merging line. Lavraud and Borovsky [2008] came to the same conclusion that for very large IMF values the geoeffective length shrinks, using a different global MHD code.

Three other points should be noted about the behavior of the reconnection potential in the nonlinear regime. First, the conductivity dependence is clear, with lower ionospheric potential for higher conductivity, and the onset of a noticeable saturation effect in the 10 mho runs is at a somewhat lower value of the solar wind electric field compared to the 5 mho runs. Second, in the nonlinear regime the density dependence is reversed compared to the linear regime, with higher density producing a larger potential. There is evidence for these two behaviors in the DMSP data [Ober et al., 2003]. Finally, the linear regime extends to larger solar wind electric field for larger densities since the saturation effect is not evident in the \( 8 \) cm \(^{-3} \) runs until \( VB_z \) is larger that \( 6 \) mV/m. On the other hand, for all of the runs in Figure 4, the reconnection potential calculated from the solar wind flow is the same as the ionospheric potential minus the viscous potential. This strongly suggests there is a single explanation for the origin of the reconnection potential that unites both the linear and nonlinear regimes, namely, that the magnetosheath flow that controls the amount of solar wind potential can be imposed on the merging region.

A single explanation for the control of the reconnection potential by the magnetosheath flow for all values of the IMF reduces the question of what produces the saturation effect to understanding why the magnetosheath flow pattern changes as it does for large IMF. Clearly, the physics is well represented by MHD; otherwise these effects would not be evident in the LFM code or the other MHD codes that have documented similar behavior. The transition between the linear and nonlinear behavior occurs for large \( B_z \) in the solar wind, and Figures 4 and 7 are essentially the behavior of the potential as a function of \( B_z \) since the solar wind speed is 400 km/s in all cases. Thus the nonlinear regime is a low magnetosonic Mach number flow [e.g., Ridley, 2005; Borovsky and Denton, 2006; Lavraud and Borovsky, 2008], and under those conditions the bow shock plays an important role in modulating solar wind–magnetosphere coupling [Lopez et al., 2004]. And it is the role of the bow shock in the coupling that we now examine.

5. Role of the Bow Shock

Most references to the bow shock in the context of solar wind–magnetosphere coupling assume a high magnetosonic Mach number shock with a steady compression ratio of 4 of the plasma and the transverse components of the field, which provides a linear relationship between upstream and magnetosheath parameters [Lopez et al., 2004]. However, the large IMF values in the nonlinear regime produce low Mach number flows. Under such conditions the compression ratio across the bow shock becomes sensitive to variations in the Mach number, and the ratio of solar wind to magnetosheath parameters becomes nonlinear, unlike the linear relationship that exists during high Mach number flow. This leads to variability in solar wind–magnetosphere coupling as the solar wind magnetosonic speed changes, so that density variations can modulate the coupling [Lopez et al., 2004; Lavraud and Borovsky, 2008].

For high magnetosonic Mach number shocks, fluid stresses are the primary force flowing the flow at the shock; the \( J \times B \) force plays a minor role. As the Alfvén speed increases and the magnetosonic Mach number decreases, the magnetic field, through the \( J \times B \) force, comes to dominate the shock interaction. Similarly, for a low Alfvén Mach number flow, the \( J \times B \) force dominates the momentum equation in the magnetosheath, controlling the magnetosheath flow. The situation is illustrated in Figure 10 for the case of purely southward IMF. For simplicity we assume a plane perpendicular MHD shock, for which the solution is a longstanding textbook problem [e.g., Boyd and Sanderson, 1969]. The compression ratio, \( r \), has an asymptotic value of 4 for large magnetosonic Mach number and it goes to 1 as the Mach number goes to 1. The compression of the magnetic field across the shock means that a magnetic shear is present. The magnetic shear across the bow shock is a simple consequence of conservation of mass, momentum, and energy as expressed in the Rankine–Hugoniot equation. This shear means that a current is flowing on the shock. The current per
unit length is given by Ampere’s Law (where $B_z$ is the field in the solar wind and $r$ is the compression ratio across the shock, which depends on the magneto-sonic Mach number):

$$J_y = \frac{\Delta B_z}{\mu_0} = \frac{B_z}{\mu_0} (r - 1) \quad (2)$$

This current produces an outward force slowing the solar wind. The ratio of the outward force per unit area to the dynamic pressure of the solar wind is

$$\frac{J_y B_z}{\rho V^2/2} = \frac{B_z^2 (r^2 - 1)}{\mu_0 \rho V^2} = \frac{(r^2 - 1)}{M_a^2} \quad (3)$$

where $B_z$ is the average field across the shock and $M_a$ is the Alfvén Mach number in the solar wind. This outward $\mathbf{J} \times \mathbf{B}$

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{image1.png}
\caption{Color-coded density in the equatorial plane with plasma flow streamlines (in yellow) that begin in the solar wind near the equatorial plane ($Z = 0.2 R_E$) at $Y = 10, 5.2, 4.6, 4, 3.7, 2.9, \text{ and } 2.5 R_E$ for a single time step in the LFM simulation run with $V = 400$ km/s, $n = 5 \text{ cm}^{-3}$, $C_s = 40$ km/s, $\Sigma_p = 5 \text{ mhos}$, and $B_z = -20$ nT. (top) View from sunward of the magnetosphere, just above the equatorial plane, and (bottom) view from above the magnetosphere.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{image2.png}
\caption{Compression of the solar wind across the sub-solar bow shock (solid vertical line) for purely southward IMF, and the resulting bow shock current and the $\mathbf{J} \times \mathbf{B}$ force exerted on the solar wind.}
\end{figure}
force is a fundamental fact due to the compression of the field across the shock. It is also important to note in Figure 10 that $\mathbf{J} \cdot \mathbf{E} < 0$ across the shock. The shock is a dynamo converting solar wind mechanical energy into magnetic energy as negative work is done on the solar wind flow by the $\mathbf{J} \times \mathbf{B}$ force. This point was made by Siebert and Siscoe [2002], who identified the bow shock current as the dynamo that powers reconnection at the magnetopause.

[32] Under the conditions when the $\mathbf{J} \times \mathbf{B}$ force is larger than 50% of the force exerted by the solar wind, the Chapman-Ferraro current cannot be the primary contributor to force balance with the solar wind since most of the force on the solar wind is exerted at the bow shock. The Siscoe-Hill model for saturation [Siscoe et al., 2002a, 2002b] is explicit on this issue to the point of claiming that the Chapman-Ferraro current disappears and that the Region 1 current flowing on open field lines usurps the role of the Chapman-Ferraro current in standing off the solar wind. This Region 1 current closes the large bow shock current in the ionosphere, and the strength of the magnetic perturbation associated with the Region 1 current on the dayside was used by Siscoe et al. [2002a] to determine under what conditions the generation of the ionospheric potential would be in the nonlinear regime.

[33] Siscoe et al. [2002b] traced current streamlines from the bow shock to demonstrate that Region 1 current actually does connect the ionosphere to the bow shock. Current streamlines, however, do not necessarily provide the overall view of the distribution of the current. Figure 11 presents the distribution of current density (or equivalently, curl $\mathbf{B}$) across the equatorial plane for a strongly driven magnetosphere simulation run. What can be seen is that the current points toward the Earth across the entire dawnside magnetosheath and it points away from the Earth across the entire duskside. This is Region 1 polarity. Moving the display plane to other $Z$ values shows the same thing. Throughout the entire magnetosheath volume there is a magnetic shear and associated current with Region 1 polarity that is generated at the bow shock. Some of this current is communicated along the magnetic field to the ionosphere, where it would appear on open field lines. Lopez et al. [2008] demonstrated the existence of such Region 1 polarity current on open field lines in the polar cap using low-altitude DMSP observations during a period of large southward IMF. Therefore we are in agreement with Siscoe et al. [2002b] that current with Region 1 polarity flows from the ionosphere into the magnetosheath, where it closes some of the current present due the magnetic shear generated at the bow shock. Since solar wind conditions that produce ionospheric potential saturation will generate a large bow shock current, the majority of the force balance will be at the bow shock, the Chapman-Ferraro current on the magnetopause will be much smaller than the bow shock current, and there will likely be a greater closure of bow shock current through the ionosphere compared to what closes through the magnetopause currents because the magnetopause current is much smaller than the bow shock current.

6. Force Balance in the Magnetosheath and the Reconnection Potential

[34] When the magnetosphere is in the nonlinear regime with low Mach number solar wind flow (both Alfvénic and magnetosonic), a large magnetic shear is transmitted across
the magnetosheath, and some part of the associated current closes in the ionosphere. This provides a direct link from the bow shock to the ionosphere. But this large magnetic shear also has profound consequences for force balance and dynamics in the magnetosheath. Figure 12 presents a schematic of the equatorial plane showing the magnetosphere under southward IMF, including the closure of the bow shock current. Given that the current is almost everywhere along the radial direction and the solar wind magnetic field is southward, the $J \times B$ force in the sheath is essentially azimuthal. This force redirects the magnetosheath flow slowed by the bow shock so that it can flow around the magnetosphere. As pointed out above, the shock is a generator with $J \cdot \vec{E} < 0$. In Figure 12 we see that $\vec{V} \cdot J \times \vec{B} = J \cdot \vec{E} > 0$, so the plasma in the magnetosheath is a load and the magnetic energy that had been extracted from the flow at the shock is converted back into mechanical energy as the flow is accelerated back up to solar wind speeds. The ionosphere is also a load for the bow shock current, with the current in the polar cap dissipating some of the energy extracted from the flow at the bow shock.

[35] Under normal circumstances the dominant force in the magnetosheath is the plasma pressure gradient. However, when the solar wind is a low Alfvén Mach number flow, the magnetic energy is a sizable fraction of the solar wind flow energy and in the magnetosheath the magnetic force may become the dominant force. Lavraud and Borovsky [2008] present an extensive study of the effects of low Mach number solar wind flow, particularly the effects on magnetosheath forces and configuration. The dominance of magnetic pressure can lead to very asymmetric magnetospheric configurations. The acceleration of magnetosheath flow to speeds in excess of the solar wind speed can occur, and the size and location of Kelvin-Helmoltz vortices can be affected. As mentioned above, they noted that the geoeffective length of the solar wind flow is much reduced in a magnetically dominated magnetosheath. However, that study did not directly compare the potential implied by the geoeffective length to the actual potential in the ionosphere.

[36] The indicator of the magnetically dominated magnetosheath is when $\beta$, the plasma energy density divided by magnetic energy density, is less than one. In order to estimate the conditions when $\beta < 1$, we have calculated $\beta$ just downstream of a plane, perpendicular 1-D MHD shock wave. Figure 13 shows the value of the solar wind electric field ($VB_z$ for $V = 400 \text{ km/s}$) at which point for a given density the value of $\beta$ just downstream of the shock drops to one. Two different solar wind sound speeds have been plotted, and there is a difference. This is because the compression ratio across the shock is dependent on the sound speed, as is the plasma pressure. This is a point to which we will return later.

[37] Figure 13 provides key information for understanding why the ionospheric potential saturates, at what point the saturation effect becomes evident, and how that point depends on solar wind conditions. The lines in Figure 13 show (for a given set of solar wind plasma conditions) the value of the solar wind electric field ($VB_z$) for which the 1-D calculation indicates that $\beta = 1$ just downstream of the shock. For the $C_s = 40 \text{ km/s}$ curve, we see that for a density of $5 \text{ cm}^{-3}$, the $\beta = 1$ condition occurs for a solar wind electric field of $4.7 \text{ mV/m}$ (that is to say at $B_z = -11.7 \text{ nT}$ since $V$ is held constant at $400 \text{ km/s}$), which is quite close to the transition point seen in the data and in the LFM simulation runs. For larger solar wind southward IMF (hence larger electric field), $\beta$ will just get smaller as the magnetic energy density and the $J \times B$ force increase. The increasing magnetic force will produce an increasing azimuthal deflection of the magnetosheath flow (as described by equation (1)), reducing the amount of flow that reaches the magnetopause and the merging region. When the magnetic forces become dominant in the magnetosheath, the amount of deflection of the magnetosheath flow limiting the amount of solar wind that reaches the merging region is proportional to the solar wind magnetic field. A greater solar wind magnetic field will produce a greater deflection in the flow, shortening the geoeffective length and reducing the amount of solar wind potential imposed on the merging region, thus limiting the reconnection potential. It is also important to note that saturation effect will not occur if one simply increases the solar wind velocity to increase the solar wind electric field above $4.5 \text{ mV/m}$. Even though the electric field increases, the $\beta$ in the magnetosheath will actually get larger and the role of the $J \times B$ force will diminish as the velocity increases. However, as discussed above, as the magnetosheath plasma pressure increases due to increasing the solar wind kinetic energy. 

Figure 12. Schematic showing the closure of the bow shock current and the $J \times B$ forces in the magnetosheath.
energy density, the geoeffective length in the solar wind will decrease, so the actual value of the potential will depend in a complicated fashion on solar wind parameters. But there will be no saturation effect because increasing the magnitude of the southward IMF under such circumstances will result in a linearly related increase in the reconnection potential.

The transition to a magnetically dominated magnetosheath explains why saturation occurs and provides a unified description of how the reconnection potential is generated. For low values of the IMF, the solar wind flow in the magnetosheath is diverted around the magnetosphere by the pressure gradient in the solar wind, reducing Y-extent of the solar wind that reaches the merging region. This is why the reconnection potential is not simply the voltage drop across the magnetosphere. If the solar wind density increases, the pressure gradients will increase but so will the inertia of the solar wind so the flow pattern will not change much. The reconnection potential, however, decreases slightly (Chapman-Ferraro scaling) because the scale size of the magnetosphere and the merging line is smaller under larger solar wind pressure. And there may be other nonlinear effects due to the increased viscous potential. Increasing the southward component of the IMF when the IMF is not large has a big effect on the ionospheric potential because the potential drop imposed on the merging region will increase. In the linear regime the increasing IMF magnitude has no effect on the geoeffective length in the solar wind because that length is determined by the magnetosheath force balance, which is dominated by the plasma pressure. However, once the magnetic field increases to the point that $\beta = 1$, the magnetic forces begin to dominate. In that case, when the IMF magnitude increases, the force on the magnetosheath flow also increases without an increase in the inertia of the flow. Thus the diversion of the flow increases, reducing the geoeffective length in the solar wind. This cancels out much of the increase in the potential that one would otherwise expect by an increase in the magnitude of the southward IMF.

For larger values of density, the transition to a magnetically dominated magnetosheath flow takes place for larger values of the southward IMF because a larger solar wind density means that there will be a larger pressure gradient in the magnetosheath. From Figure 13 we would expect that for the $8 \, \text{cm}^{-3}$ run the transition would occur at $6 \, \text{mV/m}$. Inspecting Figure 3, we see that in the $8 \, \text{cm}^{-3}$ run the reconnection potential continues to behave linearly up to $6 \, \text{mV/m}$, as predicted. Since the magnetosphere stays within the linear regime up through larger IMF values, the saturation value of the potential is larger. Changing the ionospheric conductivity shifts the point at which the magnetosheath become magnetically dominated by changing the shape of the obstacle. As can be seen in Figure 4, the effect of magnetic forces in the magnetosheath becomes evident in the $4 \, \text{mV/m}$ run for the $10 \, \text{mho}$ case, whereas the effect of magnetic forces becomes evident in the $5 \, \text{mV/m}$ run for the $5 \, \text{mho}$ case.

Figure 14 presents the value of $\beta$ in the equatorial plane for the three sets of conditions presented in Figure 4 with $B_z = -10 \, \text{nT}$ ($4 \, \text{mV/m}$). We can clearly see that the magnetosheath is still dominated by plasma pressure in the $8 \, \text{cm}^{-3}$ run, whereas in the $5 \, \text{cm}^{-3}$ run the magnetic pressure is becoming larger than the plasma pressure, except right at the subsolar point (which is really the spot where Figure 13 applies). However, in the $10 \, \text{mho}$ case, the region of the magnetosheath in which the magnetic pressure is larger that the plasma pressure is considerably thicker since the magnetopause has eroded further earthward. The greater erosion in the $10 \, \text{mho}$ case is a function of the larger Region 1 current [Sibeck et al., 1991; Wiltberger et al., 2003]. The net result is that net deflection of the solar wind flow away from the merging region is larger in the $10 \, \text{mho}$ case than the $5 \, \text{mho}$ case and the magnetosphere
enters the magnetically dominated, nonlinear regime for a somewhat lower IMF.

7. Comparison to Other Models of Saturation

Several models have been presented to explain the saturation of the ionospheric potential. Raeder et al. [2001], with a different global MHD code using a case study of the Bastille storm, suggested that saturation happens because during strong driving, while the Region 1 currents weaken the Earth’s magnetic field at the dayside magnetopause for low latitudes, it also strengthens the field at high latitudes. Such a magnetosphere with “shoulders” develops dimples that fill with plasma modifying the reconnection rate at the magnetopause. Merkin et al. [2003, 2005] argued that the saturation is due to the change in ionospheric conductance that reshapes the magnetopause, changing the magnetosheath flow and reducing the dayside reconnection potential. Those papers also pointed out the dependence of the width of the magnetosheath on the ionospheric conductance, which is an important factor in the ionospheric control of the reconnection potential. Winglee et al. [2002] proposed an explanation based on ionospheric outflow using a multifluid global model. This study showed that the mass loading from the ionosphere into the magnetosphere lowers the transpolar potential. Ridley [2007] and Kivelson and Ridley [2008] developed an alternative model, based on Alfvén wings (regions of slow flow and limited electric field). The limited electric field in the Alfvén wings results in a limited reconnection potential. In this model, neither reconnection efficiency nor magnetospheric geometry plays a role in producing the saturation of the potential.

Siscoe et al. [2002a, 2002b] have presented a model for saturation based on earlier work by Hill et al. [1976] and Hill [1984]. In the Siscoe-Hill model, transpolar saturation is a result of a feedback created by Region 1 currents, which generates an opposing magnetic field to the Earth’s magnetic field at the magnetopause, limiting the rate of reconnection. It is the weakening of the dayside field consistent with an increase in the Region 1 currents that is responsible for magnetopause erosion, but there is a limit to which the dayside field can be weakened. This feedback mechanism would thus limit the amount of Region 1 current, and hence limit the polar cap potential. They calculate this current and from that they calculate the value of the ionospheric potential at which saturation effects become evident. Siscoe et al. [2002b], using the ISM MHD code, extended this model and argued that the mechanism for saturation was related to Region 1 current system closing on the bow shock taking over of the role of the Chapman-Ferraro currents in providing the primary force balance with the solar wind. Since those currents also close (in part) through the ionosphere, limiting the Region 1 current will also limit the ionospheric potential. In this view saturation happens as a result of the solar wind ram pressure limiting the amount to which the total Region 1 current can grow. Siscoe et al. [2004] compared various models for saturation (weakened field, dimple formation, change in the magnetosheath flow, and Region 1 force balance) and found that “the role that the Region 1 current system plays turns out to be virtually indistinguishable between them, at least at the present level of the mechanism formulation.”

It is worth pointing out that the force balance between the terrestrial obstacle and the solar wind is not primarily linked to the existence of a Region 1 current system. In the Chapman-Ferraro picture, there are no Region 1 currents and the high-latitude ionosphere does not enter into the force balance. If the magnetosphere is closed but filled with plasma, there are two ways that the momentum is transferred: there is a direct pressure on the dayside ionosphere and a magnetostatic interaction between the Earth’s dipole (below the insulating atmosphere) and the external current systems. Even if the magnetosphere is open with high-latitude Birkeland currents, the ionosphere is unlikely to play much of a role in the overall momentum balance. The
force exerted on the ionosphere is the local $J \times B$. Take an ionosphere with a uniform Pedersen conductance, $\Sigma_p$. The $J \times B$ force can be written as $\Sigma_p E \times B$, or $\Sigma_p V_\perp B_\perp$, where $V_\perp$ is the ionospheric circulation velocity. The integral of that quantity over the ionosphere in this case is zero, since we have a closed circulation pattern.

[44] The model that we propose here shares many similarities with the model outlined by Siscoe et al. [2002a, 2002b], and perhaps even more so with the ideas presented by Merkin et al. [2005] and Lavraud and Borovsky [2008]. In general, the solar wind flow is diverted away from the merging line, thus limiting the amount of solar wind potential that is applied to the merging region. During periods of high Mach number flow, the magnetosheath force balance is dominated by the plasma pressure gradient and changes in the solar wind magnetic field do not impact the magnetosheath flow. This leads to a linear dependence of the reconnection potential on the magnitude of the southward IMF. As the solar wind magnetic field increases in magnitude, the magnetic forces in the magnetosheath become more important in diverting the solar wind flow and eventually dominate the force balance and flow. This leads to the saturation effect since increasing the magnitude of the southward IMF also increases the magnetic force diverting the flow. We agree with the Siscoe-Hill model that the bow shock current becomes dynamically important during periods of low Mach number flow; however, we argue that the onset of the saturation effect is controlled by the magnetosheath force balance and not by a critical value of the Region 1/bow shock current.

[45] Our position is that the magnitude of the IMF that will produce the onset of saturation is the value of the IMF that leads to the dominance of the magnetic pressure in the magnetosheath. This value is easy to calculate for a plane MHD shock, though the actual distribution in the magnetosheath of magnetic and plasma energy density is quite dependent on the actual shape of the magnetosphere and hence the ionospheric conductance [e.g., Merkin et al., 2003, 2005]. This introduces the ionospheric conductivity as a factor that regulates the value of the saturation potential, since the conductivity controls the amount of flaring and magnetopause erosion for a given potential. We note, however, that the same effect regulates the dependence of the potential on conductivity in the linear regime. We also note that Figure 13 provides evidence that the value of the IMF at which the $\beta = 1$ condition in the magnetosheath is satisfied is dependent on the solar wind sound speed. The Alfvén wing model [Ridley, 2007; Kivelson and Ridley, 2008] should have no dependence on the solar wind sound speed. Similarly, the Siscoe-Hill model as currently formulated has no dependence on the solar wind sound speed. On the other hand, the sound speed does affect the bow shock jump conditions, and thus the dynamics of the magnetosheath flow upon which our understanding of the regulation of the reconnection potential rests.

8. A Role for Solar Wind Temperature

[46] Figure 13 shows that solar wind temperature (expressed here in terms of the sound speed) can modify the transition to a magnetically dominated magnetosheath, requiring a larger IMF value to create a magnetically dominated magnetosheath and hence (in our view) to produce the saturation effect. This is because the larger upstream thermal energy of the plasma will produce a correspondingly larger downstream plasma pressure. On the other hand, the larger plasma pressure means that the pressure gradient forces will be correspondingly larger, pushing more flow away from the merging region. This would imply a lower reconnection potential.

[47] Figure 15 presents the ionospheric potential for a run with a sound speed of 120 km/s and a flow speed of 400 km/s (error bars for all points are a few kV, similar to Figure 3). When compared with Figure 3, which had a sound speed
of 40 km/s, we see that the slope of the linear regime is 44.7 kV/(mV/m), which gives us a geoeffective length of $7 \, R_E$. An analysis of LFM results for various values of the IMF shows that some solar wind plasma flow streamlines that intersect the merging line in the 40 km/s sound speed case do not do so in the 120 km/s sound speed case. This confirms that the geoeffective length is less for a hot solar wind compared to a cold solar wind, all other things being equal. We also note that the viscous potential that we calculate is larger for the hotter solar wind. Further study is required to understand why this is the case and what is the functional dependence of the viscous potential on the solar wind temperature.

Figure 15 also shows less of a saturation effect in the range of solar wind electric field for which we have run the simulation, though the nonlinear dependence is present. Figure 13 suggests that the saturation effect would become evident for $\nabla B_z$ greater than 6 mV/m, and in fact in going from 6 mV/m to 8 mV/m the reconnection potential increased only 45 kV, roughly half the increase that one would predict from the linear fit to the small solar wind electric field values. Below 6 mV/m the linear fit is a good predictor of the potential, always within 10 kV of the actual value. But above 6 mV/m the linear fit and the simulated potentials diverge, so that for 8 mV/m the predicted value of the reconnection potential is 357 kV, while the simulation generates only 299 kV. Figure 16 shows the difference between the estimated total potential from the linear fit and the actual potential across the simulated ionosphere. Above 6 mV/m the difference between the ionospheric potential and the linear potential grows steadily. This is the operational definition of potential saturation, and it becomes evident at the point where the simple plane MHD shock calculation indicates that the magnetic energy density is greater than the plasma energy density just downstream of the shock. Thus when the simulation enters the regime of the magnetically dominated magnetosheath, the ionospheric potential begins to saturate.

9. Variable Value of the Region 1 Current at Saturation

The value of the reconnection potential where the 120 km/s sound speed simulation transitions to the nonlinear regime is 245 kV (289 kV total potential, including the viscous potential). The corresponding value of the reconnection potential in 40 km/s sound speed runs is 272 kV (306 kV total potential). The Region 1 current should scale as $C_s^2 \, \Sigma_p$ [Siscoe et al., 2002a], so the saturation value of the Region 1 current in LFM with $C_s = 40$ km/s will be larger than the saturation value of the Region 1 current in LFM with $C_s = 120$ km/s. Thus it appears that there is not a singular value of the amount Region 1 current for a given solar wind dynamic pressure that is associated with saturation as the Siscoe-Hill model posits. Moreover, from equation (2) we can calculate the ratio of the $\mathbf{J} \times \mathbf{B}$ force to the solar wind dynamic pressure for the conditions that yield $\beta < 1$ just downstream of the shock, which we identify as an indicator for the onset of saturation. For a shock with a solar wind flow speed of 400 km/s and $C_s = 40$ km/s the $\mathbf{J} \times \mathbf{B}$ force is 63% of the solar wind dynamic pressure at the subsolar point while for $C_s = 120$ km/s it is 56%. This indicates that the ionospheric potential and the Region 1 currents saturate well below the value that would produce the $\mathbf{J} \times \mathbf{B}$ force needed to stand off the solar wind. Also, the ratio of the $\mathbf{J} \times \mathbf{B}$ force to the solar wind dynamic pressure is different for the two cases because, while at Alfvén speeds are the same, the compression ratios are different (see equation (3)). This shows that there is no singular value of the bow shock current or the force it produces when the saturation of the potential becomes evident.
This conclusion is reinforced by the results of Wilder et al. [2008], who found that the potential saturates for strongly northward IMF. Siscoe et al. [2002a] specifically address the question of northward IMF and state that it should take \(\cdots\)a larger \(E_{in}\) to reach saturation...\(\cdots\) compared to southward IMF because for northward IMF reconnection is less efficient and it produces a weaker Birkeland current system. But Wilder et al. [2008] found value of the electric field at which saturation is evident in for northward IMF is \(4 \text{ mV/m}\), which corresponds to \(B_z\) around \(10 \text{ nT}\). This is the value of the IMF where the magnetosheath is becoming magnetically dominated, and also the value of the IMF where saturation effects for southward IMF begin. However, the saturation potentials are much lower for northward IMF as compared to saturation under southward IMF. Thus saturation occurs for both northward and southward IMF when the magnetosheath becomes magnetically dominated, not when a critical value of Birkeland current is produced by the system. This is exactly what one expects if the control of the potential imposed at the merging line is best understood by considering the force balance in the magnetosheath. The force balance mechanism and the transition to saturation should be valid for all IMF orientations, and the Wilder et al. [2008] result is consistent with this line of argument.

The result is the same when one considers large IMF \(B_z\). We have unambiguous evidence from both data and simulations that the ionospheric potential saturates for large IMF \(B_z\), [Mitchell et al., 2010] and the saturation effect occurs when the magnetosheath becomes magnetically dominated, that is to say, at the same value of the IMF magnitude for which saturation begins for southward and northward IMF, given the identical solar wind speeds and densities. Moreover, the results of Hu et al. [2009] show that the ionospheric potential depends on the IMF clock angle as expected using the formula of Kan and Lee [1979]. Thus the value of the saturation potential and the total Region 1 is clock angle dependent, whereas the current flowing on the bow shock does not depend on clock angle for an IMF. Thus for any clock angle purely transverse to the flow (and hence a basic scaling of the potential as determined by the clock angle and the Kan-Lee formula), all of the arguments made about force balance in the magnetosheath for purely southward IMF apply. Therefore we argue that the physical mechanism originally suggested by Hill et al. [1976], then expanded upon by Siscoe et al. [2002a, 2002b] is not the mechanism that is responsible for saturation since the transition to a saturated potential occurs for a wide and variable range of total ionospheric potential, total Region 1 current, and total bow shock current.

All of the evidence points in favor of our paradigm that the force balance in the magnetosheath regulates the reconnection potential. One could posit a contrary view, that the cause and effect is reversed and that the magnetosheath flow reorganizes itself to response to changes in the reconnection rate along the merging line. In that case one is left with several basic questions. Why should the merging rate suddenly change when the IMF \(B_z\) gets to a critical value? Why is that critical value the value of \(B_z\) that makes the magnetosheath become magnetically dominated? Why does the same thing happen for other IMF orientations at different values of the potential, hence different reconnection rates and different values of the Region 1 current? Why does the geoeffective length and hence the reconnection rate go down faster than one would expect from Chapman-Ferraro scaling as the solar wind velocity increases? In fact, by trying to reverse cause and effect, so that the reconnection rate is the cause and the magnetosheath force balance and flow pattern is the effect, the critical issue of why the reconnection rate changes remains a mystery. On the other hand, by accepting cause (magnetosheath force balance and the effect on the flow) and effect (a modulated geoeffective length that translates into the modulated global reconnection rate) as proposed in this paper, one arrives at a coherent conceptual framework for understanding the generation of the reconnection potential.

This coherent framework explains both the behavior of the reconnection potential in the linear regime as well as in the nonlinear, saturation regime. Across the entire range of solar wind parameters, the geoeffective length is controlled by the forces acting on the flow in the magnetosheath. In the linear regime the magnetosheath flow is controlled by the pressure gradient, so increasing the IMF increases the flux crossing the merging line and the reconnection potential, and this happens for all IMF orientations. In the nonlinear (or saturated) regime, the magnetosheath flow is controlled by the \(J \times B\) force, so increasing the IMF decreases the geoeffective length and neither the flux crossing the merging line nor the reconnection potential increase, and this happens for all IMF orientation. Thus the transition to saturation is determined not by a critical value of the integrated Birkeland current system or a critical value of the bow shock current or the force it exerts on the solar wind. The transition to a saturated potential is governed by the solar wind and ionospheric conditions that result in a magnetically dominated magnetosheath, irrespective of the IMF clock angle.

### 10. Summary

We present a unified framework for understanding the control of the reconnection potential imposed by the solar wind on the dayside merging region by considering the role of force balance in the magnetosheath. While this paper deals with the situation where the IMF is southward (which is the easiest to understand), the arguments presented here apply to other clock angles. For all values of the IMF, only a fraction of the available solar wind potential across the magnetosphere is actually applied to the dayside merging region. This is because plasma flow lines, which are equipotentials, are diverted away from the merging regions by forces in the magnetosheath. Thus only a limited extent of solar wind flow actually reaches the merging line. Whatever affects this geoeffective length will modulate the amount of solar wind flux that reaches the merging line, thus modulating the portion of the ionospheric potential that is produced by reconnection. When the IMF is weak, the force that diverts the magnetosheath flow is the pressure gradient force. Changes in the solar wind magnetic field magnitude do not significantly alter the force balance in the magnetosheath and so do not change the geoeffective length of solar wind flow that is applied to the merging line. Therefore when the magnetosheath force balance is dominated by the plasma pressure, the portion of the ionospheric potential due to reconnection increases linearly as the solar wind magnetic...
field, magnitude increases. When the IMF is large and the solar wind Mach number is small, the $J \times B$ force in the magnetosheath becomes comparable to or even larger that the pressure gradient force. Under such conditions, increasing the magnitude of the solar wind magnetic field does impact the magnetosheath flow, causing a greater diversion of that flow and a reduction in the geoeffective length of solar wind flow that is applied to the merging line. This leads to the saturation of the ionospheric potential since an increase in the solar wind electric field is countered by a decrease in the geoeffective length. The dependence of the ionospheric potential on ionospheric conductivity, and solar wind density, velocity, and temperature (sound speed) can all be understood by considering the impact on the dynamics of the magnetosheath flow and the geoeffective length as well.

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Hill, T. W., A. J. Dessler, and R. A. Wolf (1976), Mercury and Mars: The solar wind electric field is countered by a decrease in the geoeffective length. This leads to the saturation of the ionospheric potential since an increase in the solar wind electric field is countered by a decrease in the geoeffective length. The dependence of the ionospheric potential on ionospheric conductivity, and solar wind density, velocity, and temperature (sound speed) can all be understood by considering the impact on the dynamics of the magnetosheath flow and the geoeffective length as well.

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