Effects of Surface Exchange Coefficients and Turbulence Length Scales on the Intensity and Structure of Numerically Simulated Hurricanes

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ABSTRACT

Using numerical simulations, this study examines the sensitivity of hurricane intensity and structure to changes in the surface exchange coefficients and to changes in the length scales of a turbulence parameterization. Compared to other recent articles on the topic, this study uses higher vertical resolution, more values for the turbulence length scales, a different initial environment (including higher sea surface temperature), a broader specification of surface exchange coefficients, a more realistic microphysics scheme, and a set of three-dimensional simulations. The primary conclusions from a recent study by Bryan and Rotunno are all upheld: maximum intensity is strongly affected by the horizontal turbulence length scale $l_h$ but not by the vertical turbulence length scale $l_v$, and the ratio of surface exchange coefficients for enthalpy and momentum, $C_k/C_d$, has less effect on maximum wind speed than suggested by an often-cited theoretical model. The model output is further evaluated against various metrics of hurricane intensity and structure from recent observational studies, including maximum wind speed, minimum pressure, surface wind–pressure relationships, height of maximum wind, and surface inflow angle. The model settings $l_h \approx 1000$ m, $l_v \approx 50$ m, and $C_k/C_d \approx 0.5$ produce the most reasonable match to the observational studies. This article also reconciles a recent controversy about the likely value of $C_k/C_d$ in high wind speeds by noting that simulations in a study by Emanuel used relatively large horizontal diffusion and low sea surface temperature. The model in this study can produce category 5 hurricanes with $C_k/C_d$ as low as 0.25.

1. Introduction

Atmospheric conditions near the surface of intense (category 4–5) tropical cyclones (TCs) remain difficult to characterize. Although the magnitude of peak wind speed in TCs near the surface is fairly well documented from surface observing stations and aircraft reconnaissance (via remote sensing technology and/or dropsondes), other properties like the functional form of wind gusts with height and the variance of wind speed in time and space are less certain because of the hazards involved in data collection (e.g., Harper et al. 2010). These uncertainties create difficulties for certain disciplines—perhaps most notable being the field of civil engineering, which needs to design structures that can withstand TC winds (e.g., bridges near coastlines, offshore facilities for oil and gas companies, and proposed offshore wind-energy turbines).

The interface between the ocean and the atmosphere is also difficult to characterize. This uncertainty is often cited as holding back progress of numerical forecast model accuracy (e.g., the majority report to the National Oceanic and Atmospheric Administration 2006). One particular topic of interest for model development is the values of surface exchange coefficients for enthalpy $C_k$ and momentum $C_d$ (i.e., the drag coefficient) at hurricane wind speeds, which are used to determine fluxes of heat and momentum between the ocean and atmosphere in numerical models. Maximum hurricane wind speed has long been hypothesized to be proportional to $(C_k/C_d)^{1/2}$ (see, e.g., the review by Emanuel 2004). However, the exact functional form between wind speed and $C_k/C_d$ has been called into question recently based on some numerical simulations (e.g., Bryan and Rotunno 2009b; Montgomery et al. 2010) and a new analytical model (Emanuel and Rotunno 2011).

Numerical models offer promise as a tool that can decrease uncertainty about atmospheric conditions in

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intense hurricanes because they can provide dynamically consistent profiles of temperature, moisture, and winds that are difficult to obtain observationally. However, small-scale turbulence in hurricane boundary layers cannot be resolved in numerical models unless grid spacing is less than approximately 100 m (e.g., Rotunno et al. 2009); because such resolution is onerously expensive, the vast majority of modeling studies of hurricanes use turbulence parameterization schemes. The uncertainties in near-surface winds in TCs mentioned above make it difficult to design and evaluate these turbulence schemes, although some recent studies have been making progress by using special research datasets (e.g., Nolan et al. 2009a,b).

Numerical models have also been shown recently to produce clearly unnatural results in some cases that seem to be related to parameters in surface layer and/or turbulence parameterizations. For example, Hausman (2001) and Persing and Montgomery (2003) showed, using two different codes, that numerical models could produce unnaturally strong TCs (e.g., maximum wind speed > 140 m s\(^{-1}\) and minimum pressure < 850 mb). The roles of horizontal turbulent exchange (i.e., mixing) between the eye and eyewall were considered by both Hausman (2001) and Persing and Montgomery (2003) as a potential explanation for the unnaturally large intensities in their model simulations. In a comprehensive examination of model settings, Bryan and Rotunno (2009b) showed that the horizontal turbulence parameterization in an axisymmetric model had a strong effect on maximum intensity, even more so than settings in the vertical turbulence (i.e., boundary layer) scheme and/or settings in the surface flux parameterizations.

This study examines a large set of numerical simulations to gain a better understanding of how TC intensity and structure are affected by changes in certain settings of the turbulence and surface-layer schemes. The primary methodology is to focus only on storms at maximum intensity, which removes the need to study the complex process of TC intensification, and allows for comparison with some steady-state theories for TC intensity. Another goal of this study is to determine which model settings yield realistic values of relatively well-observed properties of strong TCs such as maximum winds, minimum pressure, the surface wind–pressure relationship, height of maximum winds, and near-surface inflow angle. To test the generality of the results, two different sets of initial conditions are considered, one of which has a higher sea surface temperature than another recent modeling studies. The results herein also reconcile a recent controversy about a conclusion by Emanuel (1995) concerning the likely value of \(C_k/C_d\) in intense TCs.

### 2. Methodology

**a. Model setup**

This study uses Cloud Model version 1 (CM1), which is a nonhydrostatic numerical model described in Bryan and Rotunno (2009b, hereafter BR09). The axisymmetric version of CM1 is used for most simulations because of the low computational cost, which allows for greater exploration of the settings that affect simulated hurricane intensity and structure. The results from more than 400 simulations are shown in this article, all of which have been integrated for \(\approx 12\) days. Such a study using only 3D simulations would be computationally prohibitive. One set of 3D simulations is examined later in this article to test the generality of the results.

Two different model setups are used herein. For one configuration, referred to as setup A, the initial conditions and physical parameterizations are the same as the “default” configuration of BR09, which are identical to those used in the influential study by Rotunno and Emanuel (1987, hereafter RE87); see Table 1. The microphysics parameterization from RE87 is a simple scheme that does not account for ice processes and requires saturation (i.e., 100% relative humidity) in the presence of liquid water. The initial CAPE of the RE87 sounding is approximately 400 J kg\(^{-1}\). The nominal potential intensity (PI) for this initial environment, using the method of Bister and Emanuel (2002), is 44 m s\(^{-1}\) assuming \(C_k/C_d = 0.5\), and PI is 70 m s\(^{-1}\) assuming \(C_k/C_d = 1\).

A second configuration, referred to as setup B, uses a different sounding, higher sea surface temperature \(T_s\), and a double-moment microphysics scheme that includes ice processes (Morrison et al. 2009); see second column of Table 1. For setup B, the moist tropical (MT) sounding of Dunion (2011) is used as the initial sounding; the initial CAPE is approximately 2000 J kg\(^{-1}\). The larger CAPE of this sounding could be problematic in an axisymmetric model because, as discussed by RE87 (p. 548), convective updrafts could be artificially intense.
Table 2. Settings for the relatively low vertical-resolution configuration (Res1, the BR09 default) and the relatively high vertical-resolution configuration (Res2).

<table>
<thead>
<tr>
<th></th>
<th>Res1 (BR09)</th>
<th>Res2 (herein)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tot No. of vertical levels</td>
<td>100</td>
<td>123</td>
</tr>
<tr>
<td>No. of levels below 1 km</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>Height of first grid level (for (u, v)) (m)</td>
<td>125</td>
<td>10</td>
</tr>
<tr>
<td>(\Delta z) for (z &lt; 7) km (m)</td>
<td>250 Variable</td>
<td>250</td>
</tr>
<tr>
<td>(\Delta z) for (z &gt; 7) km (m)</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>

and might be forced to artificially cascade energy upscale; hence, 3D simulations using this setup are shown later in this article. For setup B, the sea surface temperature is 29°C, which is chosen because it gives an initial air–sea temperature difference \(\Delta T\) of 2.2°C; this value of \(\Delta T\) is similar to that of the RE87 setup (Table 1), and is near the climatological average value for hurricane environments (e.g., Cione et al. 2000). The nominal PI for this environment is 63 m s\(^{-1}\) for \(C_d/C_d = 0.5\) and 92 m s\(^{-1}\) for \(C_d/C_d = 1\).

The initial vortex for all simulations is the same as that in section 3c of BR09. The domain size and horizontal grid spacing are the same as the default setup of BR09: radial grid spacing \(\Delta r\) is 1 km for \(r < 64\) km and \(\Delta r\) gradually increases to 15 km at the lateral boundary. To address concerns about the vertical resolution used by BR09, the simulations herein use a stretched vertical grid with much smaller values of vertical grid spacing \(\Delta z\) near the surface. The differences between the relatively low vertical resolution of BR09 (Res1) and the relatively high vertical resolution used herein (Res2) are provided in Table 2.

b. Turbulence parameterization

The turbulence parameterization is the same as in BR09. Four different values of horizontal turbulence length scale \(l_h\) are examined herein: \(l_h = 3000, 1000, 300,\) and 0 m. Three different values of vertical turbulence length scale \(l_v\) are examined: \(l_v = 200\) m (which was used in section 3c by BR09), 100 m, and 50 m. These values of \(l_v\) nearly cover the range from other commonly used models.\(^1\)

With the high vertical resolution of Res2, it is difficult to maintain numerical stability for vertical diffusion without using a very small time step (of order 0.1 s). To overcome this problem, CM1 uses an implicit formulation for vertical diffusion that can be generalized as follows:

\[
\frac{\phi^{t+\Delta t} - \phi^t}{\Delta t} = (1 - \gamma) \frac{\partial}{\partial z} \left( K_v \frac{\partial \phi}{\partial z} \right) + \gamma \frac{\partial}{\partial z} \left( K_v \frac{\partial \phi^{t+\Delta t}}{\partial z} \right),
\]

where \(\phi\) represents a predictive variable in the model, \(\Delta t\) is the time step, and \(K_v\) is the vertical diffusion coefficient. The superscripts \(t\) and \(t + \Delta t\) denote values at the current time and one time step into the future, respectively. Specifically, (1) is solved at the beginning of each time step and the resulting tendencies are held fixed for the remainder of the numerical solution procedure.\(^2\) The variable \(\gamma\) can be set by the model user. Setting \(\gamma = 0\) corresponds to the “Euler-forward” method (i.e., explicit forward-in-time integration) that is used in many models, including the RE87 axisymmetric model and the “bulk PBL” scheme in the fifth-generation Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model (MM5). This method is unstable if \(K_v \Delta t/\Delta z^2 > 0.5\), and so it is only appropriate for rather coarse vertical grid spacing\(^3\) or small time steps. In fact, the RE87 model and the bulk PBL scheme in MM5 simply reduce \(K_v\) as necessary, to prevent instability. Alternatively, to maintain numerical stability with a reasonable time step, one can use an implicit method with \(0 < \gamma \leq 1\). This technique is more expensive, as it requires solving a tridiagonal matrix for (1); however, it does not force model users to change \(K_v\) or \(\Delta z\). Moreover, \(\gamma \geq 0.5\) is absolutely stable (e.g., Richtmyer and Morton 1967). The CM1 default setting is \(\gamma = 1\)—that is, the “implicit Euler” or “Euler-backward” method, which is used for all simulations in this article. Although this method is only first-order accurate in time, it is absolutely stable and nonoscillatory (Ferziger and Peric’ 2002). This setting yields smooth fields near the surface with reasonable time steps, whereas lower values of \(\gamma\) sometimes produce obviously unphysical 2\(\Delta z\) oscillations. The time step for all simulations herein is 3 s, except for the \(l_h = 0\) simulations with setup B, for which the time step is 1.5 s.

c. Surface exchange coefficients

Surface fluxes of heat and momentum in the model are calculated using bulk aerodynamic formulae applied at the lowest model level, as in RE87. The surface

\(^1\) By default, the RE87 model uses \(l_h = 200\) m and the bulk PBL model in MM5 uses \(l_v = 40\) m.

\(^2\) The same methodology is used for several of the planetary boundary layer schemes in the WRF model.

\(^3\) The MM5 user’s guide (chapter 8, p. 8) recommends \(\Delta z > 250\) m for this scheme.
exchange coefficients for enthalpy $C_k$ and momentum $C_d$ are held fixed for each individual simulation in this study to allow for comparison with past studies (e.g., Emanuel 1995; BR09), and because this approach allows for straightforward interpretation with theory. (This methodology should not imply that $C_k$ and $C_d$ are constant in nature; see, e.g., Andreas 2011.) The $C_k$ is used for surface fluxes of both potential temperature and water vapor mixing ratio, and unless stated otherwise it is $1.2 \times 10^{-3}$ (as in section 3c of BR09); this value is based on recent observational and laboratory studies (e.g., Drennan et al. 2007; Jeong 2008; Bell 2010), although an appropriate value for intense tropical cyclones remains uncertain. Most results are presented in terms of the ratio $C_k/C_d$, which is theoretically important for maximum hurricane intensity (e.g., Emanuel 2004). Dissipative heating is included in all simulations.

d. Analysis methods

All simulations are integrated for 12 days, except the $l_v = 0$ simulations with setup A are integrated for 18 days. As in BR09, the primary focus herein is maximum hurricane intensity. The maximum azimuthal velocity at any given time $v_{max}(t)$ is obtained by searching all model levels. Because intensification rate is affected by $C_d$ (e.g., Rosenthal 1971; Montgomery et al. 2010), and because an arbitrary time period for analysis may affect results (Bryan 2011, manuscript submitted to Quart. J. Roy. Meteor. Soc.), the overall maximum intensity $V_{max}$ is determined objectively by searching for the largest 2-day running-mean value of $v_{max}(t)$ using hourly output. Examples of $v_{max}(t)$ and $V_{max}$ from two cases are shown in Fig. 1. All other measures of hurricane intensity and structure are 2-day averages using the same time period.

Maximum observed intensities as a function of $T_s$ are obtained from DeMaria and Kaplan (1994) for $V_{10,max}$ (maximum total wind speed at 10 m MSL) and from Holland (1997) for minimum surface pressure $P_{min}$. Values of $P_{min}$ as a function of $T_s$ are similar in the database used to develop the Statistical Hurricane Intensity Prediction Scheme (SHIPS; M. DeMaria 2011, personal communication). As in BR09 (p. 1775), $V_{10,max}$ is multiplied by a conversion factor to obtain an estimate for maximum azimuthal velocity above the surface, $V_{max}$; based on a variety of analyses (LeeJoice 2000; Montgomery et al. 2006; Kepert 2006a; Zhang et al. 2011b), a value of 1.35 is used herein. Of course, maximum intensity is expected to vary with other environmental parameters, such as outflow temperature and air–sea temperature difference (e.g., Emanuel 1986). Maximum observed intensity is tied only to sea surface temperature herein for simplicity and because of the availability of thorough observational studies for the relationship between $V_{max}$ and $T_s$ (e.g., DeMaria and Kaplan 1994; Whitney and Hobgood 1997).

e. Reevaluation of BR09’s conclusions

Before proceeding to the primary results of this study, the results of BR09’s section 3c (“Ratio of surface exchange coefficients”) are reevaluated using the model settings of this study. The most significant differences are the change in $\Delta z$ near the surface (Table 2) and the level at which the surface exchange coefficients are specified (i.e., the lowest model level for winds; third row in Table 2). Results using either Res1 or Res2, and $l_v = 200$ m (as in BR09’s section 3c), are shown in Fig. 2a. The results are essentially the same for the two different approaches, and essentially the same as in BR09 (cf. their Fig. 6). With $l_v = 50$ m (which was not examined in BR09), values of $V_{max}$ are slightly lower (by as much as 12%) when using the Res2 (Fig. 2b); however, the difference occurs primarily for $C_k/C_d > 1$. The reason for this difference seems to be related to slightly larger vertical diffusivity and slightly smaller radius of maximum winds when using coarser vertical resolution. Results using setup B (not shown) yield the same results, although the differences for $l_v = 50$ m are not as pronounced (the maximum difference in $V_{max}$ is only 6%).

Overall, these tests demonstrate that all primary conclusions from BR09 are upheld when using the different model settings herein (i.e., higher vertical resolution, and surface exchange coefficients specified at 10 m MSL instead of 125 m MSL). Specifically, these results are as
follows: $V_{\text{max}}$ is very sensitive to $l_h$, but not to $l_v$ or $\Delta z$; the theoretical response $V_{\text{max}} \sim (C_k/C_d)^{1/2}$ (e.g., Emanuel 1995) is only found for small $l_h$ ($<100$ m); and $V_{\text{max}}$ has essentially no dependence on $C_k/C_d$ for large $l_h$ (3000 m). The higher-resolution setup (Res2) is used for the remainder of this article.

### 3. Comparison with E95

Figure 3 shows $V_{\text{max}}$ as a function of $C_k/C_d$ for the various model setups used herein. The gray horizontal line denotes the estimated maximum value from observed storms for this sea surface temperature (see section 2d). For comparison, the purple lines on Fig. 3a show results from the study by Emanuel (1995, hereafter E95), which examined the effects of the ratio $C_k/C_d$ on maximum winds in TCs. The lower line shows the results from E95 using the RE87 axisymmetric model, which did not include dissipative heating; the upper line shows these results multiplied by 1.2 to estimate the effects of dissipative heating, which theoretically increases $V_{\text{max}}$ by $\sim 20\%$ (Bister and Emanuel 1998). [Dissipative heating increases $V_{\text{max}}$ by $5\%$–$15\%$ in CM1 (not shown).] The model settings for E95’s simulations are not stated in his article, so a new set of simulations was conducted by the present author using the RE87 model; the results suggest that the model settings are the same as those in RE87 ($T_s = 26^\circ C$, $l_h = 3000$ m, and $l_v = 200$ m). Hence, the E95 results are plotted along with the setup A results from this study.

For $C_k/C_d < 1$, the E95 simulations are similar to CM1 with $l_h = 3000$ m (red lines). The response is different for $C_k/C_d > 1$, and the E95 results are more like CM1 with $l_h = 1000$ m (green lines). These differences may be attributable to different resolution, different model settings (i.e., formulation of surface exchange coefficients), or to approximations used in the RE87 model (see the appendix in BR09 for more details). Nevertheless, E95’s simulations are clearly among the set of simulations that have strong horizontal diffusion, especially for $C_k/C_d < 1$.

Several recent articles (e.g., Black et al. 2007; Zhang et al. 2008; Haus et al. 2010; Montgomery et al. 2010; Andreas 2011) have evaluated the conclusion from E95 that $C_k/C_d$ is likely $\sim 0.75$ for intense TCs in nature because, according to E95, “...otherwise, the [modeled] wind speeds would be much weaker than observed.” However, if one accepts the low-$l_h$ settings examined herein, then this lower bound on $C_k/C_d$ would be only 0.25 for setup B (Fig. 3b). Hence, the inability of E95’s simulations to produce the strongest storms in nature unless $C_k/C_d$ exceeds roughly 0.75 is likely attributable to large horizontal diffusion ($l_h \approx 3000$ m) and low sea surface temperature ($T_s = 26^\circ C$) in his simulations.

Andreas (2011) noted that neither the simulations by E95 nor BR09 produce a category 5 storm (10-m wind speed $>69$ m s$^{-1}$) unless $C_k/C_d > 0.75$. However, it

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**Fig. 2.** Sensitivity of $V_{\text{max}}$ (m s$^{-1}$) to the ratio $C_k/C_d$ using setup A with two different vertical grids. The solid lines use the same vertical resolution as BR09 (Res1) and the dashed lines use higher vertical resolution (Res2). Different colors are used for different values of $l_h$ as indicated in the legend. (a) Simulations with $l_h = 200$ m (as in BR09) and (b) simulations with $l_h = 50$ m. The gray curve shows $V_{\text{max}} \sim (C_k/C_d)^{1/2}$ for reference.
should be noted that a category 5 storm has never been observed for $T_s = 26^\circ C$ (DeMaria and Kaplan 1994; Whitney and Hobgood 1997). The $T_s = 29^\circ C$ simulations herein produce category 5 storms for $C_k/C_d = 0.25$ if horizontal diffusion is relatively weak ($l_h \approx 300$ m).

Clearly, the values for surface exchange coefficients are important for maximum intensity in numerical models, but the large set of simulations herein demonstrates that several other factors are also important, such as the intensity of horizontal turbulence (via $l_h$ in this model) and certain properties of the environment (e.g., sea surface temperature). Hence, the E95 simulations, which to this author’s knowledge used only one value for $l_h$ and $C_d$, have limited usefulness for comparison with observations. On the opposite extreme, the broader set of simulations shown in Fig. 3 is not especially useful, because it is unclear which model settings are realistic. The analyses in the following two sections aim to determine appropriate values for $C_d$, $C_k$, $l_h$, and $l_v$ by examining several metrics of TC intensity and structure.

4. Comparison with maximum observed intensity

As shown in Fig. 3, most model setups produce unnaturally strong storms. Considering first $C_k/C_d \approx 1$, then only $l_h = 3000$ m (red lines) yields reasonable intensity. However, recent observational and laboratory studies (e.g., Powell et al. 2003; Donelan et al. 2004; Drennan et al. 2007; Jeong 2008; Haus et al. 2010; Bell 2010) found that the most intense hurricanes probably have $C_k \approx 1 \times 10^{-3}$ and $C_d \approx 2 \times 10^{-3}$ (although the uncertainty is considerably large). Considering $C_k/C_d \approx 0.5$ as a best guess for conditions in nature, then $l_h = 1000$ m (green lines) yields the approximately correct maximum intensity with $C_k/C_d \approx 0.5$ for both setups A and B. For smaller values of $l_h$ (blue and black lines), the ratio $C_k/C_d$ needs to be $\leq 0.25$ to prevent unnaturally strong intensity. For a given $l_h$, the parameter $l_v$ has little influence on $V_{max}$. A detailed analysis of the effects of horizontal and vertical diffusion on $V_{max}$ is presented by Rotunno and Bryan (2011, manuscript submitted to J. Atmos. Sci.), and readers are referred to that article for more explanation.

Figure 4 shows $P_{min}$ for the various model setups. Conclusions from above are upheld when using $P_{min}$ as a metric of storm intensity. That is, realistic maximum intensity occurs only with $C_k/C_d > 1$ when $l_h = 3000$ m, but with $C_k/C_d \approx 0.5$ when $l_h = 1000$ m, or with $C_k/C_d \approx 0.25$ when $l_h \leq 300$ m. The quantity $P_{min}$ seems to be slightly more sensitive to $l_v$ than $V_{max}$, although it is difficult to generalize the relationship from these results; considering only simulations with $P_{min} > 900$ mb (i.e.,
the most realistic simulations), then lower $l_v$ usually produces weaker storms.

To better examine the interrelated effects of pressure and wind for the various model settings, Figs. 5–6 show scatterplots of minimum surface pressure $p_{\text{min}}(t)$ and maximum 10-m wind speed $v_{10,\text{max}}(t)$; each dot on these figures represents instantaneous values from a different time in the simulations. These figures use 10-m wind speed (instead of maximum tangential wind above the surface, which was used in previous figures) to facilitate comparison with observational studies. Specifically, the gray lines are wind–pressure relationships from Knaff and Zehr (2007) that are based on observed storms. Equation (8) from their study is used with two values of their normalized size parameter $S$; the upper curve on Figs. 5–6 is for $S = 0.2$, corresponding to relatively small storms; the lower curve is for $S = 1.0$, corresponding to relatively large storms.

Considering setup A (Fig. 5), there is a tendency for the model to produce winds that are too strong for a given $p_{\text{min}}$. This bias is especially pronounced for relatively large $l_y$ (Figs. 5a–c). Using $l_v = 50$ m (Figs. 5d–f), the model-produced wind–pressure relationship is close to the Knaff and Zehr (2007) small-storm curve, but only for $C_k/C_d \approx 0.5$ (green and red dots). This analysis further supports the conclusion that $C_k/C_d \approx 0.5$ is likely the best setting for axisymmetric numerical model simulations of intense TCs.

The same overall conclusions are reached when using setup B (Fig. 6), although the large-$l_v$ cases are acceptable for this setup as long as $l_h \geq 1000$ m. For $l_v = 50$ m, the model results are closer to the large-storm relation from Knaff and Zehr (2007) as long as $C_k/C_d < 0.5$. These results imply that the simulated storms are larger for setup B than setup A, which is confirmed in the next section.

The maximum azimuthal velocity at any level ($V_{\text{max}}$) is used in previous figures because it allows for comparison to previous modeling studies (e.g., E95) and to theoretical maximum wind speed (discussed in a later section). To evaluate wind speeds at the standard reporting level of 10 m MSL, the open circles on Figs. 5–6 denote the maximum 10-m wind speed from DeMaria and Kaplan (1994) with the $p_{\text{min}}$ values from Holland (1997) for the specified sea surface temperatures. For setup A, the settings ($l_h, l_v, C_k/C_d = (1000 \text{ m}, 50 \text{ m}, 0.5)$ (green dots of Fig. 5e) produce acceptable results in the sense that the model output is close to the Knaff–Zehr wind–pressure relationship and $v_{10,\text{max}}$ remains below the maximum observed value and $p_{\text{min}}$ remains above the minimum observed value. For setup B, interpretation is not quite as clear. The $l_h = 300$-m simulations with $C_k/C_d = 0.25$ (red dots in Figs. 6a,d) seem reasonable. However, as discussed earlier, recent observational/laboratory studies are finding $C_k/C_d \approx 0.5$; assuming this value, then ($l_h, l_v = (1000 \text{ m}, 50 \text{ m})$ might be the best settings (green dots in Fig. 6e), although there is a slight overestimation of maximum intensity for this case. It seems that a value of $l_h$ somewhere between 1000 and 3000 m would produce the best results for setup B.

FIG. 4. As in Fig. 3, except for minimum surface pressure (mb).
5. Comparison with observed TC structure

In this section, various measures of TC structure are evaluated. This analysis addresses whether the settings determined in the previous section yield realistic structure, and some of the following analyses help narrow down likely values for these settings.

Figure 7 shows $R_{34}$, which is the radius of gale force winds (34 kt; \(17.5 \text{ m s}^{-1}\)) at the time of $V_{\text{max}}$. Overall, by this metric, the simulated storms are slightly larger for setup B, which supports the inference about storm size drawn in the previous section. (It is unknown why a few simulations with setup B and $C_k/C_d$ differ. Larger storms for setup B (which has a larger nominal potential intensity) is also qualitatively consistent with theory, for which the systematic difference in $R_{34}$ between setups A and B is especially notable for $C_k/C_d$. $lh$ has no systematic influence on $R_{34}$. There is a slight tendency for smaller $R_{34}$ as $ly$ decreases. The dataset analyzed by Dean et al. (2009), which is based on satellite data and in situ flight-level observations, shows that most observed storms have $R_{34} < 200$ km, and almost all cases have $R_{34} > 90$ km; the model results in Fig. 7 are within this range as long as $C_k/C_d \leq 1$.

As another measure of TC size, Fig. 8 shows $R_{\text{max}}$, defined as the radius of $V_{\text{max}}$. There is a clear relationship between $l_h$ and $R_{\text{max}}$, which is explained in Rotunno and Bryan (2011, manuscript submitted to J. Atmos. Sci.). There is no clear impact on $R_{\text{max}}$ from either $l_h$ or $C_k/C_d$.
The analytic model of Emanuel and Rotunno (2011) also predicts no dependence on $R_{\text{max}}$ with $C_k/C_d$ [at least, when assuming constant outer-vortex scale; see their Eq. (42)]. The same conclusions are drawn if $R_{\text{max}}$ is defined at a common level of 2 km MSL as in Stern and Nolan (2009) and Zhang et al. (2011b). Recent studies of flight-level data (usually at 700–800 mb) by Willoughby and Rahn (2004) and Mallen et al. (2005) have found that the radius of maximum winds (RMW) for strong storms (wind $>50$ m s$^{-1}$) is usually less than $\sim 40$ km, and RMW is sometimes as small as $\sim 10$ km. The model results are reasonably consistent.

As it relates to the primary goal of this section, Figs. 7–8 demonstrate that the settings determined in the previous section—$l_h \approx 1000$ m, $l_v \approx 50$ m, and $C_k/C_d \approx 0.5$—yield reasonable values of TC size. Stated another way: there are no extraordinarily small or large storms for these model settings.

An insightful measure of TC boundary layer structure is the inflow angle, $\beta = \tan^{-1}(u_{10}/v_{10})$, where $u_{10}$ and $v_{10}$ are the radial and azimuthal velocities, respectively, at 10 m MSL at the location of $V_{\text{max}}$. From a large database of dropsonde data, Powell et al. (2009) found an average value of 23°; this value is produced in the model simulations only for low values of $l_v$ and/or low values of $C_k/C_d$ (Fig. 9). Unreasonably small values of $\beta$ are produced for $C_k/C_d > 1$.

Figure 10 shows $z_{\text{max}}$, which is the height of $V_{\text{max}}$. There are three clear relationships in this figure. First, for $l_h$ held fixed, $z_{\text{max}}$ tends to increase as $l_v$ increases (i.e., black lines tend to be near the bottom and red lines near the top). Second, for $l_v$ held fixed, $z_{\text{max}}$ tends to increase as $l_h$ increases (i.e., black lines tend to be near the bottom and red lines near the top). The gray line on Fig. 10 shows the value of $z_{\text{max}}$ from the composite analysis of dropsonde data from Zhang et al. (2011b), using the value from their category 4–5 storms only. There are multiple model settings that yield this value ($z_{\text{max}} = 0.9$ km), but for $C_k/C_d = 0.5$ most
model settings overestimate \( z_{\text{max}} \) slightly. This positive bias in \( z_{\text{max}} \) might be related to the use of a constant value of \( l_y \) in these simulations; it is widely acknowledged in studies of boundary layers that \( l_y \) should be smaller near the surface than near the top of the boundary layer (e.g., Wyngaard 2010, p. 234). Sensitivity simulations using a variable \( l_y(z) \) are shown in a later section. The use of constant \( C_k/C_d \) everywhere in these simulations may also be a problem; this ratio is well known to be relatively larger in weak winds (e.g., Black et al. 2007 and references therein). Nevertheless, for the settings \((l_h, l_y, C_k/C_d) = (1000 \text{ m}, 50 \text{ m}, 0.5)\) (green-dotted line in Fig. 10), \( z_{\text{max}} \) is 0.9 km for setup A and 1.0 km for setup B—both of which are close to the value from Zhang et al. (2011b).

Overall, this analysis of TC structure supports the conclusion that the model settings \( l_h \approx 1000 \text{ m} \) and \( C_k/C_d \approx 0.5 \) produce realistic TCs with this axisymmetric model. The setting \( l_y \approx 50 \text{ m} \) produces reasonable values for
inflow angle $\beta$, height of maximum winds $z_{\text{max}}$, and yields a reasonable wind–pressure relationship at the surface (previous section). The value $l_y \approx 50$ m is within the range of values determined from flight-level observations in hurricane boundary layers by Zhang et al. (2011a), although their mean value for $l_y$ is roughly 100 m.

The estimate $l_y \approx 1000$ m is 33% lower than the estimate of 1500 m from BR09; the difference herein is attributable mostly to the new evidence from observational and laboratory studies that find $C_k/C_d \approx 0.5$ in intense hurricanes. The estimate $l_y \approx 50$ m is half of the estimate from BR09; the difference herein is attributable

Fig. 9. As in Fig. 3, except for inflow angle $\beta$ ('') at 10 m MSL at the location of maximum wind. The gray line denotes the average value from dropsondes reported by Powell et al. (2009).

Fig. 10. As in Fig. 3, except for $z_{\text{max}}$—the height (km) of $V_{\text{max}}$. The gray line denotes the average value for category 4–5 storms reported by Zhang et al. (2011b).
to new observational studies (particularly Powell et al. 2009; Zhang et al. 2011b) that have helped refine this estimate.

6. Comparison with theory

Theoretical studies by Emanuel (1986, 1995) found that maximum azimuthal velocity at the top of the boundary layer should vary as follows:

\[ V_{\text{max}} \sim \left( \frac{C_k}{C_d} \right)^{1/2}. \]

Using two different numerical models—one a balanced model and the other a nonhydrostatic primitive equation model—Emanuel (1995) found approximate correspondence with (2). [Technically, using values reported in Emanuel’s Tables 1–2, the E95 model gives \( V_{\text{max}} \sim (C_k/C_d)^{0.59} \) and the RE87 model gives \( V_{\text{max}} \sim (C_k/C_d)^{0.43} \).] BR09 found good correspondence with (2) only when using small horizontal diffusion (i.e., \( l_h \) of order 100 m or less). As \( l_h \) increased in BR09’s simulations, the correspondence with (2) became worse, and there was no clear relationship between \( V_{\text{max}} \) and \( C_k/C_d \) for \( l_h = 3000 \) m. The higher-vertical-resolution simulations herein yield the same conclusions as in BR09. Table 3 lists the exponent \( b \) for best fits of the function \( V_{\text{max}} = a(C_k/C_d)^b \) for all setups shown in Fig. 3. As in BR09, the exponent \( b \) increases as \( l_h \) decreases. The best matches to (2) are obtained for \( l_h = 0 \), especially for setup A. The same conclusions are obtained when using maximum gradient wind speed \( V_g \) from anywhere in the domain (Table 4), where \( V_g \) is calculated in the same manner as Bryan and Rotunno (2009a, p. 3047). Montgomery et al. (2010) found no support for (2), but their conclusion seems to be attributable to their shorter model integration times (Bryan 2011, manuscript submitted to Quart. J. Roy. Meteor. Soc.).

The theoretical relationship (2) is typically derived by assuming gradient wind balance above the boundary layer, along with several other approximations (e.g., Emanuel 1986, 1995). However, simulations with weak horizontal diffusion can have large gradient wind imbalance; for example, in the control simulation of Bryan and Rotunno (2009a) the centrifugal acceleration is twice the magnitude of the horizontal pressure gradient at the location of \( V_{\text{max}} \). To examine the magnitude of gradient wind imbalance in the present simulations, the ratio \( V_{\text{max}}/V_g \) is used, where \( V_g \) for this analysis is calculated at the location of \( V_{\text{max}} \). Figure 11 shows that all simulations have at least a slight supergradient flow \( (V_{\text{max}}/V_g > 1) \) at the location of \( V_{\text{max}} \), which is consistent with previous studies (e.g., Rott and Lewellen 1966; Kuo 1971; Kepert 2001; Kepert and Wang 2001; Zhang et al. 2001; Smith et al. 2008). For fixed values of \( l_h \) and \( l_v \), the imbalance increases as \( C_k/C_d \) decreases (i.e., going from right to left on Fig. 11); this result is consistent with the increasing impact of radial inflow and unbalanced flow effects as surface drag increases, as shown by Kuo (1971) and Rotunno and Bryan (2011, manuscript submitted to J. Atmos. Sci.) (their parameter \( K \) is proportional to \( C_k/C_d \) herein). For fixed values of \( l_h \) and \( C_k/C_d \), the imbalance increases as \( l_v \) decreases (i.e., solid lines tend to be near the bottom of the figure, and dotted lines tend to be near the top). For fixed values of \( l_v \) and \( C_k/C_d \), it is difficult to draw a general conclusion about the relationship between the ratio \( V_{\text{max}}/V_g \) and \( l_h \). From the conclusions of Bryan and Rotunno (2009a), one might expect \( V_{\text{max}}/V_g \) to increase as \( l_h \) decreases, but this is not always the case in Fig. 11.

To allow a direct comparison with the results of Bryan and Rotunno (2009a), the theoretical maximum intensity (i.e., potential intensity) from Emanuel (1986), hereafter referred to as E-PI, is calculated using the relation

\[ \text{E-PI} = \left[ \alpha \frac{C_k}{C_d} (T_B - T_0) (s_{\text{surf}} - s_0) \right]^{1/2}, \]

where \( T_B \) is temperature at \( V_{\text{max}} \), \( T_0 \) is outflow temperature, \( \alpha = T_0/T_s \) is moist entropy at the pressure and temperature of the ocean surface, \( s_0 \) is moist entropy in the atmospheric surface layer, and all variables are calculated at the radius of \( V_{\text{max}} \). The ratio \( V_{\text{max}}/\text{E-PI} \) is plotted in Fig. 12. From this figure it can be concluded that the simulations herein are consistent with those of Bryan and Rotunno (2009a); that is, for fixed \( l_v \) and \( C_k/C_d \), \( V_{\text{max}} \)

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in the low-$l_h$ runs exceeds E-PI by a greater amount than in the large-$l_h$ runs (i.e., black lines tend to be at the top, and red lines at the bottom). Figure 12 also shows that for $C_k/C_d \approx 2$, $V_{\text{max}}$ does not appreciably exceed E-PI for any model setup, which is consistent with relatively weak supergradient flow. With regards to observed TCs, some studies have documented cases in which observed $V_{\text{max}}$ exceeds E-PI (e.g., Bell and Montgomery 2008), which, based on Fig. 12, further supports the conclusion that $C_k/C_d < 1$ for intense TCs in nature. Theoretically, though, it seems that E-PI should be a reasonable model for maximum intensity as long as $C_k/C_d \approx 1.5$. As mentioned above, (3) is a model for maximum gradient wind speed, and so evaluation has also been done using the maximum value of $V_g$ from anywhere in the domain; results (not shown) look much the same as Fig. 12, although the maximum value of $V_{g,\text{max}}/E$-PI is 1.75 (for $C_k/C_d = 0.25$).

A more complete analytic model that includes inertial terms (i.e., that does not assume gradient wind balance) was presented by Bryan and Rotunno (2009a). They derived the relation

$$\text{PI}^+ = [(E-\text{PI})^2 + \alpha r_h \eta_h w_h]^{1/2},$$

where $r$ is radius, $\eta$ is azimuthal vorticity, $w$ is vertical velocity, and the subscript $b$ denotes that variables are evaluated at the location of $V_{\text{max}}$. The ratio $V_{\text{max}}/\text{PI}^+$ is shown in Fig. 13, which shows that $V_{\text{max}}$ can exceed PI$^+$ by a small amount in some of these simulations (up to 20%, particularly for low $l_h$). This slight underestimation by PI$^+$ for some cases is probably attributable to one of the approximations made in the derivation of PI$^+$. Nevertheless, the ratio $V_{\text{max}}/\text{PI}^+$ is near 1 for all $C_k/C_d$ and all model configurations. Hence, (4) can be utilized to understand the relationship between $V_{\text{max}}$ and $C_k/C_d$. Re-arranging (4) using (3), one finds

$$V_{\text{max}} \sim \left(\frac{C_k}{C_d}\right)^{1/2} \left[\alpha(T_B - T_0)(s_{\text{surf}} - s_0) + \frac{\alpha r_h \eta_h w_h}{C_k/C_d}\right]^{1/2}.$$  

(5)

It is unclear how all the terms in the square brackets should vary with $C_k/C_d$. Using the model output, the bracketed term in (5) is found to be roughly constant with $C_k/C_d$ for all values of $l_h$ and $l_y$ (not shown). Hence, (5) reduces to (2) for these simulations, and therefore (2) can be valid for unbalanced as well as balanced TCs.

The reason the relation (2) is not found in all simulations (e.g., Table 3) is probably attributable to the effects of horizontal diffusion in the boundary layer, which is not considered in the derivations of (3) or (4). The new analytical model by Emanuel and Rotunno (2011) incorporates the effects of turbulence, and is evaluated in Fig. 14. Specifically, their (41) is used to estimate maximum gradient wind speed $V_m$ assuming nominal potential intensity $V_p$ of 70 and 92 m s$^{-1}$ for setups A and B, respectively (see section 2a). This analytical model reasonably predicts the maximum gradient wind in the model simulations with $l_h = 1000$ m (green lines in Fig. 14). The underprediction for $l_h \approx 300$ m may be attributable to the neglect of dissipative heating and unbalanced flow effects in the derivation of $V_m$.

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**FIG. 11.** As in Fig. 3, except for the ratio of $V_{\text{max}}$ to $V_g$ (gradient wind speed at the location of $V_{\text{max}}$).
7. Sensitivity simulations

The simulations evaluated in sections 3–6 all use $C_k = 1.2 \times 10^{-3}$. To check whether conclusions are sensitive to this value, a set of simulations has been conducted using $(L_h, L_v) = (1000 \text{ m}, 50 \text{ m})$ but using different values for $C_h$. There is negligible impact on the sensitivity of maximum 10-m wind speed ($V_{10,\text{max}}$) to $C_k/C_d$ (Fig. 15). The same conclusion holds for $V_{\text{max}}$ (not shown). (Wind speed at 10 m is used in this subsection to allow for direction comparison with the observations in DeMaria and Kaplan 1994.) Some other metrics have a small sensitivity to the choice of $C_k$; for example, $P_{\text{min}}$ tends to decrease as $C_k$ increases (not shown), but the effect is

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**Fig. 12.** As in Fig. 3, except for the ratio of $V_{\text{max}}$ to E-PI (the theoretical maximum azimuthal velocity assuming balanced flow).

**Fig. 13.** As in Fig. 3, except for the ratio of $V_{\text{max}}$ to PI$^+$ (the theoretical maximum azimuthal velocity assuming unbalanced flow).
only notable (~10 mb) for $C_k/C_d > 1$, which is likely not relevant to intense TCs in nature.

As discussed in the previous section, the height of maximum winds $z_{max}$ compares favorably with observations only when using $l_v \approx 50$ m, which is lower (by a factor of 2) than the average value determined from flight-level observations at $z = 400$ m by Zhang et al. (2011a). To check whether this result is attributable to the use of constant $l_v$, a set of simulations has been conducted using $l_h = 1000$ m and the formulation for $l_v$ from Blackadar (1962), $l_v(z) = \kappa z [1 + (\kappa z/l_w)]^{-1}$, where $\kappa = 0.4$ is the von Kármán constant. Results using three different values of $l_v$ show that maximum winds vary with $C_k/C_d$ in essentially the same manner as shown in previous sections (Fig. 16a), although values tend to be slightly lower with the Blackadar formulation for $l_v$. The settings $l_h = 1000$ m and $C_k/C_d = 0.5$ produce reasonable maximum intensity, as before, although with this test the value of $l_w$ needs to be 100 or 200 m. Using $l_w = 100$ or 200 m also yields reasonable values for $z_{max}$ (Fig. 16b).

**FIG. 14.** As in Fig. 3, except for the ratio of maximum gradient wind ($V_{g,\text{max}}$) to theoretical maximum gradient wind from Emanuel and Rotunno (2011) ($V_m$).

**FIG. 15.** Maximum wind speed at 10 m MSL $V_{10,\text{max}}$ (m s$^{-1}$) using $l_h = 1000$ m and $l_v = 50$ m with (a) setup A and (b) setup B. Different lines are for different values of $C_k$, as indicated in the legend.
The value of $l_y$ at 400 m MSL is listed at the bottom of Fig. 16; for $l_y = 200$ m, the vertical length scale $l_y$ is 90 m, which is similar to the average value determined by Zhang et al. (2011a). Because of the better correspondence with observational studies, a variable $l_y(z)$ should probably be used for future studies.

Finally, to test the sensitivity to model dimensionality, a set of three-dimensional simulations has been conducted using the same numerical model (CM1) with $(l_h, l_y) = (1000$ m, $50$ m). The 3D version of CM1 uses the same governing equations and physical parameterizations, but is integrated on a Cartesian grid. Further details are available in Bryan et al. (2010). Horizontal grid spacing is 2 km for these tests, and setup Res2 (Table 2) is used here. To allow for direct comparison with the axisymmetric model, the 3D model output is azimuthally averaged; values of wind speed at 10 m (Fig. 17) produce a similar response to $C_k/C_d$, although the 3D simulations are consistently weaker than the axisymmetric simulations. (Maximum instantaneous values of 10-m wind speed from any gridpoint in the 3D model fall between the two curves shown in Fig. 17.) This systematic difference between $V_{\text{max}}$ for axisymmetric and 3D simulations is consistent with results using the lower-CAPE initial condition of RE87 (Bryan et al. 2010). Further analysis is planned for a future article, although Bryan et al. (2010) speculated that momentum exchange between the eye and eyewall, which seems to be greater in the 3D runs because of their ability to resolve 3D features, probably leads to weaker intensity in 3D simulations; see also Schubert et al. (1999) and Hausman et al. (2006). Nevertheless, the 3D simulations herein confirm that $l_y \approx 1000$ m and $C_k/C_d \approx 0.5$ yield reasonable maximum intensity, although these simulations

![Figure 16](image16.jpg)

**Fig. 16.** Results using setup B from simulations that use different formulations for $l_y(z)$ as indicated in the legend: (a) maximum wind speed at 10 m MSL $V_{10,\text{max}}$ (m s$^{-1}$); and (b) the height of maximum azimuthal velocity, $z_{\text{max}}$ (km).

![Figure 17](image17.jpg)

**Fig. 17.** Maximum wind speed at 10 m MSL $V_{10,\text{max}}$ (m s$^{-1}$) using $l_h = 1000$ m and $l_y = 50$ m where the solid line is from axisymmetric simulations and the dashed line is from 3D simulations. The 3D results are azimuthally averaged values.
suggest that axisymmetric models require different (i.e., larger) values of \( l_h \) than 3D models.

8. Summary

This study examines how changes in surface exchange coefficients and turbulence length scales in a numerical model can affect maximum hurricane intensity and structure. Compared to other recent studies on the topic, this study uses higher vertical resolution, a double-moment mixed-phase microphysics scheme, an initial environment based on a recent climatological study by Dunion (2011), a higher sea surface temperature (29°C), more values for \( l_h \) and \( l_v \) (the horizontal and vertical length scales, respectively), more values of \( C_k \) and \( C_d \) (the surface exchange coefficients for enthalpy and momentum, respectively), and a set of three-dimensional simulations. Most results are drawn from axisymmetric model simulations because of the lower computational cost, which allows for a thorough examination of the effects on hurricane intensity and structure from changes in the parameters \( C_k, C_d, l_h, \) and/or \( l_v \). More than 400 simulations are examined herein.

Despite all the differences in model setup compared to the recent study by Bryan and Rotunno (2009b, hereafter BR09), all primary conclusions from that study are upheld. Maximum intensity, whether examined in terms of maximum winds \( V_{\text{max}} \) or minimum surface pressure \( P_{\text{min}} \), is very sensitive to \( l_h \) but not to \( l_v \). Also in agreement with BR09, it is shown that the theoretical response \( V_{\text{max}} \sim (C_k/C_d)^{1/2} \) occurs only when horizontal turbulent diffusion is weak (\( l_h < 100 \text{ m} \)). Increasingly larger values of \( l_h \) (up to 3000 m) yield much weaker responses of \( V_{\text{max}} \) to \( C_k/C_d \) (Table 3).

From a comparison of model output to relatively well-observed metrics of hurricane structure and intensity—such as maximum winds, minimum surface pressure, height of maximum winds, surface inflow angle, and the surface wind–pressure relationship—it is concluded that the settings \( l_h \approx 1000 \text{ m}, l_v \approx 50 \text{ m}, \) and \( C_k/C_d \approx 0.5 \) produce the most reasonable results. These values are reasonably consistent with recent observational and laboratory studies (e.g., Powell et al. 2003; Donelan et al. 2004; Drennan et al. 2007; Jeong 2008; Haus et al. 2010; Bell 2010; Zhang et al. 2011b). Determination of these settings is roughly independent of the different model setups used herein (Table 1), although the setup with higher sea surface temperature (i.e., higher winds) probably requires \( l_h \) somewhere between 1000 and 3000 m in the axisymmetric model to prevent excessively strong intensity. Using the variable formulation for \( l_h(z) \) from Blackadar (1962) yields some better correspondence with observations, and a value of asymptotic vertical length scale \( l_v = 200 \text{ m} \) \( (l_v = 90 \text{ m} \text{ at } 400 \text{ m MSL}) \) produces the best match to the observational metrics. The three-dimensional numerical simulations require lower values of \( l_h \) to prevent excessively strong intensity, presumably because of their ability to produce three-dimensional motions that must be parameterized in axisymmetric models.

Gradient wind imbalance (i.e., supergradient overshoot) in the model simulations generally increases as \( C_k/C_d, l_v, \) and/or \( l_h \) decreases. The inviscid, balanced model developed by Emanuel (1986, 1995) yields a reasonable upper bound on \( V_{\text{max}} \) as long as \( C_k/C_d \approx 1.5 \). The relation for maximum winds that includes unbalanced flow effects (i.e., inertial terms) presented by Bryan and Rotunno (2009a) is reasonably close to \( V_{\text{max}} \) for all axisymmetric model simulations, and the analytical model for maximum gradient wind by Emanuel and Rotunno (2011) reproduces the numerical model results with \( l_h = 1000 \text{ m} \). The simple relation \( V_{\text{max}} \sim (C_k/C_d)^{1/2} \) seems to apply to simulations with strong supergradient overshoot, despite the fact that this theoretical relation is often derived by assuming balanced flow.

This study also reconciles a recent controversy in the literature about Emanuel’s (1995) conclusion that \( C_k/C_d \) is probably \( \lesssim 0.75 \) in nature, despite recent observational/laboratory data showing \( C_k/C_d \approx 0.5 \) in intense hurricanes. Emanuel’s simulations had relatively large horizontal diffusion (probably \( l_h \approx 3000 \text{ m} \)) and relatively low sea surface temperature (26°C). The simulations herein can produce category 5 storms for \( C_k/C_d \) as low as 0.25, but only when the horizontal turbulence diffusion is weak (\( l_h \leq 300 \text{ m} \)).

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