Large-scale Rossby normal modes are studied for the Northern Hemisphere winters of 2005, 2006, 2008, and 2009 using global observational meteorological analyses spanning the 0–92-km altitude range. Spectral analysis of geopotential height fields shows pronounced peaks at westward-propagating zonal wavenumber 1 near the theoretical locations of the free Rossby waves at 25, 16, 10, and 5 days that, in some cases, have amplitudes significantly larger than the estimated background spectrum. Evidence is also found for a wavenumber-2 free mode near 4 days. A coherence analysis is used to extract the amplitude and phase of the waves, and to isolate those regions of the latitude/altitude plane where the signals are statistically significant. Although the spectral location, temporal evolution, and vertical structure of several of these waves are suggestive of the presence of Rossby normal modes, this study shows that in the real atmosphere the waves only occasionally have the global properties of classical normal modes. Moreover, no evidence is found that the amplitudes of these modes are enhanced during stratospheric sudden warmings.

1. Introduction

The existence of large-scale free Rossby waves in the atmosphere has been known for many decades (e.g., Eliasen and Machenhauer 1965, 1969), and the theory dates back to the nineteenth century [see Madden (2007) for a historical perspective]. Substantial theoretical progress was made with the calculations of Longuet-Higgins (1968), who described the solutions of Laplace’s tidal equation in an isothermal atmosphere using different equivalent depths, including structures corresponding to normal, or “free,” modes. The behavior of normal modes in the presence of a realistic background atmosphere was calculated numerically by Salby (1981b) using prescribed climatological background winds and temperatures typical of solstice and equinox. Salby (1981a) also showed that inhomogeneities in the background atmosphere can lead to spectral broadening, Doppler shifting, and distortion of the energy associated with the westward-propagating normal modes. A recent review of atmospheric normal modes, including their spatial structure and temporal behavior, is provided by Madden (2007). The existence of some of these modes in the real atmosphere has been documented by several authors. Some of the free modes that are most frequently identified in the atmosphere at zonal wavenumber 1 have characteristic periods of 25, 16, 10, and 5 days. Their theoretical horizontal structures are shown in Fig. 1 in terms of the normalized amplitude of the geopotential. Free modes at higher zonal wavenumbers were modeled by Salby (1981b) but they are difficult to identify in the atmosphere because of their

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small amplitude and their susceptibility to variations in the background winds. The gravest mode, the (1, 1) 5-day wave, has no nodal points in the meridional direction (see Fig. 1). It has been identified in radar observations of the mesosphere and the lower thermosphere by Day and Mitchell (2010) and in satellite data by Wu et al. (1994). There is also observational evidence of a 5-day oscillation in global surface pressure (Madden and Julian 1972). The 5-day wave has been investigated numerically by Salby (1981b) and Meyer and Forbes (1997).

The second gravest mode, the (1, 2) 10-day wave, corresponds to an antisymmetric structure with one node at the equator and two large (but opposite in sign) amplitude peaks at the midlatitudes of both hemispheres (Fig. 1). The behavior of the (1, 2) mode during equinox and solstice conditions was described by Salby (1981b). Hirooka and Hirota (1985) documented the intermittent synoptic structure of the 10-day mode during some Northern Hemisphere (NH) winters and its relation to the varying background atmospheric conditions.

The (1, 3) mode, or 16-day wave, has a symmetric structure with two nodes in the subtropics (Fig. 1). The 16-day wave has been observed ubiquitously in the troposphere (Madden 1978) and in the middle atmosphere during winter (Forbes et al. 1995), as well as at the summer mesopause (Williams and Avery 1992) and in the lower thermosphere (Namboothiri et al. 2002; Day and Mitchell 2010). Espy et al. (1997) suggested that the presence of the 16-day wave at high latitudes of the summer mesopause is related to a favorable wind configuration in the tropical upper stratosphere. In some years, the 16-day mode has also been linked to an amplification of wave activity during a stratospheric warming (Smith 1985), while at other times the relation to stratospheric warmings is nonexistent (Lou et al. 2000). Hirooka and Hirota (1985) document the intermittent synoptic structure of the 10- and 16-day modes and their relation to the varying background atmospheric conditions.

The (1, 4) mode, or 25-day wave, is antisymmetric with two nodes in the subtropics and a third node at the equator (Fig. 1); two amplitude peaks at high latitudes of both hemispheres are of opposite sign. Evidence of the 25-day mode is provided by Madden (2007) and references therein.

A complication arising in both observations and time-dependent numerical simulations is that no wave mode in the atmosphere is expected to appear as a pure normal (free) mode; rather, in some instances, waves are observed to exhibit behavior that resembles the normal modes. This occurs because variability of the background atmosphere and the presence of dissipation represent important departures from the conditions assumed to derive the theoretical description of normal modes. Nevertheless, the occurrence of normal modes has been linked to weather patterns and precipitation events (Madden 2007).

As noted above, Salby (1981b) discussed the effect of realistic background conditions on the behavior of the free modes using mean climatological winds in his numerical calculations. Salby’s results raise interesting questions about the behavior of the normal modes during disturbed conditions in the middle atmosphere, such as those occurring during stratospheric sudden warmings (SSWs). During SSWs, the stratospheric polar vortex reverses from its climatological eastward flow to a westward flow. As the temperature is in approximate geostrophic balance with the zonal winds at middle and high latitudes, substantial temperature changes occur both in the stratosphere and in the mesosphere. These large perturbations to the background atmosphere may affect not only the propagation through changes of the refractive index, but also the excitation of free modes. In
fact, Hirooka and Hirota (1985) suggested that normal modes might be amplified during an SSW, possible observational evidence for which was reported during SSWs by Dowdy et al. (2004), Espy et al. (2005), and Pancheva et al. (2008).

In this study, nonstationary, westward-propagating atmospheric variability is examined during four recent Northern Hemisphere winters using global meteorological analyses that extend from about 0- to 92-km altitude. Prominent zonal wavenumber-1 (wave 1) modes at periods that correspond closely to the theoretical normal modes are identified, and their structure is examined for similarity with the theoretical free modes. In addition, we were able to identify the gravest symmetric wavenumber-2 normal mode, but no other wave-2 normal modes. The goal of this study is to document their structure during quiet, moderately disturbed, and greatly disturbed Northern Hemisphere winters. A unique aspect of this study is the use of data assimilation products that extend to the mesosphere and lower thermosphere (MLT; ~92 km) that offer for the first time the possibility to study transient behavior in a deep atmospheric column. The forecast assimilation system and data assimilation products are described in section 2. A description of the time mean and transient behavior of the background atmosphere during the Northern Hemisphere winters of 2005, 2006, 2008, and 2009 is given in section 3. Spectral analyses of the filtered data products are shown in section 4. The relation to SSW is discussed in section 5. Conclusions are given in section 6.

2. Data products and methodology

The observationally constrained products that form the basis of this study are 6-hourly global meteorological analyses from about 0- to 92-km altitude issued by an Advanced Level Physics High-Altitude (ALPHA) prototype of the Navy Operational Global Atmospheric Prediction System (NOGAPS). NOGAPS is the Department of Defense’s operational global numerical weather prediction (NWP) system (Hogan and Rosmond 1991). NOGAPS-ALPHA is a high-altitude research prototype of that system that extends the vertical range and capabilities of both the forecast model and data assimilation system (DAS) components, thereby enabling the system to operate up to much higher altitudes (~92 km).

High-altitude global analyses were generated using the production NOGAPS-ALPHA configuration described in sections 2 and 3 of the paper by Eckermann et al. (2009), to which the interested reader is referred for full details. Briefly, the high-altitude forecast model component operates at a horizontal resolution of T79 with 68 vertical levels from about 0- to 92-km altitude, providing 0–9-h forecast backgrounds to the Naval Research Laboratory (NRL) three-dimensional variational data assimilation (3DVAR) DAS (NAVDAS; Daley and Barker 2001), which combines those forecast backgrounds with global observations to issue a global analysis field every 6 h. In addition to the archived suite of satellite and suborbital observations from operational meteorological sensors that the standard NOGAPS assimilates, which extend from about 0- to 40-km altitude (see Baker et al. 2007), these production NOGAPS-ALPHA runs also assimilate version-2.2 limb retrievals of temperature, water vapor, and ozone from the Microwave Limb Sounder (MLS) on NASA’s Aura satellite and version-1.07 limb retrievals of temperature from the Sounding of the Atmosphere using Broadband Emission Radiometry (SABER) instrument on NASA’s Thermosphere–Ionosphere–Mesosphere Energetics and Dynamics (TIMED) satellite. The MLS and SABER temperature observations are assimilated over the pressure range 32–0.002 hPa (~25–92-km log-pressure altitudes). Final analysis products are issued every 6 h on a uniform 1° × 1° global grid at a series of 60 reference pressure levels from the ground to about 100 km, distributed approximately every 2 km in the vertical throughout the middle atmosphere. These gridded analyses form the basis for the present study.

Several previous studies have confirmed that the zonal mean state and planetary wave structures are captured accurately in these NOGAPS-ALPHA analyses at all altitudes, including the newly analyzed upper range of altitudes from about 50 to 90 km (Eckermann et al. 2009; Stevens et al. 2010; McCormack et al. 2010; Coy et al. 2011). Of particular relevance for the present study are the quasi-5-day Rossby (1, 1) normal mode structures studied and validated by Eckermann et al. (2009) and Nielsen et al. (2010). Nielsen et al. (2010) showed how the temperature structure of the 5-day wave in the NOGAPS-ALPHA analyses during August 2007 was entirely consistent with independent observations of 5-day wave-1 modulation of polar mesospheric cloud (PMC) brightness from NASA’s Aeronomy of Ice in the Mesosphere (AIM) satellite. Given the demonstrated ability of the current NOGAPS-ALPHA configuration to capture the dominant planetary-scale motions of the atmosphere over the altitude range 0–92 km, we use these global analysis products to conduct a thorough investigation of Rossby normal modes over the entire range of analyzed altitudes for some Northern Hemisphere winter periods.

1 This elevation and all others quoted in the following are log-pressure altitudes, using a scale height of 7 km.
To determine the statistical significance of the spectra, we compare the spectrum of the data to the spectrum of an autoregressive process of first order (AR1; see Wilks 2006) whose lag-1 autocorrelation and the white noise variance are determined by fitting the data spectrum to the theoretical spectrum of the AR1 process. The variance of the AR1 spectrum is then compared to the variance of the data spectrum; the latter is deemed statistically significant at any given frequency only when it exceeds the corresponding AR1 variance by a factor that is proportional to a critical value of $\chi^2$. Details of this calculation can be found in Wilks (2006) and are summarized in the appendix. The Hayashi (1971) technique is used to calculate the amplitude and phase of the band-passed modes. Following Hayashi, a quadrature spectrum and a co-spectrum are calculated, using the spectral coefficients over a specified band, and a coherence squared analysis is used to evaluate the robustness of the wave structures by focusing on the results where the coherence-squared exceeds 0.90.

3. Climatology of the four Northern Hemisphere winters

In this section, a brief climatology of the four Northern Hemisphere winters is presented. As indicated in the previous section, the NOGAPS-ALPHA data products have been extensively validated (Hoppel et al. 2008; Eckermann et al. 2009). Moreover, various aspects of the wintertime meteorology have been documented: the tropospheric preconditioning of an SSW (Coy et al. 2009), the role of gravity wave drag in the mesosphere during different winter conditions (Siskind et al. 2010), the effect of dynamical variability on the occurrence of polar mesospheric clouds (Nielsen et al. 2010; Siskind et al. 2011), and a climatology of the 2-day wave (McCormack et al. 2009, 2010). The climatologies for each single winter case of those considered in the present study have been discussed in Manney et al. (2008, 2009), Coy et al. (2009, 2011), and Siskind et al. (2010). Here we present only the behavior of the background atmosphere that is relevant to the presence of normal modes.

We start by examining the zonal-mean winds, which influence strongly the behavior of the free modes. Figure 2 shows the zonal-mean zonal wind averaged during the four NH winters. The average is taken during the periods described in section 2. The differences among these NH winters are subtle in the time average but still important to notice. During the quiet winter of 2005 (Fig. 2a), the zonal circulation shows the strongest westerly flow in the stratosphere. The 20 m s$^{-1}$ line in the winter hemisphere closes off in the lower stratosphere (~20 km at 45°N) and a strong westerly flow is found in the mesosphere. The winter of 2006 (Fig. 2b) shows a zonal-mean flow that is substantially weaker in the stratosphere, with the 20 m s$^{-1}$ contour closing at about 40 km and 45°N. The winter of 2008 (Fig. 2c) is moderately disturbed as the zonal circulation weakens substantially below 40 km compared to 2005 and the 20 m s$^{-1}$ line retreats to 25 km. On the other hand, during the winter of 2009 (Fig. 2d), the 20 m s$^{-1}$ line retreats to even higher altitudes (above 30 km) and the mesospheric jet appears displaced and strengthened on the poleward side of the jet core.

The time-mean behavior gives only an indication of the refractive properties of the background through which planetary-scale waves propagate (Dickinson 1968), as the zonal circulation may change very rapidly during an SSW. In fact, as discussed by Salby (1981a), the location of the critical line and the strength and the vertical structure of the westerly polar jet are factors that affect the presence of a normal mode. Figure 3 shows the vertical structure as a function of time of the zonal-mean zonal wind averaged poleward of 60°N from the ground.
to the MLT during the four winters. To evaluate the state of the stratosphere, the WMO definition of SSW is adopted: an SSW occurs when the zonal-mean zonal wind at 60°N and 10 hPa becomes westward. An SSW is indicated in Fig. 3 by vertical dashed lines when it occurs in a given winter. In all years, at the beginning of winter (November and December), the polar jet strengthens up to 40–60 m s$^{-1}$ with a peak near the stratopause. During the quiet winter (2005; Fig. 3a) the middle atmosphere maintains an isolated polar vortex until March 2005, when the zonal wind becomes westward as equinox approaches. The winter of 2006 (Fig. 3b) shows the effect of the large SSW as westward zonal winds (about −30 m s$^{-1}$) appear at the stratopause in early January 2006 several days prior the SSW; the westward zonal winds descend during the following month and weaken, reaching the lower stratosphere in early February. Notice that as westward zonal winds invade the stratosphere, a strong eastward jet develops in the mesosphere during February. The winter of 2008 (Fig. 3c) is somewhat similar to the quiet winter 2005, except for brief interruptions of the eastward zonal circulation during late January and early February 2005 and the brief SSW in late February. Immediately following the SSW, the stratosphere begins to show the springtime climatology with weak eastward winds. The winter of 2009 (Fig. 3d) shows a behavior in this case more similar to the winter of 2006. Following the eastward jet established at the beginning of winter, substantial westward winds descend quickly from the mesosphere into the stratosphere in January. These anomalous westward winds continue to descend and invade the lower stratosphere, persisting as weak westward winds throughout February of 2009. As in the case of 2006, this is accompanied by a recovery of the eastward polar vortex in the mesosphere.

Figures 2 and 3 illustrate two different types of seasonal evolution of the NH winter. The quiet winter of 2005 is characterized by strong and relatively slowly varying westerly winds throughout the middle atmosphere. At the other extreme, the highly disturbed NH winters of 2006 and 2009 show a large and rapid reversal of the zonal circulation in midwinter, a prolonged disturbance in the lower stratosphere, and a rapid recovery in the mesosphere. An example of a behavior between

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2 Because of the limited temporal record in 2006, this behavior is less apparent in that year (Fig. 3b).
the disturbed and quiet winters is given in 2008, which has an SSW of short duration in late winter; the SSW in 2008 merges with the seasonal transition to springtime. In the next section, we examine how the presence of the normal modes is related to the meteorological behavior of a given period.

4. Spectral analysis

The spectra of westward-propagating waves of wavenumbers 1 and 2 for the four NH winters are investigated in this section. We identify those spectral regions that are statistically significant using the method described in the appendix; we then discuss the corresponding spatial structures using a coherence analysis. Specifically, we look for statistically significant wave-1 variance near 25, 16, 10, and 5 days, the theoretical eigenperiods of the Rossby normal modes (1, 4) through (1, 1), respectively (Longuet-Higgins 1968). For wavenumber 2, we identify significant variance near the theoretical frequency of the (2, 1) normal mode. Following on the discussion of the mean climatologies for each winter in section 3, we expect that there will be a good deal of year-to-year variability in the observed spectra. Numerical calculations (e.g., Salby 1981b; see his Fig. 4) show that the response is broadly enhanced in different spectral ranges for each normal mode. In Table 2 we list frequency bands for each mode of wavenumber 1, labeled V1–V4, based on guidance from Salby’s numerical calculations. These bands are plausible ranges of frequency within which the 25–5-day normal modes might be expected to occur. Within these bands we apply a “band a priori” hypothesis, discussed in the appendix, to evaluate statistical significance. The band a priori hypothesis reduces the value of \( \chi^2 \) required for significance within the band compared to the case with the “no a priori” hypothesis; everywhere else the more stringent value of \( \chi^2 \), appropriate to the no a priori case, is used (see the appendix).

Figure 4 shows the wave-1 spectral amplitude as a function of frequency and altitude at 60°N for each
The spectral bands identified in Table 2 are identified in each panel; unshaded areas identify those spectral regions where the amplitude is large enough to reject the null hypothesis based on the band a priori test. For the V4 band, we find three winters with large enough amplitude to be considered statistically significant: 2005, 2008, and 2009. The first two winters show statistically significant amplitude in the lower stratosphere and in 2008 also in the troposphere; in 2009 the amplitude is statistically significant only in the middle stratosphere, between about 20 and 40 km. The V3 band shows statistically significant amplitude in all winters over a greater or lesser altitude range depending on the year; while the largest amplitudes are found in the upper stratosphere and the lower mesosphere, statistically significant amplitudes are found also in the troposphere. The V2 band shows statistically significant amplitude during the winters of 2005, 2008, and 2009 in the upper stratosphere and lower mesosphere and also in the troposphere in 2009. Finally, the V1 spectral band shows statistically significant amplitude in 2005, 2006, and 2009. The amplitude is statistically significant over a deep atmospheric layer in 2006 and 2009 but is more confined to the troposphere in 2005.

The spectral bands identified in Fig. 4 are used to carry out a coherence analysis, following Hayashi (1971). In the following, we discuss the spatial coherence of the amplitude and of the phase of these spectral bands and identify those cases when the spatial structure is consistent with the nominal normal mode behavior.

### a. 25-day mode (V4 band)

The second antisymmetric Rossby mode for wave-1 has a nominal period of 25 days; it is also referred to as the (1,4) mode (Salby 1981b). Its geopotential structure with respect to latitude is antisymmetric about the equator (Fig. 1), with three zero-crossings, one at the equator and one each in the NH and Southern Hemisphere (SH; see also Madden 2007). The zonal phase speed of this wave is very slow (~9 m s⁻¹ at 60°N). This mode is difficult to identify reliably in the spectrum not only because of distortion effects occurring with changes of the background zonal wind, but also because of the limited spectral bandwidth in any single-wintertime series.

Figure 5 shows the amplitude and the phase of the band-passed geopotential heights obtained from the coherence analysis applied in the V4 band (see Table 2 and Fig. 4). Note that the amplitude and phase are shown without shading where the coherence squared is greater than 0.9 and the shaded regions indicate those locations where instead it is lower than 0.9; the reference point in the coherence analysis is at 60°N and 18 km and is indicated by a cross in each panel. The amplitude plots show the largest wave amplitude around 70°N, with substantial magnitude extending throughout the stratosphere. The winter of 2005 (Fig. 5a) has an amplitude minimum near the stratopause in winter, while in 2009 (Fig. 5c) the amplitude minimum is at about 70 km, and during 2008 (Fig. 5c) the amplitude is large throughout the stratosphere and mesosphere. The phase (Figs. 5b,d,f) conveys information on the spatial structure of the band-passed behavior. Although somewhat distorted, the phase structure in 2005 (Fig. 5b) is consistent with the (1, 4) mode in the troposphere and in much of the stratosphere, with nodal points in both northern and southern subtropical latitudes, and a third nodal point near the equator. During 2008 (Fig. 5d), there is no indication of the expected antisymmetric structure with three nodes anywhere. In fact, during 2008 the phase structure is mostly symmetric, particularly in the stratosphere, which is reminiscent of the (1, 3) normal mode. The phase structure in 2009 (Fig. 5f) shows evidence of a phase behavior that is consistent with the nominal (1, 4) mode in the troposphere. Above 20 km the phase shows a mixed response: while the NH preserves the modal response characteristic of the Rossby (1, 4) mode at least up to 40 km, the SH shows a behavior closer to a (1, 3) mode.

As can be evinced from Fig. 5, the identification of the mode from coherence analysis remains subject to uncertainty; for example, in 2005 the (1, 4) mode with the expected meridional symmetry appears in the stratosphere. On the other hand, during 2008, while the spectral location of the statistically significant variance (V4) is similar to 2005, the phase structure is only reminiscent of the (1, 3) mode with overall meridional symmetry in the stratosphere. A somewhat similar situation exists in 2009 when the structure is mostly symmetric in the

### Table 2. Wavenumber-1 spectral bands.

<table>
<thead>
<tr>
<th>Band label</th>
<th>Frequency range (cpd)</th>
<th>Period range (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V4</td>
<td>0.030–0.050</td>
<td>20–33</td>
</tr>
<tr>
<td>V3</td>
<td>0.040–0.085</td>
<td>12–25</td>
</tr>
<tr>
<td>V2</td>
<td>0.090–0.120</td>
<td>8–11</td>
</tr>
<tr>
<td>V1</td>
<td>0.160–0.220</td>
<td>4.5–6.25</td>
</tr>
</tbody>
</table>

3 Statistically significant amplitude in 2006 in the troposphere appears in a continuum of decreasing magnitudes starting at zero frequency and continuing through the V4 band. Since it does not emerge as an isolated spectral peak, we do not identify it as a potential evidence of the (1, 4) mode.

4 A sliver of statistical significance is found also in 2006 in the mesosphere around 0.11 cpd. Because of the limited spectral width of this region, we disregard it in the present discussion. It is likely that the coarse bandwidth in 2006 influences the isolation of spectral amplitude.
stratosphere, where the largest statistical significance is calculated (Fig. 4d), but there is evidence of the nominal (1, 4) normal mode behavior in the troposphere and in the lower stratosphere, where the statistical significance, however, is substantially reduced. This is an indication that the coherence analysis may be capturing a superposition of waves that includes the (1, 4) normal mode but also other modes that are Doppler shifted to appear at different frequencies than expected from theory. As noted above, the difficulty with the (1, 4) mode is also related to using single-winter data, which yields a relatively broad bandwidth, hindering discrimination of different modes. Figure 5 documents a wave-1 oscillation filtered around 25 days in the NOGAPS-ALPHA analysis that, notwithstanding the prominence of the spectral peaks in some years (Fig. 4), does not always resemble the theoretical (1, 4) Rossby normal mode. Our analysis of the periods analyzed by NOGAPS-ALPHA shows that the (1, 4) mode is clearly present only in the troposphere and stratosphere in the dynamically undisturbed winter of 2005.

b. 16-day mode (V3 band)

The 16-day mode, or (1, 3) mode, is the second symmetric Rossby mode of wavenumber 1. It has a theoretical zonal phase speed of about 14 m s\(^{-1}\) at 60°, which also makes it rather susceptible to broadening due to changes of the background winds. It was studied numerically by Salby (1981b) and Forbes et al. (1995) and extracted from reanalysis fields by Madden (2007). The meridional structure (Fig. 1) is symmetric with two nodes at low latitudes on opposite sides of the equator. Salby (1981b) noted that the spectral response under realistic background winds of the (1, 3) mode is very broad, from 20 to 12 days.

Figure 6 shows the amplitude and the phase of the band-passed behavior obtained from the coherence analysis in band V3. Again, the amplitude is shown without shading where the coherence squared, with a reference point at 60°N and 60 km, is greater than 0.9. Note that the reference point now is located at a different altitude from that used in Fig. 5, near the amplitude
maximum. In all winters the largest amplitude occurs in the upper stratosphere and the lower mesosphere. However, the largest amplitude is obtained in 2008 (~100 m; Fig. 6e), which is several times larger than the smallest, in 2009 (~30 m; Fig. 6g). In all cases, the amplitude in the summer hemisphere is small. This contrasts with the results presented by Williams and Avery (1992), where the largest amplitude of a 16-day oscillation is found instead during local summer: this difference of the amplitude structure may reflect different excitation mechanisms operating at different times. During the winter of 2009 (Fig. 6h), the phase behavior has symmetry that is clearly consistent with the expected structure of the (1, 3) normal mode in the troposphere, stratosphere, and lower mesosphere; the behavior remains overall symmetric in the mesosphere, but its structure is rather distorted, with the nodal points extending to the high latitudes in both hemispheres. The winter of 2008 (Fig. 6f) also shows evidence of the (1, 3) structure but only in the troposphere and the lower stratosphere. The winter of 2005 (Fig. 6b) shows symmetric behavior with nodal points around 30° in both hemispheres in the troposphere.
Fig. 6. As in Fig. 5, but for the V3 band, for (a) 2005, (b) 2006, (c) 2008, and (d) 2009. Contour interval is 10 m for the amplitude. Reference point is at 60°N and 60 km, indicated by the black cross.
and stratosphere, although the phase tends to be somewhat distorted in the upper levels. During the winter of 2006 (Fig. 6d), the phase structure shows limited evidence of a symmetric behavior in the stratosphere and mesosphere, between about 20 and 70 km.

In summary, for the second symmetric Rossby mode we find evidence of a modal structure that resembles the theoretical Rossby normal mode in 2005, 2008, and 2009 in the troposphere the stratosphere, and part of the mesosphere, with more limited evidence in 2006. Note, however, that the amplitude of the band-passed behavior during the winter of 2009 (a winter with a very large and persistent SSW) is several times smaller than during the winter of 2008, reflecting the different climatology of those two winters (cf. Figs. 2 and 3). The conclusion based on the limited evidence offered by the NOGAPS-ALPHA analysis fields is that, regardless of the excitation mechanism that forces the (1, 3) mode, the occurrence of the SSW in 2009 is likely responsible for the small amplitude of the normal mode in that year. This is consistent with the presence for a prolonged period of time of westward winds in the stratosphere following the SSW.

It is worth noting that the lower stratosphere and the troposphere during the winter of 2008 show the presence of a (1, 3) structure beyond the limits of the V3 spectral band as defined here. Evidence of this is shown in Fig. 5d for a spectral band around 25 days; another example is shown later for the V2 band in 2005 that also has a clear (1, 3) modal behavior in the troposphere and the stratosphere. Such spectral broadening and Doppler shifting is also present in numerical calculations (Salby 1981a); in the real atmosphere, the frequency overlap between normal modes makes the distinction between modes rather difficult and more complicated. It should also be noted that in these instances a finer spectral bandwidth would be of no help, as the overlapping modes are due to the physical properties of the background atmosphere as opposed to the limited length of the observational series.

c. 10-day mode (V2 band)

The 10-day normal mode, also referred to as the (1, 2) Rossby mode, is the first antisymmetric normal mode of wavenumber 1, with a single node at the equator (Fig. 1). Its zonal phase speed is about 23 m s\(^{-1}\) at 60°, which should make it less susceptible to moderately small changes of the background winds.

Figure 7 shows the amplitude and phase of the band-passed behavior in the V2 band. The largest amplitudes (~30–40 m) are found in 2005 and 2008 (Figs. 7a,c), and the smallest (~20 m) in 2009 (Fig. 7e). In all cases the amplitude reaches a maximum in midlatitudes of the winter hemisphere. The phase structures in 2005 and 2009 (Figs. 7b,f) are symmetric and, in fact, largely reminiscent of the (1, 3) mode. Only in 2008 (Fig. 7d) is the phase structure somewhat reminiscent of the (1, 2) mode in the troposphere. If one neglects the behavior at high southern latitudes, the two hemispheres show antisymmetric behavior that is separated by a zero phase line approximately at the equator. Note that above 20 km, the behavior in 2008 is largely symmetric, as for the (1, 3) mode. The prominent presence of symmetric structures in particular in the stratosphere and mesosphere suggests frequency overlap between different normal modes, following spectral broadening and Doppler shifting due to the unsteadiness of the background atmosphere.

d. 5-day wave (V1 band)

The 5-day wave, also known as the (1, 1) mode, is the first symmetric Rossby mode. Salby (1981b) discusses the spectral breadth of this mode, identifying a 5-day type response between 4 and 6 days. Wu et al. (1994) found evidence in satellite data of the (1, 1) mode at about 6 days (~0.16 cpd) in high-resolution Doppler imager (HRDI) data, and Pancheva et al. (2010) show that a westward wave-1, 5.5-day oscillation is present in SABER data. The signature of a 5-day wave has also been found in tracer data using output of the Canadian Middle Atmosphere Model (Pendlebury et al. 2008); these results also suggested that the interannual variability of the background winds can affect the observed amplitude of the 5-day wave. The (1, 1) mode is symmetric and lacks nodal points in the meridional direction (see Fig. 1). The theoretical mode has a sufficiently fast horizontal phase speed (46 m s\(^{-1}\) at 5 days; 33 m s\(^{-1}\) at 7 days) that it should be less susceptible to variations of the underlying winds, relative to the slower modes.

Statistically significant spectral peaks at around 0.2 cpd are found in the analysis spectra in 2005, 2006, and 2009 (Fig. 4). The coherence analysis yields the amplitude and phase plots shown in Fig. 8. In 2005 (Fig. 8a) and 2006 (Fig. 8c), two distinct amplitude peaks are found, in the winter (Northern) hemisphere around 70°N and in the summer (Southern) hemisphere in the MLT region around 60°N. The amplitude in 2009 (Fig. 8e) instead has a broad peak in the MLT with no indication of a preferential latitude. This last result does not agree with numerical calculations (Salby 1981b), which show a more pronounced amplitude in the summer (Southern) hemisphere. The different location of the amplitude maxima in 2009 between the observational analysis and theoretical numerical calculations is probably indicative of a forcing mechanism operating in the winter (Northern) hemisphere: a larger amplitude will generally occur near the location where the forcing is
present. The accompanying phase structures show a mode whose phase is uniform horizontally and with very long vertical wavelength during 2005 (Fig. 8b) and 2009 (Fig. 8f), confirming the presence of a (1, 1) Rossby normal mode in this winter. The phase structure in 2006 is reminiscent of a (1, 1) mode only in the upper mesosphere. Large amplitudes in the Southern (summer) Hemisphere are present also in numerical calculations (e.g., Salby, 1981b; see his Fig. 6), and it is worth noting that those amplitudes in 2005 and 2006 may exert a substantial control over the occurrence of polar mesospheric clouds (Nielsen et al. 2010); the wave amplitude can be enhanced locally by the occurrence of dynamical instability associated with large meridional gradients of the zonal-mean wind and the associated temperature fluctuation can favor the formation of clouds.

e. Wavenumber-2 Rossby modes

We briefly investigate now the occurrence of Rossby normal modes at wavenumber 2. Salby (1981b) discusses the gravest mode (2, 1), which in his numerical calculations occurs in the frequency band 0.21–0.26 cpd (see his
Using the band a priori hypothesis (see the appendix) in this spectral band we find that a statistically significant peak only in 2009. Figure 9 shows the amplitude spectrum at 60°N as a function of altitude during 2009 (Fig. 9a); also shown are the amplitude (Fig. 9b) and phase (Fig. 9c) of the band-passed behavior (unshaded only where the coherence squared is greater than 0.9) during the same winter. The spectral amplitude at 60°N (Fig. 9a) exceeds the null hypothesis in the 0.21–0.26 cpd band in a vertically extended region from the upper troposphere/lower stratosphere up to the upper mesosphere. The wave structure (Fig. 9b) determined from the coherence analysis (with a reference point at 60°N and 18 km) shows two broad peaks in the lower mesosphere/upper stratosphere in both the Northern (winter) and Southern (summer) Hemispheres; the winter hemisphere peak is somewhat larger than its counterpart in the summer hemisphere. The phase is symmetric above 40 km with only a nodal line at high latitudes of the Southern (summer) Hemisphere. The phase structure below 40 km is more complex. Overall, the band-passed behavior is consistent with that obtained...
from numerical calculations by Salby (1981b) above 40 km. Aside from this mode, we were unable to find statistically significant evidence for any other Rossby modes of wavenumber 2.

5. Relation to SSW

The spatial structures of the band-passed wave-1 geopotential anomalies in the analysis fields, discussed in the previous section, illustrate the occasional presence of normal modes during four recent winters. Based on the limited evidence presented in this study, the winters with SSW do not seem to favor the amplification of normal modes, contrary to some previous findings (e.g., Pancheva et al. 2008). The two winters with large SSW (2006 and 2009) show limited evidence of slow normal modes [e.g., the (1, 4) mode], and the amplitude of the modes that can be identified is substantially smaller than during the winters (2005, 2008), which are relatively quiet dynamically. The only wave-1 normal mode that is clearly identifiable during all winters is the 16-day wave (V3 band), the mode most typically reported in observations during SSWs (e.g., Dowdy et al. 2004).

The fact that the normal modes are less likely to be found in winters with SSWs is not inconsistent with theory. Salby (1981b) shows that normal modes should appear more prominently when the zonal circulation is

![Fig. 9](image-url). (a) Amplitude spectrum of the westward wavenumber-2 geopotential height at 60°N during 2009. Color scale is shown at the bottom as the $\log_{10}$ of the amplitude (m). White-out areas are not statistically significant based on the application of the band a priori hypothesis. (b),(c) Amplitude and phase obtained from coherence analysis in the 0.21–0.26 cpd spectral band for 2009. Reference point is at 60°N and 18 km. Contour interval is 1 m for the amplitude and 30° for the phase.
relatively undisturbed and eastward. The amplitude of
the normal modes is substantially biased toward the
winter hemisphere near the solstice when westward
winds are predominant in the summer hemisphere. This
picture is complicated in real atmospheric situations
where the zonal circulation is far from being steady,
which is one of the assumptions in Salby’s study, and the
presence of forced waves may result in a reversal of the
zonal circulation, inhibiting the presence of westward-
propagating normal modes in the stratosphere. Gravity-
wave drag further complicates normal mode structures
in the mesosphere and lower thermosphere (e.g., Forbes
et al. 1995). Therefore, the absence of some Rossby
normal modes in the winters with SSWs may be the result
of disturbed periods during which the zonal circulation
reverses and prevents the slower modes from reaching
the middle atmosphere.

By way of example, we show the intrinsic frequency
corresponding to the V3 (16-day wave) band in 2005 and
2009 in Fig. 10. Figure 10a shows that the intrinsic fre-
quency in 2005 is quite similar to that obtained by Salby
(1981b, his Fig. 8d) using a numerical model that employs
a stationary wind distribution. The intrinsic frequency is
negative (i.e., the zonal phase speed is westward) in the
winter hemisphere with values increasingly more west-
ward in the upper stratosphere and lower mesosphere. A
critical line is found in the summer hemisphere; starting
from the stratopause at the South Pole, it extends to the
equator in the upper stratosphere. The intrinsic fre-
quency during the highly disturbed winter of 2009 prior
to the warming (averaged between 1 November 2008
and 23 January 2009; Fig. 10b) is similar to that shown in
2005 because the zonal circulation during this time in
2009 is still westerly in the winter hemisphere as is the
case of the winter of 2005. However, the averaged
intrinsic frequency after the SSW (between 24 January
2009 and 15 February 2009; Fig. 10c) shows a sub-
stantially different structure; the intrinsic frequency is
weakly westward below 40 km in the winter hemi-
sphere and large westward values are found only in the
upper mesosphere.

In a situation like that found in 2009 (Fig. 10c), slow
normal modes, like (1, 4) and (1, 3), are inhibited from
propagating in the stratosphere by the presence of
a critical line. Particularly for the (1, 3) mode, evidence
of the high susceptibility of the slow modes to the
background wind configurations is also reported in Lou
et al. (2000) and Namboothiri et al. (2002). At the same
time, faster modes such as (1, 1) are less affected by
changes of the zonal winds even in a highly disturbed
winter, such as 2009. Figure 11 shows the intrinsic fre-
quency for the (1, 1) mode averaged prior to the SSW and
after the SSW (same averaging periods as in Figs. 10b,c).

The resulting maps of the intrinsic frequency in the two
periods are similar: although the intrinsic frequency is
somewhat less westward in the lower stratosphere after the
SSW (Fig. 11b) compared to the earlier period (Fig. 11a),
it remains substantially westward globally.
6. Summary and conclusions

Using global observationally constrained meteorological analyses that extend from the surface to the upper mesosphere and lower thermosphere, we have isolated the spectral band-passed behavior of the gravest zonal waves corresponding to the eigenfrequencies of the (1, 4), (1, 3), (1, 2), and (1, 1) normal modes during four recent winters. The spectral locations of the response are near the eigenfrequencies of those modes, but substantial broadening around those frequencies is observed and inferred. This is to be expected because, as the background atmosphere changes, the response broadens and shifts from the theoretical eigenfrequency (Salby 1981a). It should also be borne in mind that the resulting response calculated using atmospheric data always contains a mixture of waves that may confound the detection and identification of the normal modes. While we cannot offer a proof, we propose that this mixture of waves results from the spectral broadening that is caused by the nonstationarity of the background atmosphere. This difficulty is compounded by the limited record length, which coarsens the bandwidth and the ability to isolate those modes clearly.

Other authors have attempted to isolate normal modes in atmospheric data. The present work has a different emphasis from those studies that focus on lower atmospheric behavior (e.g., Madden 2007) or those that investigate the normal modes during specific wintertime conditions (Smith, 1985; Forbes et al. 1995). Instead, we exploit the deep atmospheric coverage of the NOGAPS-ALPHA data assimilation products to investigate the structure of the normal modes up to the lower thermosphere during four Northern Hemisphere winters: a quiet winter (2005), a disturbed winter (2006), a moderately disturbed winter (2008), and a highly disturbed winter (2009). At the same time, an evident shortcoming of this study is that the conclusions drawn here apply strictly speaking only to those winters. A more comprehensive climatology of these normal modes and their relevance during different atmospheric conditions is yet to be done but is beyond the scope of this study.

We offer the following conclusions:

• Statistically significant spectral peaks at the spectral locations of the (1, 4), (1, 3), (1, 2), and (1, 1) normal modes are found in some winters, but with markedly different amplitudes. The geopotential amplitudes are largest in the midlatitude winter upper stratosphere and lower mesosphere. A close inspection of the meridional behavior (via coherence analysis) yields a structure that is only occasionally reminiscent of the theoretical normal modes (Salby 1981b). The amplitude of the slower modes tends to be reduced during dynamically disturbed winters (2006 and 2009). Particularly noticeable is the presence of a symmetric structure at frequencies greater than 0.06 cpd that is reminiscent of the (1, 3) normal mode. On the other hand, the (1, 2) antisymmetric behavior is missing from most of the years analyzed, emerging with very modest amplitude in the troposphere only in the winter of 2008.

• The (1, 1) mode, or 5-day wave, is present with statistically significant amplitude during three winters (2005, 2006, and 2009). During 2009, the amplitude is significantly different from the AR1 spectrum over a deep atmospheric layer from the lower stratosphere to the upper mesosphere. This wave mode has a relatively high zonal phase speed, and it is not affected substantially by the changes of the background atmosphere during winter. The resulting spatial structure is very close to that predicted by theory for the
(1, 1) mode. We found that the amplitude in the summer MLT region is amplified in some years; this amplification has been suggested as a cause for the observation late in the season of polar mesospheric clouds.

- The slower modes, that is, the (1, 3) and (1, 4) normal modes, are more likely to be affected by the transient behavior of the background winds in particular during disturbed wintertime periods. In fact, we present some evidence of these modes in the winter of 2005, which is quiet. In particular for the (1, 4) mode, its presence is limited to the lower atmosphere.

- There is no evidence that the occurrence of an SSW amplifies normal modes, or vice versa. In fact, when an SSW is so large and persistent as to eliminate the eastward winds in the extratropical stratosphere of the winter hemisphere, a critical line can appear for the slow normal modes in the lower stratosphere that prevents them propagating to higher altitudes. It is perhaps for this reason that the normal modes are seen more clearly in the winter of 2005, which is the most dynamically quiet of all the winters analyzed and included no SSW.

- We have discussed briefly the occurrence of Rossby normal modes at wavenumber 2. These modes are much more difficult to detect with amplitudes that exceed the null hypothesis. In fact, we find statistically significant amplitude at wavenumber 2 only in the band 3.8–4.7 days, and only in the winter of 2009. This frequency band corresponds to the (2, 1) Rossby normal mode and, in this case, the band-passed behavior resembles closely the (2, 1) mode modeled by Salby (1981b). We do not know why this mode is not present with significant amplitude in the other years examined in this study.

We note that the modes identified here have generally small time-mean amplitudes in all of the winters analyzed. In particular, the (1, 3) mode, or 16-day wave, has winter-mean amplitudes (Fig. 6) that do not exceed about 100 m. This is much smaller than the transient wave amplitudes attributed to the 16-day wave in the winter of 1979 prior to the occurrence of a major SSW (as much as several hundred meters at 1 hPa; see, e.g., Madden and Labitzke 1981; Smith 1985). The discrepancy may arise from the way in which transient waves are characterized. In this study, we identify the (1, 3) normal mode (and other normal modes) with the variance within a band predicted by theory and judged to be statistically significant, as explained in section 4. On the other hand, the studies cited above define transient waves as the departure from the time mean for the period of analysis. For data with short records, this is unavoidable, but it conflates transient variance at all frequencies. If one defines the normal modes in terms of the statistically significant response only within the theoretical frequency bands where they are expected to occur, one must conclude that they achieve only modest amplitudes and may not play an important role in the generation of SSW events. In fact, recent studies of the 16-day wave yield amplitudes more in line with our results. For example, Pogoreltsev et al. (2009) estimate 16-day wave geopotential amplitudes of about 30 m at 30 hPa from National Center for Atmospheric Research (NCAR)–National Centers for Environmental Prediction (NCEP) reanalysis data, which is comparable to the results shown in our Fig. 6. Lott et al. (2009) use 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40) data to construct composite episodes of strong activity in the period band 7–30 days, and they estimate amplitudes of 30 m in the lower stratosphere (50 hPa). This is larger than what we obtain at that level by at least a factor of 2, but our results are wintertime means whereas Lott et al.’s results emphasize periods of high 16-day wave activity. When we compute the time-dependent amplitude of the 16-day wave in the winter of 2007/08, we find values at 50 hPa (not shown) that range between 10 and 30 m.

The limited frequency bandwidth due to a single winter’s length of data may have limited our ability to reliably distinguish between wave modes that are closely located in spectral space. At the same time, we find a mixture of modes with overlapping spectral bands and in this case a finer bandwidth is of limited help. In contrast to numerical, steady-state calculations in a uniform background state, the real atmosphere contains background winds and temperatures that are neither steady nor uniform and that would be expected to broaden the normal mode responses around their theoretical frequencies. Evidence presented in this study suggests that wave modes can be Doppler-shifted and spectrally broadened to overlap adjacent modes.

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APPENDIX

Statistical Significance of Signals in Selected Frequency Bands Using a “Band A Priori” Hypothesis

In the following we draw from the discussion of AR1 processes found in Wilks (2006). An autoregressive process of the first order is defined as

\[ x_{t+1} - \mu = \rho(x_t - \mu) + e_{t+1}, \quad (A1) \]

where \( x_t \) and \( x_{t+1} \) are the values of the predicted time series at times \( t \) and \( t + 1 \), \( \mu \) is the mean of the time series, \( \rho \) is the lag-1 autocorrelation parameter, and \( e_{t+1} \) is a random (uncorrelated) quantity at time \( t + 1 \) with zero mean and variance \( \sigma_e^2 \) equally distributed at all frequencies. The spectrum of the autoregressive process (A1) is

\[ S(\omega) = \frac{2\sigma_e^2/N}{1 + \rho^2 - 2\rho \cos(\omega)}, \quad (A2) \]

where \( \omega \) is the frequency in rad s\(^{-1}\) and \( N \) is the number of data points. Equation (A2) represents the background spectrum, which we use to define the null hypothesis. Wilks (2006) provides a simple formula to determine how large the spectral amplitude at frequency \( \omega_i \) needs to be in order to reject the null hypothesis: the squared amplitude \( \Psi_i^2 \) associated with the \( i \)th spectral component at frequency \( \omega_i \) is deemed statistically significant at level \( 1 - \alpha \) if

\[ \frac{n\Psi_i^2}{S(\omega_i)} > \chi^2, \quad (A3) \]

where \( \chi^2 \) is the value of the chi-squared distribution corresponding to a statistical significance level of \( 1 - \alpha \) with \( n \) degrees of freedom.

Following Fisher (1929), Wilks (2006) notes that, without an a priori hypothesis on the spectral location of the peaks of variance, one needs to use a more stringent level of statistical significance in (A3) than \( \alpha \). In essence, without knowing a priori where the spectral peaks are located, it is necessary to evaluate the statistical significance at a value of \( \alpha^* \) given by

\[ \alpha^* = \alpha/K^*, \quad (A4) \]

where \( K^* \) is the effective number of independent frequencies; that is, \( K^* = K/S_f \), where \( K \) is the number of frequencies in a band of interest and \( S_f \) is the number of frequencies that are averaged to smooth the periodogram. All spectra in this study are obtained by smoothing the periodogram using a five-point boxcar average. For example, for a 90% confidence level, \( \alpha = 0.1 \) and taking \( K = 180 \) and \( S_f = 5 \), then \( K^* = 36 \) and \( \alpha^* = 0.003 \), which implies that an effective confidence level of 99.7% is required to reject the null hypothesis with 90% confidence. The value of \( \chi^2 \) in (A3) is determined from the value of the chi-squared probability distribution corresponding to \( 1 - \alpha^* \) and 10 degrees of freedom (2 degrees of freedom per spectral component).

To apply the test (A3), one needs to evaluate (A2), where only two parameters are unknown: the “white noise” variance \( \sigma_e^2 \) and the autocorrelation \( \rho \) at lag 1. The autocorrelation is determined from the spectral coefficients using the Weiner–Khinchin theorem, while the white noise variance is easily obtained from (A1) by squaring and averaging both sides of the equation to obtain

\[ \sigma_e^2 = (1 - \rho^2)\sigma_x^2, \quad (A5) \]

where \( \sigma_x^2 \) is the variance of the time series (see also Wilks 2006).

While an evaluation of the spectra without an a priori hypothesis is certainly possible, it is unnecessarily restrictive, since in fact we have some a priori knowledge of where the normal modes are expected to occur and also of the associated spectral uncertainty. The latter may be estimated from the spectral range obtained from the numerical calculations of Salby (1981b) and summarized for each mode in Table 2. Thus, the V3 spectral band has about eight spectral frequencies with a bandwidth of 180 days. Accounting for the spectral smoothing (\( S_f = 5 \)), the effective number of frequencies \( [K^* \text{ in (A4)}] \) is then about 1.6, which requires a critical value of \( \chi^2 \) corresponding to a 94% probability to reject the null hypothesis with a confidence of 90%. It should be noted that a band a priori hypothesis can be evaluated with this reduced level of confidence only in the spectral band for which we have a hypothesis that the normal mode may exist; outside that spectral range, one needs to use the no a priori hypothesis (A,4), which requires instead a value of \( \chi^2 \) corresponding to 99.7%, as discussed above.

The RMS amplitude at 60°N for the null hypothesis thus estimated is shown in Fig. A1 for the winter of 2005 using the 90% level of statistical confidence. Spectral peaks of the data products are compared to the null hypothesis and only those peaks that exceed it are considered statistically significant. A band a priori hypothesis is used only in the V3 spectral range (see Table 2) for this example, while the no a priori hypothesis is used in the rest of the spectrum; this explains the apparent
discontinuity in amplitude between the vertical black lines, which delimit the V3 spectral band and the rest of the spectral amplitudes. Notice how narrowly peaked the null spectrum is below 20 km as opposed to the much broader spectrum above that elevation. This reflects the high autocorrelation of the time series in the troposphere and lowermost stratosphere, as well as the reduced autocorrelation and larger white noise variance in the rest of the middle atmosphere.

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