The Influence of Near-Surface, High-Entropy Air in Hurricane Eyes on Maximum Hurricane Intensity

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ABSTRACT

Using a time-dependent axisymmetric numerical model, the authors evaluate whether high-entropy air near the surface in hurricane eyes can substantially increase hurricanes’ maximum intensity. This local high-entropy anomaly is ultimately created by surface entropy fluxes in the eye. Therefore, simulations are conducted in which these surface fluxes are set to zero; results show that the high-entropy anomaly is eliminated, yet the axisymmetric tangential wind speed is only slightly weakened (by ∼4%, on average). These results contradict the hypothesis that transport of high-entropy air from the eye into the eyewall can significantly increase the maximum axisymmetric intensity of hurricanes. In fact, all simulations (with or without high-entropy anomalies) have an intensity that is 25–30 m s⁻¹ higher than Emanuel’s theoretical maximum intensity. Further analysis demonstrates that less then 3% of the total surface-entropy input to the hurricane comes from the eye, and therefore the total magnitude of entropy transport between the eye and eyewall is a negligible component of the entropy budget of the simulated hurricanes. This latter finding is consistent with a cursory comparison with observations.

1. Introduction

It has been established that numerically simulated tropical cyclone intensity can exceed the maximum intensity that is predicted by analytic theories. In particular, the steady-state analytical maximum potential intensity (MPI) theory of Emanuel (1986, 1995)—hereafter referred to as E-MPI—has recently been evaluated against time-dependent numerical model simulations by Persing and Montgomery [2003 (hereafter PM03), 2005], Hausman et al. (2006), and Cram et al. (2007). Additionally, recent observational studies (e.g., Tonkin et al. 2000; Montgomery et al. 2006; Bell and Montgomery 2008) have found cases in which observed intensities are greater than E-MPI. The fact that E-MPI can be significantly weaker (by ∼10%–50%) than simulated and/or observed tropical cyclone intensity is not challenged here.

What remains to be established is why E-MPI can be significantly weaker than the intensities of some simulated and observed tropical cyclones. To this end, a hypothesis was put forth by PM03, which has further been evaluated using observations by Montgomery et al. (2006), Aberson et al. (2006), and Bell and Montgomery (2008) and using numerical simulations by Persing and Montgomery (2005) and Cram et al. (2007). The theory posits that the locally high-entropy air at low levels in the tropical cyclone’s eye can provide an additional source of energy that is not considered in E-MPI. This process has been referred to as the “superintensity” mechanism (PM03; Cram et al. 2007) and as the “turbulence” mechanism (Montgomery et al. 2006; Bell and Montgomery 2008). We will refer to it hereafter as the PM03 mechanism.

Evidence is clear in both observations and numerical simulations that there can be anomalously (and significantly; i.e., ∼10–20 K) higher equivalent potential temperature ($\theta_e$) in the low levels of tropical cyclone eyes as compared to $\theta_e$ in the eyewall. It is also clear that this air can be transported from the eye into the eyewall. It is not clear to us that this process has any significant effect on maximum azimuthally averaged hurricane intensity. Furthermore, to our knowledge, the total magnitude of $\theta_e$ transport from the eye to the eyewall has
never been evaluated in comparison to $\theta_e$ transport from other locations of the tropical cyclone; in other words, even though some high-$\theta_e$ air from the eye is transported into the eyewall, its effect on average $\theta_e$ in the eyewall has never been quantified clearly.

The purpose of this article is to evaluate quantitatively the PM03 mechanism. To that end, we run experiments with a time-dependent axisymmetric numerical model and we conduct control volume analyses around the eye and eyewall using the model output. We demonstrate that the PM03 mechanism does happen as envisioned but that it is quantitatively small and has only a small effect on maximum axisymmetric intensity. Furthermore, we demonstrate that this mechanism cannot explain the subintensity of E-MPI compared to hurricane intensity in these numerical simulations.

2. Methodology

As did PM03, we use the nonhydrostatic axisymmetric numerical model developed by Rotunno and Emanuel (1987, hereafter RE87). Our default configuration, hereafter referred to as the Control simulation, is identical to the 4x-resolution simulations by PM03, except for the initial sounding. We use the same sounding that was used by RE87 (Fig. 1a), but we have interpolated their sounding to a higher-resolution grid; below their lowest model level and above their highest model level, we extrapolate potential temperature and mixing ratio to new grid points as needed. In contrast, the sounding used in the 4x simulations by PM03 is warmer in the boundary layer and cooler above the boundary layer (Fig. 1b); this different sounding was arrived at through interpolation error and by different assumptions near the bottom and top of the model (J. Persing 2007, personal communication). We have retained their sounding for simulations shown later in this article to allow for comparisons with results published by PM03. However, we use the RE87 sounding (Fig. 1a) for most of our analysis. The sea surface temperature ($T_s$) is 26.13°C in simulations with this sounding. All simulations reported here achieve a steady intensity after $t = 8$ days, so we report the average of maximum tangential velocity ($u_{\text{max}}$) over $t = 8–12$ days as a measure of maximum tropical cyclone intensity.

For calculations of entropy ($s$) and equivalent potential temperature ($\theta_e$), we use the approximate pseudo-adiabatic formulations that were presented by Bryan (2008). These formulations use a constant latent heat of vaporization ($L_v$), which can be adjusted to increase accuracy for a specific numerical model. Using the code from Bryan (2008), and using the equations and constants from the RE87 model for the reference solution, we find that $L_v = 2.604 \times 10^6$ J kg$^{-1}$ is appropriate for this model. The maximum error in $\theta_e$ is 0.4 K.

For the sake of reference, we have calculated estimates of E-MPI for the Control sounding using the “a priori” method of Emanuel (1986), which uses only parameters from an environmental sounding. Specifically, we use (43) of Emanuel (1986), with $r_0 = 400$ km, $f = 5 \times 10^{-5}$ s$^{-1}$, $T_b = 292$ K, $p_a = 1015.1$ mb, and $C_p/C_D = 1$ [see Emanuel (1986) for definitions of these
variables]. Given uncertainty in the outflow temperature (\(T_o\)) and the ambient boundary layer relative humidity (RH_{\text{sea}}), we use a range of values for these two parameters. For \(T_o = 26.13^\circ\text{C}\), we find a minimum value of E-MPI = 37 m s^{-1}, with \(T_o = 220\) K and RH_{\text{sea}} = 0.9, and a maximum value of E-MPI = 60 m s^{-1}, with \(T_o = 200\) K and RH_{\text{sea}} = 0.8.

Given the large inherent uncertainty in the calculation of a priori E-MPI, hereafter we use instead the “local PBL balance” method from PM03 (their p. 2352). Evaluation is performed at the radius of maximum winds using output from the numerical simulation. To obtain a value for \(T_o\), we compute a trajectory using the mean flow fields from the model (averaged from \(t = 8–12\) days); for the parcel that passes through the maximum tangential winds, we calculate the average temperature along the parcel trajectory as it passes between \(r = 200–300\) km. This calculation of \(T_o\) is consistent with the definition by Emanuel (1986, p. 587) as “the temperature the air has along \(M\) surfaces as they flare out to very large radii.”

### 3. Results with the RE87 environment

After about 5 simulated days, the tropical cyclone in the simulation with the RE87 sounding (Control) reaches an approximately steady intensity of 86 m s^{-1} (Table 1). We estimate E-MPI to be 56 m s^{-1} (Table 1). Clearly, this theoretical value is significantly less than the simulated storm’s intensity. We also note that this magnitude of subintensity is comparable to that found by PM03.

Near the surface in the eye, \(\theta_e\) is 12 K higher than \(\theta_e\) at the radius of maximum winds (Fig. 2a). The magnitude of this \(\theta_e\) difference is comparable both to that in the simulation by PM03 and to that observed in Hurricane Isabel (Bell and Montgomery 2008).

To identify the processes that allow this \(\theta_e\) pattern to exist, we have calculated a \(\theta_e\) budget using the same technique as RE87 (see p. 555), except we use the formulation of \(\theta_e\) from Bryan (2008). We confirm (following PM03, p. 2355) that the ultimate source of the high-\(\theta_e\) anomaly is the upward flux of entropy from the sea surface in the eye. All other processes (i.e., advection, horizontal turbulent diffusion, and “radiation”) act to lower \(\theta_e\) in the low-level eye.

Given that surface entropy fluxes in the eye ultimately create the high-\(\theta_e\) anomaly, we conduct a series of simulations in which the surface fluxes are set to zero in the eye. This eliminates the high-\(\theta_e\) anomaly and thus allows us to evaluate the role it plays in maximum hurricane intensity. To this end, we set the surface exchange coefficient for entropy (\(C_E\); see RE87, p. 547) to zero for \(r < 17\) km (the average radius of maximum winds from Control). We note that using the radius of maximum winds to delineate the eye is a very generous definition; that is, this choice probably makes “the eye” too large. In fact, some grid points inside the eye, thus defined, are actually cloudy (and precipitating). We make this choice primarily because of similar simulations conducted by J. Persing and M. Montgomery (2007, personal communication) and because it biases the results toward the largest effect one could expect to find from fluxes in the eye. In other words, we give the eye fluxes the best possible chance to have a significant effect.

We first analyze a simulation in which surface fluxes are zero for \(r < 17\) km for the entire simulation (re-
ferred to as NoEyeFlux). As expected, the high-$\theta_e$ anomaly in the eye does not form in this simulation (Fig. 2b). However, there is very little difference in maximum intensity compared to Control (Fig. 2c). The difference in $u_{max}$ is only 3 m s$^{-1}$ (Table 1). In Fig. 2c, shading is $\theta_e$ difference (as indicated by color bar on the right), and contours are $\theta_e$ difference every 20 m s$^{-1}$ with the zero contour excluded.

**FIG. 2.** Output averaged from $t = 8$–12 days: (a) from Control, (b) from NoEyeFlux, and (c) the difference between the two simulations (NoEyeFlux − Control). In (a) and (b), thin contours are $\theta_e$ every 1 K, shading is $\theta_e \geq 360$ K, and thick contours are $v$ every 20 m s$^{-1}$. In (c), shading is $\theta_e$ difference (as indicated by color bar on the right), and contours are $v$ difference every 2 m s$^{-1}$ with the zero contour excluded.

We also conduct a simulation in which surface fluxes are set to zero for $r < 17$ km after 7 days only (referred to as NoEyeFlux-7d). This simulation is identical to Control for the first 7 days, which is long enough into the simulation for the high-$\theta_e$ anomaly to form and reach a steady state. After the surface fluxes in the eye are turned off, the high-$\theta_e$ anomaly is quickly eradicated (in about 8 h) (Fig. 3c). Notably, the maximum sustained winds are hardly affected by this change; $u_{max}$ decreases by only 2 m s$^{-1}$ (Table 1).

In addition, we conduct a simulation in which we track the radius of maximum winds ($r_m$) at every time step during the simulation and set $C_E$ to zero inward of this point after some nominal intensity (30 m s$^{-1}$) is achieved (referred to as NoEyeFlux-rm). In this case, the zero surface fluxes are not confined to a specified region. Furthermore, when the intensity threshold of 30 m s$^{-1}$ is first exceeded (at $t = 3$ days), the region of zero fluxes extends initially to $r \approx 50$ km. Nevertheless, the same overall result is ultimately obtained. That is, $r_m$ eventually decreases to 17 km (Fig. 3d), and the steady maximum intensity is only slightly less than that from the Control simulation (Table 1).

As mentioned earlier, our definition for the eye in these simulations is quite generous and includes grid points that have precipitation. Consequently, these simulations have zero surface fluxes over part of the eyewall, conventionally defined. To investigate further, we have also conducted simulations that retain the fluxes over all of the eyewall (defined either as precipitating grid points, or as grid points with vertical velocity exceeding 1 m s$^{-1}$ at $z = 1$ km). Results (not shown) show that the high-$\theta_e$ anomaly in the eye is eliminated in these simulations but that $u_{max}$ is unchanged from Control. Thus, the small ($\sim 3$ m s$^{-1}$) change noted in Table 1 is an overestimate of the effect on maximum hurricane intensity due to the high-$\theta_e$ anomaly in the low-level eye.

**4. Simulations in other environments**

To evaluate whether our results are sensitive to the environment chosen for the initial state, we also conduct simulations with the sounding used for the 4x simulations by PM03 (Fig. 1b), with $T_s = 26.13^\circ$C, and also with the mean hurricane sounding by Jordan (1958) (Fig. 1c), with $T_s = 28.0^\circ$C. The latter environment is identical to that used by Hausman et al. (2006), and is nearly the same as the “Mid CAPE” simulation by Persing and Montgomery (2005). As in the previous section, we conduct additional simulations in which the enhancement of low-level eye entropy” (PM03, p. 2349).
high-\(\theta_e\) anomaly is eliminated by setting the surface fluxes to zero inside the radius of maximum winds. We use all three of the methodologies described in the previous section, although in these two environments the nominal value of \(r_m\) is 24 km. As before, we bias the results toward the greatest possible effect of the high-\(\theta_e\) anomaly by including all grid points for \(r < r_m\).

Consistent with the simulations in the previous section, the high-\(\theta_e\) anomaly is completely eliminated when there are no surface fluxes in the eye. In the simulations without the eye fluxes, \(u_{\text{max}}\) is again slightly weaker than in simulations that retain these fluxes (Table 1). Of all our experiments, the maximum decrease in \(u_{\text{max}}\) is 7.5 m s\(^{-1}\) and the average decrease is 4.5 m s\(^{-1}\). Nevertheless, \(u_{\text{max}}\) in all simulations is 25–30 m s\(^{-1}\) higher than E-MPI (Table 1).

Out of all simulations in which eye fluxes are included, the simulation JordanControl has the largest \(\theta_e\) anomaly; the maximum \(\theta_e\) difference is 25 K (in which \(\theta_e\) in the eye is compared to \(\theta_e\) at the radius of maximum winds). This \(\theta_e\) difference is comparable to the maximum value documented in observations (e.g., Eastin et al. 2005b; Montgomery et al. 2006; Marks et al. 2008; Bell and Montgomery 2008), which suggests that our simulations are producing features that are consistent with observations. Although some of these observational studies suggested that this feature has a significant impact on the intensity of tropical cyclones, our simulations demonstrate that this feature has a negligible impact on maximum axisymmetric intensity.

We have repeated these simulations using an axisymmetric version of the cloud model developed by Bryan and Fritsch (2002). This newer model uses more accurate numerical techniques, conserves mass and energy, and includes dissipative heating; all of these are significant improvements over the RE87 model. We also use smaller grid spacing (\(\Delta r = 1000\) m; \(\Delta z = 250\) m) for these simulations. Our overall conclusions from these additional simulations are the same. The maximum decrease of \(u_{\text{max}}\) when we remove the eye fluxes in these
simulations is 4 m s\(^{-1}\), and this occurs in the simulation JordanNoEyeFlux-rm.

5. Interpretation

We conclude, based on these simulations, that the PM03 mechanism has a negligible effect on maximum axisymmetric intensity and that it does not explain the significant subintensity of E-MPI for these simulated hurricanes. We acknowledge that the process identified by PM03 does happen [as shown, for example, by Cram et al. (2007)], but these numerical simulations demonstrate that it has a very small impact on azimuthally averaged winds. [The PM03 mechanism clearly affects the intensity of individual, unsteady, nonaxisymmetric features as measured by local buoyancy and/or updraft intensity, as shown by Braun (2002), Eastin et al. (2005a,b), and Cram et al. (2007). The focus here is on the steady, axisymmetric intensity, which these simulations show to be barely affected by the PM03 mechanism.] It also follows, based on these simulations, that the PM03 mechanism can be reasonably neglected in analytic MPI theories.

We reiterate that by shutting off the surface fluxes in the eye, we are directly evaluating the PM03 mechanism (as explained in section 3). In contrast, PM03 attempted to evaluate this mechanism by inserting a heat sink (via Newtonian relaxation) in the low-level eye (see their section 5c). By doing so, they not only countered the effect of the surface fluxes in the eye (which is the ultimate goal of such tests), but they also countered all other processes that contribute toward the warm-core structure of the low-level eye. Not surprisingly, they found a substantial decrease in intensity in their simulations (~15 m s\(^{-1}\) decrease in \(u_{\text{max}}\)) and an adjustment of the hurricane’s structure (i.e., an increase in radius of maximum winds) as the hurricane responded to this large heat sink. The weak (~4 m s\(^{-1}\), on average) decrease in intensity reported here is a more direct measure of the PM03 mechanism.

The small impact of the high-\(\theta_e\) anomaly in these simulations is ultimately attributable to several factors, which we investigate in the next several subsections.

a. Magnitude of surface fluxes

One factor involved in the weak impact of the PM03 mechanism is the small magnitude of the surface fluxes in the eye compared to surface fluxes at and beyond the eyewall. To illustrate, we examine the surface entropy fluxes from the Control simulation. The magnitude of this flux at a single grid point is given by

\[
F^s = \rho C_E (u_0^2 + \dot{u}_0^2)^{1/2} (s_{\text{surf}} - s_0),
\]

where \(\rho\) is density, \(u_0\) and \(\dot{u}_0\) are the radial and tangential velocities at the lowest model level, \(s_{\text{surf}}\) is surface entropy at saturation at temperature \(T_s\), and \(s_0\) is entropy at the lowest model level. Because \(F^s\) just inside the eye is comparable to \(F^s\) in the eyewall (Fig. 4a), we might conclude that surface fluxes in the eye are very important; in comparison, fluxes far outside of the eyewall (\(r > 40\) km) seem unimportant. However, energetically, fluxes integrated over area (as opposed to fluxes at individual grid points) are ultimately impor-
tant for tropical cyclone intensity. We therefore need to calculate the area-integrated flux, which is given by

$$\langle F^s \rangle(r) = \int_{r - \Delta r/2}^{r + \Delta r/2} 2\pi r' F^s(r') \, dr',$$

(2)

where $\Delta r$ is the radial grid spacing. This analysis, shown in Fig. 4b, suggests a much larger potential impact from the surface fluxes in the region well beyond the radius of maximum winds (i.e., for $50 \text{ km} < r < 100 \text{ km}$ in Fig. 4b).

To illustrate the relative magnitude of surface fluxes in the eye, as compared to fluxes elsewhere in these simulations, we calculate surface fluxes integrated over the entire storm. We perform this analysis over the section of the storm where $\theta_e$ increases with decreasing radius in the boundary layer; this is the region where upward flux of $\theta_e$ from the ocean is approximately balanced by inward radial advection of $\theta_e$ toward the eyewall [referred to as region II by Emanuel (1986, p. 594)]. Based on plots of $\theta_e$, as well as our $\theta_e$ budget analysis (not shown), we find the outer radius $R$ of this region to be at $r = 200 \text{ km}$. (In an analysis using the same numerical model and the same sounding, but different model resolution, RE87 also concluded that there is a region, extending beyond the eyewall, where surface fluxes balance radial advection of $\theta_e$; in their case, they found $R \approx 240 \text{ km}$.) Based on this analysis, we normalize the storm-integrated surface fluxes as a function of $r$ by the total value as follows:

$$S(r) = \frac{\int_r^R \langle F^s \rangle \, dr}{\int_0^R \langle F^s \rangle \, dr}.$$

(3)

The results (Fig. 4c) show that 97.5% of the surface entropy input into the cyclone comes from outside the eye. This analysis is consistent with the simulations NoEyeFlux, NoEyeFlux-7d, and NoEyeFlux-rm, in the sense that surface fluxes in the eye have negligible effect on the energetics of simulated axisymmetric tropical cyclones.

The analysis in this subsection has focused solely on the energetics of tropical cyclones, but it is not clear how these surface fluxes affect the intensity of tropical cyclones (other than the fact that surface fluxes in the eye have already been shown to have a negligible impact on intensity, as shown in Table 1). To investigate further, we conduct a series of simulations in which the surface entropy fluxes are set to zero from $r = 0$ to $r = r_f$ using: (a) the RE87 environment, (b) the PM03 environment, and (c) the Jordan mean hurricane environment. The dashed line marks the nominal location of the radius of maximum winds from control simulations (i.e., from the $r_f = 0$ simulations), and the gray shaded region denotes the nominal location of the eyewall from the control simulations.

![Fig. 5. Values of $u_{\text{max}}$ (m s$^{-1}$) from simulations in which surface entropy fluxes are set to zero from $r = 0$ to $r = r_f$ using: (a) the RE87 environment, (b) the PM03 environment, and (c) the Jordan mean hurricane environment. The dashed line marks the nominal location of the radius of maximum winds from control simulations (i.e., from the $r_f = 0$ simulations), and the gray shaded region denotes the nominal location of the eyewall from the control simulations.](image)
simulations. These simulations support the conclusion that surface fluxes over a broad region (specifically, from the eyewall to \( r = R \)) are important to the energetics, and thus to the intensity, of hurricanes.

b. Entropy transport

Another way to evaluate the PM03 mechanism is to calculate the total magnitude of entropy transport from the eye to the eyewall. To this end, we perform a control volume analysis around the high-\( \theta_e \) anomaly. We begin with a governing equation for \( \theta_e \) assuming steady flow:

\[
\frac{\partial \theta_e}{\partial t} + \frac{u}{r} \frac{\partial \theta_e}{\partial r} + w \frac{\partial \theta_e}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \theta_e F_{r}^{\theta_e}) + \frac{1}{r \theta_v} \frac{\partial}{\partial z} (r \theta_v F_{z}^{\theta_e}) = 0,
\]

where \( u \) and \( w \) are the radial and vertical velocities, \( F_{r}^{\theta_e} \) and \( F_{z}^{\theta_e} \) are the parameterized turbulent fluxes of \( \theta_e \) in the radial and vertical directions, \( \theta_v \) is the base-state density, and \( \theta_v \) is the base-state virtual potential temperature. We neglect the small contribution from the “radiative cooling” term in the model. To write a flux-form version of this equation, we utilize the model’s equation for mass-conservation in steady flow [following RE87’s Eq. (4)]:

\[
\frac{\partial}{\partial r} (r \theta_v u) + \frac{\partial}{\partial z} (r \theta_v w) = 0.
\]

Using (4) and (5), the model’s conservation equation for \( \theta_e \) can be expressed as

\[
\frac{\partial}{\partial r} (r \theta_v \theta_e u) + \frac{\partial}{\partial z} (r \theta_v \theta_e w) + \frac{\partial}{\partial r} (r \theta_v F_{r}^{\theta_e}) + \frac{\partial}{\partial z} (r \theta_v F_{z}^{\theta_e}) = 0.
\]

(6)

We base our analysis on the JordanControl simulation, which produces the largest \( \theta_e \) anomaly in our set of simulations. The steady flow from this simulation suggests that a convenient control volume (dashed box in Fig. 6) can be defined from \( r = 0 \) to \( r = r_m \), where \( r_m \) is the radius of maximum winds, and from \( z = 0 \) to \( z = H \), where \( H \) is the top of the control volume (at \( z = 3.5 \) km). We integrate (6) over this control volume:

\[
\int_{0}^{H} \int_{0}^{r_m} \left[ \frac{\partial}{\partial r} (r \theta_v \theta_e u) + \frac{\partial}{\partial z} (r \theta_v \theta_e w) + \frac{\partial}{\partial r} (r \theta_v F_{r}^{\theta_e}) + \frac{\partial}{\partial z} (r \theta_v F_{z}^{\theta_e}) \right] dr dz = 0.
\]

(7)

The boundary conditions in the numerical model impose \( u|_{r=0} = w|_{z=0} = F_{r}^{\theta_e}|_{r=0} = 0 \). Further, we find from analysis of model output that terms with \( F_{r}^{\theta_e}|_{r=r_m} \), \( F_{z}^{\theta_e}|_{z=H} \), and \( w|_{z=H} \) are small and therefore negligible.
compared to other terms on the boundaries. The control volume budget thus reduces to

\[ F_{\text{surf}} = - \frac{1}{r_m} \int_{0}^{r_m} \bar{u} \bar{v} \, dr \prod_{0}^{r_m} r F_{z}^{|z=0} \, dr, \tag{8} \]

which states simply that the total flux of \( \theta \) through the right side of the control volume [left side of (8)] balances the total flux of \( \theta \) from the sea surface [right side of (8)]. With (8), it is straightforward to demonstrate how we can remove the PM03 mechanism from our simulations by simply setting \( F_{z}^{|z=0} = 0 \) for \( r < r_m \); this means that the total flux through the right side of the control volume [left side of (8)] must be zero, and thus there is no net enhancement of \( \theta \) for parcels that pass through the eye and then back into the eyewall.

We obtain a quantitative measure of the PM03 mechanism using either side of (8), which we hereafter refer to as \( F_{\text{surf}} \). We use output from simulation JordanControl, which we then average over \( t = 8-10 \) days (the time period shown in Fig. 6). We find the value of \( F_{\text{surf}} \) to be \( 2 \times 10^{11} \) kg K\(^{-2}\) s\(^{-1}\). This is a quantitative measure of the PM03 mechanism, and it incorporates the total contribution to \( \theta \) in the eyewall due to the eye/eyewall exchange mechanism identified by PM03. Apparently, this is a negligible contribution to \( \theta \) in the eyewall because setting this to zero (i.e., setting the surface fluxes in the eye to zero) has negligible effect on \( v_{\text{avg}} \) (Table 1) and a negligible impact on the \( \theta \) pattern in the eyewall (e.g., Fig. 2). Theoretical studies have concluded that the \( \theta \) distribution in the eyewall of hurricanes is important for maximum hurricane intensity (e.g., Emanuel 1986, 1997). Consequently, we compare \( F_{\text{surf}} \) to other terms in the \( \theta \) budget around the eyewall (dotted box in Fig. 6). We find that \( F_{\text{surf}} \) is one order of magnitude smaller than surface fluxes in the eyewall:

\[ F_{\text{surf}} = - \frac{1}{r_m} \int_{0}^{r_m} \bar{u} \bar{v} \, dr \prod_{0}^{r_m} r F_{z}^{|z=0} \, dr, \tag{9} \]

where \( r_1 \) and \( r_2 \) are radii that encompass the eyewall updraft, as shown in Fig. 6. We also find that \( F_{\text{surf}} \) is three orders of magnitude smaller than the \( \theta \) flux on the right side of the control volume (which incorporates all surface entropy fluxes between \( r_2 \) and \( R \)). In summary, consistent with the numerical experiments (Table 1 and Fig. 2), the PM03 mechanism is a negligible component of the entropy budget of the eyewall in these simulated hurricanes.

Using a Lagrangian-based analysis, Cram et al. (2007) found that parcels that pass from the low-level eye to the eyewall tended to have \( \theta \) that was several K higher (on average) than parcels that did not enter the eye (e.g., their Fig. 9). In contrast to our findings, they concluded that the eye/eyewall exchange mechanism can “benefit the intensity of the storm.” One might wonder whether the simulated hurricane used by Cram et al. (2007) is significantly different from the simulated hurricanes used here and whether this could lead to their different conclusion. We note from JordanControl that average \( \theta \) exiting the eye (between \( z = z_m \) and \( z = H \)) is 4 K higher than average \( \theta \) entering the eye (between \( z = 0 \) and \( z = z_m \)) (Fig. 6). This average value of \( \theta \) difference is comparable to that found by Cram et al. (2007) using Lagrangian parcel analysis. Thus, it seems likely that the magnitude of the PM03 mechanism is similar in these simulations.

Even though air parcels are exchanged between the eye and eyewall (e.g., Braun (2002), PM03, Cram et al. (2007)), and these parcels do have anomalously high \( \theta \) such that they could contribute to higher mean \( \theta \) in the eyewall, it turns out that this process is just not quantitatively significant in these simulations (e.g., Fig. 2; the eyewall spans roughly 15 km < \( r < 25 \) km). It follows that there must be a relatively small amount of mass that passes between the eye and the eyewall as compared to the total mass in the eyewall. Using a control-volume analysis of mass continuity around the eyewall (i.e., dotted box in Fig. 6), we find that the mass flux from the eye to the eyewall is only 8% of the total upward mass flux at \( z = H \) in the eyewall. Considering that \( \theta \) is elevated by 4 K (on average) for the parcels that pass through the eye, this implies that average \( \theta \) in the eyewall may be elevated by 0.3 K (at most) by the PM03 mechanism (assuming linear mixing of \( \theta \)). This analysis is consistent with results from numerical experiments (e.g., Fig. 2); in fact, average \( \theta \) in the eyewall is only a few tenths of a degree Kelvin higher in simulations with the PM03 mechanism as compared to simulations without the PM03 mechanism.

c. A sensitivity test

We conclude, based on the preceding analyses, that the PM03 mechanism—and, hence, surface fluxes in the eye—would have to be much larger in magnitude to have a significant effect on the energetics of hurricanes. To estimate how much larger, we note that the total surface entropy flux in the nominal eyewall region (\( F_{\text{surf}} \)) is one order of magnitude larger than total surface flux in the eye (\( F_{\text{surf}} \)). From the sensitivity simulations shown in Fig. 5, we find that setting \( F_{\text{surf}} \) to zero results in approximately a 15% decrease in \( v_{\text{avg}} \). Therefore, we surmise that \( F_{\text{surf}} \) would probably have
to be one order of magnitude larger to have a significant effect on \( u_{\text{max}} \).

To test this hypothesis, we simulate simulations in which \( C_e \) in the eye is increased by a factor of 10. For these simulations, we track the radius of maximum winds every time step (as in the NoEyeFlux-rm simulations) and we multiply \( C_e \) by 10, as compared to the formulation from the default model. Results are shown in Table 2. We find that maximum intensity increases by 5–14 m s\(^{-1}\), with the greatest difference occurring in the Jordan mean hurricane environment. These results support our hypothesis that surface fluxes in the eye would have to be at least one order of magnitude higher than typical values to have a significant positive effect on maximum axisymmetric hurricane intensity. However, we note that the \( \theta_e \) anomalies in these simulations are much larger (in amplitude) than values that have been reported in observations (e.g., Eastin et al. 2005b; Montgomery et al. 2006; Marks et al. 2008; Bell and Montgomery 2008). In the experiment with the Jordan environment, \( \theta_e \) in the eye is 46 K higher than \( \theta_e \) at the radius of maximum winds, which is a factor of 2 larger than the maximum observed amplitude of the \( \theta_e \) anomaly.

d. Comparison to observations

One might wonder whether aspects of these simulations might be atypical compared to observed cyclones and thus whether the PM03 mechanism might be more important in natural hurricanes. To investigate further, we note that \( F_{\text{surf}}^{\text{eye}} \) can be estimated using either side of (8). The left side of (8) is especially convenient because it contains terms that are easily observed. To make the following interpretation easier, we simplify this expression. First, we divide the control volume into two parts: a bottom part where \( u < 0 \), from \( z = 0 \) to \( z = z_m \); and an upper part where \( u > 0 \), from \( z = z_m \) to \( z = H \). Next, for simplicity, we further assume that \( u \) and \( \theta_e \) in the bottom part can be represented by average values \( u_1 \) and \( \theta_{e1} \), and the same variables in the upper part can be represented by average values \( u_2 \) and \( \theta_{e2} \). By further assuming that \( \rho \theta_{e0} \approx \text{constant} \), we arrive at

\[
F_{\text{surf}}^{\text{eye}} \approx r_m [z_m u_1 \theta_{e1} + (H - z_m) u_2 \theta_{e2}] .
\]

Mass continuity requires that \( (H - z_m) u_2 = -z_m u_1 \). Thus, we arrive at an approximate relation:

\[
F_{\text{surf}}^{\text{eye}} \approx -r_m z_m u_1 (\theta_{e2} - \theta_{e1}) .
\]

The magnitude of surface fluxes in the eye can be evaluated using (11) and can thus be retrieved from a steady, azimuthally averaged flow field. Our analysis of model output (in the previous subsection) suggests that

<table>
<thead>
<tr>
<th>Environment</th>
<th>( u_{\text{max}} ) (m s(^{-1}))</th>
<th>( \Delta u_{\text{max}} ) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE87</td>
<td>91.3</td>
<td>+4.8</td>
</tr>
<tr>
<td>PM03</td>
<td>101.4</td>
<td>+8.7</td>
</tr>
<tr>
<td>Jordan</td>
<td>117.1</td>
<td>+13.8</td>
</tr>
</tbody>
</table>

\( F_{\text{surf}}^{\text{eye}} \) needs to be at least one order of magnitude larger to have a significant impact. The right side of (11) suggests that this would be difficult, if not impossible, to accomplish. The depth of the inflowing layer into the eye (\( z_m \)) seems to have little variation in observations (e.g., Bell and Montgomery 2008, their Fig. 4). The radius of maximum winds \( r_m \) certainly would not be expected to be substantially larger for very intense storms (i.e., those having intensities that greatly exceed E-MPI). Furthermore, values of \( u_1 \) and \( (\theta_{e2} - \theta_{e1}) \) from available observations (e.g., Bell and Montgomery 2008) are comparable to values from the JordanControl simulation. In summary, to raise the right side of (11) by an order of magnitude would require a drastic change in hurricane structure that seems highly unlikely. We conclude from this analysis that surface fluxes in the eye are unlikely to have a significant effect (on maximum axisymmetric intensity) in observed tropical cyclones. Further analysis with observations and/or numerical simulations in different environments, and for hurricanes with different \( r_m \), and with three-dimensional effects, would be needed to confirm this conclusion, especially because indirect effects are not considered in this relatively simple analysis.

6. Summary

We use a nonhydrostatic axisymmetric numerical model to evaluate the “superintensity” (or “turbo-boost”) mechanism put forth by PM03. The theory posits that anomalously high-$\theta_e$ air from near the surface in hurricane eyes is transported into the eyewall and that this mechanism (which is not considered in analytic theories for maximum intensity) can substantially increase the maximum intensity of hurricanes. We find that although the PM03 mechanism does occur in the numerical simulations, it is too small in magnitude to significantly affect the maximum axisymmetric intensity of hurricanes. In simulations in which we set surface fluxes in the eye to zero, which effectively and quickly eliminates the high-$\theta_e$ anomaly, the intensity of the cyclone is only slightly affected (the typical decrease in intensity is \( \sim 4\% \), despite a very generous definition for “the eye”). We also find that all of our simulations retain an
intensity that is 25–30 m s⁻¹ greater than the maximum theoretical intensity of Emanuel (1986) (E-MPI; see Table 1), regardless of whether there is a high-entropy anomaly in the low-level eye.

From further analysis, we find that total surface entropy fluxes in the eye are negligible compared to total surface fluxes near and outside the eyewall, and therefore the total magnitude of \( \theta_e \) flux between the eye and eyewall is a negligible component of the \( \theta_e \) budget in the eyewall. A cursory comparison to observations suggests that no combination of relevant parameters (e.g., radius of maximum winds, \( \theta_e \) distribution in the eye) is likely to occur that would suggest that the PM03 mechanism is important in natural hurricanes. Additional sensitivity tests with the numerical model support these conclusions.

With regard to future work on this topic, additional numerical simulations could be undertaken in different environments (i.e., with different soundings and/or \( T_s \)) to check whether our results are sufficiently general. Furthermore, a similar set of numerical simulations could be undertaken in three dimensions to investigate whether asymmetric features play any significant role that is not considered here. To that end, a recent study by Yang et al. (2007) compared results from three-dimensional simulations to comparable axisymmetric simulations. Based on figures in their article, there do not appear to be any significant changes in the relevant proxy parameters [e.g., \( r_m, z_m, u_1 \); see (11)] that would suggest any significant changes in surface fluxes from the eye in their simulations.

Finally, although the mechanism proposed by PM03 might seem like a reasonable qualitative explanation for the subintensity of E-MPI, our quantitative analysis shows that it is far too weak to explain the discrepancy. We do not propose an alternative explanation for the subintensity of E-MPI here, although we are exploring this issue for a future article. Nevertheless, based on the analyses here, we conclude that the neglect of surface fluxes in the eye is not an inherent limitation of E-MPI.

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REFERENCES


