On the Dynamics of the East African Jet. II: Jet Transients

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ABSTRACT

A barotropic, primitive equation model on an equatorial beta plane is used to investigate the transient behavior of the East African jet. Both analytic and numerical solutions provide insight into the jet response to a diurnal fluctuation in the friction coefficient over land and to temporal variations in the upstream (eastward) and southern boundary forcings.

Results indicate that the diurnal variation in the strength of the surface drag over land can account for the observed increase in the speed and westward shift of the jet core during the night. The observed large variations in the meridional wind just offshore and in the zonal wind field are not explained by the theory.

In contrast to the diurnal variations in the finest structure of the jet, time-dependent variations in the upstream and southern boundary forcings can produce changes in the large-scale features of the jet. For either type of transient perturbation, the change in the jet speed can be significant and may explain the observed jet surges. In the case of southern boundary forcing, this result demonstrates that eastward propagating, middle-latitude disturbances can have a significant effect on the flow at the equator in the presence of an impermeable western boundary.

1. Introduction

The purpose of this study is to investigate the transient behavior of the East African jet (EAJ). Specifically, the jet response to a diurnal fluctuation in the surface drag over land and to temporal variations in the upstream (eastward) and southern boundary forcings is modeled.

The recent MONSOON 77 aircraft exploration (Hart et al., 1978) of the EAJ documents the diurnal variation in the jet. During the day the maximum jet speed is reduced by ~30% over land, while the flow far over the ocean appears to be relatively unaffected. Hart et al. (1978) suggest that this phenomenon is the result of increased convective activity over land during the daytime. In comparison, the low-level jet of the Great Plains of North America [see, e.g., Bonner (1968) for a description] exhibits a strong (~100%) diurnal variation which Blackadar (1957) first interpreted as an inertial oscillation excited by the sudden reduction in surface drag at nightfall. Such a description is not applicable to the EAJ because of its equatorial location. This aspect of the EAJ is studied analytically and numerically by letting the coefficient of surface friction over land undergo a prescribed diurnal oscillation.

The motivation for the study of upstream transient forcing arises from the observation of Krishnamurti and Bhalme (1976) that the meridional pressure gradient over the Indian Ocean has a strong component with a 14-day period during the northern summer monsoon. Their spectral analysis also revealed a similar oscillation in wind data of the EAJ. Because the jet data were for a different year than the pressure observations, it was impossible to obtain the phase relation between the two. More recently, however, Raghavan et al. (1978) show that increases in the wind speed at Garissa, Kenya, lag pressure drops over central India by a few days during July 1973. This result suggests that the 14-day monsoon oscillation causes surges in the EAJ.

This upstream forcing is modeled by a prescribed variation in the source-sink term used to drive the flow. Anderson (1976), in a similar study, found a 30% variation in the jet speed for a 100% variation in the Northern Hemisphere sink with a 24-day period. He noted that his large eddy diffusivity precluded a higher frequency forcing from producing a significant response. In this study, a linear analysis is performed for an oscillatory, antisymmetric source-sink distribution of arbitrary frequency. The methodology is similar to that employed in the oceanographic literature (see, in particular, Moore and Philander, 1976; Cane and Sarachik, 1976, 1977). Here the treatment is extended to include Rayleigh damping. In addition, numerical calculations predict the nonlinear response of the EAJ to a forcing with a 14-day period.

Synoptic fluctuations to the south also appear to
be important in explaining the temporal variation of the jet. Map analysis shows that the mean flow can be severely distorted by the passage of middle latitude disturbances over the Cape of Africa. Fig. 1 illustrates one such episode during MONSOON 77. With the high-pressure center over South Africa (Fig. 1a), air is pumped vigorously northward through the Mozambique Channel. As the high moves eastward, the southerly flow diminishes (Fig. 1b) and is eventually reversed (Fig. 1c) as the propagating high reinforces the Mascarene High to the east of Madagascar. Hart et al. (1978) note that intensification of the cross-equatorial flow is correlated with strong flow up the channel though the phase relation has not been determined as yet.

Forcing of tropical motions by middle latitude disturbances has been investigated previously by a number of authors. Mak (1969) in a stochastic approach and Lamb (1973) in a mode analysis used a channel model of the tropics with a specified meridional velocity at both the northern and southern boundaries. In each case, a maximum response at the equator was obtained for forcings of ~5 days. Hayashi (1976), however, showed that this “non-singular” resonance did not correspond to a realistic tropical mode but rather to a westward propagating anti-Kelvin wave that can exist only on an equatorial beta plane of finite meridional extent. The present approach circumvents this problem by considering a semi-infinite beta plane. The flow is forced by a meridional velocity specified along the southern boundary. Analytic solutions of the linearized model equations are obtained for forcing of arbitrary frequency and zonal wavenumber. For the case of no meridional boundary, the results agree with the numerical calculations of Bennett and Young (1971); only westward propagating waves of large zonal wavelength have a significant response at the equator. The presence of a boundary is seen to alter this conclusion. In the numerical simulation of the nonlinear problem, an eastward propagating wave of a 6-day period and wavenumber 6 is used. Such a choice is consistent with the observed spectral peak (Kao et al., 1970) in the meridional geostrophic wind of middle-latitude disturbances in the Southern Hemisphere.

The remainder of this paper is divided into four parts. In Section 2 the model formulation and numerical technique are described. Section 3 presents a linear analysis of the model equations with no orography. Numerical solutions of the nonlinear equations with orography are described in Section 4. Comparison of the two solutions helps elucidate the role of advection by the mean flow and of the orography. In the last section, the main conclusions of this investigation are summarized and the limitations of the study are delineated.

2. The model

The approach used here follows that in Part I (Bannon, 1979b). The equations of motion and of continuity describing the depth-averaged flow on an equatorial beta plane subject to Rayleigh damping and to a reduced gravity are
\[ \frac{du}{dt} - \beta y v = -g' \frac{\partial h}{\partial x} - \frac{R u}{D}, \]  
\[ \frac{dv}{dt} + \beta y u = -g' \frac{\partial h}{\partial y} - \frac{R v}{D}, \]  
\[ \frac{dD}{dt} + D \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = Q, \]

where
\[ \frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}, \]
and the notation is that in Bannon (1979b). The parameter settings are \( \beta = 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \), \( g' = 0.60 \text{ m s}^{-2} \) and \( R = 1 \text{ cm s}^{-1} \). A rigid western boundary is assumed at \( x = 0 \).

The numerical model is identical to that used in Part I except for the absence of Madagascar. This choice facilitates comparison with the linear analysis of Section 3. In each numerical experiment of Section 4, the model is started from day 12 of experiment B1 in Part I and integrated forward in time for three complete cycles. Experiment B1 provides a fairly realistic simulation of the EAJ except for the absence of a wind speed maximum off the tip of Madagascar and of a trough-ridge pressure distribution over the island. In order to avoid complete flow separation (i.e., \( D \ll 0 \)) during the initial cycle, the northern boundary was placed at \( y_N = +400 \text{ km} \) for the case of southern boundary forcing only. All other experiments have \( y_N = +500 \text{ km} \). Inspection of both global and grid-point quantities (Bannon, 1979a) indicates that an approximately periodic state is achieved by the third cycle. All numerical results presented here are based on the analysis of the last cycle.

3. Linear analysis

Linearization of the model equations about a state of rest yields, with flat bottom topography,
\[ \frac{\partial u}{\partial t} - \beta y v = -g' \frac{\partial D}{\partial x} - \frac{R u}{D_0}, \]
\[ \frac{\partial v}{\partial t} + \beta y u = -g' \frac{\partial D}{\partial y} - \frac{R v}{D_0}, \]
\[ \frac{dD}{dt} + D_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = Q, \]

where \( D_0 = 2 \text{ km} \) is a representative fluid depth. The domain is taken to be semi-infinite in the \( x-y \) plane with an impermeable meridional boundary at \( x = 0 \). The general solution is separated into time-independent and time-dependent components, e.g.,
\[ v(x,y,t) = v_d(x,y) + v'(x,y,t). \]

Bannon (1979a) has solved (2) for the steady-state solution subject to the boundary layer approximation for arbitrary \( Q = Q(y) \). For the structurally simple form
\[ Q(y) = -b y e^{-ay^2}, \]

where \( b > 0 \) and \( a^{-1/2} = 2000 \text{ km} \), the solution of the velocity field is
\[ v = V_0 e^{-ay^2}(1 + A e^{-\varepsilon^2} \}
\[ u = V_0 (a \varepsilon)^{1/2} y e^{-ay^2}(1 - e^{-\varepsilon^2}) \]

where \( V_0 = b l (a D_0), \varepsilon = \beta D_0/R \) and \( A = \varepsilon^2/a \). This solution describes a western boundary current analogous to that of Stommel (1948) for the Gulf Stream. For \( R = 1 \text{ cm s}^{-1} \), however, the jet width, \( \varepsilon^{-1} = 217 \text{ km} \), is too narrow, and the amplification of the cross-equatorial flow in the boundary current to that far upstream, \( A = 85 \), is too large. Despite these shortcomings, the solution (4) is used as a standard of comparison for the transient linear analyses to be presented in the following subsections.

a. Diurnal variation of friction coefficient over land

During the day, increased insolation at the surface will result in increased convective activity and, hence, an increased frictional drag. As the diurnal variation in the surface temperature is much larger over the land than over the ocean, this phenomenon is generally confined to continental regions. Thus, let the Rayleigh drag coefficient be given by \( R(x,y,t) = R + R'(x,y,t) \), where \( R' \) describes the diurnal variation over land from a constant value \( R \). Estimates of the dependence of the drag coefficient \( C_D \) on stability (Deardorff, 1968) suggest that the magnitude of the diurnal variation of \( R \) is roughly half its value under neutral conditions. With \( R = 2 \text{ cm s}^{-1} \) over land, \( |R'| \) is taken to be 1 cm s\(^{-1}\).

In this subsection, the time-dependent variables represent the response of the flow to the variable drag coefficient. They satisfy
\[ \frac{\partial u}{\partial t} - \beta y v = -g' \frac{\partial D}{\partial x} - \frac{(R + R')u}{D_0} + F, \]
\[ \frac{\partial v}{\partial t} + \beta y u = -g' \frac{\partial D}{\partial y} - \frac{(R + R')v}{D_0} + G, \]
\[ \frac{dD}{dt} + D_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \]

where the primes on \( u, v \) and \( D \) have been dropped for convenience. Here \( F = -R' u / D_0 \) and \( G = -R' v / D_0 \) represent the perturbation of the steady-state solution from equilibrium caused by the variable bottom friction. They act as body forces to drive the transient motion. Since \( v' \gg u' \) over land, \( F \) is dropped compared to \( G \).
For simplicity, a coastline independent of $y$ and a sinusoidal time dependence are assumed. Then

$$R' = R e^{-i\Omega t}[1 - H(x - x_c)],$$

where $R_0 = 1$ cm s$^{-1}$, $2\pi/\Omega = 1$ day, $H$ is the Heaviside stepfunction and $x_c = 400$ km denotes the longitudinal extent of the land surface from the rigid western boundary at $x = 0$. Since $|\Omega D_0/R + R'| \geq 5$ for $D_0 = 2$ km, the damping terms in (5) may be ignored. The simplified set is then identical to that of Lighthill (1969) who showed that the meridional wind satisfies

$$\frac{\partial^2 v}{\partial t^3} - g'D_0 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \beta\frac{\partial v}{\partial t} = \frac{\partial^2 G}{\partial t^2} - g'D_0 \frac{\partial^2 G}{\partial x^2}. \quad (6)$$

By making the boundary layer approximation and noting that $\Omega^2 \gg \beta\gamma^2$ near the equator, Eq. (6) reduces to

$$\frac{\partial^2 v}{\partial t^3} - g'D_0 \frac{\partial^2 v}{\partial x^2 \partial t} = \frac{\partial^2 G}{\partial t^2} - g'D_0 \frac{\partial^2 G}{\partial x^2}. \quad (7)$$

The zonal variation of the steady-state meridional wind (4) has been shown not to be representative of the mean EAJ. This variation is therefore ignored so that $G$ may be written as

$$G = -G_0 e^{-\alpha y/2} e^{-i\Omega t}[1 - H(x - x_c)], \quad (8)$$

where $G_0 = V_{00} R_0 / D_0$. (The convolution theorem could be employed to obtain the response to forcing with arbitrary zonal structure.) The land-sea contrast is preserved in the forcing (8), while the finestucture of the jet over land is not. Here $V_{00}$ is a typical jet speed over land and may be taken to agree with observations (e.g., $V_{00} = 15$ m s$^{-1}$) rather than with the two fast, linear, steady-state solution (4).

Let

$$v = e^{-\alpha y/2} e^{-i\Omega t} \left[ E_0 [1 - H(x - x_c)] + E(x) \right], \quad (9)$$

where $E_0 = G_0 / (i\Omega)$. Substitution of (8) and (9) into (7) yields

$$i\Omega g'D_0 \frac{d^2E}{dx^2} - g'D_0 \frac{dE}{dx} + i\Omega^2 E = -E_0 g'D_0 \delta(x - x_c), \quad (10)$$

where $\delta$ is the Dirac delta function.

A Green’s function solution to (10) is obtainable in a straightforward manner. For $x \neq x_c$, (10) has the solution

$$E(x) = \begin{cases} E_0 e^{ik_+x} + E_0 e^{ik_-x}, & x > x_c, \\ E_0 e^{ik_+x} + E_0 e^{ik_-x}, & x < x_c, \end{cases} \quad (11)$$

where

$$k_+ = \frac{2\pi}{\lambda_+} = -\frac{\beta}{2\Omega} + \left[ \left( \frac{\beta}{2\Omega} \right)^2 + \frac{\Omega^2}{g'D_0} \right]^{1/2}. \quad (12)$$

With $g' = 60$ cm s$^{-2}$ and $D_0 = 2$ km, $\lambda_+ = 3.22 \times 10^3$ km and $\lambda_- = -2.77 \times 10^3$ km. Since $k^2 \gg a$, the boundary-layer approximation holds a posteriori.

The four constants in (11) are found by applying the following boundary conditions: 1) radiation condition at $x = \pm \infty$; 2) continuity of $E(x)$ at $x = x_c$; 3) integration of (10) across $x = x_c$ to obtain a condition on the derivative of $E$ at $x = x_c$; and 4) $u(x = 0) = 0$. The result is

$$E_2 = 0, \quad (13)$$

$$E_4 = \frac{\beta E_0}{\Omega} e^{-ik_-x_c}, \quad (14)$$

$$E_3 = \left[ \frac{i\beta E_0}{\Omega} C_+ + C_+ E_4 \right] / C_+, \quad (15)$$

$$E_1 = E_3 + E_0 e^{i(k_+-k_-)x_c}, \quad (16)$$

where

$$C_+ = i \left[ \frac{\Omega \beta + ak_- g'D_0}{\Omega^2 - k_-^2 g'D_0} \right]. \quad (17)$$

The response of the cross-equatorial flow is shown in Fig. 2. The forced meridional wind field is confined to lie over the land area and leads the drag variation by a quarter period (i.e., the maximum increase in $v$ occurs 6 h after the minimum in the drag coefficient). The magnitude of the oscillation is $|v/V_{00}| = R_0 / (\Omega D_0) = 6.85\%$ for $R_0 = 1$ cm s$^{-1}$. In contrast the response over the ocean is weaker, $|v/V_{00}| = 0.30\%$ and, just offshore, is $180^\circ$ out of phase with $R(t)$. 

Fig. 2. Linear response of $v$ at the equator to a diurnally varying drag coefficient over land.
While the response of the $v$ field is largest at the equator for the forcing (8), both the $u$ and $D$ fields are zero at $y = 0$ and reach maxima at $y = \pm a^{-1/2}$ of $|u/V_\infty| = 1.14\%$ and $|D/D_0| = 0.78\%$ (for $V_\infty = 20$ m s$^{-1}$), respectively. Both fields are continuous at $x = x_0$; there are no infinite divergences or pressure waves. They represent long wavelength ($\sim 3000$ km) inertia-gravity waves.

b. Time-dependent upstream forcing

Here the zonally symmetric source-sink forcing is assumed to undergo a sinusoidal variation in time:

$$Q(y,t) = -b'y e^{-\omega_0 t/2} e^{-i\omega t} (b' > 0).$$  \hspace{1cm} (12)

The complete solution to (2) consists of the forced response and the homogeneous solution. The former is independent of $x$. The latter consists of the free modes of (2) which may be a function of $x$. They are needed to satisfy the boundary condition $u(x = 0) = 0$. For example, let

$$v(x,y,t) = e^{-i\omega t} [v_0(y) + v_F(x,y)],$$

where the subscripts $Q$ and $F$ denote the forced and free solution, respectively.

The forced meridional wind field satisfies

$$\frac{d^2}{dy^2} v_q + \left( \frac{\omega_0}{g'D_0} - \frac{\beta^2}{g'D_0^2 \omega_0} y^2 \right) v_q = \frac{1}{D_0} \frac{dQ}{dy},$$  \hspace{1cm} (13)

where $\dot{\omega} = \omega + iR/D_0$. (The Rayleigh coefficient is assumed constant in time and uniform in space in this subsection.) Eq. (13) is solved using an eigenfunction expansion for $v_0$ and $1/D_0(dQ/dy)$. We consider the set of Hermite functions defined by

$$\psi_m(z) = \frac{1}{\left(2^m m!\right)^{1/2}} \frac{1}{\pi^{1/4}} e^{-z^2/2} H_m(z),$$

$$m = 0, 1, 2, 3 \ldots,$$

where $H_m(z)$ are the Hermite polynomials (see, e.g., Morse and Feshbach, 1953). Each $\psi_m$ is a solution to

$$\frac{d^2}{dz^2} \psi_m + (2m + 1 - z^2) \psi_m = 0,$$  \hspace{1cm} (14)

subject to the boundary conditions $\psi_m = 0$ at $z = \pm \infty$. The Hermite functions form a complete set over the complex plane and are orthonormal in the sense that

$$\int_0^{\infty} \psi_m(z) \psi_n(z) dz = \begin{cases} 1, & n = m \\ 0, & n \neq m. \end{cases}$$

Expanding $v_0$ and $(1/D_0)(dQ/dy)$ in terms of the $\psi_m$ yields

$$v_q(y) = \sum_{m=0}^{\infty} v_m \psi_m(\mu y),$$  \hspace{1cm} (15)

$$\frac{1}{D_0} \frac{dQ}{dy} = \sum_{m=0}^{\infty} Q_m \psi_m(\mu y),$$  \hspace{1cm} (16)

where $\mu = (\beta^2 g'^2 D_0^2 \omega_0^3)^{1/4}$ is complex. Substituting (15) and (16) into (13) and making use of the relation (14) and of the orthonormality of the $\psi_m$, one finds

$$v_m = \frac{Q_m}{\lambda_m},$$

where

$$\lambda_m = \frac{\omega_0 \dot{\omega}}{g'D_0} - (2m + 1)\mu^2.$$

In the inviscid problem, a singular resonance occurs for $\lambda_m = 0$. Here inclusion of Rayleigh damping precludes such a possibility. The $u$ and $D$ fields for the forced solution are

$$u_q(y) = \sum_{m=0}^{\infty} i v_m \left( \frac{g'D_0^2 \omega_0^3}{\omega_0 \dot{\omega}} \right)^{1/4}$$

$$\times \left[ \left( \frac{m + 1}{2} \right)^{1/2} \psi_{m+1}(\mu y) + \left( \frac{m}{2} \right)^{1/2} \psi_{m-1}(\mu y) \right],$$

$$D_q(y) = \frac{i}{\omega} \left[ Q(y) - D_0 \frac{d}{dy} v_q(y) \right].$$  \hspace{1cm} (17)

The free mode solutions to (2) satisfy

$$\frac{d^2}{dz^2} v_F + \left( \frac{\omega_0 \dot{\omega}}{g'D_0} - k^2 - \frac{k\beta}{\omega} - \frac{\beta^2}{g'D_0^2 \omega_0^3} y^2 \right) v_F = 0.$$  \hspace{1cm} (18)

The solution bounded at $y = \pm \infty$ is

$$v_F = C_m e^{i8_0 z} \psi_m(\mu y), \quad m = 0, 1, 2, \ldots,$$

where the zonal wavenumber is determined from the dispersion relation

$$k^2 + \frac{\beta}{\omega} k_m - \frac{\omega_0 \dot{\omega}}{g'D_0} + (2m + 1)\mu^2 = 0.$$  \hspace{1cm} (19)

The corresponding $u$ and $D$ fields are

$$u_F = C_m \left( \frac{g'D_0^2 \beta}{\omega_0 \dot{\omega}} \right)^{1/4} \left[ \left( \frac{m + 1}{2} \right)^{1/2} \psi_{m+1}(\mu y) + \left( \frac{m}{2} \right)^{1/2} \psi_{m-1}(\mu y) \right]$$

$$- \left( \frac{m + 1}{2} \right)^{1/2} \psi_{m+1}(\mu y) - \left( \frac{m}{2} \right)^{1/2} \psi_{m-1}(\mu y).$$

$$D_F = C_m \left( \frac{g'D_0^2 \beta}{\omega_0 \dot{\omega}} \right)^{1/4}$$
where

\[ k_m = k_m \left( \frac{g'D_0}{\omega \omega_0} \right)^{1/2}. \]

The quadratic (19) allows two complex roots. Here only the root with its imaginary part greater than zero is used. This choice assures a finite value at \( x = +\infty \). The \( m = 0 \) root merits special discussion. In order that the solution is bounded at \( y = \pm \infty \) (Matsuno, 1966),

\[ k_0 = \left( \frac{\omega_0}{g'D_0} \right)^{1/2} - \frac{\beta}{\omega}. \tag{20} \]

This root corresponds to the Yanai or mixed Rossby-gravity wave and has a positive imaginary part. The other root \( k_0 = -\omega \omega_0 (g'D_0)^{1/2} \) corresponds to the spurious westward propagating anti-Kelvin wave which is unbounded at \( y = \pm \infty \) and at \( x = +\infty \).

In addition, a free-mode solution exists with \( \psi \) identically zero. This eastward propagating Kelvin mode is denoted by \( m = -1 \):

\[ u_{-1} = 0, \]

\[ D_{-1} = C_{-1} \left( \frac{g'D_0 \beta^2}{\omega_0^5} \right)^{1/4} \psi_0(\mu y), \]

\[ k_{-1} = + \left( \frac{\omega \omega_0}{g'D_0} \right)^{1/2}. \]

The \( C_m \) are determined by satisfying the boundary condition \( u = 0 \) at the western wall. The \( M \)th forced mode of (17) is matched by a sum of free modes \( m \leq M \) such that the boundary condition is satisfied for all \( y \). The technique is described in detail in Moore and Philander (1976). One finds that only the even (odd) \( m \) modes are needed for \( M \) even (odd).

By symmetry of the forcing (12), \( Q_m = 0 \) for all odd \( m \). Thus only the modes with \( m \) even are present. A convenient measure of the convergence of the series (16) after \( M \) terms is given by

\[ S_M = S_M/S_m, \]

where

\[ S_M = \sum_{m=0}^{M} Q_m. \]

\[ S_m = \int_{-\infty}^{+\infty} \mu \left( \frac{1}{D_0} \right)^{2} dy = \frac{3\mu}{4} \left( \frac{b'}{c} \right)^{2} \left( \frac{1}{\alpha} \right)^{1/2}. \]

For \( S_M = 1 \), the series has converged. Note that \( S_M \)

is complex. Table 1 gives \( |S_m| \) for various values of \( R, a \) and forcing period \( P \). For these representative parameter settings, the series (16) may be safely truncated after \( m = 8 \).

Also displayed in Table 1 is a measure of the power of the zonal wind for each forced mode since

\[ \left( \int_{-\infty}^{+\infty} u_{m} \mu dy \right)^2 = \frac{(2m + 1)}{2} \left( \frac{g'D_0 \beta^2}{\omega_0^5} \right)^{1/2} |v_m|^2. \]

Table 1 shows that the Yanai \((m = 0)\) mode is the dominant forced response. Because \( Q \) is zonally symmetric, only gravity waves are excited directly by the forcing. For the relatively long-period fluctuations considered, the Yanai mode is the gravity mode closest to resonance (i.e., \( |\lambda_0| < |\lambda_m| \), \( m > 0 \)). In addition, the projection of \((1/D_0)(dQ/dy)\) onto the \( \psi_m \) modes for large-scale forcing \((a \approx 0.25 \times 10^{-12} \text{ m}^2)\) is largest for \( m = 0 \). For forcing of narrower meridional extent \((a = 0.50 \times 10^{-12} \text{ m}^2)\), the \( m = 2 \) projection is greatest. The explanation of the unusual dependence of power on \( R \) (power increasing for increasing \( R \), except for \( m = 0 \) and \( P = 6 \) days) lies in the fact that while \( |Q_m| \) decreases for increasing \( R \), the decrease in \( |\lambda_m| \) is greater.

Table 1 suggests that a fairly accurate solution will be given by including the \( m = 0 \) mode only. The zonal wind forced by the time-dependent \( Q \) may then be written approximately as

### Table 1. Modal power and series convergence for \( Q(y,t) \) given by (12).

| \( R \) (cm s\(^{-1}\)) | \( a \) (\( \times 10^{12} \text{ m}^2 \)) | \( (2m + 1) |v_m|^2 \) [arbitrary units] |
|-----------------|-----------------|----------------------------------|
|                 | \( m = 0 \)     | \( m = 2 \)     | \( m = 4 \)     | \( m = 6 \)     | \( m = 8 \)     |

**A. \( P = 14 \) days**

| \( \frac{1}{2} \) | 0.25 | 1.561 | 0.082 | 0.043 | 0.013 | 0.003 | 1.010 |
| 1               | 0.25 | 1.790 | 0.155 | 0.063 | 0.017 | 0.004 | 1.049 |
| 2               | 0.25 | 2.212 | 0.396 | 0.116 | 0.025 | 0.005 | 1.130 |
| 4               | 0.25 | 0.30  | 0.283 | 0.019 | 0.001 | 0.000 | 1.042 |
| 8               | 0.25 | 2.910 | 0.033 | 0.047 | 0.035 | 0.021 | 0.937 |

**B. \( P = 6 \) days**

| \( \frac{1}{2} \) | 0.25 | 2.000 | 0.065 | 0.038 | 0.012 | 0.003 | 0.994 |
| 1               | 0.25 | 2.064 | 0.080 | 0.042 | 0.013 | 0.003 | 1.005 |
| 2               | 0.25 | 2.055 | 0.135 | 0.058 | 0.016 | 0.004 | 1.037 |
Westward flow in the Southern Hemisphere lags the forcing by a quarter period. The boundary response to (21) is a free Yanai mode

$$v_p(\chi, y, t) = -v_0(1 - k_0)\psi_1(\chi y)e^{-\gamma t}.$$  

At the boundary, the cross-equatorial flow is roughly $180^\circ$ out of phase with $Q$. Because $k_0$ is complex, the zonal structure resembles the time dependence of an underdamped harmonic oscillator. The dependence of wavelength $2\pi/k_0$ and $e$-folding distance $d_0$ on $P$ and $R$ is shown in Table 2. In all cases $d_0$ is greater than the steady-state jet width from linear theory, $R/\beta D_0$, and comparable to the actual mean jet width of $\sim 1000$ km. In addition the wavelengths of the boundary response are large compared to the actual mean jet width. Thus the small-scale ($\sim 100$ km) variation in jet position observed by Hart et al. (1978) is not explained by these results.

A convenient measure of the magnitude of the boundary response (22) is obtained by comparison with the steady-state solution (4). Here we define the intensification factor $I$ as

$$I = \left[ \frac{v_p(0, 0, 0)}{v_s(0, 0)} \right]^{1/2} \int_0^\infty u_q(0, y) dy.$$  

The numerator (denominator) is the ratio of the meridional wind along the equator at the boundary to the net zonal flow incident on the boundary in the Southern Hemisphere for the transient (steady-state) flow. For example, if the net inflow for the transient and steady-state forcings are the same, $I$ measures the strength of the transient cross-equatorial response relative to that for the steady state. For the solutions (21) and (22), and (4), $I$ may be written

$$I = \left( \frac{\omega_0 D_0}{g' D_0} \right)^{1/2} \frac{R}{\beta D_0} (1 - k_0).$$  

For high-frequency oscillations, the Yanai wave is essentially a gravity wave, $k_0 \approx 1$ and $I \approx 0$. Physically the boundary response to the incident mass flux for short periods is an increase in fluid depth and no meridional flow. Using the dispersion relation (20), (23) may be rewritten as

$$I = \frac{1}{\gamma + i},$$  

where $\gamma^{-1} = R/(\omega D_0) = (2\pi)^{-1} \times (\text{oscillation period/ frictional spin down time}, \tau_s = D_0/R)$. For long-period oscillations, the fluid surface acts as a rigid lid. In order to conserve mass, the boundary response is a meridional flow. Thus, as $\omega \to 0$, $\gamma \to 0$ and $I \to 1$. For $R = 1$ cm s$^{-1}$, $2\pi\tau_s \approx 14$ days. A 14-day oscillation will therefore cause almost twice as great a change in the jet as a 6-day oscillation (see Table 3) if the net zonal flow excited is the same in each case.

The effect of including the other terms (i.e., $m \leq 8$) is depicted in Fig. 3 for $P = 6$ and 14 days. In each case, the magnitude of the forcing $b'$ and $a = 0.25 \times 10^{-12}$ m$^{-2}$ are the same. Because a long-period oscillation excites a stronger zonal wind fluctuation, the cross-equatorial flow is stronger for $P = 14$ than $P = 6$ days. In agreement with Table 2, the longer period response is more closely trapped to the wall.

c. Time-dependent zonally asymmetric southern boundary forcing

In this subsection the linear response of the $\eta AJ$ to motion forced along a southern latitude is determined. Mathematically the problem consists of solving the homogeneous form of (2) subject to the inhomogeneous boundary condition of a specified meridional velocity at $y = -y_0$, i.e.,

$$v(x, y_0, t) = V_s e^{i(k_0 x - \omega t)}.$$  

The solution is obtained in two steps. First the response to (24) is calculated in the absence of a meridional boundary. Then the free mode solutions required to satisfy the condition $u = 0$ at the western boundary are determined. The solutions of interest are characterized by the growth rate $\gamma$ or $\gamma$-factor $\gamma^{-1}$ defined in Table 3. For $\gamma^{-1} = (P/2\pi\tau_s)$, the boundary response is independent of $\gamma$

| $\gamma^{-1}$ | $|I|$ |
|--------------|------|
| $0$          | $0$  |
| $1/3$        | $0.196$ |
| $1/2$        | $0.447$ |
| $1$          | $0.707$ |
| $2$          | $0.894$ |
| $5$          | $0.981$ |
| $\infty$     | $1$  |
Solutions to (25) which are bounded as \( y \to +\infty \) are the Weber or parabolic cylinder functions (see, e.g., Whittaker and Watson, 1958)

\[
v(\xi) = D_\alpha(\xi).
\]  

(27)

The solution forced by the inhomogeneous boundary condition is

\[
v = \tilde{V}_S e^{ikx - \omega t} D_\alpha(\xi),
\]

where \( \xi_0 = \xi(y_0) = 2.88 \) for \( y_0 = 2.5 \times 10^3 \) km, \( \tilde{V}_S = V_S/D_\alpha(-\xi_0) \), and \( \alpha_S \) is given by (26) with \( k = k_S \).

The quantity \( n(\omega, k, \xi) \) is the index of refraction of the tropical atmosphere. As such it measures the ability of a disturbance to propagate in the meridional direction. For \( n^2 > 0 \), the solution is oscillatory in \( y \). For \( n^2 < 0 \), the index of refraction is imaginary and the solution is evanescent. In such a case, motion forced at middle latitudes will have little effect at the equator. The minimal requirement for a propagating solution is that \( \alpha > -\frac{1}{2} \). For waves of meteorological importance, \( \omega \) is small, and this criterion can be written approximately using (26) as

\[
-\left( \frac{k^2 + \frac{k}{\beta}}{\omega} \right) > 0.
\]

(28)

This constraint is met only for relatively long-wavelength, westward propagating \( (k/\omega < 0) \) waves (see Table 4). Bennett and Young (1971) reached a similar conclusion by treating the problem as a numerical eigenvalue problem. In addition they considered the effect of a mean flow with latitudinal shear. Here the condition (28) may also be applied to cases with uniform mean flow \( U_0 \) provided \( \omega \) is replaced by the Doppler-shifted frequency \( \omega_D = \omega - kU_0 \).

The \( u \) and \( D \) fields corresponding to (27) are

\[
u(\xi) = \frac{i}{\sqrt{2}} D_\alpha \left[ \frac{1}{\alpha + \frac{k}{\beta}} \right]^{1/4} \left[ \frac{D_{\alpha+1}(\xi)}{1 - k^*} + \frac{\alpha}{\alpha + k^*} \right] ^{1/4} \left[ \frac{D_{\alpha+1}(\xi)}{1 - k^*} - \frac{\alpha}{\alpha + k^*} \right] ^{1/4},
\]

where \( k^* = k(g'D_0/\omega^2)^{1/2} \). Examples of the latitudinal structure of \( u, v \) and \( D \) are given in Fig. 4. The

<table>
<thead>
<tr>
<th>Wavenumber*</th>
<th>Period (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12</td>
<td>-0.40</td>
</tr>
<tr>
<td>-6</td>
<td>+0.27</td>
</tr>
<tr>
<td>0</td>
<td>-0.41</td>
</tr>
<tr>
<td>+6</td>
<td>-2.42</td>
</tr>
<tr>
<td>+12</td>
<td>-5.76</td>
</tr>
</tbody>
</table>

* \((+(-))\) denotes eastward (westward) phase propagation.
Philander (1977) plots solutions of (29) as a function of $\xi_0$. For $\xi_0 = \infty$, $\alpha_m = m$, and the solutions (27) reduce to the (unnormalized) Hermite functions used in the preceding subsection.

Eq. (26) is the dispersion relation which gives $k_m = k_m(\omega, \alpha_m)$. Here $\omega$ is set by the forcing and $\alpha_m$ by (29). The equation is quadratic in $k_m$ and allows two roots. For a real radicand, the negative root is taken. Such a choice is consistent with that made if Rayleigh damping is included. For $m = 0$, this method leads to the selection of a Yanai-type solution. Thus the spurious, anti-Kelvin wave branch is not utilized. For complex roots, the one whose imaginary part is greater than zero is used. This choice assures a bounded response at $x = +\infty$.

The set of free-mode solutions is completed with the inclusion of the equatorial Kelvin wave. Because this mode has $v$ identically zero, it is unaffected by the presence of the longitudinal boundary at $y = -y_0$. As in Section 3b, this mode is denoted by $m = -1$.

The magnitude of the free mode response is determined by satisfying the boundary condition $u(x = 0) = 0$. As in the previous subsection, the zonal wind components of the free modes $u_{\alpha_m}$, $m = -1, 0, 1, 2, \ldots$, form a complete set but are not orthogonal. Here the $\alpha_m$'s are not integers. Thus the $u_{\alpha_m}$'s are not simply related to each other as in the case of the Hermite functions. As a consequence, the approach used in Section 3b is not applicable.

Here the method of Mosfied and Rattray (1971) is employed. The zonal wind fields of both the forced and free waves are expanded in a series of Weber functions $D_{\beta_n}(\xi)$ which satisfy

$$\frac{d^2 D_{\beta_n}(\xi)}{d\xi^2} + \left(\delta_n + \frac{1}{2} - \frac{\xi^2}{4}\right)D_{\beta_n}(\xi) = 0,$$

subject to the boundary conditions

$$\frac{d D_{\beta_n}}{d\xi} = 0 \quad \text{at} \quad \xi = -\xi_0$$

$$D_{\beta_n} = 0 \quad \text{at} \quad \xi = +\infty$$

As this is a Sturm-Liouville problem, the $D_{\beta_n}$, $n = 0, 1, 2, \ldots$, form a complete orthogonal set. Thus

$$u_{\alpha_n}(\xi, k_m) = \sum_{n=0}^{\infty} F_{mn}(k_m)D_{\beta_n}(\xi),$$

$$u_{\alpha_n}(\xi, k_3) = \sum_{n=0}^{\infty} G_n(k_3)D_{\beta_n}(\xi),$$

where

$$F_{mn} = \int_{-\xi_0}^{\xi_0} u_{\alpha_n}D_{\beta_m}d\xi \int_{-\xi_0}^{\xi_0} D_{\beta_n}D_{\beta_m}d\xi,$$

$$G_n = \int_{-\xi_0}^{\xi_0} u_{\alpha_1}D_{\beta_1}d\xi \int_{-\xi_0}^{\xi_0} D_{\beta_1}D_{\beta_n}d\xi.$$
Table 5. Boundary response to eastward propagating lateral forcing with a 6-day period and wavenumber 6.

| $m$ | $\alpha_m$ | $|v_{\alpha_m}(0,0,\xi)|/V_S$ | Phase of $v_{\alpha_m}(0,0,\xi)$ |
|-----|-------------|-----------------------------|-----------------------------|
| -1  | -1          | 0.353*                      | 100*                        |
| 0   | 0.016       | 1.000                       | -177                        |
| 1   | 1.192       | 0.065                       | 135                         |
| 2   | 2.299       | 0.227                       | -56                         |
| 3   | 3.600       | 0.148                       | -72                         |
| 4   | 4.981       | 0.044                       | 115                         |
| 5   | 6.420       | 0.075                       | 96                          |
| 6   | 7.905       | 0.074                       | 113                         |
| 7   | 9.426       | 0.035                       | 87                          |
| 8   | 10.977      | 0.002                       | -67                         |
| 9   | 12.553      | 0.023                       | -101                        |
| 10  | 14.149      | 0.042                       | -68                         |
| 11  | 15.764      | 0.021                       | -108                        |
| 12  | 17.395      | 0.021                       | -70                         |
| 13  | 19.040      | 0.001                       | -114                        |
| 14  | 20.697      | 0.015                       | 106                         |
| 15  | 22.366      | 0.007                       | 61                          |
| 16  | 24.045      | 0.028                       | 102                         |
| 17  | 25.734      | 0.004                       | 59                          |
| 18  | 27.431      | 0.016                       | 97                          |

$\hat{G}_N = \sum_{n=1}^{N} G_n \int_{-\xi_0}^{\xi_0} D_{\alpha_n}D_{\alpha_n}d\xi \int_{-\xi_0}^{\xi_0} u_{\alpha_n}u_{\alpha_n}d\xi.$

For $\hat{F}_{mN} = 1$ and $\hat{G}_N = 1,$ the series have converged.

Table 5 gives the magnitude and phase of the cross-equatorial flow at $x = 0$ for the free modes excited at the boundary for an eastward propagating forcing of a 6-day period and zonal wavenumber 6. The convergence of the series expansion of the free modes (30a) is excellent ($|\hat{F}_{mN}| = 1.00,$ $m = 1,$ 20; for the forced motion $G_{N0} = 0.98.$ The dominant response in the cross-equatorial flow is the $m = 0$ mode with a magnitude equal to that of the meridional wind specified at the southern boundary but $\sim 180^\circ$ out of phase. The contribution of the other modes is to reduce slightly the magnitude of the $v$ field near the boundary. For $m > 1,$ the modes are zonally trapped having complex wavenumber with e-folding distances which decrease monotonically from $10^4$ km for $m = 1$ to $167$ km for $m = 18.$ In contrast, the $m = 0$ mode has a real wavenumber with a wavelength of $4.1 \times 10^3$ km. The fact that the reflected waves have a different wavelength than the incident wave is a consequence of the asymmetric dispersion relation for waves on the beta plane. (Similarly, in the case of upstream forcing discussed in the previous subsection, the incident wave with wavenumber zero excited reflected waves with nonzero wavenumber.) Fig. 5 summarizes the physical situation.

The results for lateral forcings of various periods and wavenumbers are summarized in Table 6. In each case the $m = 0$ mode is the dominant response. Its wavelength decreases to $1.5 \times 10^3$ km for a period of 14 days. For $k_S \geq 0, (<0),$ the cross-equatorial flow is $\sim 180^\circ$ ($0^\circ$) degrees out of phase with $v_S.$ This result is a consequence of the phase

![Fig. 5. Schematic illustration of southern boundary forcing with a meridional boundary. Eastward propagating middle-latitude disturbance of wavenumber 6 and period 6 days exciting a westward propagating equatorial response of wavenumber 9 due to the presence of a meridional boundary at $x = 0.$ Thin arrows denote the meridional wind of the forcing at $y = -y_0,$ and of the response at $y = 0.$ Thick arrows give the zonal wind associated with the extratropical wave. Centers of high and low pressure are also noted.](image-url)
relation of the zonal wind relative to the lateral forcing (see Fig. 4 and discussion). The increased response for longer periods is due to the free surface acting more rigid as the period increases. The peak response at wavenumber zero for a given period arises because the magnitude of the zonal wind induced by the lateral forcing is a maximum.

It is important to emphasize that these results are based on calculations assuming no background flow. The addition of a uniform zonal wind $U_0$ may be incorporated using a Doppler-shifted frequency $\omega_0 = \omega - kU_0$. For easterly flow the effective frequency is increased for eastward forcing. A critical layer would develop only for westward forcing (assuming $U_0 < 0$, as observed). Investigations of such phenomena are, however, outside the scope of the present study.

The results of the jet response to time-dependent source-sink and southern forcing may be summarized together. The major role of either forcing is to create a variation in the zonal mass flux incident on the meridional boundary. The boundary response depends critically on the frequency of the forcing. For high-frequency oscillations, the fluid surface acts freely, and the incident mass flux accumulates at the boundary. At low frequencies, the fluid surface acts rigidly, and the incident mass flux is deflected into a meridional current. For forcings of period ≥6 days, a significant increase in the cross-equatorial flow is predicted to occur a quarter period after an increase in the incident zonal mass flux. The latter will occur roughly a quarter period after an increase in the subsidence in the Southern Hemisphere or in the lateral forcing at the southern boundary (for eastward propagating forcing). The zonal variation of the response of the cross-equatorial flow is of the same order as the observed jet width (~500–1000 km) and decays with distance from the boundary.

4. Nonlinear calculations

a. Diurnal variation of surface drag over land

Here the Rayleigh coefficient of bottom friction over land undergoes an oscillation of the form

$$ R = R_0 \left(1 + e^{-i\Omega t}\right), $$

where $R_0 = 1 \text{ cm s}^{-1}$ and $\Omega$ is the angular rotation rate of the earth. Thus $0 < R < 2 \text{ cm s}^{-1}$. Over the ocean, the coefficient assumes the constant value of $1 \text{ cm s}^{-1}$.

Fig. 6 depicts the variation of the cross-equatorial flow over the third day of the simulation. The magnitude of the variation is large over the land ($\pm 1.5 \text{ m s}^{-1}$) and relatively insignificant ($\pm 0.1 \text{ m s}^{-1}$) out over the ocean. These results are in good agreement with the linear analysis of Section 3a (cf. Figs. 6 and 2). The linear theory predicts a variation in $v$ of ±1.2 m s$^{-1}$ over land and ±0.05 m s$^{-1}$ over the ocean for a representative mean meridional wind ($V_{00}$) of 18 m s$^{-1}$. The large variation of the transient

| Period (days) | Wavenumber* | $|\hat{G}_{20}|$ | Magnitude | Phase (deg) |
|---------------|-------------|----------------|-----------|------------|
| 6             | -12         | 0.99           | 0.48      | 3          |
| -6            | 1.00        | 2.63           | 1         |
| 0             | 0.99        | 3.66           | -178      |
| 6             | 0.98        | 1.00           | -177      |
| 12            | 0.96        | 0.57           | -175      |

* (+) denotes eastward (westward) phase propagation.

![Fig. 6](image-url) (a). Nonlinear response of $v$ at the equator to a diurnally varying drag coefficient over land. Also shown is the model bottom topography $z_B$ at the equator (such that 10 m s$^{-1} = 2 \text{ km}$). (b). Deviation of the meridional wind, $\Delta v$, from the time average of the fields in Fig. 6a.
\[ S = \frac{\partial v}{\partial \tau} \left( \frac{u}{\partial x}, \frac{v}{\partial y} \right) \]

or
\[ S = \frac{\Omega}{(U/L_x, V/L_y)}. \]

For the representative values of \( U = 1 \text{ m s}^{-1}, V = 20 \text{ m s}^{-1}, L_x = 200 \text{ km}, \) and \( L_y = 2000 \text{ km}, \) one finds \( S = (15.7) \gg 1. \) Thus nonlinearities are small compared to the local time derivative, and the linear analysis is a good approximation.

Fig. 7 presents the diurnal variation in the meridional wind at \(-2^\circ\)S from the MONSOON 77 aircraft data (see Hart et al., 1978). Here "day" refers to the depth and longitude average of all flights from 0700–1900 GMT and "night" from 1900–0700 GMT. (Local time is 3 h ahead of GMT.) This figure should be compared to Fig. 6 with day (night) being the profile at \( \Omega t = \pi/2 \) \((3\pi/2)\). Both figures show that the peak velocity moves laterally \(-50\)–\(-100 \text{ km} \) to the west from day to night and that the maximum variation (\(-5 \text{ m s}^{-1}\)) lies over the higher orography. In each case, the variation decreases toward the coast. The observations exhibit a large \((-2 \text{ m s}^{-1}\) fluctuation near the coast and over the ocean that eventually changes sign at \( x = 650 \text{ km}. \) Because of the paucity of data for \( x > 600 \text{ km} \) (only one flight went beyond \( x = 600 \text{ km} \) during the day), the significance of this feature is not known. Analysis of the east-west component (Bannon, 1979a) indicates that the depth-averaged zonal wind over land increases 1–1.5 m s\(^{-1}\) from day to night. This variation is much larger than that predicted by the nonlinear model.

b. Fortnightly variation in upstream forcing

Here the source-sink forcing is assumed to undergo a sinusoidal variation in time of a period of 14 days:
\[ Q(y, t) = Q(y)(1 + e^{-i\omega t}). \]

The meridional structure \( Q(y) \) is identical to that for profile 1 in Table 5 of Part I.

The time-dependent, zonally symmetric model equations with (34) as the forcing were solved numerically in parallel with the two-dimensional numerical model. The domain for the one-dimensional problem extended from \( y = -y_0 \) to \( y = +y_0 \) with rigid northern and southern boundaries. This solution provided the necessary inflow boundary conditions (see Table 1 of Part I) at the eastern end of the two-dimensional domain. The maximum east-
ward flow of \(\sim 10 \text{ m s}^{-1}\) occurs about a quarterperiod after the maximum subsidence in the Southern Hemisphere. The minimum was 4.6 \text{ m s}^{-1}.

The response of the cross-equatorial flow to this fluctuation in the incident flow is shown in Fig. 8. As in the steady-state calculations of Part I, the location of the maximum meridional wind does not change with a change in the strength of the inflow. In addition, the magnitude and width of the time-dependent cross-equatorial flow (Fig. 8a) increases with increasing mass influx in a fashion similar to that displayed in the steady-state calculations (see cases C1 and C2 in Table 7 of Part I). Fig. 8 also shows that the cross-equatorial flow is in phase with the eastward inflow but lags the subsidence in the Southern Hemisphere by a quarter-period. This result is in qualitative agreement with the observation of Raghavan et al. (1978) which indicates that increases in the jet over Kenya lag behind a pressure drop over India by a few days.

Fig. 8b, which depicts the zonal variation of the deviation of the cross-equatorial flow, should be compared to that predicted by linear analysis shown in Fig. 3b. The zonal structure of the transient \(\nu\) field is in good agreement. However the nonlinear response leads the linear one by a quarter-period. This discrepancy arises because advective accelerations in the jet dominate local time derivatives for long-period fluctuations. For \((U,V)\) of order \((10,20) \text{ m s}^{-1}\) and \((L_x,L_y) = (1,2) \times 10^{9} \text{ km}\), the Strouhal number \(S = (0.52, 0.65) \gg 1\) for oscillations of a 14-day period.

In contrast, the phase of the zonal wind predicted by the nonlinear solution agrees with that of linear analysis. Far upstream, \((U,V) = (10,1) \text{ m s}^{-1}\) and \((L_x,L_y) = (\infty,2) \times 10^{6} \text{ km}\). Thus \(S = (\infty,6.5) \gg 1\) and local tendencies dominate the nonlinear field accelerations in the time-dependent, zonally symmetric solutions.

A final comparison with the linear analysis can be made in terms of the intensification factor \(I\) [see Eq. (23)]. For periods of 14 days, \(R = 1 \text{ cm s}^{-1}\) and \(D_0 = 2 \text{ km}\), \(|I| = 0.71\) (see Table 3 for \(\gamma^{-1} = 1\)). A nonlinear intensification factor may be defined analogously as

\[
I_{NL} = \frac{\Delta v_{\text{max}}(y = 0)/\bar{v}_{\text{max}}(y = 0)}{\Delta M_{in}/M_{in}},
\]

where \(M_{in}\) is the net mass influx and the overbar denotes a time average over the cycle. Here \((\bar{v}_{\text{max}}, \Delta v_{\text{max}}) = (17.5,3.0) \text{ m s}^{-1}\) and \((M_{in},\Delta M_{in}) = (29.7, 13.0) \text{ km}^2 \text{ s}^{-1}\). Thus \(I_{NL} = 0.39\), and the linear measure \(I\) overestimates the increase in the maximum jet speed by roughly a factor of 2.

c. Six-day southern boundary forcing

Along the southern boundary, \(y = -y_0\), the meridional wind field is given by

\[
v(x, -y_0, t) = V_S S(x) \exp(i k_x x - \omega_d t), \tag{35}
\]

with

\[
S(x) = \text{sech}^2(x/\sigma_S),
\]

where \(V_S = 5 \text{ m s}^{-1}\), \(2\pi/\omega_0 = 6 \text{ days}\), \(k_x = (6/\text{ radius of earth})\) and \(\sigma_S = 3000 \text{ km}\). The eastward propagating sinusoidal disturbance is modified by the envelope \(S(x)\) which reduces the magnitude of the forcing to \(\sim 0\) at the eastern end of the computational domain. This modification is required by the assumption that the flow satisfy the steady, zonally symmetric equations there.

A plot of the nonlinear response of the cross-equatorial flow to the forcing (35) is not presented. The constant position of the peak speed and the variation of the magnitude and width of the flow are similar to that displayed in Fig. 8a for the case of upstream forcing. Here the variation of the maximum wind speed is from 13.5 to 23.0 \text{ m s}^{-1} with a phase lag of about 120°. In comparison, the linear, frictionless analysis of Section 3c predicts that the
variation of the cross-equatorial flow at the western boundary should be 180° out of phase with the forcing (35) and have roughly the same magnitude. The phase discrepancy may be a consequence of Rayleigh damping or nonlinearity, both present here but absent in the linear theory. Following Section 4b, the Strouhal number is $S = (1.2, 1.5) \approx 1$ for oscillations of period 6 days. Thus advective and local time derivatives are of equal importance in this case. Physically one would expect the inertial terms to advect the perturbations excited at the southern boundary rapidly into the jet. This mechanism is a possible explanation for the decrease in the phase lag between the southern boundary forcing and the equatorial response observed in the nonlinear results.

The deviation of the cross-equatorial flow from the mean over the last cycle (not shown) is qualitatively similar to that in Fig. 3a which depicts the flow response for upstream forcing of period 6 days. (As previously noted, the linear analysis for southern forcing does not include Rayleigh friction and thus does not predict an exponential decay with distance of the $v$ field response. As both the southern and upstream forcing cases both predict the $m = 0$ term to be dominant, qualitative comparison with Fig. 3a is justifiable.) Here the decay rate is greatest near the boundary and larger than that exhibited in Fig. 3a. Both orographically enhanced friction and nonlinearities should produce greater trapping of the response.

The velocity pattern and pressure field associated with the flow are depicted in Figs. 9 and 10, respectively. Fig. 9 shows that while the maximum re-

**Fig. 9.** Velocity vectors representing the deviation of the flow from its time average for the case of southern boundary forcing. The display points do not correspond to model grid points. The maximum vector length corresponds to a speed of 5.6 and 8.0 m s$^{-1}$ in (a) and (b), respectively. (a) $\omega t = 0$; (b) $\omega t = \pi/2$.

**Fig. 10.** Free surface height (km) for the case of southern boundary forcing. Contour interval is 50 and 70 m in (a) and (b), respectively. (a) $\omega t = 0$; (b) $\omega t = \pi/2$. 
sponse of the cross-equatorial flow lags the maximum forcing at \((x = 0, y = -y_0)\) (Fig. 9a), it is approximately in phase with the peak mass inflow at \(\omega_{df} = \pi/2\) (Fig. 9b). The latter figure shows a surge in the meridional wind extending along the entire African coast. This is in agreement with the cases of southern boundary forcing described by Findlater (1969, p. 368):

On some occasions the south-easterly jet from the Mauritius area \([58^\circ E, 23^\circ S]\) is joined by, or even replaced temporarily by, low-level jet streams moving northward through the Mozambique Channel after bursts of cooler air come round the tip of southern Africa.

The pressure field of the flow agrees with this description and with the sequence presented in Fig. 1. The maximum cross-equatorial flow occurs as the cold high pressure center moves eastward over the Mozambique Channel.

5. Summary and conclusion

The diurnal oscillation in the strength of the surface drag due to changes in the vertical stability of the flow is modeled by a prescribed variation in the magnitude of the coefficient of surface friction \((0 < R < 2 \text{ cm s}^{-1})\). Unlike the situation in middle latitudes, the equatorial response is not an inertial oscillation. Variations in the drag coefficient act to generate a time-dependent bottom stress which, in the case of the EAJ, drives predominantly a forced oscillation in the meridional wind over land and a set of weak gravity waves. Comparison of linear and nonlinear calculations indicates that the linear theory contains the essence of the jet response. Enhancement of the surface drag over the orography (where \(D\) is small) must be included, however, for better agreement with the observed magnitude of the oscillation. Both observations and the nonlinear calculations exhibit a 3 m s\(^{-1}\) increase in the maximum of the depth-averaged meridional wind from day to night as well as a lateral shift westward of \(-50\text{ to }-100 \text{ km}\) of the jet core. The model also predicts that the oscillation is confined to the landward portion of the jet. In contrast, the MONSOON 77 data set shows a 2 m s\(^{-1}\) increase in the meridional wind from day to night just offshore and a 1–1.5 m s\(^{-1}\) increase in the zonal wind field. These discrepancies suggest that local baroclinic circulations, ignored in this investigation, are important. For example, a land-sea breeze circulation could cause displacements in the jet near the coast and account for the zonal wind variation.

In contrast to the diurnal fluctuations in the fine structure of the EAJ, time-dependent variations in the upstream and southern boundary forcings produce changes in the large-scale features of the jet (e.g., the width of the outer flank). For either type of periodic perturbation, the change in the jet speed can be large. Linear theory overpredicts, by roughly a factor of 2, the nonlinear intensification of the jet to a 14-day upstream forcing. In the case of southern forcing, the variation in the cross-equatorial flow is of the same magnitude as the meridional wind forcing for both the linear and nonlinear calculations. This result demonstrates that eastward propagating, middle-latitude disturbances of the Southern Hemisphere can have a significant effect on the flow at the equator in the presence of a meridional boundary.

The analysis of Findlater (1966) of 18 wind profiles when the maximum jet speed exceeds 30 m s\(^{-1}\) indicates that the magnitude of the depth-averaged flow during surge conditions is roughly half the peak speed. This observation coupled with the results of this investigation suggests that either upstream or southern forcing may provide the mechanism for the observed surges in the jet. More detailed synoptic analysis could be utilized to distinguish between the two types of surges and to provide better documentation of their occurrence. In particular, it would be useful to monitor the flow over South Africa and in the Mozambique Channel more closely.

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