On the uncertainty of radio occultation inversions in the lower troposphere

S. Sokolovskiy, C. Rocken, W. Schreiner, and D. Hunt

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[1] Development of radio-holographic inversion methods that solve for multipath propagation of radio occultation signals in the moist lower troposphere resulted in significant reduction of inversion errors of the bending angle and refractivity. Still, inversion errors depend on the length of recorded radio occultation signals, additive noise, and some tunable inversion parameters. These errors have components with nonzero mean (biases) and thus must be understood and quantified for weather and climate applications. In this study a physical explanation of the above mentioned inversion biases is given and their magnitude is evaluated (about 1% in the tropical lower troposphere). Assuming data with 50 Hz sampling rate and a noise level that is typical for the COSMIC GPS radio occultation observations, this magnitude can be considered as the measure of uncertainty of radio holographic inversions below ~5 km in the moist tropical troposphere.


1. Introduction

[2] Bending angles, refractivity and meteorological parameters retrieved from GPS radio occultation (RO) signals are used for weather forecasting [Cucurull et al., 2007] and considered for climate monitoring [Ho et al., 2009]. Evaluation of accuracy and precision of the RO-retrieved parameters is an important and complicated task. Methods and algorithms used for retrieval of these parameters are based on certain assumptions, approximations and depend on tunable processing strategy parameters. This introduces the uncertainty in the RO-retrieved parameters. The level of uncertainty is different at different heights. It is minimal between approximately 8 and 30 km, and increases in the upper stratosphere and lower troposphere.

[3] Increase of the uncertainty in the upper stratosphere is related to a reduction of the neutral atmospheric signal below the noise level in terms of the phase. The phase noise includes the residual ionospheric effects, receiver noise, and orbit determination errors. Integration (Abel inversion) requires reduction of error propagation downward for retrievals of refractivity and meteorological parameters. The choice of the error reduction method introduces an uncertainty in the stratospheric inversion results. Evaluation of this uncertainty is a subject of separate studies [Ao et al., 2006].

[4] Increase of the uncertainty in the lower troposphere (LT), especially in the moist LT, is related to reduction of the neutral atmospheric signal below the noise level in terms of the amplitude. This also reduces the accuracy of the measurement of the phase. Information about refractivity in the lowest several kilometers is spread over a long section of the RO signal with low signal-to-noise ratio (SNR). Unlike in the upper troposphere and stratosphere, the RO observable in the LT is not a precisely calibrated connected phase but a noisy complex signal (reliable connection of the phase between samples is not always possible).

[5] Significant advances in inversions of RO signals in the LT were achieved with the development of the methods that use the integral transform of the full RO signal from the time or coordinate to the impact parameter representation under the assumption of spherical symmetry of the refractivity (hereafter referred to as radioholographic (RH) methods). This assumption reduces the dimension of the inverse RO problem (otherwise underdetermined) and is inherent to all RH and geometric optical (GO) inversion methods. The RH methods are the Canonical Transform (CT) [Gorbunov, 2002], Full Spectrum Inversion (FSI) [Jensen et al., 2003], Phase Matching (PM) [Jensen et al., 2004] and CT2 [Gorbunov and Lauritsen, 2004]. The CT/FSI/CT2 methods reduce the transform to the Fourier transform (this allows application of the Fast Fourier Transform (FFT)) by propagating the complex signal from orbit to a straight line (CT) or by approximations that use a model of the frequency of the RO signal (FSI, CT2). In all RH methods, the length of the RO signal used for the inversion is a tunable parameter.

[6] In this study we investigate the dependence of the RO-retrieved bending angle and refractivity in the LT on the length (truncation height) of the RO signal at the bottom of an occultation and on additive noise of the RO signal. We base our study on real COSMIC RO signals (though we can’t subtract noise but we can add it and estimate the response of the inversion). This approach has certain advantages over simulations where realistic 3-D modeling of the RO signal is...
Figure 1. Layout of GPS radio occultation (RO) observations.

difficult by itself and because a realistic 3-D moisture structure in the LT is not well known while the effects considered in this study depend substantially on the structure of RO signals (in particular, the out-of-plane bending contributes to the spectrum of RO signal). As opposed to the simulations, in our study the truth is not known. However, the goal of our study is the estimation of not the accuracy but the uncertainty of the GPS RO in the LT and this can be achieved with real RO signals without knowing the truth. In this paper we use the European Centre for Medium-Range Weather Forecasts (ECMWF) analysis only as a reference but not for the validation. We focus on the mean differences (biases). Since the biases may change with the change of data processing algorithms, noise level, and the atmospheric structures, they are critical for weather and, especially, for climate applications.

Section 1 provides overview of the problem considered in this study. Section 2 outlines the PM method developed in other’s studies, used for inversions of RO signals in the LT in this study. Section 3 demonstrates dependence of the inversion bias on the RO signal cutoff height and additive noise, using statistical approach. In section 4, the biases introduced in section 3 are explained and investigated based on analysis of individual occultations. Section 5 outlines the radio-holographic filtering developed in other’s studies, and investigates the dependence of the results on tunable filter parameters. Section 6 introduces dynamic cutoff approaches for RO signals and investigates the inversion biases for different approaches. Section 7 shows statistical difference of the inversion biases for COSMIC occultations with low and high noise level. Section 8 demonstrates examples of the occultations where the signal is observed well above the noise level deep below the limb. Section 9 discusses the results of this study as well as some other sources of the biases of RO in the LT and outlines problems for further studies. Section 10 concludes the study. Appendix A demonstrates the asymmetry of local spectrum of noise of the RH-transformed signal and dependence of the asymmetry on the sampling rate of RO signal by numerical simulation. Appendix B gives possible explanation of the observations of RO signals deep below the limb (shown in section 8) by modeling wave propagation for the horizontally inhomogeneous surface duct.

2. Phase Matching

In this study we use the PM method for inverting GPS L1 RO signals in LT. Here we briefly outline the PM method following Jensen et al. [2004] where more details can be found.

The measured complex RO signal \( u \), which is a composition of subsignals (rays) arriving concurrently at receiver, is considered a function of the central angle \( \theta \) between transmitter and receiver located at distances \( r_1(\theta) \) and \( r_2(\theta) \) from the local center of sphericity of the refractivity (see Figure 1). For the spherically symmetric refractivity \( N = N(\rho) \) each ray is uniquely characterized by an impact parameter \( a \) (see Figure 1). The signal \( u(\theta) = A(\theta)\exp[i\Phi(\theta)] \) is multiplied by \( \exp[-ikS(a, \theta)] \), where \( k \) is the wave number and \( S(a, \theta) \) is the geometric optical phase model

\[
S(a, \theta) = a\theta + \sqrt{r_1^2(\theta) - a^2} + \sqrt{r_2^2(\theta) - a^2} \\
- a\arccos(a/r_1(\theta)) + \arccos(a/r_2(\theta))
\] (1)

whose frequency \( k\beta S(a, \theta)/c \theta \) for a given impact parameter \( a \) is equal to the frequency \( d\Phi/da \) of the subsignal with the same impact parameter, and integrated

\[
v(a) = B(a) \exp[i\Psi(a)] = \int A(\theta) \exp[i\Phi(\theta) - ikS(a, \theta)]d\theta
\] (2)

Since in the spherically symmetric \( N(\rho) \) there is only one ray with a given impact parameter \( a \), the integral (2) has only one stationary point \( \theta_s(a) \) where the frequency of any of the subsignals composing the signal \( u(\theta) \) matches the frequency of the model \( S(a, \theta) \)

\[
d\Phi/da_s - k\beta S(a, \theta_s)/\theta_s = 0
\] (3)

For a given \( a \), the location of stationary point \( \theta_s(a) \) and thus the bending angle \( \alpha \) of the subsignal can be determined from the equation obtained by differentiating the phase \( \Psi(a) \) of the transformed signal \( v(a) \) which, with account for (3), is

\[
d\Psi/da = -k\beta S(a, \theta_s)/\partial a
\] (4)

and, with account for (2), the bending angle is

\[
\alpha(a) = k^{-1}d\Psi/da
\] (5)

Instead of the impact parameter \( a \) it is convenient to use the impact height \( h = a - r^2 \), where \( r_s \) is the local curvature radius of the Earth at the location of an occultation.
10 It is important to note that RO signals sampled at 50 Hz (this rate covers the local spectral width of RO signal) are undersampled for the purpose of the phase transform (2). This means that the phase function is changing by many cycles between the 50 Hz samples. In the methods using FFT (CT/FSI/CT2) the RO signal is upsampled prior to application of the FFT [Jensen et al., 2003]. We perform the upampling concurrently with the calculation of the phase transform (2) by linearly interpolating the phase and amplitude functions in (2) and calculating the integrals analytically between the 50 Hz samples.

3. Dependence of the Refractivity Inversion Bias on the Cutoff Height and Noise

[11] We represent RO signals as functions of the height of a straight line between GPS and LEO

\[ H = \frac{r_1 r_2 \sin \theta}{\sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta}} - r_e \] (6)

as shown in Figure 1. COSMIC RO receivers are set to track GPS L1 C/A signals in the open-loop mode below \( H \sim -10 \) km down to at least \( H_{\text{min}} \sim -150 \) km. Figure 2 shows actual minimal tracking height (depth) \( H_{\text{min}} \) as a function of latitude for 1 d of COSMIC observations.

[12] We perform the following test. We terminate the RO signals by cutting them off at heights \( H_{\text{cut}} = -150 \) km and \(-100 \) km. The signals are inverted as they are, and after adding random uncorrelated Gaussian noise on both in-phase (I) and quadrature (Q) components. As shown in section 6, the background noise level on COSMIC RO signals is about 9–10 V/V. We add noise with the RMS magnitude 17 V/V which approximately doubles the background noise level (because of summation with squares). We compare the retrieved profiles \( N(z) \), where \( z \) is the height over the geoid, with those from ECMWF analysis and calculate mean differences. Figure 3 shows the results obtained from
COSMIC observations for certain days between 1 and 10 July 2007 in different latitude bands (different numbers of days for different latitude bands are used in order to have close numbers of occultations). It is seen that increase of the $H_{\text{cut}}$ from $-150$ to $-100$ km results in a reduction the mean refractivity (negative $N$ bias) while additive noise results in the positive $N$ bias. The positive bias induced by noise is substantially larger for the lower than for the higher $H_{\text{cut}}$, and is larger in the summer than in the winter hemisphere. At the bottom of some of the profiles for $H_{\text{cut}} = -100$ km (Figures 3a, 3c, and 3d) the additive noise introduces small negative bias. These biases are explained in section 4 and Appendix A.

In order to evaluate the statistical significance of the results in Figure 3 we repeated calculations for different day intervals in July 2007. The mean differences between the inversion results obtained with different $H_{\text{cut}}$ and noise levels (but not the differences with the ECMWF model) calculated independently for different days have maximum spread at $\approx 1$ km height of about 0.1%. This magnitude characterizes the level of uncertainty of the estimates of the biases obtained in this study.

4. Analysis of the Dependence of the Refractivity Inversion Bias on the Cutoff Height and Noise

The subsignals (rays) composing a RO signal have different bending angles. The larger the bending angle of the subsignal the deeper (i.e., at lower $H$) it is received. Increasing the truncation height of RO signals removes the subsignals with the largest bending angles and thus introduces the negative bias in the inverted profiles $\alpha(h)$ and $N(z)$. This effect was described by Sokolovskiy [2003]. The negative bias because of higher truncation is smaller in high latitudes because on account of lower moisture and smaller vertical gradients of refractivity there are fewer subsignals with large bending angles propagating to low $H$.

The effect of noise on RO signals is more complicated. When refractivity is spherically symmetric the transformed RO signal is quasi-monochromatic and the additive noise does not introduce a bias [Jensen et al., 2006]. This is approximately true in the atmosphere with a small amount of water vapor (high latitudes and winter months) because the temperature fields commonly have large horizontal scales. Water vapor fields commonly have smaller horizontal scales. This results in broadening of the local spectrum of $v(a)$. As the result, calculation of the frequency (i.e., bending angle $\alpha$) by differentiation of the phase $\Psi(a)$ loses physical sense and may depend on the shape of the local spectrum of $v(a)$. In particular, asymmetry of the spectrum may result in the bias of the determined frequency [Sirmans and Bumgarner, 1975].

For undersampled signals (such as the RO signals sampled at 50 Hz used in this study), the asymmetry of the local spectrum of noise of the transformed signal $v(a)$ is related to up-sampling (or interpolation) of the RO signal for application of the phase transform (2) (mentioned in section 2). The additive observational noise is distributed in the band $\Delta f$ limited by the sampling frequency $f_s$. After the up-sampling the local spectrum of noise is shifted and centered at the mean frequency of the up-sampled signal. Thus in the time–frequency representation the noise is distributed in the band $\Delta f$ around the mean frequency as this is schematically shown in Figure 4 (left). Transformation to the impact parameter representation maps the uncertainty of the frequency to the uncertainty of the impact parameter $\Delta a = 2\pi\Delta f/k\Omega$ where $\Omega = d\theta/dt$ (as this follows from the phase function (2) in case of circular orbits which may be considered for simplicity). Thus in the bending angle–impact parameter representation the noise is distributed in the band of the width $\Delta a$ around the mean impact parameter as this is schematically shown in Figure 4 (right). As the result of convexity of the function $\alpha(a)$, the local distribution of
noise for a given $a$ becomes asymmetric, i.e., $\Delta a_+ \neq \Delta a_-$. This asymmetry results in the positive bias in $\alpha(a)$ when local spectrum of the RH-transformed signal $v(a)$ is broad. When RO signal is truncated high enough (this is shown schematically by dashed lines in Figure 4) then the asymmetry is reduced or even reversed and may result in the negative bias at the bottom of the retrieved profile $\alpha(a)$ (this negative bias is mentioned in the discussion of Figure 3). The asymmetry of the local spectrum of noise is demonstrated in Appendix A by RH-inversions of the simulated signal with added noise (also, another possible reason for asymmetry is considered which is important for RO signals sampled at high rate). Below we demonstrate the effects discussed above and seen in Figure 3, on inversions of individual occultations.

[17] Figure 5 (left) shows the amplitude of the RO signal (high latitude, winter) which apparently is below the noise level at $H < -80$ km. Figure 5 (right) shows the fractional difference in $N(z)$ retrieved with $H_{cut} = -80$ and $-150$ km. As seen, the mean difference is close to zero. The $H_{cut}$ higher than $-80$ km results in the higher cutoff of the inverted signal (which is based on amplitude of the transformed RO signal $B(a)$) but otherwise does not introduce a mean difference in $N(z)$.

[18] Figure 6 shows the same as Figure 5 but for the tropical occultation. Truncation of the RO signal at $H_{cut} = -115$ km

Figure 5. (left) Amplitude of RO signal for a polar winter occultation. (right) Fractional difference of refractivities retrieved from the signal truncated at the height indicated by dashed line in Figure 5 (left), and from the full signal.

Figure 6. Same as Figure 5 but for a tropical occultation.
results in some negative mean difference in the retrieved $N(z)$ at $z < 3$ km (compared to retrieval of the full signal) although the signal at $H < -115$ km is mostly noise. This can be explained by an increase of the asymmetry of the local spectrum of noise with an increase in length of the RO signal, as demonstrated in Appendix A. Thus the negative bias seen in Figure 6 at $z < 3$ km, basically, is equivalent to reduction of the positive bias induced by noise when the noisy section of the signal at $-190 \text{ km} < H < -135$ km is not used.

Figure 7 shows the same as Figures 5 and 6 but for the subtropical occultation characterized by strong refractivity gradient on the sharp top of the atmospheric boundary layer (ABL) at 1.5–2 km, resulting in large bending angles and the multipath propagation. The subsignals above the noise level are seen down to the bottom of the RO signal. Significant negative difference, observed below the ABL top, between $N(z)$ retrieved with $H_{cut} = -90$ km and $H_{cut} = -170$ km is related to mainly removal of those subsignals from the inversion.

Figure 8 (left) shows raw amplitude of the RO signal for the same polar occultation as in Figure 5 (black line) and of the signal width added noise 17 V/V (red line). Figure 8 (right)
shows fractional differences between \( N(z) \) retrieved with and without the added noise when the whole RO signal is used and truncated at \( H_{cut} = -80 \) km. As seen, the mean differences in both cases are close to zero.

[21] Figure 9 shows the same as Figure 8 but for the tropical occultation from Figure 6. Figure 9 (right) shows fractional differences between \( N(z) \) retrieved with and without the added noise when using the whole RO signal (solid line) and truncated at \( H_{cut} = -115 \) km (dashed line). It is seen that for this occultation the additional noise results in significant positive bias in retrieved \( N(z) \) at \( z < 6 \) km when the signal is used down to \( H \sim -190 \) km while there is almost no bias (and even small negative difference at \( z \sim 2 \) km) when the signal is used down to \( H = -115 \) km.

[22] To explain the effect of noise in Figures 8 and 9 we calculate sliding spectrograms of the RH-transformed signals. Figure 10 (left) shows the spectrograms for the polar winter occultation and the tropical occultation (Figure 10, right). Each spectrogram consists of the multiple amplitude spectra \( \nu(\alpha) = \int \nu(h) \exp(-i\alpha h) dh \) calculated in a running window of fixed length in \( h \). The level of zero amplitude corresponds to position of the window in \( h \) while the amplitude itself is represented using some scale convenient for visualization (the scale is the same for Figure 10, left and right). The polar winter signal (Figure 10, left) has a single-tone structure expressed by a single sharp maxima in the spectra while other fluctuations are mainly related to noise. The tropical signal (Figure 10, right) has a multitone structure in the LT and is not as well distinguishable from the noise at low \( h \) as the polar signal. When RO signals are used down to the end (\( H = -150 \) and \( -185 \) km) the local spectra of noise of the RH-transformed signals are clearly asymmetric, extended to larger bending angles (Figures 10a, 10b, 10e, and 10f show spectrograms obtained from the original signals without and with the additional noise). This asymmetry explains the positive bias of the bending angle induced by noise for the tropical occultation (see Figure 9). When RO signals are truncated \( (H_{cut} = -80 \) and \( -115 \) km) the asymmetry is reduced and reversed at the bottom of the RH-transformed signals (Figures 10c, 10d, 10g, and 10h show spectrograms obtained from the truncated signals without and with the additional noise). The reversed asymmetry explains the small negative bias induced by noise for the truncated tropical occultation (see Figure 9). Both positive and negative biases are observed for the tropical occultation with broad local spectrum of \( \nu(h) \) and not observed for the polar occultation with the sharp spectrum.

[23] To further explore the effect of noise in Figures 8 and 9, we first smooth the bending angle calculated by differentiation of the phase of the transformed signal \( \langle \alpha(h) \rangle = k^{-1} \langle d^b \Psi/dh \rangle \). Then we multiply \( \nu(h) \) by the model function \( \exp[-\Psi_m(h)] \) where \( \Psi_m(h) = ik \langle \alpha(h) \rangle dh \). Thus the mean frequency of the function \( \tilde{\nu}(h) = B(h) \exp[i\Psi(h) - i\Psi_m(h)] \) is close to zero.

Next we calculate the Fourier spectrum of the function \( \tilde{\nu}(h) \) by using this function in a limited impact height interval, \( 2 < h < 6 \) km (the lower and upper limits approximately correspond to the minimal impact height of the ray touching the surface and to the height above which the biases found in this study become insignificant), \( \tilde{\nu}(\beta) = \int \tilde{\nu}(h) \exp(-ik\beta h) dh \).

Parameter \( \beta \) is the deviation of the bending angle from the one obtained by differentiation of the reference function \( \Psi_m(h) \). For an ideal RO signal corresponding to smooth \( N(z) \), the spectrum \( \tilde{\nu}(\beta) \) is close to delta function. For a real RO signal the spectrum \( \tilde{\nu}(\beta) \) first, is broadened because of the effect of nonspherically symmetric irregularities in \( N \). Second, it contains noise. Third, it contains subsignals with large bending angles (corresponding to strong gradients in \( N(z) \)) that do not pass the smoothing applied for the model \( \Psi_m(h) \). The spectrum \( \tilde{\nu}(\beta) \) was introduced by Gorbunov and Lauritsen [2006] and Gorbunov et al. [2006] for the purpose of the filtering of RO signals, which is discussed in section 5.

[24] Figure 11 shows amplitudes of the spectra \( \tilde{\nu}(\beta) \) for the polar winter (Figure 11, left) and the tropical (Figure 11, right) occultations. Red and black colors correspond to the...
Figure 10. Spectrograms of the RH-transformed RO signals for (a–d) the polar winter occultation and (e–h) the tropical occultation for different truncation heights, with and without the additional noise (indicated in the panels).

Figure 11. Amplitudes of the spectra $\tilde{w}(\beta)$ for (left) the polar winter and (right) the tropical occultations.
RO signals with and without the added noise. Figure 11 (bottom) show zoomed parts of the spectra below the horizontal lines in Figure 11 (top). First, it is seen that the spectrum of the tropical occultation is wider than that of the polar winter occultation. Second, the spectra are asymmetric in agreement with the explanation above (see Figure 4) and in Appendix A. The asymmetry is present in the spectra of raw signals (black) and is increased with adding noise (red). For the polar winter occultation, because of the narrow spectrum, the asymmetry does not affect determination of the frequency (bending angle) by differentiation of the phase in agreement with Jensen et al. [2006] and Figure 8. For the tropical occultation, because of the broader spectrum, the asymmetry results in the positive bias of the frequency (bending angle) determined by differentiation of the phase in agreement with Figure 9.

5. Use of Radio Holographic Filtering

Gorbunov et al. [2006] and Gorbunov and Lauritsen [2006] suggested filtering of RO signals in the impact parameter domain hereafter referred to as radioholographic filtering (RHF). The idea behind the RHF is that the multipath propagation in the lower troposphere broadens the spectrum of RO signal in time/coordinate representation thus making noise filtering inefficient. RH transform compresses the spectrum of the RO signal since, under the assumption of spherical symmetry, the signal transformed to impact parameter representation must have a single-tone structure and this shall enable efficient noise filtering. The filtering consists in the windowing of the spectrum \( \tilde{w}(\beta) \) with the following inverse transform to the impact parameter space, calculation of the frequency and up-conversion. Gorbunov et al. [2006] apply a smoothing window \( a_w = 250 \text{ m} \) for \( \alpha(a) \) used in the reference function \( \exp[-ikR_h \alpha(a) da] \) and Gaussian windowing of the spectrum \( \tilde{w}(\beta) \). We note that in the special cases of infinitely wide and infinitely narrow spectral windows \( \beta_w \) the RHF does not have an impact because it yields raw and smoothed \( \alpha(a) \) calculated by differentiation of raw \( \Psi(a) \).

Figures 12 and 13 (left) show raw amplitudes of the signals from Figures 6 and 7 (black), and of the signals with added noise 17 V/V (red). Figures 12 and 13 (top right) show the spectra \( \tilde{w}(\beta) \) with and without the additional noise (red and black), and Gaussian windowing functions of different widths (other colors). Figures 12 and 13 (bottom right) show differences of the refractivities retrieved from the RO signal with added noise without RHF (red line) and with RHF with the windowing functions shown in Figure 12 (top right) (colors are corresponding) from the refractivity retrieved from the original RO signal.
the RHF effectively eliminates the noise-induced positive $N$ bias concurrently with introducing some negative bias at $z < 3$ km. This negative bias (which does not significantly depend on the window $\beta_w$) apparently is related to suppression of the effect of noise in the raw signal. For the subtropical occultation (Figure 13) the results substantially depend on the window $\beta_w$. The subsignals with large bending angles are clearly seen both in the raw signal at low $H$ and in the spectrum $\hat{w}(\beta)$ at large positive $\beta$ where they are mixed with noise. The negative bias that increases with decreasing $\beta_w$ is caused to a large extent by the suppression of those subsignals (similarly to the bias which is caused by truncation of the subsignals at low $H$ in Figure 7).

Thus, generally, RO signals are not fully separable from noise not only in the time/coordinate but also in the impact parameter domain. Suppression of noise in the RH-transformed RO signals comes at the expense of the suppression of subsignals with large bending angles. However, RHF has certain advantages that will be discussed in sections 6 and 7.

Next we investigate sensitivity of the inversion results (in terms of biases) to tunable RHF parameters, such as the window $\alpha_w$ for smoothing of the raw $\alpha(h)$ and the window $\beta_w$ for the spectrum $\hat{w}(\beta)$. In fact, the length of the RH-transformed RO signal $\nu(h)$ subjected to RHF also can be a tunable parameter; however, in this study we apply RHF in the fixed interval $2 \text{ km} < h < 6 \text{ km}$ (for the reasons mentioned earlier, when introducing $\hat{w}(\beta)$ in section 4).

Figure 14 shows mean fractional differences of COSMIC RO and ECMWF $N(z)$ in the tropics ($30^\circ\text{S} - 30^\circ\text{N}$), for 1 d, for different RHF tunable parameters. For the cutoff of the RO signals we use the approach (D) introduced in section 6. Figure 14 (left) shows results for different $\alpha_w$ and fixed $\beta_w = 0.016$ rad; Figure 14 (right) shows results for different $\beta_w$ and fixed $\alpha_w = 250$ m. It is seen that the sensitivity of the inversion results to $\alpha_w$ is not very strong while the sensitivity to $\beta_w$ is stronger. Increase of $\beta_w$ from 0 to infinity results first in decrease and then increase of the mean $N(z)$ (as mentioned earlier, $\beta_w = 0$ or infinity are both equivalent to no RHF). The maximal mean difference introduced by RHF, approximately 0.75% at 2 km, corresponds to $\beta_w \sim 0.008$ rad. Though around $\beta_w \sim 0.008$ rad the results of RH filtering are least sensitive to the value of $\beta_w$ it is not clear whether this window is optimal. For example, for the subtropical occultation $\beta_w = 0.008$ rad corresponds to significant negative bias because of suppression of subsignals with large bending angles (see Figure 13). In principal, the tunable RHF parameters can be determined dynamically, based on the structure of the RH-transformed RO signal, but this requires a separate study.

### 6. Use of Dynamic Cutoff Height

Since excessively high or low $H_{\text{cut}}$ can introduce negative or positive inversion biases, respectively, it is reasonable to determine $H_{\text{cut}}$ individually for each occultation. Such determination, based on comparison of the signal amplitude to background noise level $A_{\text{bg}}$, is subjective (depends on tunable parameters). Since COSMIC receivers do not provide an estimate of the background noise level, we define it ad hoc.
as the lowest mean amplitude in a continuous time interval (we use a 3 s interval). Figure 15 shows distributions of $A_{bg}$ versus latitude and versus azimuth of the occulted GPS wrt antenna boresight (for Figure 15 (top and bottom) we use only the occultations with $H_{\text{min}} < -150$ km). It is seen that $A_{bg}$ is grouped around $9 - 10$ V/V. There are no significant dependencies on latitude and on azimuth, there is only minor increase of $A_{bg}$ off the antenna boresight. Existence of large values $A_{bg}$ (mainly in low latitudes but also in midlatitude and high latitude in the summer hemisphere, i.e., correlated with the water vapor) means that for some occultations the amplitude of the RO signal never reduces to noise level and that $H_{\text{min}}$ currently used in the COSMIC receivers is not sufficient for all occultations.

[31] We define a tunable parameter $C_{\text{cut}}$ which specifies the cutoff amplitude $A_{\text{cut}} = C_{\text{cut}} A_{bg}$. Figure 16a–16c show distributions of the heights $H_{\text{cut}}$ where the amplitude of RO signals, smoothed by sliding averaging (in this study we use a 3 s time window), first time increases above $A_{\text{cut}}$ (from bottom to top of an occultation) for $C_{\text{cut}} = 1.5, 2, 3$, respectively. Larger $C_{\text{cut}}$ results obviously in greater cutoff heights and in more pronounced latitudinal dependence of $H_{\text{cut}}$. This is caused by deeper penetration of RO signals in low latitudes. Smaller $C_{\text{cut}}$ retains more information in RO signals, concurrently with passing more noise, and also it reveals a set of very deep RO signals with latitude-independent distribution. These weak signals may be caused by nonocculted GPS because of the relatively short period of the C/A code [Bonnedal et al., 2010].

[32] We also use the following empirical cutoff approach. First we determine an intermediate height $H_{\text{int}}$ where the smoothed amplitude first time increases above $A_{\text{int}} = C_{\text{int}} A_{bg}$ (from bottom to top of an occultation), next we determine $H_{\text{cut}}$ where the smoothed amplitude first time decreases below $A_{\text{cut}}$ (from $H_{\text{int}}$ to bottom of the occultation). Latitudinal distribution of thus obtained $H_{\text{cut}}$ for $C_{\text{int}} = 3$ and $C_{\text{cut}} = 1.5$ is shown in Figure 16d. This approach cuts off RO signals deeper compared to approach (Figure 16c), but excludes most of the deep weak latitude-independent signals and concomitant noise compared to approach (Figure 16a).

[33] In order to investigate sensitivity of low-latitude tropospheric inversions to noise with different cutoff approaches we invert RO signals as they are and after adding noise 17 V/V. Adding noise would increase the dynamically determined $H_{\text{cut}}$. This would result in mixing of the two inversion biases.
discussed in sections 3 and 4: the positive bias due to noise and the negative bias due to higher cutoff. In order to isolate the effect of noise we use the $H_{\text{cut}}$ determined before adding of the noise. Figure 17 shows the mean fractional differences between COSMIC RO refractivities retrieved with different cutoff approaches (Figures 17a–17d) explained above and ECMWF refractivities in the tropics (30°S–30°N) for 1 July 2007. Solid and dashed lines correspond to inversions without and with the added noise, red and green colors correspond to inversions without and with the RHF. For the RHF we use $a_w = 250$ m, $b_w = 0.016$ rad. It is seen from Figure 17 that different cutoff criteria result in different inversion biases; the RHF substantially reduces sensitivity of inversions to noise and slightly to cutoff height; and there is a systematic difference between the inversions with and without the RHF.

7. Inversions of COSMIC Occultations With Different Signal-to-Noise Ratios

[34] COSMIC GPS receivers represent the RO signal amplitude as the so-called signal-to-noise ratio (SNR) in units of V/V. The SNR is scaled to 1 Hz band, i.e., is √50 ~ 7 times lower for 50 Hz sampled RO signals than for 1 Hz observations. For amplitudes well above the noise level, the SNR is a true signal-to-noise ratio. However, this is not true at the bottom of occultations where the amplitude is close to zero while SNR converges toward a background noise level $A_{bg}$ (distributions of the $A_{bg}$ are shown in Figure 15). Figure 18 (bottom) shows the distribution of the mean SNR on azimuth of the occulted GPS wrt antenna boresight (0 and 180°–180 degrees correspond to boresight in the velocity and antivelocity direction). We define mean SNR by averaging between $H = 60$ and 80 km because this interval is high enough to neglect the defocusing effect of the neutral atmosphere and is below the E layer of the ionosphere. Though mean SNR as defined in this way is not representative of the whole occultation because of antenna pattern and attitude variations, we characterize the noisiness of occultations using this parameter in a statistical sense. As seen from Figure 18, the mean SNR depends substantially on azimuth, as opposed to $A_{bg}$. For reference, Figure 18 (top) shows the distribution of the azimuth on latitude. Peculiarities of this distribution reflect inclinations of the GPS and COSMIC orbits. In particular, occultations with maximal SNR (i.e.,
close to the antenna boresight) never occur close to equator and poles.

[35] Figure 19 shows mean fractional differences between COSMIC RO and ECMWF refractivities in the tropics (30°S–30°N) for 1–3 July 2007, constrained by SNR > 700 V/V (solid lines) and SNR < 700 V/V (dashed lines). Red and green lines correspond to no RHF and RHF ($a_w = 250 \text{ m}, \beta_w = 0.016 \text{ rad}$). Figure 19 (left) corresponds to the fixed cutoff height $H_{\text{cut}} = -150 \text{ km}$, and Figure 19 (right) to the dynamic cutoff by use of the approach (D) discussed in section 6. It is seen that there is a significant difference in the inversion bias in the tropics between low and high noise occultations, which is reduced but still remains significant with the application of RHF. With the fixed cutoff, higher noise results in positive $N$ bias, which is consistent with previous results. With the dynamic cutoff there is a combination of two effects, positive $N$ bias due to the noise (dominates above 1–2 km) and negative $N$ bias due to the increase of $H_{\text{cut}}$ with the increase of noise (dominates below 1–2 km). Overall, above ~1 km, the use of the dynamic cutoff reduces the difference between the inversion results for low and high noise occultations.

8. Deep RO Signals (Observed at Very Low Heights)

[36] As it follows from Figure 16, some occultations have signals strengths exceeding the background level at heights $H$ substantially below $-150 \text{ km}$. Figure 20 shows several examples of amplitudes of such RO signals. Sometimes RO signals submerge into and then reemerge from the background noise, sometimes they never submerge into noise ($C004.2007.182.20.52.G23$), this corresponds to large $A_{bg}$ in Figure 15. As it follows from Figure 20 (also mentioned in discussion of Figure 15), the currently set minimal tracking height $H_{\text{min}}$ in COSMIC receivers is not sufficient for all occultations.

[37] Here we consider inversion of the deepest occultation in Figure 20 ($C005.2007.182.16.57.G17$) by cutting off the RO signal at different heights. Figure 21 (left) shows amplitude of this RO signal and different cutoff heights denoted by horizontal lines. Figure 21 (right) shows the differences between the $N(z)$ retrieved with $H_{\text{cut}} = -190 \text{ km}$ and $-135 \text{ km}$ and the $N(z)$ retrieved from the full signal. It is seen that removal of the subsignals observed at $H < -190 \text{ km}$ introduces a significant negative bias below the top of the boundary layer at ~1.5 km. On the other hand, removal of a rather long section of RO signal between $-135$ and $-190 \text{ km}$ (where the RO signal is below the background noise) introduces a much smaller difference related to the effect of noise. This suggests...
that for this occultation the inversion bias related to truncation of the RO signal is mainly related to removal of the real subsignals observed at very low height.

### 9. Discussion

[38] The moist LT is a challenge for RO remote sensing. Despite significant progress, development of the RH methods that disentangle multipath, and elimination of the receiver tracking errors by using the open-loop tracking, still other error sources remain. Reduction of the RO signal amplitude with the increase of the bending angle and the gradual mixing of the signal with noise introduces an uncertainty in the RO inversion results. Using the RO signal down to lower observational heights accounts for larger bending angles but increases the effect of noise. The effect of noise is related to
the asymmetry of the local spectrum of noise of the RH-transformed signal due to the up-sampling of the observed signal. This results in a bias in the bending angle and refractivity when the local spectrum of the RH-transformed signal itself is broad (due to nonspherically symmetric N irregularities induced mainly by water vapor).

[39] The biases of RO inversions in the moist LT depend on the signal processing method and the receiver noise, but they also depend on the structure of the troposphere itself and this is important for climate applications. Given the amount of water vapor in the LT, a change of the structure of its distribution may result in a change of the inversion biases and their aliasing in the retrieved amount of water vapor. For example, a sharper ABL top (or other inversion layers) results in propagation of RO signals deeper behind the limb and thus in a negative bias in the bending angle and refractivity (given the noise level and the tracking depth of the receiver). The N gradients exceeding critical result in an additional refractivity bias due to super-refraction, which also may change with time. Increase of the intensity of the nonspherically symmetric irregularities broadens the spectrum of the transformed RO signals and thus increases the bias due to noise as investigated in this study. Increase of the intensity of horizontal gradients also increases the out-of-plane component of the bending angle investigated by Healy [2001]. This component reduces the projection of the wave vector on the occultation plane and thus introduces a bias. Though this is a second-order effect, its magnitude in the LT has not been investigated yet.

[40] The above discussed uncertainty, in other words, means that the RO signal and noise are not fully separable in the LT. Different filtering strategies may reduce one bias by introducing another. Gorbunov and Lauritsen [2006] performed a comparison of two filtering strategies of RO signals in different representations. Such studies may be further needed in order to find an optimal filtering, which generally may be different for different applications. For example, RH filtering reduces the uncertainty of the inversion bias related to noise and this is advantageous for climate applications. However, this comes at the expense of removal of sub-signals with large bending angles. For an application such as monitoring of the ABL depth, preserving the sub-signal with large bending angles is most important (to ensure reconstruction of the sharp ABL top) but the uncertainty in the bias is less important, and thus different filtering or different tuning of the filtering may be applied.

[41] Reduction of the receiver noise certainly is useful because it allows deeper tracking and reduction of the inversion biases, but it is difficult to quantify this effect (this study imposes additional noise on real RO signals and estimates the response but the existing noise cannot be removed). Increase of the sampling rate of the RO signals allows for the reduction of the asymmetry of the local spectrum of noise of the RH-transformed signal by applying RHF with larger spectral window, and thus reduction of the bias induced by noise without increase of the bias due to removal of sub-signals with large bending angles. In order to preserve the sub-signals with the largest bending angles by keeping local spectrum of noise of the RH-transformed RO signal symmetric, the sampling rate must cover the full frequency change of RO signal during an occultation (from one to several kHz). However, an increase of the sampling rate beyond what is required for capturing the local spectrum of RO signal (50–100 Hz) will result in transmission of an excessive amount of data. Another option is performing RH transform (with the use of FFT) of the high-rate signal in the receiver. This would reduce the amount of transmitted data because the RH transform compresses a long section of RO signal of 100–200 km length in the H representation into a short section of several kilometer length in the h representation, which corresponds to the LT and where only the RH-transformed RO signal has to be sampled and transmitted at high rate.

10. Conclusions

[42] We investigated the uncertainty (in terms of the mean difference) of the inversions of RO signals sampled at 50 Hz with SNR ~ 400–800 V/V, such as the COSMIC RO signals, in the moist LT. The uncertainty is related to different truncation height of the RO signals and different noise level. Increase of the truncation height results in a negative inversion bias related to removal of the sub-signals with the largest bending angle from the inversion. Decrease of the truncation height results in a positive inversion bias extending up to the height 5–6 km because of the asymmetry of the local spectrum of noise of the RH-transformed RO signal. Application of a dynamic cutoff height reduces the uncertainty. The largest uncertainty is about 1% in the summer hemisphere, reaching about 1.5% in the tropics and reducing to about 0.1% (i.e., below the level of significance obtained in this study) in the polar winter. Application of the RH filtering further reduces the uncertainty to about 0.5% concurrently with resulting in smaller retrieved refractivities. The latter is due to removal of the effect of noise and removal of the sub-signals with the largest bending angles that are not fully separable from the noise. The estimated uncertainties must be taken into account for climate applications of RO in the LT especially when using results obtained with different processing methods.

Appendix A: Distribution of the Local Spectrum of Noise of the RH-Transformed RO Signal

[43] Here we demonstrate the distribution of the local spectrum of noise of the RH-transformed RO signal, briefly explained in section 4, based on inversions of the simulated RO signal sampled at different rates and used down to different heights H. For the demonstration we have simulated the RO signal (phase and amplitude) for the refractivity profile $N(z) = 400 \exp(-z/3 \text{ km})$ (representing the tropical LT) using multiple phase screen (MPS) propagation [Sokolovskiy, 2001], by sampling the signal with 1 m step on the trajectory normal to the direction of the incident plane wave at a distance $l = 3000 \text{ km}$ from limb, as shown in Figure A1. We invert the RO signal using the CT method [Gorbunov, 2002] (the effect of down-sampling of RO signal on the distribution of noise of the RH transformed signal is similar for all RH methods). Because the simulated signal does not result in multipath propagation at the observational distance, it is convenient to express the observational Doppler frequency in terms of the bending angle $\alpha$ as a function of the observational height $H$, and the CT-retrieved bending angle $\alpha$ as
a function of the impact height \(h\) as shown in Figure A1. Before the inversion, we impose pseudorandom Gaussian additive noise on the simulated complex signal sampled at 1 m step, then down-convert the noisy signal (by using the frequency of the original signal as the reference model) and down-sample by averaging I and Q in the intervals of fixed length. Next we up-sample RO signal to the original 1 m step by zero-padding of the spectrum (linear interpolation of the connected phase and amplitude yields identical results), up-convert and subject to CT.

Figure A2 (left and right) shows the spectrograms of the RO signals in the \(H\) and \(h\) representations, i.e., before and after the CT. Each spectrogram consists of the multiple amplitude spectra calculated in a running window of fixed length (similarly to those shown in Figure 10). The level of zero amplitude corresponds to the position of the window in \(H\) or \(h\) while the amplitude itself is represented using some scale convenient for visualization (the scales are the same on the left and right). Since the used signal is a single-tone both in the \(H\) and \(h\) representations, the single maximum in each spectrum corresponds to the signal and other fluctuations to the noise. All observed signals were used down to the height \(H = -262.144\) km. All CT-transformed signals are shown in the impact height interval \(3 < h < 9\) km. Figures A2a and A2f show observed and CT-transformed signals without resampling. The uniform distribution of noise in the observed signal remains uniform after the CT. The local spectrum of noise is symmetric with respect to the signal frequency both in \(H\) and \(h\) representations (this is achieved by down-conversion of the signal to mean zero frequency used for determination of the frequency by differentiation of the phase). Figures A2b, A2c, A2d, and A2e show results obtained with resampling, using averaging intervals of 2, 4, 64 and 128 m. It is seen that the observational noise is distributed in a band centered at the frequency of the observed signal. This results in that the local spectrum of the CT-transformed signal generally becomes asymmetric (Figures A2g, A2h, A2i, and A2j). When the sampling band is considerably smaller than the change of the frequency of RO signal (this is the case of 50 Hz sampling of RO signals), the shape of the distribution of noise around the CT-transformed signal is consistent with that qualitatively explained in section 4 (Figure 4). This is seen in Figures A2i and A2j (64 and 128 m averaging intervals). This explains the asymmetry of the spectra in Figure 10 and the positive bias induced by noise for the occultations with broad spectrum of the transformed signal.

[45] The structure of Figure A3 is similar to that of Figure A2. Figures A3e and A3f show the spectrograms of the CT-transformed signal obtained with resampling interval 64 m, when the observed signal was used down to \(H_{\text{cut}} = -131.072\) km (Figure A3a) and zero-filled between \(H = -131.072\) and \(262.144\) km (Figure A3b). It is seen that increase of the cut-off height \(H_{\text{cut}}\) results in reduction of the asymmetry of the local spectrum of the CT-transformed signal and reversal of the asymmetry at low \(h\). This is consistent with the explanation in section 4 and results in Figure 3 where the positive bias induced by noise is significantly smaller for \(H_{\text{cut}} = -100\) km than for \(H_{\text{cut}} = -150\) km (and small negative bias induced by the noise can be noticed at the bottom of some profiles for \(H_{\text{cut}} = -100\) km).

[46] For a low sampling rate the asymmetry of the local spectra of the RH-transformed signal does not depend on whether the RO signal was truncated or zero-filled. Figures A3c, A3d, A3g, and A3h show the spectrograms obtained without resampling. Here it is seen that truncation of RO signal (Figure A3c) results in the uniform spectrum of noise (Figure A3g) while zero filling of RO signal (Figure A3d) results in the asymmetry of the local spectrum of noise of the transformed signal (Figure A3h).

[47] For the methods that are based on calculation of the phase integral without the use of FFT, like PM, it is possible to independently define the sampling rates of the observed and transformed signals. The effect similar to that in Figure A3h is obtained if the transform (2) is oversampled. For example, in the special case \(\alpha = \text{const}\) and \(\phi_r = \text{const}\), the constant impact parameter step \(\Delta \alpha < 2\pi/|\theta_2 - \theta_1|\) will result in the oversampling and nonuniformity of the distribution of noise in the RH-transformed signal. In other cases (arbitrary orbits,
nonuniform sampling in a) the distribution of the local spectrum of noise may need to be investigated individually.

Appendix B: Effect of Horizontally Inhomogeneous Surface Ducts on RO Signals

[48] While elevated ducts (such as at the top of boundary layer) result in deep propagating RO signals, it is also interesting to mention a mechanism that potentially may be responsible for the appearance of deep RO signals induced by surface ducts. The surface duct is characterized by $r_n(r_n) > r_n(r)_{\text{min}}$ where $n(r)$ is refractive index. The horizontally homogeneous surface ducts are “invisible” in RO because there are no rays with tangent points below the duct and the diffractional effects are small. However, surface ducts of limited horizontal extension, located at the side of LEO from the limb can trap and release radio waves, resulting in deeper propagation than without the duct. Here we demonstrate such an effect by simulating RO signal using MPS propagation similarly to that used in Appendix A (Figure A1).

We use the following two-dimensional distribution of refractivity $N(z, \theta) = N_0(z)[1 + D(z)W(\theta)]$ where: $N_0(z) = 300 \exp(-z/7.5 \text{ km})$ is the background exponential refractivity profile; $D(z) = 0.03(\pi/2 - \arctan((z-0.1 \text{ km})/0.03 \text{ km}))$ is the vertical modulation modeling the surface duct at the height of 0.1 km; $W(\theta) = [1 \pm \sin((\pi(\theta - \theta)_{1,2}/2\Delta \theta))]/2$ when $\theta_{1,2} - \Delta \theta < \theta < \theta_{1,2} + \Delta \theta$; $W(\theta) = 1$ when $\theta_{1,2} - \Delta \theta < \theta < \theta_{1,2} + \Delta \theta$; $W(\theta) = 0$ otherwise, is the horizontal windowing function providing transition of the width $\Delta \theta$ between the background profile and the duct located between $\theta_1$ and $\theta_2$.

Figure B1 shows the background profile $N_0(z)$ and the profile with the surface duct $N_d(z) = [1 + D(z)]$.

[49] We place the duct between $r_1 = 100 \text{ km}/r_e$ and $r_2 = 400 \text{ km}/r_e$ (here central angle $\theta$ is counted from $z$ direction in Figure A1), i.e., approximately between the distances 100 and 400 km from limb toward the receiver. The amplitudes of the simulated RO signals are shown in Figure B2 (the amplitude in a vacuum is 1; different signals are shifted in

Figure A2. (a–e) Spectrograms of the simulated RO signal with additive noise sampled at different rates. (f–j) Spectrograms of the noisy RO signal transformed to impact parameter representation. The simulated RO signal is used down to $H = -262.144 \text{ km}$. For details see text.
amplitude by 0.1 for better visualization). The signal A corresponds to the background profile $N_0(z)$. The signals B–F correspond to the duct with different horizontal transitions to the background: $r_c \Delta \theta = 1, 10, 50, 100, 150$ km. The effect of surface duct of limited horizontal extension is well seen in RO signal even when transition between the duct and

Figure A3. Similar to Figure A2 but the RO signal is used down to $H = -131.072$ km. For details see text.

Figure B1. Background refractivity profile (solid line) and the profile modeling surface duct (dashed line).

Figure B2. Amplitudes of the simulated RO signals (shifted in vertical by 0.1 for better visualization) for horizontally inhomogeneous surface duct with different parameters (for details see text).
the background extends over the horizontal distance comparable to the horizontal extension of the duct itself (in this case 300 km). The signal \( G \) corresponds to the duct of infinite horizontal extension. Though this signal differs from the background signal \( A \), this difference is not as significant as for the signals \( B-F \). If the surface ducts extend horizontally over distances of about several hundred kilometers, they may explain some of the deep signals observed in COSMIC data. Statistical analysis of these phenomena with existing RO data is difficult and needs further study which shall include modification of the RO receiver tracking firmware to acquire RO signals at lower \( H \) (with a sufficient SNR).

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D. Hunt, C. Rocken, W. Schreiner, and S. Sokolovskiy, University Corporation for Atmospheric Research, Boulder, CO 80301, USA. (sergey@ucar.edu)