Test of a non-local excess phase delay operator for GPS radio occultation data assimilation

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Abstract. A physically-sound, non-local excess phase delay observation operator is developed for simulating excess phase delay measurements from GPS radio occultation (RO) missions. By approximating an observed ray by a straight line, the refractivity gradient information along an observed ray path is included in the simulated excess phase delay. This observation operator is used to simulate observations from the German CHAllenging Minisatellite Payload (CHAMP) RO mission based on large-scale analysis. The need to use such an observation operator for GPS RO data assimilation in spherically asymmetric regions is shown by results from a set of forward simulation and data assimilation experiments. A modification that renders the non-local excess phase delay observation operator more suitable for parallel implementation of GPS RO data assimilation is proposed.

Keywords: GPS radio occultation technique, excess phase delay, refractivity, atmospheric data assimilation, non-local GPS observation operator.

1 INTRODUCTION

Unlike conventional and radiance-based satellite observations, the radio occultation (RO) remote sensing technique provides a different way to see through the atmosphere. The technique (so called air-borne or Global Positioning System (GPS) RO technique) is able to track the radio signals transmitted from the existing constellations of higher orbiting GPS satellites to a GPS receiver onboard a low Earth orbit (LEO) satellite. When the Earth occults an LEO satellite and a GPS satellite, the measured signals are retarded due to refraction and bending, which is determined by the thermodynamic state of the atmosphere of the Earth. The relative motion of the occulted satellites enables the measurements to be taken at different heights. The GPS RO raw measurements are self-calibrating, stable in all weather and have high vertical resolution in the troposphere and stratosphere. More details on GPS RO technology can be found in [1-3].

The unique features of the GPS RO observations make it appealing for their being used together with other satellite data to improve global analyses and forecasts. Data assimilating is a necessary and important step to make it happen. In this study, the NCEP 3-dimensional variational data assimilation system [4,5] is used for the assimilation of GPS RO observations. To include GPS RO data in the NCEP 3D-Var system, an observation operator that calculates the GPS observed quantity for a given model state must be developed to obtain the simulated value. The model state is adjusted to obtain an analysis of the atmospheric state that results in a minimum difference between simulated and observed values of GPS ROs.

Then, which GPS RO variable to simulate? The basic observables detected by the GPS RO technique are phase measurements at two radio-wave frequencies ($f_1=1575.42$ MHz and

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In a vacuum, the phase measurement would be retarded (delayed) due to the relative satellite motions only. With the presence of the atmosphere, an extra phase delay is introduced to the measurement due to the refraction and bending. The difference between the measured phase delay and the “vacuum” phase delay is the excess phase delay. The GPS RO bending angle can then be derived from excess phase delay through a known geometric relationship of the velocity vectors of the GPS and the LEO satellites that holds true under the spherical symmetry assumption. Using an Abel inversion, GPS RO refractivity can finally be calculated from bending angle under the same spherical symmetry assumption. Atmospheric refractivity is a function of temperature, moisture and pressure. By using the equation of state and the hydrostatic equation, temperature and pressure can be derived from GPS RO refractivity when the water vapor content is small. In areas where the moisture contribution to refractivity cannot be neglected, the derivation of temperature and/or water vapor is an underestimated problem and requires an independent knowledge of one of the three state variables (e.g., temperature, moisture and pressure) [6, 7].

There are several ways to simulate GPS RO observations by a numerical model. One way is to solve for a 3D ray-tracing model. Given modeled atmospheric refractivities and observed positions and velocities of occulted GPS and LEO satellites, the Doppler shift of a radio signal can be obtained following a ray trajectory simulated in a three-dimensional (3D) space, and thereafter, the bending angle of the ray-path can be derived. This approach is physically consistent with the GPS bending angle observations. The ray path that intersects both the GPS and LEO satellites is found using a ray-shooting method through an iterative procedure, until the expected accuracy at the end point of the ray path, compared with the actual LEO satellite (receiver) position, is less than 1 mm. One such model is the Radio Occultation Simulation for Atmospheric Profiling (ROSAP) described by Ref. 8. Though very accurate, the 3D ray-shooting model is computationally expensive. Another way to simulated GPS RO bending angle is to solve for a 2D ray-tracing model by assuming that all rays of GPS signal are confined in a 2D occultation plane defined by the positions of the LEO satellite and its occulted GPS satellite and the center of the local curvature at the specific tangent point of the ray-path. The gradient of the refractivity perpendicular to the plane is thus ignored in a 2D ray-tracing model. The 3D ray-tracing model takes about 9 times more CPU seconds than a 2D ray-tracing model for simulating a single occultation with 380 rays on a Cray J90 [9]. The third way to simulate GPS bending angle is to use the forward Abel integral, for which the vertical profile of refractivity serves as the input atmospheric state [6]. The forward Abel integral method does not take into account the integrated effect of the atmospheric refractivity and its gradient along the ray-path and is thus a local scheme. It may be inaccurate over areas of strong refractivity gradients. But it has the lowest computational cost among the three bending angle operators. Both the 2D ray-tracing model and the local bending angle scheme were used for GPS RO data assimilation, see [11-15] and [1, 16-17], respectively. An even simpler approach for GPS RO data assimilation is to assimilate GPS refractivity through the so-called "local" refractivity assimilation. The refractivity computed from given values of pressure, temperature and water vapor at the tangent point of an observed GPS RO are directly compared with the GPS RO refractivity retrieval. The local refractivity assimilation is computationally least expensive. GPS RO data assimilation studies using this method include [19-22].

Since the simulated local refractivity is a point value and the GPS RO refractivity represents an integrated effect of the atmospheric refractivity along the ray-path to the ray trajectory, differences resulting from such an inconsistency between the local refractivity and the GPS RO refractivity could be large over regions where strong gradients of refractivity exist. The analysis obtained from using this approach could be much less accurate than that obtained from the GPS bending angle assimilation over these regions.

A non-local refractivity or excess phase delay scheme is a compromise of the simulation accuracy and computational efficiency [23-26]. In such a scheme, the simulated GPS
refractivity is expressed as a weighted sum of the atmospheric refractivity along the ray path, accounting for the along-track gradient of the refractivity. In this study, a non-local scheme similar to [26] is developed and utilized for simulating GPS RO data from CHAMP using a global model. An excess phase delay is calculated by integrating refractivities along a straight line that coincides with the tangent direction of the ray-path at the tangent point. In a discrete form, the line integration is approximated by a weighted summation. Unlike those in [27], the weighting functions are obtained by simply finding the segment lengths of the tangent line intercepting the model grid boxes. The performance of this scheme is examined for forward simulation and data assimilation. A proposed simplification makes this scheme suitable for effective implementation in parallel data assimilation systems.

The paper is arranged as follows: Section 2 introduces the basic concepts of GPS excess phase delay and points out several features of the observation operator. Section 3 describes numerical experiments. Numerical results from the forward simulation and data assimilation of GPS excess phase delay are presented in Sections 4 and 5, respectively. Finally, Section 6 provides a summary and conclusions.

2 MATHEMATICAL FORMULATION OF EXCESS PHASE DELAY

Along each ray path that goes from an occulted GPS satellite to an LEO satellite, the following relationship holds true:

\[ \int N_{\text{obs}} \, dr = \int N_{\text{loc}} \, dr \]  

(1)

where \( \int \) represents an integration along the ray-path and \( N_{\text{obs}} \) and \( N_{\text{loc}} \) are the GPS RO refractivity observation and local refractivity of the atmosphere, respectively. The local refractivity \( N_{\text{loc}} \) in the neutral atmosphere can be calculated from model variables via the following formula [27]

\[ N_{\text{loc}} = 77.6 \frac{p}{T} + 3.73 \times 10^5 \frac{pq}{T^2 (0.622 + 0.378q)} \]  

(2)

where \( T \) is the air temperature (K), \( q \) is the specific humidity (kg kg\(^{-1}\)) and \( p \) is the air pressure (hPa). It is noticed that the right-hand-side of eq. (1) is proportional to the excess phase delay, which is defined as

\[ L = \int (n_{\text{loc}} - 1) \, dr = 10^{-6} \int N_{\text{loc}} \, dr , \]  

(3)

where \( n_{\text{loc}} = 10^{-6} N_{\text{loc}} \) is the refractivity index.

Eq. (1), in principle, provides a link of the local refractivity of the atmosphere to GPS RO refractivity observations. However, both \( N_{\text{obs}} \) and \( N_{\text{loc}} \) are available at discrete gridpoints. Numerical calculations of (3) involve two steps. First, the integrals along the ray-path in (1) is replaced by integrations along a so-called tangent link \( s \):

\[ \int_s N_{\text{obs}} \, ds = \int_s N_{\text{loc}} \, ds . \]  

(4)

Here, the tangent link \( s \) is a straight line that passes the tangent point of the observed ray, is tangent to the local curvature of the Earth at the tangent point and is coplanar to the occultation plane.

The line integrals in (4) are approximated by the following summations:

\[ \sum_{i=1}^{M} \sum_{j=1}^{N} b_{m,i} N_{ij}^{\text{obs}} = \sum_{n=1}^{N} a_{m,n} N_n^{\text{loc}} . \]  

(5)
where $N$ is the total number of the model grid boxes, $M+1$ is the total number of vertical levels of the numerical model (on which $N^{loc}$ is defined), the $l$th component of the vertical profile of a GPS RO is assumed to locate at the middle of the $l$th model layer, $a_{m,n}$ is the length of the $m$th tangent link intercepted by the $n$th model grid box (Fig. 1a) and $b_{m,l}$ is the length of the $m$th link inside the $l$th layer bounded by levels $l$ and $l+1$. A schematic illustration of both $a_{m,n}$ and $b_{m,l}$ is provided in Fig. 1.

In matrix form, (5) can be written as

$$BN^{obs} = AN^{loc},$$

where $B$ is an $M \times M$ matrix and $A$ is an $M \times N$ matrix. Since the $m$th link does not go through any layer below the layer containing the tangent point, the matrix $B$ in Eq. (6) is an upper triangular square matrix. The element on the $m$th row and $n$th column of $A$, $a_{m,n}$, is equal to zero if the $n$th tangent link has no interception with the $n$th model grid box.

Equation (6) is a discrete analogy of (1). Based on (6), we define its left-hand-side as the observed excess phase delay and will serve as the GPS observable:

$$L^{obs} = BN^{obs},$$

and the right-hand-side of (6) the modeled excess phase delay:

$$L = AN^{loc}.\tag{8}$$

(8) is the excess phase delay observation operator. The local refractivity $N^{loc}$ is calculated by Eq. (2) from model state variables ($p$, $T$, $q$). A simplified excess phase delay observation operator that uses the matrix $B$ only is provided in Appendix A. Since $B$ only involves summation in the vertical direction and is thus more suitable for an implementation of in parallel data assimilation systems.

Based on Eq. (6), the GPS refractivity ($N^{obs}$) and the local refractivity ($N^{loc}$) are related through the following equation

$$N^{obs} = B^{-1}AN^{loc},\tag{9}$$

where $K = B^{-1}A$ is called the kernel function which depends on how the tangent link intercept with model grid boxes. Therefore, using the proposed excess phase delay assimilation strategy, the GPS RO refractivity is in fact a weighted sum of the local refractivity. The excess phase delay assimilation is thus equivalent to a non-local refractivity assimilation. The reason for choosing the excess phase delay as the observable over the GPS refractivity is to avoid the calculation of the inverse matrix $B^{-1}$.

### 3 MODEL FIELDS, OBSERVATIONS AND EXPERIMENTAL DESIGN

Large-scale model fields used in this study are the NCEP background fields, consist of a set of 6-hour model forecasts valid at 00UTC, 06UTC, 12UTC and 18UTC during a one-week period from 24-31 May 2002. The spectral resolution of the model is T170L42, and the gridpoint resolution is $1^\circ \times 1^\circ$ with 26 vertical levels distributed from 1000 hPa to 10 hPa.
Fig. 1. Schematic illustrations of (a) a relationship between $a_{mn}$ and the $m$th tangent link intercepting the $n$th model grid boxes, and (b) a relationship between $b_{ml}$ and the $m$th tangent link intercepting the $l$th vertical layer of the model. Points A and B indicate two choices of the grid point at which the local refractivity value is used for evaluating the excess phase delay (8).

The GPS RO refractivity observations used in this study are provided by the Constellation Observing System for Meteorology Ionosphere and Climate (COSMIC) Data Analysis and Archival Center (CDAAC). There are a total of 1158 ROs within this one-week period, nearly uniformly distributed over the global. CHAMP data have been smoothed vertically to a resolution similar to that of the NCEP SSI system. Specifically, the vertical resolution is less than 0.2 km below 4 km, 0.2 km between 4 and 6 km, 0.4 km between 6 and 8 km, 0.6 km between 8 and 14 km, 1.0 km between 14 and 26 km, 2.0 km between 26 and 30 km and 5.0 km above 30 km. The total number of vertical levels of the smoothed GPS observations is comparable to that of the model (L42).

Three data assimilation experiments are carried out: (i) PHA in which excess phase delay is used as GPS RO observables, (ii) REF which is the local refractivity assimilation and (iii) NOGPS which include no GPS data. Conventional and satellite observations are included in all three data assimilation experiments except for those radiosonde observations that are within 300-km distance from GPS RO locations. These radiosonde data are excluded from for the purpose of analysis verification. The conjugate gradient algorithm is used for finding the minimum of the cost function. Each minimization consists of two outer loops and 100 inner loops. A 6-hour assimilation window is used. For PHA, excess phase delay observations are calculated from GPS refractivity observations using (7). The vertical error covariance matrices for refractivity and excess phase delay observations are calculated using the method (R1) described in [14]. Due to an improved quality control procedure implemented in 2005 at COSMIC/CDAAC, the vertical error correlation of the CHAMP refractivity observations becomes quite small and is thus ignored in the experiments PHA and REF.

4 NUMERICAL RESULTS

4.1 Kernel Function Structures

Matrices $A$ and $B$ depend on model grids as well as tangent links, and so does the kernel function $K$. Some preliminary ideas of these matrices can be obtained by assuming a simple model grid configuration and some hypothetical tangent links. Let’s assume (i) a numerical model has 1° horizontal resolution globally and 1-km vertical resolution from 0 to 30 km; (ii) a vertical profile of GPS RO refractivities is observed at 0° longitude, 30°N latitude and half levels of the model; (iii) the tangent orientation at each of the tangent points is parallel to the equator (i.e., the azimuth angle is either 90° or 180°); and (iv) the tangent links are truncated.
at the uppermost model level at 30 km. Figure A1a shows the values of all elements in B. The values of the diagonal elements of B are largest, which implies that the interception lengths of all the tangent links are largest in the layers containing the corresponding tangent points. Showing all components in the matrix A is difficult due to its large dimension including all grid boxes (≈ 359x179x 30 = 1,927,839). As an example, only non-zero elements in the second row of the matrix A are shown in Fig. A1b. As mentioned above, these values are the interception lengths of the tangent link in successive model grid boxes for a tangent link with its tangent point at 1-km altitude (within the second model layer). The x-axis indicates the distance between the tangent point and each of the intercepted grid boxes. The total length of the link is about 1200 km. Generally speaking, the components in A are largest for the grid boxes at the tangent points and their values decrease with the increasing distances of the grid boxes to the tangent point.

Fig. 2. (a) B matrix for a numerical model with 1° horizontal and 1-km vertical resolutions simulating a RO located at (0°W, 30°N) with 30 rays (the model top is at 30 km). The tangent points located at half model levels and are parallel to the equator. The x-axis (index l) is the vertical layer that a tangent link goes through. The y-axis (index m) represents the vertical layer within which the tangent point of each ray is located. (b) The second row of the A matrix along the tangent link. The x-axis indicates the distance between the tangent point and the interception grid boxes of the tangent link.

Fig. 3. The interception points of the eight tangent links with the λ-planes (solid squares) and with the z-planes (solid triangles) when the tangent points (solid circles) is located at 0° longitude, 1.5-km height and different latitudes (10°N, 20°N, …, 80°N). Model grids are same as in Fig. 2. The x-axis indicates the distance of the points on the tangent link to the tangent point.
Because the distance of 1° longitudinal is different at different latitudes, the matrix $A$ varies with latitude. Taking the same idealized model configuration in Fig. 2, Fig. 3 shows the interception points of the eight tangent links with their tangent points located at 0° longitude, 1.5 km height and eight different latitudes (10°N, 20°N, …, 80°N). Since the tangent links are assumed parallel to the latitudinal planes, each of the interception points is constructed by the tangent links either intercepting a longitudinal plane or a vertical plane. The distance between two intercepting points (i.e., the element in $A$) for one tangent link is not evenly distributed, which explains the fluctuations of the curve seen in Fig. A1b. Generally speaking, the distance is larger in the low latitudes and smaller in the high latitudes. Consequently, the distributions of the kernel functions along the eight tangent links vary with latitude as shown in Fig. 4. The kernel functions are sharper in the tropics than in higher latitudes. The negative values of the kernel functions result from the inverse of the matrix $B$.

Having obtained coefficients for the forward observation operator of excess phase delay, the local refractivity within each grid box intercepted by the tangent link is required. There are two ways for obtaining the refractivity values: one is to use the refractivity value from a corner point (e.g., Point A in Fig. 1) and another is to obtain the value at the middle point of the intercepted link (Point B in Fig. 1). The later scheme is used in the PHA experiment.

![Fig. 4. Kernel functions (see Eq. (9)) along the tangent links described in Fig. 3.](image-url)
4.2 Forward Simulation

Results from PHA and REF forward simulations are examined in terms of the relative observational increments:

\[ \Delta_{rel} L = \frac{L - L^{\text{obs}}}{L^{\text{obs}}} \times 100\% \], \[ \Delta_{rel} N^{\text{loc}} = \frac{N^{\text{loc}} - N^{\text{obs}}}{N^{\text{obs}}} \times 100\% \]. (10)

Figure 5 shows the spaghetti plots of the above two quantities for all 1158 CHAMP ROs. The mean profiles for both quantities are also shown. Local refractivity has a small positive bias (< 0.4%) below 4 km altitude. The values are more sporadic. Individual values of \( \Delta_{rel} N^{\text{loc}} \) can reach 20%, although standard deviations (STDs) are less than 2%. Excess phase delay has a small positive bias below 4 km (< 0.2%) and negative bias above 4 km altitude (< 0.4%). All values of \( \Delta_{rel} L \) are less than ±5%. The STD of \( \Delta_{rel} L \) is smaller than 1% at all vertical levels.

![Spaghetti plots and vertical profiles](image)

Fig. 5. (a) Spaghetti plots of \( \Delta_{rel} L \) (gray) and \( \Delta_{rel} N^{\text{loc}} \) (black). (b) Vertical profiles of the mean and STD values of \( \Delta_{rel} L \). (c) Same as (b) except for \( \Delta_{rel} N^{\text{loc}} \). A total of 1158 GPS ROs is included.

The difference of the GPS RO observation operator between excess phase delay and local refractivity is that the former includes the effect of along-track refractivity as the observations do and the later doesn't. To further illustrate the relationship between the refractivity gradients and the differences between the relative observation increments of the excess phase delay and refractivity, a pair of ROs is selected with the first one in a region of relatively smaller refractivity gradients (RO1) and the second one in a region with stronger refractivity gradients (RO2), respectively. RO1 was observed at 0723UTC 30 May 2002 at (25.863°E, 23.050°N), and RO2 was observed at 0643UTC 30 May 2002 at (81.878°E, 51.850°S). Their geographic locations and tangent link orientations (straight line "A-B" for RO1 and "C-D" for RO2) are indicated in Fig. 6. Since the tangent point locations for one occultation shift vertically due to the motions of the GPS and LEO satellites, the normal directions of the occultation planes and therefore, the tangent link orientations for a GPS RO vary slightly in the vertical direction. Fig. 6 shows their vertical averages. The cross-sections of the
refractivity and refractivity gradient fields are plotted in the averaged occultation planes as shown in Fig. 7. RO1 went through an area with a horizontal gradient of ~ 0.1 N unit km\(^{-1}\) and a vertical gradient of ~ 40 N unit km\(^{-1}\) near the 700 hPa surface. RO2 went through an area with smaller refractivity gradients with a horizontal gradient of ~ 0.02 N unit km\(^{-1}\) and a vertical gradient of ~ 30 N unit km\(^{-1}\) near the 700 hPa surface.

Fig. 6. Two CHAMP ROs and their averaged tangent link orientations observed at (258.63°E, 23.05°N) (RO1) on 0723UTC 30 May 2002 and (81.27°E, 51.85°S) (RO2) on 0643UTC 30 May 2002.

Fig. 7. Cross-sections of the distributions of refractivity (contour lines, unit: N unit) and its horizontal gradient (shaded, unit: N unit km\(^{-1}\), left panels) and vertical gradient (shaded, unit: N unit km\(^{-1}\), right panels) in the occultation planes for RO1 (upper panels) and RO2 (lower panels). The tangent points in the vertical direction are marked by the solid dots.

Relative observational increments (\(\Delta _{\text{obs}} L\) and \(\Delta _{\text{obs}} N^{\text{occ}}\)) are plotted for these two ROs in Fig. 8. Comparing two ROs, the observational increments of both \(N^{\text{occ}}\) and \(L\) for RO1 (located in the area with stronger refractivity gradient) have generally larger magnitudes than those of RO2. For RO1, the largest values for \(\Delta _{\text{obs}} N^{\text{occ}}\) and \(\Delta _{\text{obs}} L\) are 5% and 2% which are located at around 5 km, respectively. For RO2, the magnitude of \(\Delta _{\text{obs}} N^{\text{occ}}\) is less than 1.4% and that of \(\Delta _{\text{obs}} L\) is less than 0.8%. In other words, the simulated values of \(N^{\text{occ}}\) deviate more from GPS RO observations than those of \(L\), and both values deviate more from GPS RO observations in an area with larger refractivity gradients. The curves for \(L\) are smoother than for \(N^{\text{occ}}\) as expected from the summation operation in excess phase delay observation operator.

4.3 Data Assimilation

To ensure that the minimization in each of the data assimilation experiments works, we performed the following three verifications: (1) The correctness of the tangent linear and adjoint operators of the excess phase delay is checked; (2) the correctness of the gradient of the cost function is verified, and (3) a sufficient decrease of the cost function and the norm of the gradient of that cost function is achieved. For example, assimilation of GPS observations within any 6-hour window centered at 00UTC, 06UTC, 12UTC and 18UTC during 24-31 May 2002 converged in 200 iterations, with a decrease in the norm of the gradient of the cost function by about 5 orders of magnitude for all experiments.

One way to assess the accuracy of the analysis obtained using different GPS RO observation operators is to examine the differences between the so-called non-local refractivity ($N_{\text{GPS}}$) and the GPS RO observations ($N_{\text{obs}}$). Here, the non-local refractivity $N_{\text{GPS}}$ is the model-simulated GPS RO refractivity that is physically consistent with the GPS RO observations ($N_{\text{obs}}$). It is obtained by a two-step procedure [14]: (i) Simulating GPS RO bending angle ($\alpha_{\text{GPS}}$) from $N_{\text{loc}}$ calculated from the analysis fields via a ray-tracing procedure [12], and (ii) Computing $N_{\text{GPS}}$ from analysis-simulated $\alpha_{\text{GPS}}$ using the Abel inversion. The values of $N_{\text{loc}}$ and $N_{\text{GPS}}$ are different since the former depends only on the atmospheric thermodynamic state at the tangent point while the latter one is a complicated nonlinear function of the atmospheric thermodynamic state along the observed ray-path. It is expected that the more accurate the GPS RO observation operator is, the smaller the difference $\Delta_{\text{obs}} N_{\text{GPS}}$ ($= N_{\text{GPS}} - N_{\text{obs}}$) is. Figure 9 plots the mean and STD of $\Delta_{\text{obs}} N_{\text{GPS}}$ for the three data assimilation experiments (NOGPS, REF and PHA) for over 1158 GPS ROs. It is seen that using $\Delta_{\text{obs}} N_{\text{GPS}}$ as a measure, the analysis from PHA is most accurate when compared to GPS observations and the REF analysis is more accurate than that of NOGPS. It is noticed that NOGPS analysis deviates from GPS RO data slightly more than the background field.

The mean and STD of the difference $\Delta_{\text{obs}} N_{\text{loc}}$ ($= N_{\text{loc}} - N_{\text{obs}}$) calculated from three analyses are shown in Fig. 10, along with $\Delta_{\text{obs}} N_{\text{GPS}}$. Since $N_{\text{GPS}}$ is consistent with $N_{\text{obs}}$, if observations assimilated in NOGPS or PHA experiments include accurate information of GPS
Fig. 9. Vertical profiles of the (a) mean and (b) STD values of $\Delta_{\text{obs}} N^{\text{GPS}}$ calculated from the background field (thick dashed) and the analyses from assimilation experiments NOGPS (thick solid), PHA (thin solid) and REF (thin dashed).

Fig. 10. Vertical profiles of the (a-c) mean and (d-f) STD values of $\Delta_{\text{obs}} N^{\text{GPS}}$ (solid line) and $\Delta_{\text{obs}} N^{\text{loc}}$ (dashed line) calculated from analyses obtained from experiments NOGPS (left panels), PHA (middle panels) and REF (right panels).

RO refractivity, the experiment is expected to produce an analysis with values of $\Delta_{\text{obs}} N^{\text{GPS}}$ closer to and/or smaller than those of $\Delta_{\text{obs}} N^{\text{loc}}$. For NOGPS (Fig. 10a,d), both the mean and STD of $\Delta_{\text{obs}} N^{\text{GPS}}$ are larger than those of $\Delta_{\text{obs}} N^{\text{loc}}$. This suggests that GPS observations are not redundant to conventional and satellite observations already included in NOGPS and can provide useful information to large-scale analyses. Assimilations of the GPS excess phase delay in PHA (Fig. 10b,e) reduced the means and STDs of both quantities: $\Delta_{\text{obs}} N^{\text{GPS}}$ and $\Delta_{\text{obs}}$.
The REF experiment resulted in smaller values of $\Delta_{\text{loc}} N^\text{loc}$ (Fig. 10c,f) than those of PHA, implying that an over-fitting to GPS observations might have occurred in REF, in which the refractive contribution of the atmosphere along the ray-path to the total bending is accounted for entirely by the refractive contribution of the atmospheric state at the tangent point.

Differences between the experiments with GPS RO data (PHA or REF) and without GPS RO data (NOGPS) for the analysis of temperature and specific humidity at 1158 GPS ROs locations are shown in Figs. 11 and 12. The mean and STD of the analysis differences are grouped into three latitudinal zones: the tropics (30°S-30°N), the mid-latitudes (30°S-60°S and 30°N-60°N) and the high-latitudes (60°S-90°S and 60°N-90°N). In the low troposphere, the mean and STD of the temperature difference are the smallest in the tropics and the largest in high-latitudes. On average, assimilation of GPS RO data (e.g., both PHA and REF) tend to produce a warmer model atmosphere at most levels, with the PHA temperature adjustments being significantly larger than those of REF. The temperature analysis differences peak at the middle and low troposphere in the middle and high latitudes. In the tropics, however, large analysis differences of temperature are found in the upper troposphere. The analysis differences with and without GPS RO data for specific humidity fields are shown in Fig. 12. It is seen that the largest (smallest) differences occur in the tropics (the high-latitudes). Both experiments tend to reduce the specific humidity value below 600 hPa. Compared to NOGPS, modifications made by PHA to the analysis of moisture are smaller than those by REF, especially in the tropics. The moisture differences between PHA and NOGPS are positive above 600 hPa in the tropics. Therefore, inclusion of GPS RO data tends to produce a drier and warmer tropospheric atmosphere than the NOGPS analysis.

Fig. 11. Vertical profiles of the (a-c) mean and (d-f) STD values of temperature difference between PHA and NOGPS (PHA-NOGPS, solid line) and between REF and NOGPS (REF-NOGPS, dashed line) in the tropics (left panels), the mid-latitudes (middle panels) and the high-latitudes (right panels).
Theoretically speaking, the GPS observation operator in PHA imposes direct modification to the model state not only at the tangent point location but also along the tangent link of each observed ray-path. This effect of having along-track information included in excess phase delay operator can be seen in the horizontal distributions of analysis increments of specific humidity from PHA. As an example, Fig. 13a presents the analysis increment of specific humidity at 06 UTC 30 May 2002 at 850 hPa in a sub-domain (100°E-140°E, 0-30°N). There are two GPS ROs in this domain at this time. For comparison purpose, we also plotted the analysis increments of specific humidity from REF (Fig. 13b). Analysis increments from REF near the two GPS RO locations are much smaller in both magnitude and horizontal extent than those from PHA, although the same background error covariance matrix is used in all three experiments. This confirms that the excess phase delay assimilation did what it is supposed to do. The maximum analysis increment is only about 0.1 g kg⁻¹ for REF but is as large as 0.6 g kg⁻¹ for PHA. The influence radius of GPS ROs data is much larger in PHA than in REF due to a combined effect of along-track refractivity and background error covariances. The analysis increments from PHA decreases with increasing distance from the GPS RO observation location, reaching 0.1 g kg⁻¹ at a distance of about 500 km from the observation locations.

6 SUMMARY AND CONCLUSIONS

This paper develops and tests a new scheme for GPS RO data assimilation. The excess phase delay is chosen as the assimilation variable, enabling the along-track refractivity and refractivity gradient information to be included in an observation operator that is simple and computationally inexpensive.

The observation operator for the excess phase delay is incorporated into the NCEP SSI system (T170L42). The forward simulation of GPS RO observables and data assimilation experiments (PHA) are conducted for the time period from 24 to 31 May 2002. A total of 1158 CHAMP ROs were observed in this time period. The local refractivity simulations and
data assimilations (REF) and a control data assimilation experiment excluding GPS data (NOGPS) are also carried out. The results from PHA, REF and NOGPS are then compared. Using the same background fields of refractivity, the excess phase delays simulated by PHA have less bias from GPS RO observations than the local refractivity. Assimilation of GPS excess phase delay by PHA results in the closest fit to $\Delta N_{\text{GPS}}$ compared to assimilations of GPS refractivity by REF and assimilations of conventional observations by NOGPS, indicating that the analysis from PHA is more accurate than those from the other two experiments. It is also found that PHA tends to produce a warmer and wetter atmosphere, with a more detailed structure and a larger radius of influence than REF.

The proposed excess phase delay scheme for GPS RO data assimilation is computationally much more efficient than the bending angle assimilation through a ray-tracing procedure and is more accurate than the local refractivity assimilation. Further tests are needed and more significant impacts are expected if higher resolution models are used. It is anticipated that a mixed use of excess phase delay and a symmetric excess phase delay as GPS data assimilation variables is needed. Over regions where the spherical symmetry assumption is not greatly violated, the observation operator for the excess phase delay is simplified into a local scheme which takes a single vertical profile of model refractivity as a required input variable. The impact of the vertical gradient of refractivity on GPS RO data is contained in the simulated excess phase delays using the simplified forward operator, which is a scheme more favorable for implementation in parallel 3D-Var/4D-Var systems.

**APPENDIX A: A LOCAL SCHEME FOR EXCESS PHASE DELAY**

Assuming that the simulated local refractivity at the tangent point is spherically symmetric, then the left-hand-side of (6) can be used as an observation operator:

$$L_{\text{sym}} = B N_{\text{loc}} = \begin{bmatrix} \vdots \\ \sum_{j=1}^{M} b_{w_j} N_{j}^{\text{loc}} \\ \vdots \end{bmatrix}. \quad (A1)$$

The excess phase delay $L_{\text{sym}}$ differs from $L$ in the sense that the former one includes only the refractivity gradients in the vertical direction, while the latter one integrates local refractivities along the tangent link and thus includes the refractivity gradients in both the horizontal and vertical directions at points distributed along the tangent link. Elements in $L_{\text{sym}}$
is calculated by a weighted sum of the local refractivity at all the vertical levels above the
tangent point and below the model top.

The observation operator for \( L^{\text{sym}} \) can be used instead of \( L \) when the horizontal gradient
of refractivity is either small or cannot be resolved by the model resolution. Since \( L^{\text{sym}} \) is
calculated at the tangent point without involving any averaging in the horizontal direction, it
is attractive for implementing it into a parallel operational data assimilation system that
divides the model space into sub-domains horizontally for parallel processing. A mixed use of
both \( L \) and \( L^{\text{sym}} \) may be a best choice according to the spherical symmetry of the atmospheric
state.

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References

Meehan, L. E. Young, and T. P. Yunck, "The Application of spaceborne GPS
toatmospheric limb sounding and global change monitoring," JPL Publication 94-
18, National Aeronautics and Space Administration and Jet Propulsion Laboratory,
"Observing Earth's atmosphere with radio occultation measurements using the
[doi:10.1029/97JD01569].
GPS radio occultation data for numerical weather prediction," Terr. Atmos. Ocean.
[8] P. Hoeg, A. Hauchecorne, G. Kirchengast, S. Syndergaard, B. Belloul, R. Leitinger,
and W. Rothleitner, "Derivation of the atmospheric properties using radio
occultation technique," Scientific Report 95-4, Danish Meteorological Institute,
of radio-occultation bending angles caused by a 2D approximation of ray tracing and
the assumption of spherical symmetry of the atmosphere," J. Atmos. Oceanic.
C. Chang, J. G. Sela, and R. A. Anthes, "A raytracing operator and its adjoint for the
of GPS limb sounding data into NCEP's global data assimilation system," Office note


