Combining Outputs from the NARCCAP Regional Climate Models Using a Bayesian Hierarchical Model

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Summary. We investigate the 20-year-average Boreal winter temperatures generated by an ensemble of six regional climate models (RCMs) in Phase I of the North American Regional Climate Change Assessment Program (NARCCAP). We use the long-run average (20-year integration) to smooth out variability and capture the climate properties from the RCM outputs. We find that although the RCMs capture the large-scale climate variation from coast to coast and from south to north similarly, their outputs can differ substantially in some regions. We propose a Bayesian hierarchical model to synthesize information from the ensemble of RCMs, and we construct a consensus climate signal with each RCM contributing to the consensus according to its own variability parameter. The Bayesian methodology enables one to make posterior inference on all the unknowns, including the large-scale fixed effects and the small-scale random effects in the consensus climate signal and in each RCM. The joint distributions of the consensus climate and the outputs from the RCMs are also investigated through posterior means, posterior variances, and posterior spatial quantiles. We use a Spatial Random Effects (SRE) model in the Bayesian hierarchical model and, consequently, we are able to deal with the large datasets of fine-resolution outputs from all the RCMs. Additionally, our model allows a flexible spatial covariance structure without assuming stationarity or isotropy.

Keywords: Bayesian analysis, downscaling, North American Regional Climate Change Assessment Program (NARCCAP), regional climate model (RCM), Spatial Random Effects (SRE) model

1. Introduction

Climate models have become an important tool for understanding climate change and its potential impact. They are essentially a series of discretized differential equations that attempt to represent physical relationships such as the flows of energy and water within and between the atmosphere, oceans, land, sea ice, etc. With anthropogenic forcings incorporated, climate models can be run under different scenarios (e.g., various \(CO_2\) levels) and thus provide information with which to assess human impact on climate change.

Since the late 1960s, atmosphere-ocean general circulation models (GCMs) have been developed to simulate the climate over the entire globe. They link an atmospheric model

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and an oceanic model and generate outputs typically on a coarse scale of 200 to 500 km. Although GCMs simulate the large-scale processes well, they are limited in describing local climate effects because of their global perspective. GCMs usually oversimplify the climate processes at regional and smaller scales and may miss the impacts of local features, such as complex terrain (lakes, coastlines, mountains, etc.). Consequently, GCMs provide only limited information at local scales where natural-resource management or environmental-policy decisions are made. To study the climate in a given region of interest, regional climate models (RCMs) have been developed with much finer spatial resolution (20 - 50 km). Computational constraints limit an RCM to a given region, and boundary conditions are required to run the model. Usually, the outputs from a GCM are used to provide such boundary conditions for RCMs; this is also referred to as “dynamic downscaling” of the GCM outputs (e.g., Fennessy and Shukla, 2000; Xue et al., 2007). The Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report contains a great deal of information, background, and references concerning RCMs and climate models in general; see Solomon et al. (2007).

Generally speaking, climate models are complicated, and their outputs are subject to many sources of uncertainty. For example, uncertainty may be due to complexity arising from interaction between atmospheric circulation and orography, from discretization, and from parameterizations of the physical-forcing processes; and for RCMs there is also uncertainty in the GCMs that provide the boundary conditions. Therefore, when GCM/RCM outputs are used to project future climate change, it is crucial to examine the uncertainties in them (e.g., Schneider, 2001; Wigley and Raper, 2001; Tebaldi et al., 2005). Meanwhile, an individual climate model usually has its own strengths and weaknesses, and it is important to synthesize the results from a number of such models (McAvaney et al., 2001). Bayesian analysis provides a natural tool to quantify the uncertainties in climate models and to produce probabilistic (consensus) climate projections with “votes” from each individual climate model. In this article, we propose a hierarchical Bayesian model that combines outputs from multi-model experiments from the North American Regional Climate Change Assessment Program (NARCCAP).

NARCCAP is an international program that provides fine-resolution (approximately 50 km) climate-output data for the United States, Canada, and northern Mexico (Mearns et al., 2009; Mearns et al., 2010; http://www.narccap.ucar.edu/). Its ultimate goal is to investigate uncertainties in regional-scale climate projections, generated by running RCMs constrained by GCM outputs, and to assess regional climate change. It consists of two phases: During Phase I, various RCMs were run with boundary conditions supplied by the NCEP Reanalysis II data produced by the National Centers for Environmental Prediction (NCEP) (e.g., Kanamitsu et al., 2002); the variability due to the RCMs under such idealized conditions is explored in this phase. In Phase II of the program, RCMs will be coupled with different GCMs, and the additional variability due to the boundary conditions from GCMs will be investigated.

In this article, we consider the outputs of mean surface temperature for the Boreal winter (December, January, and February) from all six RCMs in Phase I of NARCCAP, simultaneously, and we make optimal predictions for a “hidden” climate process. Although all RCMs capture the large-scale variation similarly, their outputs differ in small-scale details in regions of the domain. We synthesize the outputs from the different RCMs using a hierarchical Bayesian model, which allows us to capture the heterogeneous within-RCM variability and to quantify uncertainties of all unknowns conditional on the data.

Application of Bayesian hierarchical modeling to the climate sciences is not new (e.g.,
Berliner et al., 2000; Tebaldi et al., 2005), and there has been a lot of work done to combine outputs/ensembles from different climate models using statistical methodology (e.g., Tebaldi et al., 2005; Furrer et al., 2007; Berliner and Kim, 2008; Smith et al., 2009). In this context, Bayesian analysis based on our hierarchical statistical model can be viewed as a meta-analysis (e.g., DuMouchel, 1990) for combining results from different experiments or combining experts’ opinions. We propose to call the underlying process of interest the “consensus” process, and our goal is to predict/construct it based on outputs (or opinions) from various climate models (or experts). From the posterior distribution of the unknowns, including the consensus climate signal and the climate signal from the individual climate models, we can obtain optimal predictions of the unknowns along with associated uncertainties, and we can investigate the unknowns’ joint behavior.

Although there have been concerted efforts made to combine multi-model outputs within a Bayesian framework, most of them focus on GCMs, where the process of interest is the climate average over some region, and no spatial dependence is accounted for in the model. Specifically, Berliner and Kim (2008) combine data (computed from ensembles of future projections from different GCMs) to predict future northern-hemisphere and southern-hemisphere temperatures. Tebaldi et al. (2005) and Smith et al. (2009) investigate future temperature increase over 22 regions by combining GCM outputs: For each region, they separately predicted the temperature increase (a scalar); they also investigated the 22 regions overall in a multivariate model of the temperature increases, but with no spatial-dependence structure between regions. Tebaldi and Sansó (2009) generalized the scalar models for temperature given in Tebaldi et al. (2005) and Smith et al. (2009), to bivariate models for temperature and precipitation, but they only considered global temperature and precipitation, as well as western North American temperature and precipitation, in separate analyses.

In this article, we study the outputs from RCMs, which give much more local or regional details, compared to GCMs. Thus, in contrast to previous research on combining climate models where regional averages are analyzed, we view the outputs and the consensus signal as spatial fields whose spatial variability is modeled. Due to the fine resolution of RCM outputs, the size of datasets (or model outputs) is very large. This has the potential to make standard (spatial) statistical procedures hard to implement (e.g., Cressie and Johannesson, 2008; Banerjee et al., 2008). To deal with this, we use the Spatial Random Effects (SRE) model proposed by Cressie and Johannesson (2006, 2008) as a component in our Bayesian hierarchical model, which results in dimension reduction and very fast computations. The SRE model also allows a more flexible spatial covariance function, without assumptions of stationarity or isotropy (as made by Furrer et al., 2007).

In Section 2, we give a description of the basic notions of Bayesian modeling and analysis, and we introduce a more complete description of the RCM outputs. We present our Bayesian hierarchical model in Section 3, with the distributional assumptions for the data model, the process model, and the prior described. Section 4 applies the model to the outputs of mean Boreal winter temperature from the six RCMs in NARCCAP. This includes inference on the consensus climate signal, as well as investigation of the joint distribution of all the unknowns. Conclusions and discussion are given in Section 5, and Appendix contains details on the priors.
2. Basic notions and background to RCMs

We give a brief presentation of Bayesian hierarchical statistical modelling in Section 2.1. The RCMs and their outputs are described in Section 2.2.

2.1. Basic notions

As presented in recent literature in climate science (e.g., Reilly et al., 2001; Webster, 2003), compared to a single-value prediction, a probabilistic distribution for unknowns of interest is preferred, since the latter is more flexible for uncertainty quantification and making decisions. Although not the only option, Bayesian hierarchical methodology is a natural solution to developing a probabilistic distribution for the unknown quantity under study. It incorporates information in a coherent way (using Bayes' Theorem) from different sources, such as observations, (multi-)model outputs, experts’ opinions, and physical theories. In the resulting hierarchical statistical models, the data, the process, and the parameters are all treated as random quantities.

To apply these ideas to output from a climate model, we represent the output as observable data \( Z \), which depends on an unknown hidden process \( Y \). A statistical model, \( p(Z|Y, \theta) \), is formed, which is usually referred as the data model. Then, the process \( Y \) is expressed as the conditional probability distribution, \( p(Y|\theta) \), referred to as the process model. The models’ parameters are denoted by \( \theta \). The Bayesian part of the model is to treat \( \theta \) as random and model the probability distribution with a parameter model (or prior), \( p(\theta) \). The Bayesian hierarchical statistical model can be summarized as follows:

\[
\begin{align*}
&\text{data model: } p(\text{data}|\text{process, parameters}) = p(Z|Y, \theta), \\
&\text{process model: } p(\text{process}|\text{parameters}) = p(Y|\theta), \\
&\text{parameter model: } p(\text{parameters}) = p(\theta),
\end{align*}
\]

where the data model is typically a measurement-error model, and the process and parameter models can be formulated by incorporating scientific theories and experts’ knowledge whenever possible. The unknowns in this model are \( Y \) and \( \theta \). Hence, this Bayesian hierarchical model has substantial flexibility and provides a natural vehicle for scientists and statisticians to work together.

To make optimal predictions of the unknowns with associated uncertainty, Bayesian inferences rely on Bayes’ Theorem, which is used to compute the posterior distribution of the unknowns given the observed data. In this article, the observed data are the model output \( Z = z \). Specifically, the posterior distribution is,

\[
p(Y, \theta|z) \propto p(z|Y) \cdot p(Y|\theta) \cdot p(\theta),
\]

where “\( \propto \)” denotes “is proportional to,” and the proportionality constant in (1), which is a function only of the observations \( Z = z \), ensures that the left-hand side is a probability distribution.

Typically, the complexity and high-dimensionality of Bayesian hierarchical statistical models prohibit the direct computation of the posterior distribution. However, with simulation procedures, such as Markov chain Monte Carlo (MCMC), an empirical estimate of the posterior distribution can be obtained (e.g., Gilks et al., 1996).
Combining RCM Outputs

2.2. The NARCCAP RCMs and their outputs

As an international program involving climate-research groups all over the world, NARCCAP is designed to explore the uncertainties in RCM outputs and provide high-resolution (approximately 50 km) climate-output data for most of Canada, the 48 contiguous states in the United States, and northern Mexico, as well as the adjacent Atlantic and Pacific Oceans. A set of six RCMs are included in NARCCAP. In this article, we use the 4-letter abbreviation as summarized in Table 1 to refer to them. These RCMs have been developed at different research centers and have different major characteristics (e.g., lateral boundary treatments and parameterizations). The details regarding all the models included in NARCCAP and references concerning their developments are available in Mearns et al. (2009), Mearns et al. (2010), and at http://www.narccap.ucar.edu/.

We now review recent studies using NARCCAP and other RCM outputs. Kaufman and Sain (2010) give a functional ANOVA analysis using Gaussian spatial processes for main effects, to study different sources of uncertainties in RCM outputs. From the similar perspective of functional ANOVA, Sain et al. (2011b) examine the differences between two RCMs, using a conditional autoregressive (CAR) model. Sain and Furrer (2010) and Christensen and Sain (2011) use two different classical spatial correlation models to capture the dependence between the six NARCCAP RCMs. Schliep et al. (2010) build a hierarchical model based on the generalized extreme value distribution to study extreme precipitation, but each of the six NARCCAP RCMs is analyzed separately. Sain et al. (2011a) construct a multivariate Markov random field model to combine ensembles of RCM outputs on a subregion in the western United States and part of western Canada. Buser et al. (2010) study the uncertainty in multivariate climate projections from several RCMs by modeling the regional means for each variable simultaneously, using both frequentist and Bayesian methods.

In this article, we study a 20-year average (1981-2000) of surface temperature in the Boreal winter (December, January and February), produced by the set of six RCMs in Phase I of NARCCAP. The RCMs are run at a spatial scale of approximately 50 km, and their outputs were interpolated to a standard grid of 98 × 120 points, unequally spaced in longitude and latitude. The points are centered at grid cells whose union we call $D$. This spatial domain $D$ includes regions with small irregular land masses (e.g., the Caribbean), complex structures (e.g., the Rocky Mountains and the Great Lakes), and complex coastlines. In Phase I of NARCCAP, all six RCMs were run using the NCEP Reanalysis II data as boundary conditions. The NCEP Reanalysis II data is a gridded assimilation data product from the National Centers for Environmental Prediction (NCEP) and the National Center for Atmospheric Research (NCAR). It is generated by incorporating observations and numerical weather prediction (NWP) model outputs with state-of-the-art data assimilation methods (e.g., Kanamitsu et al., 2002) on a much coarser resolution grid ($2^\circ \times 2^\circ$), compared to that of the RCM outputs. Given the (coarser) boundary conditions provided by the NCEP Reanalysis II data, outputs from RCMs are generated by discretizing the complicated coupled systems of differential equations spatially and temporally. It should be noted that the RCMs don’t directly use the NCEP Reanalysis II data on the interior of the domain.

With the differential equations describing the physical dynamics such as energy flows, the six RCMs simulate 3-hourly “weather” over long time periods and generate a vast array of outputs, from which the the long-run average is commonly used as a summary of how a climate model approximates the Earth’s climate. Therefore, we created 20-year seasonal
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(Boreal winter) summaries, from 1981 through 2000, in order to capture climate properties of the RCMs. That is, for the $i$-th RCM for each pixel $s_j$ in $D$, define:

$$Z_i(s_j) = \frac{\sum_{t=1981}^{2000} Z_{i,t}(s_j)}{20},$$

(2)

where $Z_{i,t}(s_j)$ is the $i$-th RCM’s Boreal winter temperature (average over December, January, and February temperatures) in pixel $s_j$ and year $t$, for $i = 1, \ldots, 6$, $\{s_j : j = 1, \ldots, n\}$, and $n = 98 \times 120 = 11,760$. Although (winter) temperatures are expected to vary over years, it is common to take a long-term average to indicate the (winter) climate. In this article, these 20-year-averaged fields from the RCMs are referred to as “data” and are notated as $n$-dimensional vectors:

$$Z_i = (Z_i(s_1), \ldots, Z_i(s_n))',$$

for $i = 1, \ldots, 6$. Fig. 1 shows the 20-year-average winter temperature fields from all six RCMs. Compared to the regridded NCEP Reanalysis II data $\{T(s) : s \in D\}$ (left panel of Fig. 2), the RCM outputs provide a more detailed description of local climate in $D$. Landscapes at a finer resolution are characterized in RCMs, including the Great Lakes and the major mountain ranges in the Rocky Mountains such as the Bitterroot Range, the Wasatch Range, and the Colorado Rockies. Although it can be seen that the RCM outputs are similar in large-scale pattern, there are certainly different small-scale patterns for different RCMs. Our goal is to properly account for spatial dependence and ultimately find the common spatial patterns and the ones that are different (with associated uncertainties).

3. A Bayesian hierarchical statistical model for RCM outputs

We present our spatial statistical methodology in general terms and describe the Bayesian hierarchical statistical model for NARCCAP outputs in analytical form. All three levels of the hierarchical statistical model, namely the data, process, and parameter models are introduced.

3.1. Data model

Climate models use complicated coupled systems of differential equations to model the different processes that make up the Earth’s climate. These models are run over long time periods, and statistics such as long-run averages are extracted and modeled as one single realization equal to the underlying process plus noise (e.g., Furrer et al., 2007; Sain et al., 2011a). In our analysis, these long-run averages, $\{Z_i : i = 1, \ldots, 6\}$, are computed from a 20-year integration of the RCM outputs; to represent the variability that is inherent in such constructions, we consider the data model as follows:

$$Z_i = Y_i + \varepsilon_i; \quad i = 1, \ldots, 6,$$

(3)

where $Y_i = (Y_i(s_1), \ldots, Y_i(s_n))'$ represents the climate signal from the $i$-th RCM, and $\varepsilon_i = (\varepsilon_i(s_1), \ldots, \varepsilon_i(s_n))'$ represents the noise vector, for $i = 1, \ldots, 6$. The noise component
is assumed to characterize the uncertainty of the model output. We assume additionally that they are independent of each other and follow Gaussian \((\text{Gau})\) distributions:

\[
\varepsilon_i \sim \text{Gau}(0, \sigma_i^2 V_i); \ i = 1, \ldots, 6, \text{ independently}
\]

(4)

where \(\{V_i\}\) are known \(n \times n\) diagonal matrices, and \(\sigma_i^2\) is an unknown parameter. Notice that the spatial dependence in a given RCM output is captured by its climate signal. While it is possible to model further spatial dependence in the noise component, it is difficult to justify in our purely spatial analysis, and it results in a very large computational overhead.

The matrices \(\{V_i\}\) are determined by variability of replications of the outputs over time. That is, the \(j\)th diagonal element of \(V_i\) is specified as: 

\[
V_i(s_j) = \frac{\sum_{t=1981}^{2000} (Z_{i,t}(s_j) - Z(t,s_j))^2 / (19 \times 20)}{\text{variance in the climate signal, specifically from the } i\text{-th RCM. Additionally, } \{\nu_i\}\text{ are smooth. Similar models to (7) have been applied, for example, in Wikle and Cressie (1999), though in the SRE model, the basis functions are not necessarily orthogonal as required in other models. The underlying assumption that the 20-year-average winter-temperature outputs from these RCMs have a Gaussian distribution, has been supported by Meehl et al. (2000) and Tebaldi et al. (2005). It should also be noticed that the assumption of conditional independence of \(\{Z_i\}\) does not mean that they are unconditionally independent. In fact, since all RCMs are generating outputs for the same region \(D\) in the same period, we expect that, unconditionally, they are highly correlated.}

### 3.2. Process model

The climate signals from the RCMs, \(\{Y_i\}\), and the consensus climate signal, \(Y\), are modeled statistically within the process model. We first model the climate signal from the \(i\)-th RCM as follows:

\[
Y_i = X\beta + \nu_i; \ i = 1, \ldots, 6,
\]

(6)

where \(X\beta\) is a linear function of \(p\) known covariates that represents the \textit{large-scale} variation common in all RCM climate signals. The second term, \(\nu_i \equiv (\nu_i(s_1), \ldots, \nu_i(s_n))'\), represents the \textit{small-scale} variation in the climate signal, specifically from the \(i\)-th RCM. Additionally, we model \(\{\nu_i\}\) with a Spatial Random Effects (SRE) model (Cressie and Johannesson, 2006, 2008), for \(i = 1, \ldots, 6\):

\[
\nu_i(s) = S(s)' \eta_i; \ s \in D,
\]

(7)

where \(S(\cdot) \equiv (S_1(\cdot), \ldots, S_r(\cdot))'\) is a vector of \(r\) deterministic, known, multi-resolutional spatial basis functions.

In some versions of the SRE model, there is a component of pixel-scale variability (e.g., Cressie and Kang, 2010), but because we are analyzing outputs from numerical models averaged over time, it is expected here that \(\{\nu_i\}\) are smooth. Similar models to (7) have been applied, for example, in Wikle and Cressie (1999), though in the SRE model, the basis functions are not necessarily orthogonal as required in other models. The \(r\)-dimensional vectors, \(\eta_i \equiv (\eta_{i,1}, \ldots, \eta_{i,r})'\), for \(i = 1, \ldots, 6\), are Gaussian \textit{random effects} and will be described later. Define \(S\) as the \(n \times r\) matrix whose \(j\)th column is \(S_j(\cdot)\); then we can write (6) equivalently as follows:

\[
Y_i = X\beta + S \eta_i; \ i = 1, \ldots, 6.
\]

(8)
We model the consensus climate signal of interest as:

$$ Y = X\beta + S\eta, $$

(9)

where the vector of random effects \( \eta \) is modeled with a Gaussian distribution that has mean zero and \( r \times r \) covariance matrix \( K \). The consensus random effects are related to the individual random effects by assuming that,

$$ \eta_i = \eta + \xi_i; \quad i = 1, \ldots, 6, $$

(10)

where \( \{\xi_i\} \) represents a collection of zero-mean random processes, assumed independently distributed as \( r \)-dimensional Gaussian distributions, \( \{\text{Gau}(0, \omega_i^2 I)\} \), respectively. By centering \( \eta_i \) at \( \eta \), we assume a priori that the RCMs do not exhibit systematic errors from the consensus climate signal \( Y \). Additionally, the parameters \( \{\omega_i^2\} \) reflect different amounts of variability in various RCMs after accounting for the underlying consensus climate signal. It should be noticed that if a simple arithmetic mean is used to combine the outputs from the six RCMs, this amounts to assuming \( \omega_1^2 = \omega_2^2 = \cdots = \omega_6^2 \). Allowing different \( \omega_i^2 \) for different RCMs, our model captures the “vote” that each RCM has for the consensus climate. We show how to make inference on \( \{\omega_i^2\} \) in Section 4.

Combining (8) and (10), \( Y_i \) is said to follow a Spatial Mixed Effects (SME) model; that is,

$$ Y_i = X\beta + S\eta + \zeta_i; \quad i = 1, \ldots, 6, $$

(11)

where \( \beta \) is the \( p \)-dimensional vector of fixed-effects coefficients; \( \eta \sim \text{Gau}(0, K) \) is the \( r \)-dimensional vector of random effects coefficients; and \( \zeta_i = Y_i - Y = S\xi_i \sim \text{Gau}(0, \omega_i^2 \Omega) \) is an \( n \)-dimensional vector of errors, where \( \Omega \equiv SS' \).

In terms of \( \{Y_i\} \) and \( Y \), the process model can be written equivalently as,

$$ Y_i|Y, \omega_i^2 \sim \text{Gau}(Y, \omega_i^2 \Omega); \quad i = 1, \ldots, 6, $$

$$ Y|K, \beta \sim \text{Gau}(X\beta, SKS'), $$

(12)

where \( \text{var}(Y|K, \beta) \) is positive-semi-definite. While the dimension reduction means that some linear combinations of \( Y \) have zero variance, because \( K \) is not fixed, these linear combinations are not fixed either. Furthermore, notice that \( Y \) and \( \{Y_i\} \) are completely recoverable from \( \eta \) and \( \{\xi_i\} \), since \( S \) is known.

### 3.3. Parameter model

To complete the Bayesian hierarchical model, we specify the prior distribution for the various parameters, specifically the fixed-effects coefficients \( \beta \), the \( r \times r \) covariance matrix \( K \), and the variance parameters \( \omega_1^2, \ldots, \omega_6^2 \), and \( \sigma^2 \). First, we assume independence:

$$ [\beta, K, \omega_1^2, \ldots, \omega_6^2, \sigma^2] = [\beta] \cdot [K] \cdot [\omega_1^2] \cdots [\omega_6^2] \cdot [\sigma^2]. $$

(13)

Then we give prior models for each of the components, as follows. We assume that the coefficients of the fixed effects \( \beta \) have a Gaussian prior,

$$ \beta \sim \text{Gau}(\mu_\beta, \sigma^2_\beta I), $$

(14)

where \( \mu_\beta \) and \( \sigma^2_\beta \) are known hyperparameters (see Appendix).
We then assign the prior distribution for the $r \times r$, positive-definite covariance matrix $K$. Although there has been a lot of research devoted to priors for covariance matrices (see Massam, 2009, and Massam, 2010, for a list of references), in the context of random effects models, it is common to assume a very simple structure, such as $K$ is a multiple of the identity matrix (e.g., Zhao et al., 2006; Baladandayuthapani et al., 2008) or $K$ is a diagonal matrix (e.g., Furrer et al., 2007; Lopes et al., 2008). Then priors such as the inverse-gamma prior are put on the associated scale parameters. For more complicated Bayesian spatial models, such as the multivariate predictive process model discussed in Banerjee et al. (2008), the inverse-Wishart prior is assigned to a low-rank positive-definite matrix.

For the SRE model, Kang and Cressie (2011) show that the Givens-angle prior allows for much more flexibility and has better prediction performance, compared to the previously mentioned priors. In our analysis of RCM outputs, we chose to use the Givens-angle prior on $K$. Specifically, we express $K$ based on the spectral decomposition,

$$K = P \Lambda P',$$  \hspace{1cm} (15)

where $\Lambda$ is the diagonal matrix, $\text{diag}(\lambda_1, \ldots, \lambda_r)$, whose diagonal elements are the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$, and $P$ is the corresponding orthogonal matrix of eigenvectors such that $P'P = PP' = I$. As in Kang and Cressie (2011), we express $P$ in terms of the $r(r-1)/2$ Givens angles, denoted by $\theta_{ij}$, for $i = 1, \ldots, r - 1$ and $j = i + 1, \ldots, r$:

$$P = (G_{12}G_{13} \cdots G_{1r}) \cdot (G_{23} \cdots G_{2r}) \cdots G_{(r-1)r}. \hspace{1cm} (16)$$

In (16), $G_{ij}$ is the Givens rotation matrix corresponding to $\theta_{ij}$, defined as a modification of the $r \times r$ identity matrix with $i$th and $j$th diagonal elements replaced by $\cos(\theta_{ij})$ and the $(i, j)$ and $(j, i)$ elements replaced by $-\sin(\theta_{ij})$ and $\sin(\theta_{ij})$, respectively.

We assume independence of $\lambda = (\lambda_1, \ldots, \lambda_r)'$ and

$$\theta \equiv \left(\theta_{12}, \ldots, \theta_{1r}, \theta_{23}, \ldots, \theta_{2r}, \ldots, \theta_{(r-1)r}\right)'.$$  \hspace{1cm}

That is,

$$[\lambda, \theta] = [\lambda] \cdot [\theta].$$

Priors on $\lambda$ and $\theta$ imply a prior distribution on $K$ that is very flexible. We assign priors on $\lambda$ and $\theta$ as suggested by Kang and Cressie (2011), so that the prior on $K$ has a multi-resolutional structure brought about by the choice of multi-resolutional basis functions in the SRE model. Other than this, the prior aims to be noninformative; more details can be found in Appendix (see also Kang and Cressie, 2011).

We assign an inverse-gamma-distribution prior on $\{\omega_i^2\}$:

$$\omega_i^2 \sim IG(a_1, b_1); \ i = 1, \ldots, 6, \ \text{independently}, \hspace{1cm} (17)$$

where $IG(a, b)$ denotes the inverse-gamma distribution with shape parameter $a$ and scale parameter $b$, whose density is proportional to,

$$(\omega^2)^{-(a+1)} \exp\{-b/\omega^2\}. \hspace{1cm} (18)$$

Similarly, we assign an inverse-gamma-distribution prior, $IG(a_2, b_2)$, on $\sigma^2$. Details on the hyperparameter specifications are given in Appendix.
4. Results

4.1. Implementation

We now fit the Bayesian hierarchical model described in Section 3 and use the outputs from all six NARCCAP RCMs, namely \( (Z_1', \ldots, Z_6') \), to predict the consensus climate signal. Recall that all the RCMs summarized in Table 1 are driven by the same boundary conditions set by the NCEP Reanalysis II data, and from their output we calculate 20-year-average winter temperatures on a 98 × 120-resolution grid. Noticing that the RCM outputs have a similar large-scale pattern and differ in small-scale details (including the regions, such as coastlines, Hudson Bay, the Rocky Mountain, and the Great Lakes), we choose to use a single covariate common to all six RCMs, namely the RCM-scale, regridded NCEP Reanalysis II data, \( T(\cdot) \). This allows us to center the RCM outputs around the same large-scale pattern so we can investigate small-scale differences and variabilities over subregions of \( D \) using random effects terms. Specifically, we model the trend term \( X(s)\beta \) in (6), with:

\[
X(s) = (1, T(s))'; \ s \in D,
\]

where \( T(s) \) denotes the 20-year-average winter temperature field obtained from the NCEP Reanalysis II data, interpolated to the 98 × 120-resolution grid. This is visualized in the left panel of Fig. 2.

To model the small-scale-variation terms \( \{\nu_i\} \) in (6), we need to specify the \( r \)-dimensional vector of spatial basis functions \( S(\cdot) \). Shi and Cressie (2007) used W-wavelets for the analysis of a large spatial remote-sensing dataset; Cressie and Johannesson (2008) and Cressie and Kang (2010) used local bisquare functions. In what is to follow, we also use the local bisquare functions, whose generic form is:

\[
S_{jl}(s) = \begin{cases} 
1 - \left( \frac{||s - v_{jl}||}{r_l} \right)^2, & ||s - v_{jl}|| \leq r_l, \\
0, & \text{otherwise},
\end{cases}
\]

where \( v_{jl} \) is the \( j \)th center point of the \( l \)th resolution, \( l = 1, 2, \ldots \); \( ||s - u|| \) is the great-circle distance between two locations \( s \) and \( u \); and

\[
r_l = (1.5) \max_{u \in N_l, v \in N_l \setminus \{u\}} ||u - v||.
\]

In the expression above, the factor 1.5 is chosen to allow overlap between the multi-resolutional spatial basis functions, and \( N_l \) denotes the set of center points of the \( l \)th resolution, \( l = 1, 2, \ldots \). For example, for \( l = 1 \), the maxi-min distance is 1173 km, and hence \( r_1 = (1.5)1173 = 1759.5 \). For \( l = 2, r_2 = (1.5)605 = 907.5 \), where the maxi-min distance is 605 km. There are 16 and 64 functions from the first two resolutions, respectively, for \( S(\cdot) \). The right panel of Fig. 2 shows all the center points for these two resolutions.

Temperature fields differ over regions with different elevation levels and regions with a lot of land-water exchange of energy, such as the Great Lakes and Hudson Bay; the RCM outputs reflect these patterns in different ways, as can be seen in Fig. 1. Such differences are not surprising, because although all the RCMs attempt to describe the same complicated climate system based on the same physical laws, they vary in many aspects of their implementation (for example, different thermal/water layers are applied). Therefore, we also include a random effects term (that is, a basis function), equal to the elevation \( A(s) \)
and four other basis functions equal to the indicator functions, $1_{\text{land}}(s)$, $1_{\text{lake}}(s)$, $1_{\text{bay}}(s)$, and $1_{\text{coast}}(s)$, for land, the Great Lakes, Hudson Bay, and coastline, respectively. Fig. 3 gives a visualization of the indicator functions. These considerations result in $r = 16 + 64 + 1 + 4 = 85$ spatial basis functions in all.

Fig. 3 here

4.2. Bayesian predictions

From the data, process, and parameter models described in Section 3, we obtain Bayesian predictions for the unknowns, including the consensus climate signal and the parameters, via an MCMC simulation (a Gibbs sampler with Metropolis-Hastings steps; see Supporting Information online). We ran three parallel chains for 12,500 iterations each, for a total of 30,000 iterations, discarding the first 2,500 to account for the Markov-chain burn-in. All the computations were carried out in Matlab on a dual core 2.88 GHz Intel Xeon processor with 96 Gb of memory running Linux; our results took 11 CPU hours to compute. Because of the large number of parameters involved in the entire model, it is not possible to perform formal tests of convergence of the sampled chain simultaneously, so we checked convergence of the MCMC on selected quantities through trace plots. Specifically, we first looked at each element of $\beta$. We examined the convergence of the posterior distribution of the $r$-dimensional vectors $\{\eta_i\}$ and $\eta$ as follows: For the basis functions, elevation $A(s)$ and the four indicator functions, we checked all the corresponding elements in $\{\eta_i\}$ and $\eta$. Then, for the $l$th ($1 \leq l \leq L$) resolution of the local bisquare functions, we randomly chose several elements in $\eta_i$ (and $\eta$) and checked them for convergence. For $K$ expressed through eigenvalues $\lambda$ and Givens angles $\{\theta_{ij}\}$, we also randomly chose several of the elements within each resolution to check for convergence. Based on these procedures, we were satisfied that we were sampling from the stationary distribution of the Markov chain.

Fig. 4 here

The consensus climate signal given by (9) depends on the random effects $\eta$, to which the $i$-th RCM contributes information through $\eta_i$. The information in the $i$-th RCM is “weighted” according to its precision, as summarized by (10). Specifically, the difference between the random effects for the $i$-th RCM and $\eta$ has a variance equal to $\omega_i^2$; $i = 1, \ldots, 6$. One advantage of Bayesian methodology is that we can investigate behaviors of important parameters, like $\{\omega_i^2\}$, which summarize the influence of the individual RCMs in determining a consensus climate. The posterior distributions for $\{\omega_i^2\}$ can be used to assess the six RCMs’ influence. To illustrate the influence more clearly, we show the posterior distributions of each $|\omega_i|^{1/2}$; see the boxplots in Fig. 4. The most striking difference between these posterior distributions is the mean level. That is, some RCMs’ random effects vary little about the consensus random effects, and some vary a lot; for example, MM5I ($i = 4$) varies the least. We also checked the posterior bivariate distributions of all pairs of $\{\omega_i^2\}$ based on their samples from the MCMC procedure, and we found no evidence of posterior dependence.

In Fig. 5, we present the posterior mean of the consensus climate signal $Y$ (the optimal predictor of $Y$ under squared-error loss) and its posterior standard deviations. Compared to the winter temperature field $T(\cdot)$, namely the interpolated NCEP Reanalysis II data (left panel in Fig. 2), the posterior mean of $Y$ (upper-left panel in Fig. 5) is much less smooth and shows more detailed local features, such as the Great Lakes and the mountainous western
United States and Canada. The difference between the posterior mean of \( Y(\cdot) \) and \( T(\cdot) \) is substantial, especially between the Pacific coast and the Rocky Mountains, and between the Rocky Mountains and the Great Lakes. These detailed regional features in the consensus-climate signal are due to regional physical-forcing processes described within the RCMs. The posterior standard deviations of the elements of \( Y \) are shown in the upper-right panel in Fig. 5, which indicates that our consensus-climate prediction is most uncertain in the north or in regions with large land-water energy exchanges, such as areas around the Gulf of Alaska, Hudson Bay, Labrador Sea, and the Great Lakes. The uncertainty in the Rocky Mountains is also higher compared to neighboring regions.

Fig. 5 here

Our Bayesian hierarchical approach enables us to decompose the consensus climate signal into two parts, namely the fixed effects, \( X\beta \), and the spatial random effects, \( S\eta \). The lower-left panel of Fig. 5 shows the posterior mean of \( S\eta \). As expected, the small-scale variation, \( S\eta \), is of much smaller magnitude than \( Y \). Some local geographical features such as the Great Lakes and the Rocky Mountains are observed in the spatial random effects, \( S\eta \). The lower-right panel in Fig. 5 plots the associated posterior standard deviations for \( S\eta \). It can be seen that most of the uncertainty in \( Y = X\beta + S\eta \) is from the small-scale variation, \( S\eta \), since the posterior standard deviations of \( Y \) and \( S\eta \) look similar.

To look for individual RCM departures from the consensus climate, the posterior means of \( \{ Y_i - Y \} \) are plotted in Fig. 6. Recall from Fig. 4 that MM5I (\( i = 4 \)) varies the least around the consensus climate. From among the other five RCMs, CRCM (\( i = 1 \)) shows minor departures, although its prediction of temperature at most of the coastline is higher than that of the consensus climate. ECPC (\( i = 2 \)) tends to produce higher temperatures in the north around Hudson Bay. HRM3 (\( i = 3 \)) shows systematically higher temperatures in the United States and lower values in central Canada. The last two RCMs (lower panels in Fig. 6), RCM3 (\( i = 5 \)) and WRFP (\( i = 6 \)), both show lower temperatures around the Great Lakes, while they differ considerably in the north: The temperature field from RCM3 tends to be higher around Hudson Bay, and WRFP shows the opposite.

Fig. 6 here

Our Bayesian analysis also allows us to consider the posterior distribution of the joint spatial field, \( Y \) and \( Y_i \), for a given RCM. In order to calibrate the posterior probability calculations, we consider the spatial quantiles of \( \{ Y(s) : s \in D \} \), proposed by Zhang et al. (2008). A spatial quantile is defined through the spatial cumulative distribution function (SCDF):

\[
S_D(y; Y(\cdot)) \equiv \frac{1}{|D|} \int_D I(Y(s) \leq y)ds,
\]

where \( I(\cdot) \) is the indicator function. Readers are referred to Lahiri et al. (1999) for more details on the SCDF. For \( 0 \leq \alpha \leq 1 \), let \( S_D^{-1}(\alpha; Y(\cdot)) \) denote the inverse SCDF of \( Y(\cdot) \):

\[
S_D^{-1}(\alpha; Y(\cdot)) \equiv \arg\min\{y \in \mathbb{R} : S_D(y; Y(\cdot)) \geq \alpha\}.
\]

Then \( Q^\alpha_p \equiv S_D^{-1}(\alpha; Y(\cdot)) \) denotes the \( \alpha \)-th spatial quantile of the SCDF of \( Y(\cdot) \).

For a given \( p \), the \( p \)-th spatial quantile of \( \{ Y(s) : s \in D \} \) has posterior expected value, \( Q_p \equiv E(Q^\alpha_p | \mathbf{Z}_1, \ldots, \mathbf{Z}_6) \). We then define the following posterior probability:

\[
P(s; p, q) \equiv P(Q_p < Y(s) < Q_q | \mathbf{Z}_1, \ldots, \mathbf{Z}_6); 0 \leq p < q \leq 1,
\]
where \( P(s; 0, q) \equiv P(Y(s) < Q_q|Z_1, \ldots, Z_6) \) and \( P(s; p, 1) \equiv P(Y(s) > Q_p|Z_1, \ldots, Z_6) \). Similarly, we define

\[
P_i(s; p, q) \equiv P(Q_p < Y_i(s) < Q_q|Z_1, \ldots, Z_6); \ i = 1, \ldots, 6.
\]

Thus, comparisons of \( Y \) and \( \{Y_i\} \) can be made on the probability scale; such probabilities can be computed from the MCMC simulations. In the rest of this section, we consider the two RCMs, MM5I and RCM3, to illustrate how Bayesian methodology enables us to study joint distributions of elements within the temperature vectors.

We first consider the very cold, cold, and cool temperatures, by choosing \((p, q)\) to be \((0, 0.05)\), \((0.05, 0.25)\), and \((0.25, 0.5)\), respectively. Fig. 7 plots the corresponding \( P(s; p, q) \), \( P_{MM5I}(s; p, q) \), and \( P_{RCM3}(s; p, q) \) in the left, middle, and right columns, respectively. The first row is for \((p, q)\) chosen as \((0, 0.05)\), and it shows that the two RCMs chosen differ from the consensus climate signal in describing the very cold and cold regions (in the north). The consensus climate and MM5I (left and middle columns) have similar results for all these choices of \((p, q)\). RCM3 behaves differently from MM5I (and the consensus climate) in the Great Lakes region. Finally, all three have similar results in the Rocky Mountain region.

We complete our analysis by considering warm, hot, and very hot regions, by choosing \((p, q)\) to be \((0.5, 0.75)\), \((0.75, 0.95)\), and \((0.95, 1)\), respectively. Fig. 8 plots the corresponding \( P(s; p, q) \), \( P_{MM5I}(s; p, q) \), and \( P_{RCM3}(s; p, q) \) in the left, middle, and right columns, respectively. Apart from a small difference between the two RCMs in describing the warm region around the Great Lakes, the panels in each row look similar. In conclusion, it seems that the RCMs have the least consensus in the very cold and cold regions (in the north) and for moderate temperatures around the Great Lakes.

5. Discussion and conclusions

In this article, we use a Bayesian hierarchical model to combine spatial outputs from a set of six regional climate models (RCMs) from Phase I of NARCCAP. Although the RCM outputs in this ensemble are similar in large-scale variation, they differ in small-scale details in regions. It is of interest to know how to deal with the discrepancy between RCMs. Using Bayesian methodology, we construct a consensus climate signal with “votes” from each individual RCM. Compared to the coarser NCEP Reanalysis II data, the optimally predicted consensus climate shows much more detail. The Bayesian methodology also enables us to make inference on each random component of the underlying unknown process and their departures from the consensus climate, on both the temperature scale and the probability scale.

We would also like to mention that the use of an SRE model allows a flexible class of nonstationary spatial covariance functions to describe the underlying spatial statistical dependence, and the computations are efficient. In contrast to previous work, such as Furrer et al. (2007), we are not assuming either stationarity or isotropy for the spatial covariance structure in the process model. To illustrate this, consider the covariance function,

\[
C(s_0, s) \equiv \text{cov}(Y(s_0), Y(s)|K) = S(s_0)'K S(s); \ s \in D.
\] (21)
The corresponding correlation can be obtained straightforwardly as:

\[
\text{corr}(s_0, s) \equiv \text{corr}(Y(s_0), Y(s)|K) = (S(s_0)'KS(s_0))^{-1/2}S(s_0)'KSS(s)(S(s)'KS(s))^{-1/2}; s \in D. \tag{22}
\]

Based on the loss function discussed in Yang and Berger (1994):

\[
L(K_0, K) \equiv \text{tr}(K_0K^{-1}) - \log|K_0K^{-1}| - r,
\]

the Bayes estimator is \( \hat{K} \equiv \{E(K^{-1}|Z_1, \ldots, Z_6)\}^{-1} \). By substituting \( \hat{K} \) in (22), we obtain the resulting correlation with respect to \( s_0 \). Two locations \( s_0 \) are chosen, and Fig. 9 shows the two contour plots of \( \text{corr}(s_0, s) \) as a function of \( s \). The nonstationary and anisotropic structure of the fitted consensus climate signal is clear. Additionally, Kang and Cressie (2011) indicate that, compared to the method-of-moments estimator \( \tilde{K} \), the Bayes estimator based on the Givens-angle prior, \( \hat{K} \), is superior. They give a figure based on a replication from their simulation experiment that shows \( \hat{K} \)’s eigenvalues are much closer to the true values than are \( \tilde{K} \)’s. Further details on the Givens-angle prior on \( K \) can be obtained from their article.

**Fig. 9 here**

We have presented a viable spatial statistical methodology that could be applied further in climate research. Future research involves a spatio-temporal model with dynamical large-scale and small-scale variations. Such a spatio-temporal model could combine present and future climate projections from various RCMs, enabling better understanding of regional climate change under different scenarios. In Phase II of NARCCAP, all six RCMs are driven by boundary conditions set by different GCMs. Specifically, let \( Z_{i,j}(s) \) be the output at location \( s \) from the \( i \)-th RCM with boundary conditions from the \( j \)-th GCM; then we can generalize (6) and (7) as follows:

\[
Z_{i,j}(s) = X(s)'\beta_j + S(s)'\eta_{i,j} + \varepsilon_{i,j}(s); s \in D. \tag{23}
\]

As well as temperature, RCMs produce outputs for many other variables (e.g., precipitation), to which our methodology could be applied. More generally, the multivariate outputs from RCMs can be related to other variables in regional ecosystems, or to public-health issues, such as local aerosol alerts, heat waves, or mosquito-borne diseases. Linking such processes would allow proper statistical inference on quantities used for environmental protection or policy decisions.

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Regional Climate Change Assessment Program (NARCCAP) for providing the data used in this article. NARCCAP is funded by the NSF, the U.S. Department of Energy (DoE), the National Oceanic and Atmospheric Administration (NOAA), and the U.S. Environmental Protection Agency (EPA) Office of Research and Development. The National Center for Atmospheric Research (NCAR) is managed by the University Corporation for Atmospheric Research under the sponsorship of the NSF. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the authors and do not necessarily reflect the views of the NSF. This research was completed while Kang was a Postdoctoral Fellow and Cressie was a long-term visitor at the Statistical and Applied Mathematical Sciences Institute (SAMSI).

Appendix

Priors on parameters

In this section, we give details on the priors on parameters, including hyperparameter specification; sometimes we use data to guide us in the choice of hyperparameters.

Noticing the covariates are scaled very differently, we first carried out a standardizing procedure with respect to $X(\cdot)$ and $S(\cdot)$ to make the scales of the covariates comparable and to ensure that the MCMC procedure will perform appropriately. That is, we subtracted the spatial mean and divided by the spatial standard deviation for each covariate and each basis function. For simplicity, we still use the same notation for the standardized fixed and random effects covariates in the following description.

We choose a Gaussian prior, $\text{Gau}(\mu_\beta, \sigma^2_\beta I)$, for the fixed-effects coefficients $\beta$. The hyperparameter $\mu_\beta$ is chosen to be 0, and $\sigma^2_\beta$ is chosen to be $10^6$ so that the prior is approximately noninformative. Notice that because $\sigma^2_\beta$ is so large, the choice of $\mu_\beta$ is largely immaterial. This was confirmed by rerunning the MCMC using other choices.

For the prior on $K$, we wish to capture the multi-resolutional structure of the spatial basis functions; we put priors on $\lambda$ as in Kang and Cressie (2009):

$$[\lambda_1, \ldots, \lambda_r] = [\lambda_{1,1}, \ldots, \lambda_{1,q_1}], \ldots, [\lambda_{L,1}, \ldots, \lambda_{L,q_L}], [\lambda_{L-1,1}, q_{L-1}],$$

(24)

where $\lambda_{l,1}, \ldots, \lambda_{l,q_l}$ are eigenvalues corresponding to the $q_l$ basis functions from the $l$th resolution, $l = 1, \ldots, L$, and $\sum_{l=1}^L q_l = r$. In (24), $\lambda_{l,1}, \ldots, \lambda_{l,q_l}$ are distributed as the order statistics corresponding to i.i.d. lognormally distributed random variables with hyperparameters consisting of the mean $\mu_l$ and the variance $\sigma_l^2$; $l = 1, \ldots, L$.

The prior put on $\theta_{ij}$ is defined indirectly through a prior on a logit transformation of the angle:

$$k(\theta_{ij}) \equiv \log \left( (\pi/2 + \theta_{ij})/(\pi/2 - \theta_{ij}) \right).$$

(25)

That is, independently,

$$k(\theta_{ij}) \sim \text{Gau}(c_l, \tau^2_l),$$

(26)

if $(i, j) \in N_l \equiv \{(i, j) : \text{the } i\text{-th and } j\text{-th basis functions are both of the } l\text{th resolution}, 1 \leq i < j \leq r, l = 1, \ldots, L$. Otherwise, independently,

$$k(\theta_{ij}) \sim \text{Gau}(0, \tau^2_0),$$

(27)

for $(i, j) \in N_0 \equiv \{(i, j) : \text{the } i\text{-th and } j\text{-th basis functions are of different resolutions}, 1 \leq i < j \leq r\}.$
In (26) and (27), \( \{c_l\}, \{\tau^2_l\}, \) and \( \tau_0^2 \) are hyperparameters. We specify them in the same manner as suggested by Kang and Cressie (2011), so that the multi-resolutional structure of the basis functions is maintained.

To obtain a method-of-moments estimate \( \tilde{K} \) of \( K \), we first detrend the data, by performing a weighted-least-squares linear regression analysis. That is, we fit the model

\[
Z_i = X \beta + \varepsilon_i, \quad i = 1, \ldots, 6,
\]

where recall that the errors \( \{\varepsilon_i\} \) are distributed independently with mean zero and covariance matrix \( \{\sigma^2_i V_i\} \), respectively. Then the detrended data are obtained by removing the weighted-least-squares-based estimate of trend. We then compute a method-of-moments estimate \( \tilde{K} \) of \( K \) by binning the detrended data as in Cressie and Johannesson (2008). The mean hyperparameters are specified through \( \tilde{K} \), and the variance hyperparameters are “inflated,” so that the priors are approximately noninformative. Specifically, we first obtain the spectral decomposition of \( \tilde{K} \),

\[
\tilde{K} = \tilde{P} \tilde{\Lambda} \tilde{P}',
\]

where \( \tilde{\Lambda} \equiv \text{diag}(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_r) \), \( \tilde{\lambda}_1 \geq \tilde{\lambda}_2 \geq \cdots \geq \tilde{\lambda}_r > 0 \), and \( \tilde{P} \) is the corresponding orthogonal matrix of eigenvectors. The Givens angles of \( \tilde{\Lambda} \) are then calculated:

\[
\theta \equiv (\theta_1, \ldots, \theta_{r-1})'.
\]

For \( l = 1, \ldots, L \), we specify:

\[
\mu_i = \frac{\sum_{i=1}^{q_l} \log(\tilde{\lambda}_{i,l})}{q_l},
\]

\[
\sigma^2_i = \gamma \left[ \sum_{i=1}^{q_l} (\log(\tilde{\lambda}_{i,l}) - \mu_i)^2 / (q_l - 1) \right],
\]

where \( \gamma \) is a positive inflation factor for the variance, chosen as \( \gamma = 6 \) in this article. Similarly, we specify \( \{c_l\}, \{\tau^2_l\} \), and \( \tau_0^2 \) as follows:

\[
c_l = \sum_{(i,j) \in N_l} k(\theta_{ij}) / |N_l|,
\]

\[
\tau^2_l = \gamma \sum_{(i,j) \in N_l} (k(\theta_{ij}) - c_l)^2 / (|N_l| - 1),
\]

\[
\tau_0^2 = \gamma \sum_{(i,j) \in N_0} k(\theta_{ij})^2 / |N_0|,
\]

where \( k(\cdot) \) is given by (25).

We choose inverse-gamma distributions, \( IG(a, b) \), for the priors on \( \{\omega^2_l\} \) and \( \sigma^2_l \). We choose \( a = b = 0.001 \), so that the prior distributions are approximately noninformative.

References


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Table 1. Summary of RCMs in NARCCAP.

<table>
<thead>
<tr>
<th>Model</th>
<th>Research Center</th>
<th>Full Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRCM</td>
<td>OURANOS / UQAM</td>
<td>Canadian Regional Climate Model</td>
</tr>
<tr>
<td>ECPC</td>
<td>UC San Diego/Scripps</td>
<td>Experimental Climate Prediction Center Regional Spectral Model</td>
</tr>
<tr>
<td>HRM3</td>
<td>UK Hadley Centre</td>
<td>Hadley Regional Model 3</td>
</tr>
<tr>
<td>MM5I</td>
<td>Iowa State University</td>
<td>MM5 - PSU/NCAR mesoscale model</td>
</tr>
<tr>
<td>RCM3</td>
<td>UC Santa Cruz</td>
<td>Regional Climate Model version 3</td>
</tr>
<tr>
<td>WRFP</td>
<td>Pacific Northwest National Laboratory</td>
<td>Weather Research &amp; Forecasting model</td>
</tr>
</tbody>
</table>
Fig. 1. The 20-year-average Boreal winter temperatures from the set of six NARCCAP RCMs (in degrees Celsius).
Fig. 2. Left panel: Regridded NCEP Reanalysis II data for the 20-year-average Boreal winter temperatures over $D$ (in degrees Celsius). Right panel: Center points of local bisquare functions from two resolutions, marked with crosses and circles, respectively.

Fig. 3. Four regions used to define (indicator) basis functions. Left panel: Land. Middle panel: The Great Lakes and Hudson Bay in different shadings. Right panel: Coastline.

Fig. 4. Boxplots of the MCMC samples from the the posterior distributions for each of $\{|\omega_i|^{1/2}\}$. 
Fig. 5. Upper-left panel: Posterior mean of $Y$. Upper-right panel: Posterior standard deviation of $Y$. Lower-left panel: Posterior mean of $S_\eta$. Lower-right panel: Posterior standard deviation of $S_\eta$. 
Fig. 6. Posterior means of $\zeta_i \equiv Y_i - \bar{Y}$, for $i = 1, \ldots, 6$ (in degrees Celsius).
Fig. 7. Plots of $P$, $P_{\text{MM5I}}$, and $P_{\text{RCM3}}$. Very cold (first row) is $(p, q) = (0, 0.05)$; cold (second row) is $(p, q) = (0.05, 0.25)$; cool (third row) is $(p, q) = (0.25, 0.5)$. 
Combining RCM Outputs

Fig. 8. Plots of $P$, $P_{\text{MM5I}}$, and $P_{\text{RCM3}}$. Warm (first row) is $(p, q) = (0.5, 0.75)$; hot (second row) is $(p, q) = (0.75, 0.95)$; very hot (third row) is $(p, q) = (0.95, 1)$. 
Fig. 9. Contour plots for the correlations \{\text{corr}(s_0, s) : s \in D\}, for \(s_0\) chosen in the western United States (upper panel) and the eastern United States (lower panel). The location \(s_0\) is marked with a red star.