Broadening of Modeled Cloud Droplet Spectra Using Bin Microphysics in an Eulerian Spatial Domain

HUGH MORRISON, MIKAEL WITTE, AND GEORGE H. BRYAN
National Center for Atmospheric Research, Boulder, Colorado

JERRY Y. HARRINGTON
Department of Meteorology and Atmospheric Science, The Pennsylvania State University, University Park, Pennsylvania

ZACHARY J. LEBO
Department of Atmospheric Science, University of Wyoming, Laramie, Wyoming

(Manuscript received 16 February 2018, in final form 5 September 2018)

ABSTRACT

This study investigates droplet size distribution (DSD) characteristics from condensational growth and transport in Eulerian dynamical models with bin microphysics. A hierarchy of modeling frameworks is utilized, including parcel, one-dimensional (1D), and three-dimensional large-eddy simulation (LES). The bin DSDs from the 1D model, which includes only vertical advection and condensational growth, are nearly as broad as those from the LES and in line with observed DSD widths for stratocumulus clouds. These DSDs are much broader than those from Lagrangian microphysical calculations within a parcel framework that serve as a numerical benchmark for the 1D tests. In contrast, the bin-modeled DSDs are similar to the Lagrangian microphysical benchmark for a rising parcel in which Eulerian transport is not considered. These results indicate that numerical diffusion associated with vertical advection is a key contributor to broadening DSDs in the 1D model and LES. This DSD broadening from vertical numerical diffusion is unphysical, in contrast to the physical mixing processes that previous studies have indicated broaden DSDs in real clouds. It is proposed that artificial DSD broadening from vertical numerical diffusion compensates for underrepresented horizontal variability and mixing of different droplet populations in typical LES configurations with bin microphysics, or the neglect of other mechanisms that broaden DSDs such as growth of giant cloud condensation nuclei. These results call into question the ability of Eulerian dynamical models with bin microphysics to investigate the physical mechanisms for DSD broadening, even though they may reasonably simulate overall DSD characteristics.

1. Introduction

Bin microphysics schemes explicitly evolve the drop size distribution (DSD) by dividing it into several Eulerian radius or mass bins and prognosing one or more quantities in each bin (e.g., Berry and Reinhardt 1974; Tzivion et al. 1987; Kogan 1991; Feingold et al. 1994; Ackerman et al. 1995; Stevens et al. 1996a; Reisin et al. 1996; Geresdi 1998; Khain et al. 2004; Flossmann and Wobrock 2010; Lebo and Seinfeld 2011; Onishi and Takahashi 2012). Quantifying and reducing errors associated with numerically calculating DSD evolution in bin schemes has been a significant challenge since their inception. Numerous studies over the past several decades have focused on improving the numerical treatment of drop growth caused by condensation and collision–coalescence (e.g., Kovetz and Olund 1969; Bleck 1970; Young 1974; Soong 1974; Tzivion et al. 1987; Stevens et al. 1996a; Liu et al. 1997; Bott 1998; Tzivion et al. 1999; Alfonso 2015; Lkhamjav et al. 2017). For example, a simple method that conserves bulk mass by treating drop growth as first-order upwind advection in mass space was proposed by Kovetz and Olund (1969), but this led to rather inaccurate solutions for condensational growth (Liu et al. 1997) and rapid, artificial generation of precipitation (Ochs and Yao 1978). A much more accurate method was
developed by Egan and Mahoney (1972) that conserves the zeroth, first, and second moments of the mass distribution. Young (1974) proposed a method that provides information on the drop distribution within bins by solving separate conservation equations for mass and number. Similarly, Tzivion et al. (1987) developed a method for solving collision–coalescence growth that separately prognoses the mass and number in each bin but achieves closure of the equations by diagnostically relating high-order moments to the two prognostic low-order moments. The method of Stevens et al. (1996a) also separately prognoses mass and number in each bin for condensational growth, and utilizes a remapping technique assuming a top-hat distribution similar to Egan and Mahoney (1972) except that the width of the top hat is diagnosed rather than predicted. Liu et al. (1997) proposed a variational optimization method for condensational growth that prognoses only a single variable in each bin but conserves any number of DSD moments as needed.

For condensation and evaporation, analytic or quasi-analytic solutions are readily available for Lagrangian box or parcel models to provide a benchmark for evaluating these numerical methods. This approach was employed by many of the studies cited above. Multiple moment-conserving techniques such as Egan and Mahoney (1972) and Liu et al. (1997) provide relatively high accuracy compared to simpler methods in Lagrangian models (e.g., Fig. 3 in Liu et al. 1997). For cloud modeling, however, bin schemes are typically coupled with Eulerian dynamical models. Understanding and quantifying the accuracy of condensational growth and transport calculations using bin microphysics in Eulerian dynamical models is important because the DSD characteristics from condensational growth, particularly the spectral width and tail of the large end of the spectrum, exert a critical influence on the timing and location of precipitation development in warm clouds (Beard and Ochs 1993, and references therein). Only a few studies have assessed the accuracy of numerical methods for drop growth using bin microphysics in conjunction with advection in Eulerian physical space. Clark (1974) found that the implicit diffusion from Eulerian advection led to only a small broadening of the bin-modeled droplet spectra in one-dimensional tests, and convergence could be achieved with a “moderate” increase in spatial resolution; this result is discussed in more detail in the current paper. Stevens et al. (1996a) later evaluated the effects of condensation/evaporation and Eulerian advection in physical space using bin microphysics in a large-eddy simulation (LES). They compared the LES with a Lagrangian model forced with parcel trajectories from the LES. However, they focused on the horizontal mean and dispersion of bulk DSD quantities such as liquid water content and mean droplet diameter, and not on the droplet spectra.

It is well known that DSDs in real clouds are much broader than those that would occur theoretically by condensation growth alone. The mechanisms responsible for this broadening and production of large drops that initiate rain have been a subject of debate for the past several decades. Work since the 1950s has focused on the role of giant cloud condensation nuclei (CCN) in generating large drops and subsequent rain formation (e.g., Ludlam 1951; Woodcock et al. 1971; Feingold et al. 1999; Jensen and Nugent 2017). Ostwald ripening can also broaden DSDs; this occurs by the preferential growth of large drops compared to small ones owing to differences in saturation vapor pressure over the drop surfaces from curvature and solute effects (e.g., Korolev 1995; Wood et al. 2002). Droplet clustering may also drive localized regions of high supersaturation and accelerated drop growth (e.g., Shaw 2000; Vaillancourt et al. 2002). Several other studies have examined the role of turbulent mixing of different droplet populations. Entrainment of dry air will evaporate some fraction of droplets, depending on the homogeneity of mixing. This can lead to the growth of large drops owing to reduced competition for vapor during subsequent ascent (Telford and Chai 1980). Asymmetry in DSD evolution due to condensation in updrafts and evaporation in downdrafts can lead to horizontal inhomogeneity even in adiabatic clouds, broadening DSDs upon isobaric mixing (Korolev et al. 2013; Pinsky et al. 2014). More generally, mixing of droplet populations that have undergone different growth histories can broaden DSDs; this is the “eddy hopping” mechanism described by Cooper (1989), Lasher-Trapp et al. (2005), and Grabowski and Abade (2017). For instance, different vertical velocities along the cloud base can lead to horizontal variability of droplet concentrations and hence radius growth rates; subsequent horizontal mixing will broaden the DSDs above the cloud base. The key point is that in order for bin microphysics to be useful for investigating these physical mechanisms for DSD broadening, it must be able to accurately simulate condensational growth and droplet transport without excessive influence on broadening from numerical errors.

The goal of this study is to investigate and quantify the effects of numerical errors on DSD broadening from the combination of condensational growth and Eulerian transport in physical space. This study utilizes a hierarchy of models. We first present LES of a stratocumulus case using bin microphysics but with droplet activation, condensation, and evaporation as the only microphysical processes included. To isolate the effects of unphysical numerical diffusion associated with vertical advection on
DSD broadening, tests using an idealized one-dimensional (1D) Eulerian spatial domain are compared to those using a rising parcel framework without Eulerian transport in physical space. These tests use a constant vertical velocity, allowing a direct comparison with Lagrangian microphysical solutions that serve as a numerical benchmark. A wide range of conditions and configurations are tested using the parcel and 1D models, including changes in bulk droplet concentration, vertical velocity, grid spacing, time step, mass bin grid, condensational growth method, and numerical method for advection in physical space. Results are discussed in the context of physical DSD broadening mechanisms, with implications for the ability of Eulerian dynamical models coupled with bin microphysics to investigate these mechanisms.

The rest of the paper is organized as follows. Section 2 describes LES results. The design of the idealized parcel and 1D experiments are presented in section 3. Results of the parcel and 1D tests are detailed in section 4. Section 5 provides a summary and conclusions.

2. LES results

We use the University of California Los Angeles (UCLA)-LES model (Stevens et al. 2005) to simulate the Second Dynamics and Chemistry of the Marine Stratocumulus field study (DYCOMS II) RF02 stratocumulus case. We follow the LES intercomparison study of Ackerman et al. (2009) with the same initial conditions, large-scale forcing, and model configuration except using: 1) the Tel Aviv University bin microphysics scheme (Tzivion et al. 1987, 1989) as applied in Stevens et al. (1996a), with droplet activation and condensation/evaporation as the only microphysical processes (i.e., no collision–coalescence, breakup, or sedimentation); 2) a unimodal lognormal CCN distribution with constant number mixing ratio $N_{CCN} = 125 \text{ mg}^{-1}$ and standard deviation $\sigma_s = 1.7$. Condensation/evaporation is calculated using the two-moment top-hat method of moments (Stevens et al. 1996a), referred to hereafter as TH-MOM. This method prognoses both mass and number mixing ratios in each bin and calculates growth by a remapping assuming top-hat distributions with a diagnosed width in each bin. Aerosol is not explicitly tracked in the model, and all aerosol particles activated as cloud droplets are added to the first (smallest) cloud droplet bin similar to other bin schemes that do not explicitly track aerosol. Solute effects on cloud droplet growth and evaporation are neglected, while curvature and kinetic effects are included but likely have little impact on the results presented herein. Simulations are 6 h with a 1-s time step on a $7.2 \times 7.2 \times 1.5 \text{ km}^3$ grid with 50-m uniform horizontal grid spacing and variable vertical grid spacing with 5 m near the surface and inversion, and no greater than 25 m in the boundary layer (Fig. 1, left panels). Momentum advection is computed with a fourth-order centered scheme, scalar advection with a second-order monotonic flux-limited scheme, and a Smagorinsky approach is used for explicit subgrid-scale mixing. Following the standard setup of the UCLA-LES model, subgrid-scale mixing is neglected for microphysical scalar variables; for these quantities, mixing occurs implicitly from numerical diffusion. Simulations use either the standard mass doubling grid “Log(2),” or a high-resolution grid “Log(2^{1/4})” with a ratio of neighboring bin masses equal to $2^{1/4}$. For both mass grids, model output falls within the range of results presented in Ackerman et al. (2009, see their Figs. 1 and 3) for all bulk quantities including liquid water mixing ratio and drop concentration (not shown).

We focus on characteristics of the modeled DSDs. Examples of simulated DSDs at various heights within the cloud at 3 h are shown in Fig. 1. Representative DSD profiles in updraft and downdraft conditions are shown. These profiles are at the locations indicated by the red arrows in Fig. 2, which shows a vertical cross section of the vertical velocity field at 3 h. The spectral width and tail are quantified by the standard deviation (in radius), $\sigma$, and the difference between the 99th-percentile cumulative distribution radius and the median radius $\Delta R_{99}$. These are analyzed at 3 h (other times give similar results). Vertical cross sections of $\sigma$ and $\Delta R_{99}$ through the center of the domain are shown in Fig. 3. Contoured frequency by altitude diagrams of $\sigma$ and $\Delta R_{99}$ calculated at each grid cell are shown in Figs. 4 and 5.

The DSDs are fairly broad. Median values of $\sigma$ are near 2 $\mu m$ near cloud base (~500 m), decrease to about 1.5 $\mu m$ at ~700 m, and increase sharply but with large horizontal variability near cloud top (Figs. 3a, 4). The latter is likely due to the effects of entrainment and mixing on the DSDs. Overall, these $\sigma$ values are similar to observations in marine stratocumulus over spatial scales similar to the LES grid spacing (e.g., Pawlowska et al. 2006; Snider et al. 2017). This occurs even though the LES neglects collision–coalescence, which likely contributed at least somewhat to broadening of the real DSDs. The term $\Delta R_{99}$ has behavior similar to $\sigma$, with values of 4–6 $\mu m$ in the lower half of the cloud, decreasing to 2–3 $\mu m$ between ~750 and 800 m, and increasing near cloud top (Figs. 3b, 5). The simulation using the mass doubling Log(2) grid has greater horizontal variability of $\Delta R_{99}$ compared to using the Log(2^{1/4}) mass grid, which is simply a direct consequence of coarser bin resolution. Otherwise, there are few differences using the two mass grids for either $\sigma$ or $\Delta R_{99}$. 

\[ \text{median (radius), } \sigma, \text{ and the difference between the 99th-percentile cumulative distribution radius and the median radius } \Delta R_{99}. \text{ These are analyzed at 3 h (other times give similar results). Vertical cross sections of } \sigma \text{ and } \Delta R_{99} \text{ through the center of the domain are shown in Fig. 3. Contoured frequency by altitude diagrams of } \sigma \text{ and } \Delta R_{99} \text{ calculated at each grid cell are shown in Figs. 4 and 5.} \]
Adiabatic growth in an ascending parcel for the conditions of this case would theoretically produce a maximum drop radius of approximately 15 μm, but the DSD tails from the LES extend to much larger sizes over nearly the entire depth of the cloud layer (Fig. 1). This broadening to large sizes can only occur via mixing processes because other potential broadening processes (e.g., collision–coalescence, giant CCN, small-scale droplet clustering, and supersaturation fluctuations) have been neglected. The role of entrainment followed by mixing in broadening DSDs is evident from the relatively large values of $s$ and $D_{99}$ near cloud top.

Fig. 1. (left) Profiles of cloud mass mixing ratio $q_l$ (red and blue lines) at 3 h and vertical grid spacing $\Delta z$ (black line) for the DYCOMS-II RF02 LES. (right) DSD profiles at various heights through the cloud layer from the LES using the (top right) Log(2) mass grid and (bottom right) Log($2^{1/4}$) grid. DSDs at various heights are shown, alternating between solid and dashed lines for visual clarity. In all panels blue and red lines indicate values for representative updraft and downdraft columns, respectively. The locations of these columns are indicated by the red arrows in Fig. 2.
(Figs. 4, 5). This is consistent with the idea that mixing of dry air and evaporation will reduce droplet concentration, which means that droplets can grow to larger sizes during subsequent ascent compared to undilute parcels (Telford and Chai 1980). As noted in the introduction, previous studies have described other processes that can drive horizontal variability even in adiabatic clouds, thereby broadening DSDs upon mixing.

Isolating the mechanisms that broaden DSDs is extremely challenging because DSD mixing in the LES is controlled by flow-dependent numerical diffusion associated with advection of bin microphysical variables by the resolved-scale winds. Importantly, this includes numerical diffusion associated with both horizontal and vertical advection. Horizontal numerical diffusion is in line with the mixing that underpins many of the physical DSD broadening mechanisms discussed in the introduction. Nevertheless, horizontal variability and the associated isobaric mixing and DSD broadening processes are likely underresolved even in our LES with a horizontal grid spacing of 50 m, given observational evidence for DSD variability occurring on much smaller scales (e.g., Gerber et al. 2005; Burnet and Brenguier 2007; Beals et al. 2015). In contrast to horizontal mixing, vertical air motion in real clouds involves droplet condensational growth or evaporation associated with the adiabatic cooling or warming of rising or sinking air parcels. Besides driving variability of droplet activation followed by vertical mixing, fluctuations of vertical velocity alone do not contribute to DSD broadening in the absence of horizontal mixing, microscale supersaturation fluctuations, or other physical broadening mechanisms (e.g., Manton 1979; Cooper 1989; Korolev et al. 2013). Thus, vertical numerical mixing of DSDs in models is unphysical because it neglects the droplet condensation or evaporation that occurs with adiabatic expansion or compression. Note that although it is not included in our simulations, explicit vertical subgrid-scale mixing of bin variables would have the same issue.

Another potential source of artificial DSD broadening is numerical diffusion across bins during condensational growth, which can be thought of as advection in mass space (Kovetz and Olund 1969; Clark 1974; Stevens et al. 1996a).

In the remainder of the paper we focus on the role of these unphysical DSD broadening mechanisms. This is done by analyzing idealized 1D simulations that only include vertical advection and adiabatic droplet condensation growth. The next section describes the design of these 1D experiments, as well as parcel tests that only include condensational growth.
3. Description of the idealized 1D and parcel tests

a. Cloud microphysics

We use idealized parcel and 1D frameworks to test the role of unphysical numerical diffusion associated with vertical advection in physical space and condensational growth in mass space on DSD broadening. These frameworks allow a direct comparison to Lagrangian microphysical solutions that serve as a numerical benchmark. Consistent with the LES, the focus is on liquid condensational growth and transport in physical space; all other microphysical processes are neglected. However, offline calculations of

![Image](image.png)

**Fig. 4.** Contour frequency by altitude diagrams of the DSD standard deviation $\sigma$ from the LES at 3 h using the (a) Log(2) and (b) Log(2$^{1/4}$) mass grids. Dashed lines indicate the mean profiles.

![Image](image.png)

**Fig. 5.** Contour frequency by altitude diagrams of the difference between the 99% cumulative distribution radius $D_{R_{99}}$ and the median radius $D_{\text{median}}$ from the LES at 3 h using the (a) Log(2) and (b) Log(2$^{1/4}$) mass grids. Dashed lines indicate the mean profiles.
collision–coalescence growth are presented to estimate the potential effects of numerical errors on DSD evolution and precipitation formation. We emphasize that these tests are not intended to be a realistic representation of DSD evolution, especially because horizontal variability and mixing are not considered, but rather to test the effects of numerical diffusion compared to the benchmark solutions.

The mass growth rate of a droplet from condensation is calculated as

\[
\frac{dm}{dt} = 4\rho_w \pi R^2 \frac{dR}{dt} = 4\rho_w \pi RS \left( \frac{L_v \rho_w}{K R_v T^2 + \rho_w R_v T} \right)^{-1}, \tag{1}
\]

where \( m \) is drop mass, \( t \) is time, \( R \) is drop radius, \( \rho_w \) is the density of liquid water, \( L_v \) is the enthalpy of vaporization, \( \kappa \) is the thermal conductivity of air, \( R_v \) is the gas constant for water vapor, \( T \) is temperature, \( e_s \) is the equilibrium vapor pressure, \( D_v \) is the diffusivity of water vapor in air, and \( S = q_v - q_s \) is the ambient vapor excess (\( q_v \) and \( q_s \) are the water vapor and equilibrium vapor mixing ratios, respectively).

To test condensational growth on an Eulerian mass grid, two different methods are tested: 1) a one-moment approach (MPDG) prognosing the mass mixing ratio in each bin and treating condensational growth as 1D advection in mass space using the Multidimensional Positive Adveective Transport Algorithm (MPDATA; Smolarkiewicz and Grabowski 1990; Smolarkiewicz and Margolin 1998) except that their first-order upwind scheme is replaced by the more accurate MPDATA advection scheme. Because TH-MOM is two moment, for the same mass grid it requires twice as many prognostic variables as the one-moment MPDG approach.

MPDG and TH-MOM are tested because they are representative of the wide range of approaches that have been used to solve condensational growth in bin schemes; as will be shown, MPDG is quite diffusive, whereas TH-MOM retains narrow distributions in rising parcel tests without Eulerian transport in physical space. Tests of various mass grid configurations are also presented. These include grids that have a linear increase in radius and grids with a logarithmic increase in mass, including the commonly used mass doubling grid. A list of all the bin model configurations tested is given in Table 1.

### b. Spatial advection schemes

For any microphysical variable \( \psi \) expressed as a mixing ratio, the conservation equation can be written generally as

\[
\frac{\partial \psi}{\partial t} + \nabla \cdot (\rho \mathbf{u} \psi) = \frac{1}{\rho} \frac{\partial (\rho V \psi)}{\partial z} + S_{\psi} + D(\psi), \tag{2}
\]

where \( \mathbf{u} \) is the wind vector, \( \rho \) is the air density, \( V \) is the fall speed, \( D \) is an explicit subgrid-scale diffusion operator, and \( S_{\psi} \) is the microphysical source–sink term, in our case, caused by condensational growth.

Several numerical methods have been developed to solve the problem of hydrometeor transport by air motion [i.e., the first term on the right-hand side of (2)]. A comprehensive evaluation of the various advection schemes used in all cloud models is not feasible; instead we test a handful of schemes that have been implemented in two widely used models: the Weather Research and Forecasting (WRF) Model (Skamarock et al. 2008) and the Cloud Model, version 1 (CM1; Bryan and Fritsch 2002). While some details differ, the WRF-based schemes are very similar to the scalar advection scheme used in the UCLA-LES; they both employ Eulerian flux-based finite-volume methods with flux limiters to retain monotonicity. As will be shown in section 4c, there is limited sensitivity of the DSD broadening caused by vertical numerical mixing to a wide range of advection methods.

We use the third-order Runge–Kutta time stepping approach from Wicker and Skamarock (2002), given by

\[
\begin{align*}
\psi_i^{*} &= \psi_i - \frac{\Delta t}{2\Delta z} (F_{i+1/2} - F_{i-1/2}), \\
\psi_i^{*\ast} &= \psi_i^{*} - \frac{\Delta t}{2\Delta z} (F_{i+1/2}^{*} - F_{i-1/2}^{*}), \\
\psi_i^{\ast\ast} &= \psi_i^{*\ast} - \frac{\Delta t}{\Delta z} (F_{i+1/2}^{*\ast} - F_{i-1/2}^{*\ast}), \\
\psi_i^{*+\Delta t} &= \psi_i^{\ast\ast} + S_{\psi} \Delta t, \tag{3}
\end{align*}
\]

where spatial and time levels are indicated by the subscript and superscript, respectively; \( F \) is the advective flux; \( \Delta z \) is the vertical grid spacing; \( \Delta t \) is the time step;
and *, **, and *** indicate provisional time levels after the first, second, and third Runge–Kutta steps for advection, respectively. Following WRF and CM1, we use a sequential splitting approach to determine $\psi^{i+\Delta t}$ whereby $S_0$ is calculated from the state variables after advection (at time level ****) and the result added to $\psi^{***}$; this is the last equation in (3). Note that droplet sedimentation and explicit diffusion [second and fourth terms on the right-hand side of (2)] are neglected for simplicity, and for consistency with the LES configuration.

Different time-stepping methods have little impact on the results presented herein. Using the method that Grabowski and Jarecka (2015) applied in their bin microphysics model, which calculates $S_0$ using the state variables at time $t$ (before advection) and advects $\psi^t + S_0 \Delta t$, shows almost no differences compared to using the sequential splitting method given by (3). Moreover, tests replacing the third-order Runge–Kutta stepping for the advection in (3) with a simple forward Euler approach are very similar. Thus, the particular choice of time stepping method does not affect our main findings.

Various schemes for calculating the 1D advective fluxes in (3) are tested; these are listed in Table 2. The WRF-based schemes (WRF-3RD and WRF-5TH) approximate the advective flux divergences in (3) using spatial Taylor series expansions following Wicker and Skamarock ([2002], see their (4a)–(4d)], combined with the monotonic flux corrected transport (FCT) scheme of Wang et al. (2009). The FCT scheme is only applied at the third Runge–Kutta step. We also test a simple first-order upwind scheme (FIRST). FCT is not applied using this scheme because it is already monotonic.

Several weighted essentially nonoscillatory (WENO) advection schemes are also tested. A fifth-order scheme (WENO-5TH) is based on Jiang and Shu (1996), although we use the modified formulation for the WENO “smoothness indicators” from Borges et al. (2008). The seventh- and ninth-order schemes (WENO-7TH and WENO-9TH) are based on Balsara and Shu (2000) and also use modified smoothness indicators following the approach of Borges et al. (2008). Similar to the FCT schemes, WENO is only applied during the third Runge–Kutta step, whereas flux calculations based on standard Taylor series expansions of a given order are applied during the first two steps (this follows from the approach used in WRF and CM1).

c. Diagnostic collision–coalescence growth rate

We diagnose the collision–coalescence growth rate $G_{cc}$ of the largest drops within the DSD occurring at a “significant” concentration. A significant number is defined here as $dN/dt$ larger than $10^{-8}$ kg$^{-1}$ m$^{-1}$, or about five orders of magnitude smaller than the distribution peak in the tests below. The simulations presented here include vapor growth only; the collision–coalescence growth rates are diagnostic with no feedback to the DSD. This allows us to identify the time and/or height at which collision–coalescence growth would become significant, for example, compared to the condensational growth rate.

The coalescence growth rate is diagnosed following Bott (1998), whereby the collision kernel $K_c$ is calculated as

$$K_c(i,j) = \pi E_c(i,j)(R_i + R_j)^2|V_i - V_j|,$$

where $R_i$ and $R_j$ are the radii of the interacting ($i$, $j$) droplet pair, $V_i$ and $V_j$ are the fall speeds of the pair, and $E_c(i,j)$ is the collection efficiency that is found by linearly interpolating the tabulated values of Hall (1980). The fall speed is calculated following Stokes’s formulation (neglecting the density of air relative to liquid water):

$$V = \frac{2g\rho_w R^2}{9\mu},$$

where $g$ is the acceleration of gravity and $\mu$ is the dynamic viscosity of air. Following Bott (1998) the collision kernel for the interaction of drops in bin $i$ with those in bin $j$ is found by a weighted average that includes $K_c$ from neighboring bin pairs:

$$K_{ij} = 0.25[K_c(i - 1, j) + K_c(i, j - 1) + K_c(i + 1, j) + K_c(i, j + 1)] + 0.5K_c(i, j).$$

It follows that the radius growth rate of a drop in bin $j$ is given by

$$G_{cc} = \sum_{i=1}^{N_b} K_{ij} \frac{M_i}{4\pi \rho_w R_{ij}^2},$$

where $N_b$ is the number of bins, and $M_i$ is the mass mixing ratio in bin $i$. Here, $R_i$ is taken as the mean of the radius at the boundaries of bin $j$, where bin $j$ is the largest bin with a “significant droplet number” as defined above.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WRF-5TH</td>
<td>WRF fifth-order monotonic scheme</td>
</tr>
<tr>
<td>WRF-3RD</td>
<td>WRF third-order monotonic scheme</td>
</tr>
<tr>
<td>FIRST</td>
<td>First-order upwind</td>
</tr>
<tr>
<td>WENO-9TH</td>
<td>WENO ninth order</td>
</tr>
<tr>
<td>WENO-7TH</td>
<td>WENO seventh order</td>
</tr>
<tr>
<td>WENO-5TH</td>
<td>WENO fifth order</td>
</tr>
</tbody>
</table>

TABLE 2. List of the advection schemes tested. A description of the schemes is given in section 3b.
4. Numerical test results

a. 1D tests analogous to the LES configuration

We first discuss results for a configuration of the 1D model that follows the LES setup described in section 2. These tests use a cloud-base temperature of 286 K; cloud-base and cloud-top heights of approximately 500 and 850 m, respectively; and a droplet concentration of 61 cm\(^{-3}\); these are equal to the mean values from the LES. The water vapor mixing ratio at the lower boundary (i.e., cloud base) is specified to give a supersaturation of 0.3%; this is based on the maximum supersaturation of an ascending parcel for these conditions. A constant vertical velocity of 0.5 m s\(^{-1}\) is specified, which is roughly similar to the updraft strength in the LES. As shown in section 4c, there is limited sensitivity of DSD broadening in the 1D tests to \(w\). The same vertical grid as the LES is utilized. As in the LES, these tests use the TH-MOM condensation method and two different mass grids: the mass doubling Log(2) grid and the high-resolution Log(2\(1/4\)) grid.

Potential temperature and water vapor mixing ratio are prognostic variables that are advected with sources/sinks caused by condensation and latent heating. Thermodynamic conditions at the lower boundary are held constant at the initial values given above. The cloud droplet population at the lower boundary is specified by placing the total concentration of 61 cm\(^{-3}\) in the smallest mass bin, consistent with the LES in which newly activated droplets were put in the smallest bin. Above cloud top, all droplets are simply removed from the distribution each time step; this leads to some small oscillations in cloud quantities at cloud top but otherwise has a limited impact on the results. Simulations are run for 1 h with a 1-s time step; near-steady-state conditions are achieved within a few minutes near cloud base and a few minutes longer near cloud top, corresponding to the time scale for advective transport through the cloud depth. These tests use the WRF-5TH monotonic scheme (see Table 2), for advection in physical space. Because higher-order advection schemes require information for multiple upwind (and downwind) grid levels, the lower boundary is treated by assuming constant conditions below the lower boundary (height of maximum supersaturation). Additional tests using reduced-order flux calculations along the boundary, as is often done in models (e.g., WRF and CM1), or simply zero cloud water below cloud base, show little impact on the results.

Vertical profiles of bulk cloud mass mixing ratio (summed over the bins), mean radius, \(\sigma\), and \(\Delta R_{99}\) at 1 h are shown in Fig. 6. The profiles are nearly adiabatic, as expected, except for some oscillations at cloud top. The DSDs are broad even though the model only includes vertical advection and condensational growth, with \(\sigma\) of \(\sim 2 \mu m\) just above cloud base decreasing to \(\sim 1–1.5 \mu m\) higher in the cloud. The term \(\Delta R_{99}\) is \(\sim 6 \mu m\) just above cloud base, decreasing to \(\sim 4 \mu m\) near cloud top. These wide DSDs occur despite drops being present in only a single bin at the lower boundary. The Log(2) and Log(2\(1/4\)) mass grids give similar results, with somewhat more vertical variability in \(\sigma\) and \(\Delta R_{99}\) using Log(2).

A key point is that the profiles of \(\sigma\) and \(\Delta R_{99}\) in the 1D tests are similar to those from the LES (cf. Figs. 6 and 4–5). Thus, the 1D model, with only vertical advection and condensational growth, gives DSDs that are nearly as broad as the LES and comparably broad to observed DSDs in stratocumulus clouds. As shown in section 4c, this finding is general and similar results are evident over a wide range of conditions and configurations of the 1D model. There are only two possible explanations for the broad DSDs above cloud base in the 1D tests, and both are related to the numerics: 1) broadening from numerical diffusion across bins during condensational growth and 2) broadening owing to numerical diffusion associated with vertical advection. We discuss each of these in detail in the remainder of this section.

b. Comparison of bin scheme and Lagrangian microphysical solutions in a parcel framework

This subsection presents tests droplet condensational growth within a rising parcel. These tests exclude Eulerian transport in physical space so that we can focus solely on DSD broadening from numerical diffusion across bins during condensational growth. Feedbacks between the microphysics, temperature, supersaturation, and hydrostatic pressure are included as the parcels rise. The numerical benchmark is a Lagrangian microphysical solution with 200 initial drop sizes distributed uniformly between 1 and 3 \(\mu m\). In contrast to the bin model solutions, the Lagrangian solution does not use an Eulerian grid in mass space. Instead, the growth rate of each initial drop size is calculated directly by integrating (1) together with equations for evolution of temperature and water vapor mixing ratio using a simple Euler forward-in-time method. For both the Lagrangian benchmark and bin numerical solutions, the time step is 1 s. The Lagrangian numerical benchmark solution is well converged with a 1-s time step; reduction of the time step to 0.25 s has almost no impact on the results. Unlike the 1D tests in section 4a, here all of the bin model configurations are initialized with a top-hat distribution of drops with sizes between 1 and 3 \(\mu m\) consistent with the Lagrangian benchmark. This ensures more consistent initial size distributions using the various mass grids than simply placing all droplets in the smallest bin. The initial supersaturation, temperature, and pressure are 0.6\%, 288.15 K, and 900 hPa,
respectively. For simplicity, the vertical velocity is constant at 1 m s$^{-1}$. For all configurations, the supersaturation decreases monotonically from its initial value. The initial value of supersaturation is based on additional parcel tests indicating a maximum supersaturation of approximately 0.6% under these conditions (1 m s$^{-1}$ updraft and 50 mg$^{-1}$ droplet number mixing ratio). Results are analyzed for 700 m of parcel ascent.

Figure 7 presents DSDs for each bin model configuration and the Lagrangian benchmark at various heights (converting time to height using the 1 m s$^{-1}$ vertical velocity). The Lagrangian benchmark shows a narrowing of
the DSD with increasing time and/or height, which is a well-known feature of condensational growth (e.g., Pruppacher and Klett 1997). All of the TH-MOM configurations do a reasonable job of reproducing narrow DSDs from the Lagrangian numerical benchmark, insofar as allowed by the bin spacing. However, one-moment MPDG growth produces significant numerical diffusion and DSD broadening relative to the Lagrangian benchmark and all of the TH-MOM configurations, including those with relatively coarse bin spacing. All of the bin model configurations reproduce supersaturation evolution well compared to the Lagrangian benchmark (not shown) as well as evolution of bulk mass and mean radius (Figs. 8a,b).

Vertical profiles of $\sigma$, $\Delta R_{99}$, and the collision–coalescence growth rate of the largest droplet size with a significant concentration $G_{cc}$ are shown in Figs. 8b–d. There are some differences in the three quantities among the different mass grid configurations for TH-MOM simply as a result of differences in bin width. TH-MOM with the 1-μm-radius grid [“Linear(1 μm)’’] gives $\sigma$ and $\Delta R_{99}$ about 3–4 times that of the Lagrangian benchmark.

**Fig. 7.** Drop size distributions at various heights $z$ from the Lagrangian microphysical benchmark (black) and the bin model simulations (colored lines) for the parcel test with a bulk drop number mixing ratio of 50 mg$^{-1}$. Different colored lines illustrate results using different bin mass grid configurations and growth methods, as listed in Table 1.
whereas the mass doubling grid Log(2), which has the coarsest bin spacing tested for drops larger than about 7 μm, gives σ about 20–50 times larger and ΔR\textsubscript{99} about 10 times larger. Similarly, values of G\textsubscript{cc} are about 2–3 times larger than the Lagrangian benchmark for the Linear(1 μm) grid and approximately 10 times larger for Log(2). All of the TH-MOM configurations result in smaller G\textsubscript{cc} than the condensational growth rate (approximated

![Figure 8: Vertical profiles of (a) bulk mass mixing ratio Q, (b) mean drop radius R (solid lines) and standard deviation σ (dotted lines), (c) 99% cumulative ΔR (ΔR\textsubscript{99}), and (d) collision–coalescence radius growth rate G\textsubscript{cc} (as defined in the text) for the parcel test with a bulk droplet number mixing ratio of 50 mg\textsuperscript{-1}. The dotted black line in (d) indicates the radius growth rate from condensation for the mean droplet size from the numerical benchmark solution, which is similar to the condensational growth rate at the mean droplet size for all of the bin model configurations. Different colored lines illustrate results using different bin mass grid configurations and growth methods, as listed in Table 1.](image-url)
by the dotted line in Fig. 8d). Using the one-moment MPDG method with the linear 1-μm-radius grid for condensational growth leads to values of $\Delta R_{99}$ about a factor of 2 larger and $G_{cc}$ an order of magnitude larger than Log(2), and there are even larger differences compared to the other TH-MOM configurations. This illustrates considerable numerical diffusion across bins that results from a one-moment Kovetz and Olund (1969)-type approach for condensational growth.

Additional tests specifying different bulk droplet numbers give similar overall results. For example, specifying a bulk droplet number mixing ratio of 250 mg$^{-1}$ gives similar differences among the various TH-MOM and MPDG configurations, with reduced supersaturations and growth rates compared to the tests with bulk number of 50 mg$^{-1}$ as expected.

### c. Additional tests using the 1D model

In this subsection, several additional tests using the 1D model are analyzed to further investigate numerical DSD broadening. For the numerical benchmark, we employ the same rising parcel Lagrangian microphysical calculations described in section 4b, and translate time evolution to height using the specified constant vertical velocity to compare with the 1D simulations. Tests using a range of time steps, grid spacings, and vertical velocities are presented below. In contrast to the LES-like setup of the 1D model, a constant vertical grid spacing is used here to further simplify the model configuration and interpretation of results. We first show results using a 1-s time step, 20-m grid spacing, and 1 m s$^{-1}$ vertical velocity. A top-hat distribution of droplets between 1 and 3 μm with a bulk number mixing ratio of 50 mg$^{-1}$ is assumed at the lower boundary following initial conditions for the parcel tests in section 4b. Using narrower or broader DSDs at the lower boundary does not affect the main findings. Cloud quantities are set to zero above a height of 1400 m, with virtually no impact on the analysis region between 0 and 700 m.

Similar to the parcel tests described above, all of the bin model configurations well reproduce the Lagrangian numerical benchmark bulk mass and mean radius profiles (Figs. 10a,b) and supersaturation (not shown). However, in contrast to the parcel tests, all model configurations give significant spectral broadening relative to the numerical benchmark, as illustrated in Fig. 9. DSD broadening relative to the benchmark occurs using either TH-MOM or MPDG as the growth method. The DSDs are skewed toward larger sizes indicating a tendency to broaden toward bigger, not smaller, drops. This broadening is evident by values of $\sigma$ and $\Delta R_{99}$ that are about 10–100 times larger than the benchmark, and values of $G_{cc}$ that are approximately 1000 times larger (Figs. 10b,c,d). Values of $G_{cc}$ for the bin model tests exceed the diffusional growth rate of the mean drop size at heights greater than about 400–650 m, implying significant precipitation production starting around these heights. In contrast, values of $G_{cc}$ remain two orders of magnitude smaller than the diffusional growth rate for the Lagrangian numerical benchmark.

A key result is that the spectral broadening relative to the numerical benchmark is similar regardless of the bin resolution, so that all of the TH-MOM configurations give similar values of $\sigma$, $\Delta R_{99}$, and $G_{cc}$. This result is notably different from the parcel tests but consistent with the LES and LES-like configuration of the 1D model; finer bin resolution gives results closer to the numerical benchmark for the parcel tests, but it does not lead to reduced broadening when condensational growth is combined with Eulerian vertical transport. Similarly, although the MPDG method for droplet growth is clearly much more diffusive than using TH-MOM in the parcel tests, the differences are much smaller when growth is combined with Eulerian advection. In particular, the DSDs are similar using MPDG and TH-MOM growth methods in the lowest ~200 m in the Eulerian framework, and only somewhat broader using MPDG above this level (Figs. 9, 10).

The DSD broadening in the 1D tests relative to the numerical benchmark is explained conceptually as follows. In a flux-based Eulerian framework, there is a flux of drops at level $k$ to the level above on each time step when there is upward motion, and a flux of drops from the level below up to $k$. Because mean drop size increases with height, there is an upward flux of relatively small drops into $k$ compared to the flux of larger drops out of $k$. The divergences of these fluxes therefore tend to increase the drop concentrations in smaller bins but decrease the concentrations in larger bins at level $k$. Subsequent condensational growth shifts the DSDs to larger bins. This is repeated in subsequent time steps, leading to drops being present across several bins at a given level and hence DSD broadening. This highlights the tight coupling of transport in physical space and the “transport” of drops in mass space from condensational growth. Fundamentally, this close coupling means that numerical diffusion in the vertical in effect leads to diffusion across bins in mass space, and hence DSD broadening. In the absence of vertical numerical diffusion there is no broadening except from diffusion across mass bins during condensational growth calculations. This is demonstrated by tests with the 1D model using first-order upwind advection and flow conditions such that the Courant–Friedrichs–Lewy (CFL) number is equal to 1. This gives no vertical numerical diffusion because all drops from a level are transported exactly to
the next level above, and hence there is no DSD broadening beyond that seen in the parcel tests (not shown). Similarly, all drops at a given level are transported a distance \( w \Delta t \) during each time step in the Lagrangian framework. Thus, drops of different sizes from different levels are not vertically mixed artificially by numerical diffusion, allowing the DSDs to remain narrow and avoiding broadening.

Using a bulk droplet number mixing ratio of 250 mg\(^{-1}\) at the lower boundary instead of 50 mg\(^{-1}\) leads to reduced supersaturation and smaller growth rates as expected (Fig. 11). Because of the reduced growth rate, vertical gradients of mean drop radius are smaller (Fig. 11b), leading to smaller errors relative to the benchmark than the tests with 50 mg\(^{-1}\) in terms of \( \sigma \), \( \Delta R_{99} \), and \( G_{cc} \). This is particularly true for the TH-MOM configurations (cf. Figs. 11b,c,d and 10b,c,d). Nonetheless, values of \( \sigma \), \( \Delta R_{99} \), and \( G_{cc} \) are still an order of magnitude larger than in the benchmark, and the TH-MOM configurations give similar results to one another regardless of bin resolution. Differences relative to the benchmark for MPDG are 2–5 times larger than they are for any of the TH-MOM configurations. Additional tests indicate that DSD broadening is limited, relative to the parcel tests.
when the specified droplet number is increased to greater than about 500 mg\(^{-1}\) (not shown). The feedback between drop growth and supersaturation is not responsible for broadening; additional tests that specify a height-varying supersaturation from the Lagrangian numerical benchmark give very similar results to the tests with predicted supersaturation.

To investigate the effects of explicitly including droplet activation instead of specifying the DSD at the lower boundary, additional tests apply a droplet activation rate near cloud base. In these tests, a droplet activation rate of 2.5 mg\(^{-1}\) s\(^{-1}\) is applied within the lowest model level (with a depth of 20 m) above cloud base, and a constant supersaturation of 0.6% is specified.

**Fig. 10.** As in Fig. 8, but for the combined growth and Eulerian spatial advection test with a bulk droplet number mixing ratio of 50 mg\(^{-1}\), vertical grid spacing of 20 m, time step of 1 s, and vertical velocity of 1 m s\(^{-1}\).
for simplicity. This activation rate is chosen because it gives a bulk droplet number mixing ratio through the cloud layer of approximately 50 mg. Newly activated droplets follow a top-hat distribution between 1 and 3 μm. The numerical benchmark is a Lagrangian solution that activates droplets with a radius between 1 and 3 μm at a rate of 0.125 mg s⁻¹ at 1-m intervals in the lowest 20 m above cloud base, giving the same net activation rate as the Eulerian tests. Overall results are similar to the 1D tests with a fixed lower boundary DSD (not shown), with σ and ΔRPg about a factor of 4–20 larger for the bin model solutions than the numerical benchmark. Values of Gcc are two to three orders of magnitude larger than the benchmark.
As mentioned above, there is no DSD broadening from vertical numerical diffusion if the CFL number is 1, using first-order upwind advection. This implies sensitivity of broadening to the CFL number, which we explore by varying the time step, vertical grid spacing, and prescribed vertical velocity. These parameters are each varied to give a range of vertical CFL numbers from 0.025 to 0.2, which may be considered fairly typical for LES of stratocumulus or shallow cumulus (e.g., Ackerman et al. 2009; vanZanten et al. 2011). Tests that vary the time step $\Delta t$ are described first. Values of $\Delta t$ of 0.5, 1, 2, and 4 s are tested, which covers the range typically used in cloud and LES models with bin microphysics (e.g., Stevens et al. 1996a; Khain et al. 2004; Ackerman et al. 2009; Xue et al. 2017). These tests use a grid spacing $\Delta z = 20$ m and 1 m s$^{-1}$ updraft. Results are summarized by averaging $\Delta R_{99}$ between heights of 400 and 700 m. As shown in Fig. 12a, there is little sensitivity despite the fairly wide range of $\Delta t$ and hence CFL numbers tested.

To examine the sensitivity to $\Delta z$, additional tests are performed with $\Delta z$ equal to 5, 10, 20, and 40 m using $\Delta t = 1$ s and a 1 m s$^{-1}$ updraft. In contrast to $\Delta t$, there is considerable sensitivity of the DSDs and specifically $\Delta R_{99}$ to $\Delta z$ (Fig. 12b). Spectral broadening is notably reduced as $\Delta z$ is decreased using TH-MOM for droplet growth, resulting in $\Delta R_{99}$ about two orders of magnitude smaller using $\Delta z = 5$ m compared to 40 m. This is further illustrated by DSD plots for the Log(2$^{1/4}$) mass grid using various $\Delta z$ (Fig. 13). In contrast, there is little sensitivity to $\Delta z$ using MPDG as the growth method because it gives considerable DSD broadening without Eulerian spatial advection, as evident from the parcel tests (see Figs. 7 and 8). Thus, results using MPDG appear to be reasonably well converged using $\Delta z$ of 20–40 m. This likely explains the results of Clark (1974), who found that DSD broadening from numerical diffusion in physical space was fairly small and convergence could be achieved with only a moderate increase in spatial resolution. However, he used a one-moment Kovetz and Olund (1969)-type condensational growth method roughly similar to MPDG, which was presumably already quite diffusive even without Eulerian advection in physical space.

The vertical velocity affects both transport in physical space and the supersaturation. This sensitivity is examined by tests with $w$ equal to 0.5, 1, 2, and 4 m s$^{-1}$ using $\Delta z = 20$ m and $\Delta t = 1$ s. For $w$ equal to 0.5, 1, 2, and 4 m s$^{-1}$, the maximum supersaturations are approximately 0.3%, 0.6%, 1.1%, and 1.7%, respectively. These values of supersaturation are prescribed at the lower boundary. Overall there is a tendency for mean $\Delta R_{99}$ to decrease with $w$ increased from 0.5 to 4 m s$^{-1}$, except

![Fig. 12. Sensitivity of the mean difference in 99th-percentile cumulative distribution radius and the median radius $\Delta R_{99}$ to (a) time step $\Delta t$, (b) vertical grid spacing $\Delta z$, and (c) vertical velocity $w$. Results shown are for the tests with bulk droplet number mixing ratio of 50 mg$^{-1}$. The presented $\Delta R_{99}$ is an average between heights of 400 and 700 m; $\Delta R_{99}$ from the Lagrangian numerical benchmark is shown by the horizontal black line/symbols and differs slightly with changes in $w$ due to changes in the supersaturation.](image-url)
using the Log(2) mass grid. However, this sensitivity is considerably smaller than the sensitivity to $\Delta z$ over the same range of CFL number.

Finally, results using the various spatial advection schemes listed in Table 2 are compared. These tests use $\Delta z = 20$ m, $\Delta t = 1$ s, and a $1$ m s$^{-1}$ updraft velocity. For brevity, we only describe tests using the logarithmic bin mass grids [Log(2) and Log($2^{1/4}$)] with the TH-MOM growth method; tests using the linear-in-radius grids give similar overall results. To isolate the sensitivity of cloud transport to advection method we use the WRF-5TH scheme to advect potential temperature and water vapor mixing ratio in all tests and only vary the advection scheme used for the bin microphysical variables.

To provide context for the 1D growth plus transport tests described below, we first illustrate the performance of the advection schemes for the problem of pure transport. This is done by comparing results for 1D linear advection tests with a constant CFL number, allowing evaluation against an analytic benchmark solution. Figures 14 and 15 show results using the advection schemes described in section 3b, although here we use the FCT algorithm of Zalesak (1979), which is slightly different from the FCT algorithm in WRF (Wang et al. 2009). Two different initial
scalar distributions are tested. The first is well resolved, with a Gaussian distribution that is roughly 10 grid points wide (10\(\Delta x\)), as shown in Fig. 14. The second is poorly resolved, with a distribution that is only 2 grid points wide (2\(\Delta x\)), as shown in Fig. 15. In all of these linear advection tests, we use 100 total grid points, a CFL number of 0.1, and periodic boundary conditions, and we run the test for 1000 time steps. This means the final results would be the same as the initial state in the absence of advection errors. For the well-resolved 10\(\Delta x\) tests, numerical diffusion is much less problematic as the order of the advection scheme is increased (Fig. 14). In this case, the WENO-5TH scheme produces slightly better results than FCT-5TH (cf. Figs. 14c,d), and very little diffusion occurs with the WENO-9TH scheme (Fig. 14f). In contrast, for the poorly resolved 2\(\Delta x\) tests all of the schemes lead to considerable

![Fig. 14. Linear scalar transport tests using various advection schemes with the well-resolved initial scalar distribution covering approximately 10 grid points (10\(\Delta x\)). Gray lines show the benchmark analytic solution, and blue lines show the numerical solutions. The maximum value of the advected scalar is shown in the upper right of each panel; the maximum value is 1 for the analytic solution. These results are after 1000 time steps for a constant CFL number of 0.1. FCT-5TH and FCT-3RD are analogous to the WRF-5TH and WRF-3RD schemes in Table 2, differing only slightly in how they apply FCT.](image-url)
diffusion, and differences using the high- and low-order schemes relative to the analytic solution are fairly small (Fig. 15).

For transport in physical space plus condensational growth there are few consistent trends and the different advection schemes produce broadly similar results, as illustrated by profiles of $\Delta R_{\text{hy}}$ (Fig. 16), although WRF-5TH produces somewhat greater DSD broadening for the Log(2) mass grid. Based on these results and the linear advection tests described above, we attribute this lack of sensitivity to the advection method in part to the inability of the $\Delta z = 20$-m vertical grid to resolve narrow regions with mass (and number) in a given bin, even with the higher-order schemes (similar to the $2\Delta x$ linear advection tests). The rising parcel tests show that without vertical numerical diffusion, for example, drops are present across a depth of only 16 m for bin 12 (5.8-μm mean radius), 74 m for bin 14 (9.2-μm mean radius), and 311 m for bin 16 (14.5-μm mean radius), using the Log(2) mass grid (orange lines in Fig. 17). These regions are even narrower for the Log($2^{1/4}$) mass grid (purple lines in Fig. 17).

Regions with mass/number in larger-size bins are wider because the growth rate decreases with drop size.

**Fig. 15.** As in Fig. 14, but for the poorly resolved initial scalar distribution covering two grid points ($2\Delta x$).
and because the mass doubling grid means that larger bins are relatively wide in radius space. Although wider for larger bins, based on the rising parcel tests, these regions still have sharp vertical gradients of the number and mass mixing ratios (Fig. 17) that are difficult to resolve even with higher-order advection schemes. Difficulty resolving these features also helps to explain the sensitivity to $D_z$; reducing $D_z$ means that these narrow regions and sharp vertical gradients are better resolved and spatial advection errors are reduced. Because regions with mass/number in a given bin become narrower with sharper gradients as the bin resolution is increased (cf. the purple and orange lines in Fig. 17), it is even more difficult to resolve these features. This helps to explain why increasing bin resolution does not reduce DSD broadening. On the other hand, using MPDG as the growth method instead of TH-MOM leads to much broader vertical regions of mass/number for a given bin and weaker vertical gradients in the parcel tests (not shown), which are easier to resolve. This occurs because MPDG growth produces considerable numerical DSD broadening even without Eulerian spatial advection in the parcel tests. Overall, these results suggest that there is compensation between spatial advection and condensational growth errors, leading to broad DSDs even with better growth methods or increased bin resolution because of larger spatial advection errors.

The results discussed above highlight the close coupling between transport in the physical space dimensions and transport in mass space for condensational growth. In essence, a coupled four-dimensional advection problem must be solved for bin microphysics implemented in a 3D dynamical model, with transport in nearly incompressible flow in the three spatial dimensions, and in compressible “flow” in the mass dimension. Transport in each dimension is closely linked to transport in the other dimensions. Thus, numerical diffusion in physical space inevitably leads to diffusion in mass space, producing the spectral broadening seen here. In contrast, condensation and spatial transport for bulk microphysics is a 3D problem. Even though Eulerian models separately calculate spatial advection and condensation using bulk

Fig. 16. Profiles of the mean difference in 99th-percentile cumulative distribution radius and the median radius $\Delta R_{99}$ using the (a) Log(2) and (b) Log($2^{1/4}$) mass grids. Colored lines show results using the various spatial advection schemes defined in Table 2. Black lines indicate the Lagrangian numerical benchmark.
microphysics, this is of little consequence except for modeling the peak supersaturation just above cloud base (Clark 1974; Morrison and Grabowski 2008). Condensation is a fast process, and the thermodynamics and bulk cloud mass adjust rapidly with limited error by separately advecting quantities and then calculating bulk condensation at each time step. Therefore, the problem of unphysical, numerical DSD broadening is not a result of inconsistencies in the thermodynamic variables or advection of nonconservative quantities but rather is due to the unique, coupled four-dimensional advection problem that bin microphysics in a 3D Eulerian dynamical model presents.

5. Discussion and conclusions

In this study, we investigated numerical errors that arise from the combination of droplet condensation growth and vertical advection using a bin microphysics scheme coupled with Eulerian dynamical models. This was done using a hierarchy of models including parcel, 1D, and 3D dynamical models.

We first presented results from 3D LES of the DYCOMS II RF02 case using the Ackerman et al. (2009) model intercomparison study setup. Droplet activation, condensation, and evaporation were the only microphysical processes included in the LES, but nonetheless it produced wide spectra at all cloud levels, much broader than would be expected theoretically from condensation growth alone. Values of the DSD standard deviation $\sigma$ from the LES were similar to those observed in marine stratocumulus (e.g., Pawlowska et al. 2006; Snider et al. 2017). Several mixing mechanisms were potential contributors to DSD broadening in the LES. These mechanisms included numerical diffusion associated with horizontal advection, which is conceptually in line with various mixing processes that have been previously proposed to explain DSD broadening in real clouds (e.g., Cooper 1989; Lasher-Trapp et al. 2005; Korolev et al. 2013; Pinsky et al. 2014; Grabowski and Abade 2017). However, they also included broadening from numerical diffusion associated with vertical advection, which is unphysical and does not contribute to broadening in real clouds.

To isolate this unphysical DSD broadening, we utilized a 1D framework that included only vertical advection and condensation and neglected all other processes. The simplicity of this framework allowed direct comparison with Lagrangian microphysical solutions that provided a numerical benchmark. Tests using the 1D model in a setup analogous to the LES produced spectra that were about as wide as in the LES, quantified by the standard deviation $\sigma$ and the difference between the 99th-percentile cumulative distribution radius and the median radius $D_{99}$, which characterizes the DSD tail. We also performed tests using a rising parcel model without Eulerian advection in physical space. The parcel tests showed that the top-hat method of moments (TH-MOM) used in the LES for condensational growth captured the DSD characteristics reasonably well compared to the Lagrangian microphysical calculation, particularly when using fine bin resolution. These results therefore suggest that numerical diffusion across bins from the condensation calculation did not contribute significantly to DSD broadening in the LES, in contrast to numerical diffusion associated with the vertical advection. On the other hand, a simpler one-moment approach that treated growth as 1D advection on the mass grid (MPDG) produced considerable diffusion across mass bins and relatively wide spectra in the parcel tests.

Several additional tests using the 1D model were then presented. In contrast to the parcel tests, the spectra were much broader than the Lagrangian microphysical benchmark, highlighting the critical role of vertical numerical diffusion in DSD broadening. In contrast to the

---

1 Separately advecting a temperature-based quantity and water vapor mixing ratio can lead to generation of spuriously large supersaturations. This is due to the nonlinear dependence of supersaturation on temperature and water vapor combined with linear or semilinear advection, but primarily occurs at cloud edge rather than in the cloud interior (Stevens et al. 1996b; Grabowski and Morrison 2008).

---

**FIG. 17.** The vertical distribution of bin mass mixing ratio (x-axis) as a function of height (y-axis) from the rising parcel tests described in section 3a. Here, time is converted to height using the specified $1\;\text{m}\;\text{s}^{-1}$ vertical velocity. Results are shown for the Log(2) and Log($2^{1/4}$) mass grids. Solid, dotted, and dashed lines illustrate results for bins with mean radii of approximately 5.8, 9.2, and 14.5 $\mu$m, respectively.
much broader DSDs using the MPDG growth method compared to TH-MOM for the parcel tests, the DSDs were similar below ~200 m in the 1D Eulerian tests, and only somewhat broader using MPDG compared to TH-MOM above ~200 m. Because TH-MOM performed well in reproducing the benchmark solutions for the parcel tests, especially with fine bin resolution, it is unlikely that other numerical methods for condensational growth would lead to improvement in the Eulerian framework.

Based on these results, we propose that numerical diffusion associated with resolved-flow vertical advection is a key contributor to DSD broadening in LES models coupled with bin microphysics, even when the condensational growth calculations themselves contribute little to broadening (as when using TH-MOM). Although not included in the simulations here, explicit vertical subgrid-scale mixing would contribute to unphysical broadening similarly to vertical numerical diffusion. We emphasize that LES with bin microphysics do not necessarily produce overly broad spectra compared to those in real clouds, but that the DSD broadening may be dominated by an unphysical mechanism. This may explain why LES are able to produce reasonably broad spectra compared to observations (e.g., Pawlowska et al. 2006; Snider et al. 2017), despite neglecting or underresolving the physical processes that studies have shown are important for DSD broadening in real clouds, including giant CCN (e.g., Ludlam 1951; Woodcock et al. 1971; Feingold et al. 1999; Jensen and Nugent 2017), droplet clustering and microscale supersaturation fluctuations (e.g., Shaw 2000; Vaillancourt et al. 2002), and various physical DSD mixing processes (e.g., Cooper 1989; Lasher-Trapp et al. 2005; Korolev et al. 2013; Pinsky et al. 2014; Grabowski and Abade 2017). In particular, horizontal DSD variability is likely underresolved for typical LES grid resolutions (tens of meters), implying that DSD broadening from isobaric mixing of different droplet populations will be underrepresented. This underrepresentation is likely to be even more severe for cloud-resolving and convection-permitting models with horizontal grid spacing of order 1 km. It is reasonable to think that unphysical broadening from vertical numerical diffusion compensates for underrepresenting or neglecting these physical broadening mechanisms. Thus, although bin microphysics is a useful tool for many applications, it may be of less value for investigating physical mechanisms for DSD broadening when coupled with Eulerian dynamical models using typical vertical grid spacings (~10 m or larger).

Besides reducing the vertical grid spacing, minimizing unphysical DSD broadening from vertical numerical diffusion may require new methods calculating growth and physical transport in a single step given the close coupling of growth in mass space and transport in physical space. The development of such methods will be a focus of future research. We speculate that reducing unphysical DSD broadening may expose the underrepresentation of physical broadening mechanisms in LES with bin microphysics, possibly leading to unrealistically narrow DSDs. The results documented in this paper also help to further motivate the development and use of Lagrangian microphysics schemes coupled to Eulerian dynamical models that track representative droplet samples within the modeled flow (e.g., Shima et al. 2009; Andrejczuk et al. 2010; Riechelmann et al. 2012; Arabas et al. 2015; Hoffmann et al. 2017; Grabowski et al. 2018). Such an approach does not suffer from unphysical DSD broadening caused by the vertical numerical mixing. Moreover, Lagrangian droplet trajectories can be modeled using the resolved-scale flow plus a stochastic subgrid-scale component, and hence they provide a “natural” framework for representing physical DSD broadening from mixing of droplet populations that have undergone different growth histories (Grabowski and Abade 2017; Grabowski et al. 2018).

Acknowledgments. This work was partially supported by U.S. DOE Atmospheric System Research (ASR) Grant DE-SC0016579. JYH acknowledges support from the U.S. DOE ASR Grant DE-SC0012827. ZJL acknowledges support from the U.S. DOE ASR Grant DE-SC0016354. MKW acknowledges support from NCAR’s Advanced Study Program. High performance computing was provided by NCAR’s Computational Information Systems Laboratory. The National Center for Atmospheric Research is sponsored by the National Science Foundation. We thank J. Jensen for discussion and W. Grabowski for discussion and comments on an earlier draft of the paper. Comments and suggestions from A. Igel and two anonymous reviewers improved the paper. We also thank the UCLA-LES team for developing and maintaining the LES code, including B. Stevens, T. Heus, A. Seifert, and C. Hohenegger.

REFERENCES


Wood, R., S. Irons, and P. R. Jonas, 2002: How important is the spectral ripening effect in stratiform boundary layer clouds?


