Dynamic error estimation for radio occultation bending angles retrieved by the full spectrum inversion technique

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Application of any measurement requires reliable error estimates; this is particularly the case when meteorological observations are assimilated into numerical weather prediction models. Radio occultation measurement errors vary considerably between different observations. It is therefore desirable to estimate these errors dynamically on a profile-to-profile basis. In this study it is demonstrated that fluctuations in the full spectrum inversion (FSI) log amplitude can be mapped into profiles of error standard deviations for FSI retrieved bending angles on a profile-to-profile basis without using any external data. The performance of this technique is assessed by applying it to simulated GPS radio occultation signals sampled at 50 Hz with both severe additive noise and significant phase noise. In both cases, good agreement between predicted error profiles and the “true” error profiles is achieved. Comparisons between power spectra of predicted errors and “true” errors also show good agreement. However, a small part of the simulated signals is distorted by aliasing due to the downsampling to 50 Hz, which affects bending angles retrieved for a narrow range of impact parameters. For these bending angles, the predicted errors are too small as compared with the “true” errors. This shows that the proposed technique cannot predict errors related to poor signal acquisition.


1. Introduction

Radio occultation (RO) remote sensing of the Earth atmosphere allows for retrieval of temperature, pressure, and humidity. The basic observable in the RO technique is the Doppler shift imposed on a RO signal by refraction of radio waves propagating through the atmosphere between two satellites (e.g., a Global Positioning System (GPS) satellite and a low Earth orbiting (LEO) satellite) [Melbourne et al., 1994; Kursinski et al., 1997, 2000; Rocken et al., 1997]. The bending angles of the rays are directly related to the measured Doppler shifts through the occultation geometry, and may be inverted into a profile of refractivity using the Abel transform [Fjeldbo et al., 1971]. By applying the hydrostatic equation and the equation of state, so-called dry temperature profiles can be retrieved from profiles of refractivity, whereas retrieval of specific humidity for GPS occultation requires auxiliary data [e.g., Kursinski et al., 1997, 2000; Hajj et al., 2002].

GPS signals received by a LEO satellite are affected by phase modulations from the ionosphere that must be removed for reconstruction of the bending angle and refractivity profiles of the neutral atmosphere. At GPS frequencies the neutral atmosphere is nondispersive, whereas the ionosphere is dispersive. This allows for removal of the ionospheric effects to first order through a linear combination of pairs, corresponding to the two frequencies in the GPS system, of either signal phase [Spilker, 1980], phase modulation [Melbourne et al., 1994], bending angle [Vorob’ev and Krasil’nikova, 1994], or Doppler shift [Ladreiter and Kirchengast, 1996]. These techniques are referred to as ionospheric calibration.

For RO as for any observation system it is essential for the users of the data products to know the precision and the accuracy of these data. The vertical resolution of RO is diffraction limited to approximately 60 m [Gorbunov et al., 2004] at GPS frequencies, whereas the horizontal resolution is approximately 200 km. Because of the fairly low horizontal resolution of RO it is important to distinguish between measurement errors, being the errors related to measuring the Doppler fre-
quency, and the modeling errors, being errors related to the interpretation of retrieved bending angle and refractivity, e.g., the errors introduced by considering these observables as local measurements, according to definitions by Sokolovskiy et al. [2005].

In recent years a number of studies have evaluated the error statistics of RO. Kursinski et al. [1997, 2000], performed a thorough study of different RO error sources, including the modeling error related to treating the Abel retrieved refractivity as a local measurement, on the basis of first-order statistics deriving a complete error budget for retrieved refractivity, geopotential height, and temperature. Rocken et al. [1997] evaluated the accuracy of RO observation by comparing observations from the GPS/Meteorology MicroLab-1 (GPS/MET) satellite with correlative data from numerical weather prediction (NWP) models, radiosonde observations, and the GOES, TOVS, UARS/MLS and HALOE orbiting atmospheric sensors. Steiner et al. [1999] assessed the quality of GPS/MET RO data employing a Monte Carlo technique and by comparing the GPS/MET RO data to the analysis from the European Centre for Medium-Range Weather Forecasts (ECMWF). Syndergaard [1999] computed measurement errors statistics for bending angle, refractivity, pressure, and temperature on the basis of an error covariance analysis of the impact of background noise. A similar study was performed by Rieder and Kirchengast [2001] using Bayesian error analysis. Gorbunov and Kornblueh [2003] evaluated RO measurement errors from the GPS/MET and the Challenging Minisatellite Payload (CHAMP) satellites by comparing RO and ECMWF data on the basis of forward modeling and inversion of artificial radio occultation data. Kuo et al. [2004] estimated RO observation errors for the CHAMP and the Satellite de Aplicaciones Científicas-C (SAC-C) RO missions by comparing the measured refractivities to corresponding refractivities from a forecast and by jointly estimating the forecast errors. In that study measurement errors were also assessed by comparing soundings that fall within 300 km and two hours from each other. Hajj et al. [2004] assessed the precision and accuracy of RO measurements by comparing dry temperature profiles between pairs of CHAMP and SAC-C occultations occurring within 30 min and 200 km of one another. Steiner and Kirchengast [2005] computed error statistics for RO retrievals of bending angle, refractivity, temperature, pressure, and specific humidity on the basis of simulations.

Though the statistical properties of RO measurement errors can be estimated as a function of latitude and even season, such error estimates cannot distinguish between the large variations in the magnitude of the RO measurement errors between different occultations. For instance, the residual errors from the ionospheric calibration may vary by more than an order of magnitude between different occultations [Lohmann, 2005]. The magnitude and the vertical variations of errors due to small-scale atmospheric irregularities will also vary significantly on a profile to profile basis as these errors are related to the presence of water vapor. For those reasons it is desirable to estimate the measurement errors dynamically.

For assimilation of RO data into NWP models, the measurement errors must be added to the modeling and the representativeness errors. For assimilation of local refractivity and local bending angle (i.e., by assuming that the RO measurements represent point observations) the modeling errors are caused mainly by horizontal gradients in the atmosphere. As shown by Sokolovskiy et al. [2005] it is particularly important to include errors caused by horizontal gradients when assimilating local refractivities or local bending angles into high-resolution models (horizontal resolution <100 km). For low-resolution models (horizontal resolution of about 300 km or more), or when so-called nonlocal observation operators [see, e.g., Syndergaard et al., 2004] are applied, errors from horizontal gradients are greatly reduced [Sokolovskiy et al., 2005]. However, it should be noted, that it has not yet been demonstrated whether the reported error reductions by nonlocal observation operators will translate into similar impacts when RO observations are assimilated into an operational forecast system.

The scope of this study is dynamic error estimation of RO measurement errors. The first attempt to estimate RO measurement errors dynamically was proposed by Hocke et al. [1999] who dynamically estimated bending angle error profiles by relating the bending angle errors to the spectral line widths of the running spectra in the radioholographic method [Pavelyev, 1998]. Gorbunov [2002] and Lohmann [2004, 2005] applied dynamically error estimation in connection with statistical optimization of RO bending angles. Recently, Gorbunov et al. [2005] introduced a technique for dynamic error estimation of signal tracking errors for bending angle profiles retrieved from canonical transform CT2 [Gorbunov and Lauritsen, 2004] based on the width of the running spectra of the transformed wave field multiplied with a reference signal. The reference signal is obtained by smoothing the phase of the transformed field. The degree of smoothing applied to obtain the reference signal is basically a tunable parameter, which must be chosen as a trade-off between the possibilities of overestimating or underestimating the errors. If the transformed field contains contributions from real atmospheric structures with scales smaller than the smoothing window used to obtain the reference signal, the computed errors may be too large. On the other hand, if the smoothing window is too narrow errors with scales larger than the width of the smoothing window may be underestimated. Gorbunov et al. [2005] used a 250 m smoothing window.
In this study an approach is presented for dynamical error estimation of bending angle measurement errors that does not rely on any tunable parameters; the only limitation is that SNR (signal-to-noise ratio) of the transformed field must be larger than about five. This approach is based on full spectrum inversion (FSI) retrieval of bending angles [Jensen et al., 2003] and relies on the mapping of fluctuations in the FSI log amplitude to bending angle errors. Though this study focuses on FSI inversion the method presented here can also be applied to CT2 and other Fourier integral operator (FIO) based methods, as the error estimation technique proposed by Gorbunov et al. [2005] can also be applied to FSI.

This study is organized in the following form. In section 2, the mapping of log amplitude fluctuations to bending angle errors is described. Section 3 gives a description of the simulation system and the noise scenarios applied in this study. Results from the simulations are presented in section 4 where estimated RMS errors are compared to “true” RMS errors. Finally, derivations are given in Appendix A.

2. Methodology

FSI retrieval of bending angle profiles from RO signals can be divided into the following main processing steps (for details, see Jensen et al. [2003]).

1. Reconstruction of the “original” signal by adding the geometrical Doppler shift and signal upsampling to at least twice the Nyquist frequency.
2. Adjustment for the effects from noncircular orbits and higher-order variations in the opening angle between the GPS and the LEO satellite.
3. Global Fourier transform $X(\omega) = \text{FFT}(x)$ of the adjusted signal $x$.
4. Computation of pairs of bending angle and impact parameter from the Fourier transform as

$$a(t) = \frac{\omega}{k} \frac{d}{dt}$$

$$t(a) = -\frac{1}{\omega} \frac{d \arg(X)}{d\omega}$$

$$\alpha(t) = \theta(t) + \sin\left(\frac{a}{R_{TX}(t)}\right) + \sin\left(\frac{a}{R_{RX}(t)}\right) - \pi,$$

in which

$a$ impact parameter;
$\omega$ angular frequency in radians;
$k$ wave number of the transmitted signal;
$t$ time;

$R_{TX}$ distance from center of curvature to the transmitter/GPS satellite;
$R_{RX}$ distance from center of curvature to the receiver/LEO satellite;
$\theta$ angle between the two radius vectors.

From the procedure above it follows that errors in the FSI phase caused by measurement noise will propagate to errors in the FSI retrieved bending angles. Consequently, if the errors in the FSI phase can be predicted it is straightforward to predict the corresponding bending angle errors.

In this study two different noise scenarios are considered, namely additive noise and multiplicative noise. For RO measurements additive noise is mainly due to thermal noise in the instrument. Multiplicative noise can arise from noise in the reference link [Beyerle et al., 2005], but neutral atmospheric and ionospheric scintillations may also be modeled as multiplicative noise for so-called weak fluctuations [Ishimaru, 1997]. In the former case the multiplicative noise will be pure phase noise, whereas in the latter case both phase and amplitude are distorted.

2.1. Additive Noise

A RO signal, $s(t)$, contaminated with additive noise, $w(t)$, may be written as

$$x(t) = s(t) + w(t)$$

with the corresponding Fourier transform

$$X(\omega) = S(\omega) + W(\omega),$$

where $S(\omega)$ and $W(\omega)$ are the Fourier transforms of the noise-free signal and the noise signal, respectively. By applying first-order statistics it can be shown that the variance, $\sigma_{\phi}^2$, of the log amplitude, $\chi(\omega) = \ln|X(\omega)|$, and phase, $\varphi(\omega) = \arg(X(\omega))$ can be expressed as (detailed derivations are given in Appendix A)

$$\sigma_{\phi}^2 \approx \frac{1}{2SNR(\omega)},$$

since

$$\varphi(\omega) \approx \arg(S(\omega)) + \frac{\text{Im}\{W(\omega)\} \text{Re}\{S(\omega)\} - \text{Re}\{W(\omega)\} \text{Im}\{S(\omega)\}}{|S(\omega)|^2}$$
and
\[ \chi(\omega) \approx \ln(|S(\omega)|) + \frac{\text{Re}\{W(\omega)\} \text{Re}\{S(\omega)\} + \text{Im}\{W(\omega)\} \text{Im}\{S(\omega)\}}{|S(\omega)|^2}, \]

provided that

[20] 1. The signal-to-noise ratio (SNR(\omega)) is larger than about 5:
\[ \text{SNR}(\omega) = \frac{|S(\omega)|^2}{|W(\omega)|^2} \geq 5.\]

[21] 2. \( W(\omega) \) is a zero mean random process, \( \langle W(\omega) \rangle = 0 \).
[22] 3. \( W(\omega) \) and \( S(\omega) \) are uncorrelated: \( \langle W(\omega)S(\omega) \rangle = 0 \iff \langle W(\omega)s(t) \rangle = \langle w(\omega) \rangle s(t) = 0 \).
[23] 4. The imaginary and real part of \( W(\omega) \) are uncorrelated: \( \langle \text{Re}\{W(\omega)\}\text{Im}\{W(\omega)\} \rangle = 0 \).
[24] 5. \( \text{Re}\{W(\omega)\} \) and \( \text{Im}\{W(\omega)\} \) have the same statistical properties: \( \langle \text{Re}\{W(\omega)\}\text{Re}\{W(\omega) + \omega_0) \rangle = \langle \text{Im}\{W(\omega)\}\text{Im}\{W(\omega) + \omega_0) \rangle \).
[25] 6. \( |S(\omega)| \) is constant.

Here \( \langle \rangle \) represents ensemble average, whereas \( \text{Re}\{\} \) and \( \text{Im}\{\} \) mean real and imaginary part, respectively. The threshold value of 5 has been established through simulations; it ensures that the higher-order contributions to the phase and log amplitude errors are negligible. A sufficient, but not a necessary criterion, for conditions 2–5 to be satisfied is that \( w(t) \) is an ergodic zero mean signal, which is often assumed for thermal noise.

[35] Equation (6) shows that when the conditions listed above are (approximately) fulfilled the time series of the log amplitude will have exactly (approximately) the same statistical properties as the time series of the phase errors defined as the deviations between \( \text{arg}(S(\omega)) \) and \( \text{arg}(\lambda(\omega)) \). Consequently, by applying exactly the same operations to \( \chi \) as are applied to derive bending angles from the Fourier phase, a synthetic time series of bending angle errors, \( \Delta \alpha_\chi \), can be generated, which will have the same statistical properties as the time series of the ‘true’ bending angle errors. From (1) and (3) it follows that the samples of \( \Delta \alpha_\chi \) can be expressed as
\[ \Delta \alpha_\chi = \alpha(t) - \alpha(t + \Delta t_\chi), \quad \Delta t_\chi = -\frac{d\chi}{d\omega}. \] (9)

For realistic orbits the relative variations in \( \delta \theta dt \), \( r_{tx} \), and \( r_{rx} \) are generally small [see, e.g., Lauritsen and Lohmann, 2002], and (9) can therefore be well approximated as
\[ \Delta \alpha_\chi \approx \Delta t_\chi \frac{d\theta}{dt}. \] (10)

Assuming that this time series is ergodic within a sufficiently long impact parameter interval, the bending angle error autocorrelation function, \( r_\tau \), may then be estimated as a function of impact parameter as [Proakis and Manolakis, 1996]
\[ r_\tau(a_i, \tau) = \langle \Delta \alpha_\chi(a_i) \Delta \alpha_\chi(a_i + \tau) \rangle \approx \frac{1}{2M + 1} \sum_{i=-M}^{M} \Delta \alpha_\chi(a_i) \Delta \alpha_\chi(a_i + \tau), \] (11)

where \( 2M + 1 \) is the number of points used to estimate the autocorrelation function for lag \( \tau \). Subsequently, \( r_\tau \) can be propagated to dynamic error estimates for refractivity and temperature using error covariance analysis [see, e.g., Syndergaard, 1999].

2.2. Multiplicative Noise

[27] A RO signal, \( s(t) \), contaminated with multiplicative noise may written as
\[ x(t) = s(t)(1 + v(t)) = s(t) + s(t)v(t), \] (12)

with the corresponding Fourier transform
\[ X(\omega) = S(\omega) + S(\omega) \ast V(\omega), \] (13)

where the asterisk represents convolution. Equation (13) shows that it is possible also to model multiplicative noise as additive noise. The additive noise term \( S(\omega) \ast V(\omega) \) may be written as
\[ \text{Re}\{S(\omega)\} \ast \text{Re}\{V(\omega)\} - \text{Im}\{S(\omega)\} \ast \text{Im}\{V(\omega)\} \]
\[ + \sqrt{-1} \langle \text{Re}\{S(\omega)\} \ast \text{Im}\{V(\omega)\} + \text{Im}\{S(\omega)\} \ast \text{Re}\{V(\omega)\} \rangle, \] (14)

which shows that if \( V(\omega) \) fulfills conditions 2–5 listed above then \( S(\omega) \ast V(\omega) \) will also satisfy these conditions. This will be the case for weak fluctuations and for noise in the reference link. For these noise sources (6) may be used for estimation of the corresponding bending angle errors in exactly the same way as for additive noise.

3. Simulations

[28] The simulated satellite system consists of two counterrotating satellites orbiting the Earth in the same plane with the transmitter and receiver at heights of 20,200 km and 850 km, respectively. The transmitter frequency is set to 1.57542 GHz corresponding to the L1 frequency in the GPS system.

[29] The simulated signal was generated using asymptotic direct modeling [Gorbunov, 2003; Gorbunov and Lauritsen, 2004] in the form of an inverse FSI. The signal was first sampled at its full bandwidth and subsequently downsampled to 50 Hz by use of a phase
model to mimic open loop tracking of real RO signals [Sokolovskiy, 2001, 2004].

[30] Downsampling was performed by first removing all phase variations related to the movements of the satellites (these phase variations are sometimes referred to as the geometrical Doppler) followed by downconversion of the signal’s mean frequency to zero. The phase model used for the downconversion was constructed by smoothing the signal phase with a 0.02 s sliding window. Finally, the signal was integrated over 0.02 s, sampled at 50 Hz, and the trend removed from the phase by the phase model was added to the signal phase.

[31] The bending angle profile used in the simulations was derived from a high-resolution radio sonde profile (1 October 1995, 7.1°N, 171.4°E) by assuming the refractivity to be spherically symmetric. This assumption, which is not fully realistic, allows for generating a “worst case” scenario in terms of small-scale structures and signal bandwidth. This profile is depicted in Figure 1 together with the corresponding time-frequency variation of the total Doppler frequency (i.e., excess Doppler frequency plus geometrical Doppler frequency) for the simulated signal. The time-frequency variation was computed from the bending angle profile and the orbits by simultaneously solving (1) and (3) with respect to time and Doppler frequency. The amplitude variations are depicted in Figure 2. From Figures 1 and 2 it follows that the simulated signal experiences extensive multipath propagation after 24 s and has a maximum instantaneous bandwidth of 23 Hz at 24 s.

[32] To assess the performance of the error prediction technique described in the preceding section, three different scenarios are considered. The first scenario serves as reference case where the inversion is performed on the noise free signal. In the second scenario, additive Gaussian white noise with a signal-to-noise density of 40 dBHz is added to the simulated signal. This results in a fairly low SNR; for comparison the signal-to-noise density for CHAMP is approximately 50 dBHz. In the third scenario, multiplicative noise in the form of white Gaussian phase noise is added to the simulated signal phase sampled at 50 Hz. The standard deviation of the phase noise is 0.05 × 2π, also representing severe noise contamination.

[33] As the first step in the applied retrieval scheme, the total signal phase is reconstructed in order to restore the “original” signal. This is done by frequency shifting the signal to base band, followed by upsampling of the signal through interpolation of amplitude and phase to a sampling rate of 679 Hz corresponding to twice the signal bandwidth/Nyquist frequency.

![Figure 1](left) Bending angle profile used in simulation to generate RO signal and (right) time-frequency representation of the total Doppler frequency for the simulated RO signal.
The phase derivatives needed for retrieving the bending angles are computed using a two point differentiation. The corresponding bending angle profiles are derived using (1)–(3) and smoothed using a moving average over an impact parameter interval of 60 m, in agreement with the expected resolution due to diffraction.

For each scenario a bending angle RMS error profile is computed from the predicted bending angle errors derived from the FSI log amplitude variations as described in the previous section. For comparison the “true” RMS error profile is also computed from differences between the “true” bending angles and the retrieved bending angles. In both cases the RMS values are computed using a moving window of 500 m. To exclude contributions from scales smaller than 60 m, the “true” bending angle profile is smoothed using a moving average over 60 m before it is subtracted from the retrieved bending angle profile.

### 4. Results

Figure 3 depicts the FSI amplitude variations for the three scenarios and Figure 4 shows comparisons between the predicted fractional RMS errors derived from these amplitudes variations as well as the “true” fractional RMS error profiles. The RMS errors are shown relative to the “true” bending angle profile smoothed with a 500 m window to remove small-scale variations that would otherwise result in rather jagged error profiles. From Figure 4 it is seen that there is good agreement between the predicted and the “true” RMS error profiles for all three scenarios except at impact heights around 4.7 km where the predicted RMS errors are 5–10 times smaller than the true errors.

For the no-noise scenario, the errors are small (generally between 0.1 and 1 percent in the height range from 2.5–25 km impact height) with a maximum error of 1 per cent at 4.7 km impact height. The dominant error source for this scenario is small errors from the upsampling. For the additive noise scenario the errors are between 0.5 and 5% with the largest errors occurring above 20 km, excluding the large errors of 13 percent at 4.7 km. The dominant error source in this scenario is the combined effect of additive noise and upsampling errors. This combined effect is particularly important for impact parameters corresponding to rays experiencing strong defocusing, as the main contributions to the corresponding Fourier components will come from segments of the RO signal with a low SNR. In the current simulations this effect is seen below approximately 15 km where the fractional error becomes constant with height as the absolute error starts to increase with height. It is worth noting that defocusing of the RO signal does not by itself result in an increase in the bending angle measurement errors retrieved by FSI. If the upsampling had been unaffected by the SNR or if upsampling had not been necessary at all, the absolute errors would have been constant at all heights. This is the case for the multiplicative noise, where the upsampling errors are constant for the entire signal and the absolute error is, conse-

**Figure 2.** Amplitude variations of simulated signal without noise.
Figure 3. FSI amplitude profiles for the three noise scenarios.

Figure 4. Comparison between "true" RMS errors (dashed line) and predicted RMS errors (solid line) for the three noise scenarios.
quent, constant with height. The corresponding fractional error varies between 0.3% at 2.5 km to 25% at 25 km.

\[38\] To assess how well the spatial scales of the errors are predicted the power spectrum of the “true” error time series and the predicted error time series are compared for the three scenarios. For this comparison no smoothing is applied to allow for comparison for spatial scales smaller than 60 m. The power spectra are computed for errors in the height range between 5 and 25 km impact using the Welch method [Proakis and Manolakis, 1996] with 50 segments, 50% overlap, and a boxcar window function. Errors below 5 km were excluded to prevent the large localized errors at 4.7 km from contaminating the power spectra.

\[39\] Figure 5 shows comparisons between power spectra of predicted and true errors; for all three scenarios there is good agreement between the power spectra of the predicted errors and the power spectra of the “true” errors. For the additive noise, the shape of the power spectra resembles the power spectrum of differentiated white noise with damping of low-frequency components and amplification of high-frequency components. This indicates that even though the magnitude of the bending angle errors varies with height the corresponding phase errors in the Fourier transform are uncorrelated with impact height. For the no-noise scenario and the multiplicative noise scenario, the shape of the power spectra are similar. For these two scenarios most of the energy is located at spatial frequencies smaller than 0.12 m\(^{-1}\) indicating that the error power spectra are related to the power spectrum of the signal itself as implied by (13). That the error power spectra for the no-noise scenario and the multiplicative noise scenario are alike is not surprising as the signal is upsampled by independent upsampling of phase and amplitude, which means that the upsampling errors are indeed multiplicative. Figure 6 shows the autocorrelation functions corresponding to the power spectra in Figure 5. The autocorrelation functions were computed as the inverse Fourier transforms of the power spectra since the power the spectrum and the autocorrelation function of a random signal constitute a Fourier transformed pair. It should be noted that the autocorrelation functions could also have been computed directly from (11). From Figure 6 it follows that there is hardly any correlation between errors separated by more than about 5 m in the impact parameter domain in all three scenarios.

\[40\] The above discussion focused on the 5 km to 25 km range, excluding the large localized error at 4.7 km. The errors at 4.7 km impact height are related to the dip in the bending angle profile at the same impact height, which leads to high signal bandwidths of 23 Hz at 24 s. Though this bandwidth is small enough to satisfy
the sampling theorem the applied phase model fails to frequency shift the signal to (close to) zero mean frequency which results in aliasing and damping of the affected frequency components as described by [Sokolovskiy, 2001, 2004]. The impact is most significant at 4.7 km but because of aliasing the effect is also seen at higher altitudes as bumps in the error profiles and spikes in the amplitude profiles, most notable for the no-noise scenario. The amplitude profiles also show that, besides from distorting the signal, the improper signal acquisition at 24 s, and after, also leads to considerable damping of the FSI amplitudes for the affected impact parameters. This makes retrieval of bending angles in that height range extremely sensitive to noise; partially because the SNR will be very low for the affected impact parameters but also because the low SNR in the time domain will result in very large errors from the upsampling. The conclusion to be drawn here is that errors related to improper signal acquisition cannot be predicted from the FSI log amplitude variations alone. However, it is worth noting that the absolute magnitude of the predicted errors increases significantly at 4.7 km. Hence a sudden increase in the magnitude of the predicted errors may be used as a quality control criterion, particularly together with the FSI amplitude, which is also sensitive to poor signal acquisition. Such a quality control criterion can be used to “flag” bending angle error estimates which may be too small. Finally, it should be noted that the aliasing described above can be corrected for by using additional downconversion as described by Sokolovskiy [2001]. This was not attempted in this study.

5. Summary and Conclusion

Application of any measurement requires reliable error estimates. This is particularly the case when meteorological observations are assimilated into NWP models. Since RO measurement errors vary considerably between different soundings, it is desirable to estimate RO measurement errors dynamically.

In this study it was demonstrated that variations in the FSI log amplitude can easily be mapped into profiles of error standard deviations for FSI retrieved bending angles on a profile-to-profile basis without using any external data. The performance of this technique was assessed by applying it to simulated GPS RO signals sampled at 50 Hz with both severe additive noise and significant phase noise. In both cases excellent agreement between the predicted error profiles and the true error profiles was achieved. Comparisons between power spectra and autocorrelation functions of predicted errors and “true” errors also showed good agreement and it was found that in all three scenarios there is hardly any
correlation between errors separated by more than about 5 m in the impact parameter domain.

[43] A small part of the simulated signals was distorted by aliasing due to the downsampling to 50 Hz, which affected a narrow range of impact parameters. For these impact parameters the predicted errors were found to be too small as compared to the “true” errors. This shows that the proposed technique cannot predict errors that are related to poor signal acquisition. However, for the affected impact parameters an increase in the predicted errors were observed as well as a significant dip in the FSI amplitude. This implies that it may be possible to predict poor signal acquisition from the predicted bending angle errors and/or the FSI amplitude.

Appendix A: Derivation of Relation
Between Phase Errors and Log Amplitude Errors

[44] Consider a complex deterministic signal, \( S \), which is contaminated with zero mean additive noise, \( W \), so that the combined signal, \( X \), can be written as

\[
X = S + W
\]  

(A1)

The phase error, \( \Delta \phi = \arg(X) - \arg(S) \), and the log amplitude error, \( \Delta \chi = \ln(|X|) - \ln(|S|) \), of the signal \( X \) may be estimated by a first-order Taylor expansion of \( \Delta \phi \) and \( \Delta \chi \) with respect to \( \text{Re}(W) \) and \( \text{Im}(W) \) as

\[
\Delta \phi \approx \text{Re}(W) \frac{\partial}{\partial \text{Re}(S)} \text{atan} \left( \frac{\text{Im}(S)}{\text{Re}(S)} \right) + \text{Im}(W) \frac{\partial}{\partial \text{Im}(S)} \text{atan} \left( \frac{\text{Im}(S)}{\text{Re}(S)} \right)
\]

\[
= \frac{-\text{Re}(W) \text{Im}(S)}{\text{Re}(S)^2 + \text{Im}(S)^2} + \frac{\text{Re}(S) \text{Im}(W)}{\text{Re}(S)^2 + \text{Im}(S)^2}
\]

\[
= \frac{\text{Im}(WS)}{|S|^2}
\]  

(A2)

\[
\Delta \chi \approx \text{Re}(W) \frac{\partial}{\partial \text{Re}(S)} \ln(|S|) + \text{Im}(W) \frac{\partial}{\partial \text{Im}(S)} \ln(|S|)
\]

\[
= \frac{\text{Re}(W) \text{Re}(S)}{\text{Re}(S)^2 + \text{Im}(S)^2} + \frac{\text{Im}(S) \text{Im}(W)}{\text{Re}(S)^2 + \text{Im}(S)^2}
\]

\[
= \frac{\text{Re}(WS)}{|S|^2}
\]  

(A3)

in which the overbar denotes complex conjugate. These expressions show that the phase errors arise mainly from the components of \( W \) that are perpendicular to \( S \), whereas the errors in the log amplitude are due mainly to the components of \( W \) that are parallel to \( S \). From (A2) and (A3) it follows that the variance of the phase fluctuations and the log amplitude fluctuations can be expressed as

\[
\sigma_\phi^2 = \langle \Delta \phi^2 \rangle
\]

\[
\approx \frac{(\text{Re}(W))^2 \text{Im}(S)^2 + \text{Re}(S)^2 (\text{Im}(W))^2 - 2 \text{Re}(S) \text{Im}(S) \text{Im}(W) \text{Re}(W)}{|S|^4}
\]  

(A4)

and

\[
\sigma_\chi^2 = \langle \Delta \chi^2 \rangle
\]

\[
\approx \frac{(\text{Im}(W))^2 \text{Im}(S)^2 + \text{Re}(S)^2 (\text{Im}(W))^2 + 2 \text{Re}(S) \text{Im}(S) \text{Im}(W) \text{Re}(W)}{|S|^4}
\]  

(A5)

If \( \langle \text{Re}(W) \text{Im}(W) \rangle = 0 \) and \( \langle \text{Re}(W) \text{Im}(W) \rangle = 0 \), (A4) and (A5) simplify to

\[
\sigma_\phi^2 \approx \sigma_\chi^2 = \frac{|W|^2}{2|S|^2} = \frac{1}{2 \text{SNR}^2}
\]  

(A6)

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