Comparative analysis of radio occultation processing approaches based on Fourier integral operators

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[1] We analyze and compare two approaches to processing radio occultation data: (1) canonical transform method and (2) full spectrum inversion method. We show that these methods are closely related and can be explained from two view points: (1) both methods apply a Fourier transform like operator to the entire radio occultation signal, and the derivative of the phase of the transformed signal is used for the computation of bending angles, and (2) they can be explained from a signal processing view point as the location of multiple tones constituting the complete signal. The full spectrum inversion method is a composition of phase correction and Fourier transform, which makes the numerical algorithm computationally more efficient as compared to the canonical transform method. We investigate the relative performance of the two methods in simulations using a wave optics propagator. We use simple analytical models of the atmospheric refractivity as well as radiosonde data in order to reproduce complex multipath situations. The numerical simulations as well as the analytical estimations indicate that a resolution of 60 m (or even higher) can be achieved.

INDEX TERMS: 0394 Atmospheric Composition and Structure: Instruments and techniques; 3360 Meteorology and Atmospheric Dynamics: Remote sensing; 6969 Radio Science: Remote sensing; KEYWORDS: radio occultation, inversion, radio holography


1. Introduction

[2] Until recently, deciphering the geometric optical ray structure of wave fields in multipath zones was considered one of the main problems in processing radio occultation (RO) data. The following radio holographic techniques for solving this problem were developed: (1) back propagation (BP) [Marouf et al., 1986; Gorbunov and Gurvich, 1998a, 1998b; Mortensen et al., 1999], (2) Fresnel inversion [Melbourne et al., 1994; Mortensen and Hoeg, 1998], (3) sliding spectral, focused synthetic aperture methods [Lindal et al., 1987; Pavelevy, 1998; Igarashi et al., 2000, 2001; Pavelevy et al., 2002]. An attempt was also made to combine the back propagation and radio holographic analyses [Gorbunov, 2002c]. However, it was recognized that all of these approaches have strong restrictions [Gorbunov et al., 2000], which impeded their application for processing lower tropospheric data with severe multipath [Sokolovskiy, 2001a, 2001b].

[3] The introduction of the canonical transform (CT) method [Gorbunov, 2001, 2002a, 2002b] practically solved the problem. The CT method has the following advantages: (1) it allows for the achievement of very high resolution (30–60 m), which is not limited by the Fresnel zone size, even under very severe multipath conditions, (2) it has no tuning parameters (such as back propagation distance in the BP approach), and (3) its only restriction is the requirement that the refraction angle is a single-valued function of impact parameter. This made the CT algorithm a reference method indicating the highest possible accuracy and resolution, and this algorithm is already very effectively used in operational applications.

[4] The CT method uses BP as a preprocessing tool, which reduces the observation trajectory to a vertical line. A Fourier integral operator maps the back-propagated wave field from the representation of the spatial vertical coordinate to the representation of the ray impact parameter. Under the assumption that impact parameter is a unique ray coordinate (i.e., there is no more than one ray for a given value of impact parameter), the refraction
angle can be found in geometric optical approximation from the derivative of the phase of the transformed wave field. This is similar to the standard derivation of impact parameter from the derivative of the phase (Doppler frequency) of the measured wave field under no multipath conditions [Vorob’ev and Krasil’nikova, 1994]. The Fourier integral operator allows for a very effective FFT-based numerical implementation.

[5] The main drawback of this approach is the necessity of the BP preprocessing, which is the most time-consuming part of the numerical algorithm. Accurate BP preprocessing uses diffractive integrals, and for an arbitrary occultation geometry it defies a fast FFT-like numerical implementation. However, the best numerical implementations of the accurate BP algorithm are reasonably fast. For processing one radio occultation with a standard 50 Hz sampling rate, the BP part of the complete numerical algorithm may take as little as 10 s on a Pentium-based system. A very fast approximate numerical implementation of BP can be based on geometrical optics (S. Sokolovskiy, personal communication, 2003).

[6] The use of a global Fourier transform in the retrieval of radio occultation signals with a circular geometry (i.e., circular satellite orbits in the same vertical plane, spherical Earth and spherically symmetrical atmosphere) was introduced in the work of Høeg et al. [2001] and Jensen et al. [2002]. A complete development of the full spectrum inversion (FSI) method, including realistic orbits, has been presented in the work of Jensen et al. [2003]. This method uses the Fourier transform of the complete radio hologram, i.e., the record of the complex field \( u(t) \) as function of observation time \( t \). In the case of a circular geometry, the derivative of the phase, or Doppler frequency, \( \omega \) of the wave field \( u(t) \) is proportional to ray impact parameter. We can introduce the (multivalued) dependence \( \omega(t) \), which is by definition equal to the Doppler frequency (or frequencies) of the ray(s) received at time moment \( t \). In multipath zones, where the dependence \( \omega(t) \) is multivalued, it cannot be found by the differentiation of the phase of the wave field \( u(t) \). Unlike \( \omega(t) \), the inverse dependence \( t(\omega) \) is single-valued, if we assume that each impact parameter and therefore frequency \( \omega \) occurs not more than once. Using stationary phase derivation, it can be easily shown that the derivative of the phase of the Fourier spectrum \( \tilde{u}(\omega) \) is equal to \( -t(\omega) \). Thus for each impact parameter we can find the time \( t \) and therefore the positions of the GPS satellite and low-Earth orbiter (LEO) for which this impact parameter occurred. This allows for finding the corresponding refraction angle. This method is much more numerically effective as compared to CT. FSI does not require a computationally expensive BP, it uses only one FT instead of two and does not need interpolation of the spectrum on another grid.

[7] For a generic satellite orbit, which deviates from a circle, Doppler frequency \( \omega \) may not be an unique ray coordinate, and then some modification of this approach is necessary. Jensen et al. [2003] showed that the parameterization of the observation trajectory by the satellite-to-satellite angle \( \theta \) instead of time \( t \) together with subtracting a correction model from the phase is a very effective solution. The phase model corrects for the radial velocities of the satellites, and it is an approximate and numerically very efficient form of short distance BP from the real orbit to a circle. Lauritsen and Lohmann [2002] investigated an integral linear transformation of the phase followed by Fourier transform over time \( t \). This procedure also results in unique frequencies of different rays. However, this transform is nonlinear with respect to complex field \( u(t) \), which impairs the quality of amplitude reconstruction.

[8] In this paper, we compare the CT and FSI methods. In section 2 we show that the two methods are closely related. They both process the wave field by means of a Fourier integral operator. The transformed wave function is a function of ray impact parameter for the CT method, and a function of Doppler frequency, approximately proportional to the impact parameter, for the FSI method. The derivation of refraction angles uses the differentiation of the phase of the transformed wave function.

[9] The stationary phase method (which was mainly used when introducing the FSI method) is also the basic computational means in theory of Fourier integral operators [Egorov et al., 1999]. This can be used to show that the CT and FSI methods are different modifications of the same approach.

[10] Section 3 contains an analysis of the resolution of the two methods. The Fresnel zone size, which is related to diffraction on a big propagation distance from the planet limb to the LEO satellite, does not limit the resolution of these methods because they both transform the wave function to the representation of impact parameter and in this representation diffraction effects are reduced. The basic limitation factors are the synthesized aperture and the diffraction inside the atmosphere. These conclusions are confirmed by numerical simulations on the material of simple analytical profiles and high-resolution radiosonde profiles, presented in section 4. We also investigate the sensitivity of both methods to random noise. Section 5 contains the conclusions.

2. Stationary Phase Method and Canonical Transforms

[11] Consider a wave field \( u \) measured along some observation curve parameterized by some coordinate \( y \), which is linked to the observation time \( t \). It is convenient not to specify what exactly this coordinate is because it
can be chosen in different ways (e.g., use of time $t$ or satellite-to-satellite angle $\theta$ in the FSI method [Jensen et al., 2003]). The wave field can be represented as $u(y) = A(y) \exp \left( i k \Psi(y) \right)$, where $A(y)$ is the amplitude and $\Psi(y)$ is the optical path. We can introduce the frequency as $\eta = d\Psi/dy \equiv \Psi$. If $y$ is equal to time $t$ then the Doppler frequency shift is defined as $\Delta \omega = -k \Psi = -k \eta$, because we assume a temporal dependence of the wave field in the form $A(y) \exp \left( ik \Psi(y) - i \omega t \right)$. We can develop two complementary views of the complex signal $u(y)$: 1) signal processing view used in the formulation of the FSI method; 2) canonical view used in the formulation of the CT method.

2.1. Full Spectrum Inversion Method

[12] From the signal processing view point, the signal is composed of multiple tones associated with interfering rays. For each point $y$ there are, generally speaking, multiple frequencies $\eta(y)$, which we need to detect. One of the possible means of doing that is the sliding spectral (short-term Fourier) analysis [Lindal et al., 1987; Pavelyev, 1998; Igarashi et al., 2000, 2001; Pavelyev et al., 2002]. However, this approach has resolution limitations [Gorbunov et al., 2000]. If there is no a priori information about the multiple tones composing the signal, then, given the sliding aperture $\delta y$, the following uncertainty relation between the aperture and the spectral resolution $\delta \eta$ can be established: $\delta y \delta \eta \gtrsim 2 \pi k / \omega$. If $y = t$, and Doppler frequency $\Delta \omega = -k \eta$, then we have the uncertainty relation $\delta \eta / \omega \gtrsim 2 \pi$. If the sliding aperture $\delta t$ is equal to one characteristic oscillation period of the signal, $2 \pi / \omega$, where $\omega$ is the down-converted frequency of the signal approximately in the middle of its spectrum, then we obtain $\delta \omega / \omega \gtrsim 1$, which means that we cannot retrieve any information about multiple tones using such a small aperture. If, however, we a priori know that there is a single tone, then even signal measurement during one oscillation period allows for the determination of the period length and thus the frequency.

[13] The underlying idea of the FSI method is the use of the global (long-term) Fourier transform of the complex signal instead of the sliding (short-term) Fourier transform:

$$\bar{u}(\eta) = \sqrt{\frac{k}{2\pi}} \int u(y) \exp(-i k y \eta) \, dy.$$  \hspace{1cm} (1)

This expression is an oscillating integral, and it can be estimated using the stationary phase method [Born and Wolf, 1964]. For our analysis it is sufficient to only evaluate its phase. The wave field can be approximately represented as a sum of components of the form $u_{s}(y) = A_{s}(y) \exp \left( i k \Psi(y) \right)$, corresponding to interfering rays, with index $r$ enumerating the rays. For each component, the phase of the integrand in equation (1) is equal to

$k(\Psi_{s}(y) - \eta \eta)$. The stationary phase point $\eta_{s}(\eta)$ is determined from the equation

$$\frac{d\Psi_{s}}{dy} - \eta = 0.$$ \hspace{1cm} (2)

The spectrum (1) can be represented in the form $\bar{u}(\eta) = A(\eta) \exp \left( i k \Psi(\eta) \right)$. The eikonal of the spectrum is then approximately equal to

$$\Psi(\eta) = \Psi_{s}(\eta_{s}(\eta)) - \eta_{s}(\eta) + \gamma / k,$$ \hspace{1cm} (3)

where $\gamma = \pm \pi / 4$ is a piecewise-constant addition to the phase, where the sign depends on the argument of $(d^{2} \Psi / dy^{2})^{1/2}$. This addition asymptotically vanishes and it can be neglected. The derivative of $\Psi(\eta)$ is then calculated as follows:

$$\frac{d\Psi'}{d\eta} = \frac{dy_{s}}{d\eta} \left( \frac{d\Psi_{s}(\eta_{s}(\eta))}{dy} - \eta_{s}(\eta) \right) - \eta_{s}(\eta) = -\eta_{s}(\eta).$$ \hspace{1cm} (4)

If we now assume that each frequency $\eta$ occurs once, then $\eta_{s}(\eta)$ is a single-valued function, which identifies the coordinate of the point of the observation curve, where this frequency occurred. Equation (4) shows that the function $\eta_{s}(\eta)$ can be found from the derivative of the phase of the spectrum $\bar{u}(\eta)$. Under the assumption above, $\eta_{s}(\eta)$ is a single-valued function and the spectrum $\bar{u}(\eta)$ can be looked at as a signal that has a single local tone. Therefore we can choose a small aperture for differentiating its phase which enables a high resolution.

[14] For a radio occultation signal we have a general expression for the derivative of the optical path

$$\Psi = \eta(p, y) = \hat{p} \hat{\theta} + \frac{\hat{r}_{G}}{r_{G}} \sqrt{r_{G}^{2} - p^{2}} + \frac{\hat{r}_{L}}{r_{L}} \sqrt{r_{L}^{2} - p^{2}},$$ \hspace{1cm} (5)

where $\hat{\theta}$ is the satellite-to-satellite angle, and $r_{G}$ and $r_{L}$ are the radii of the GPS and LEO satellite orbits, and $p$ is the ray impact parameter. Using orbit data, satellite radii $r_{G}$ and $r_{L}$ can be expressed as functions of trajectory parameter $y$. If we assume the circular geometry of a radio occultation with a spherical Earth and circular satellite orbits ($r_{G} = r_{L} = 0$), then we can choose $y = \theta$, $\hat{\theta} = 1$, and impact parameter $p$ is equal to $\Psi = \eta$. Computing the Fourier transform $\bar{u}(\eta)$ of the wave field $u(\theta)$, and differentiating the phase of the spectrum, we can compute function $\eta_{s}(p)$. For each impact parameter $p$, this function is equal to the corresponding $\theta$, where this impact parameter occurred. Refraction angle can then be found as follows:

$$\epsilon(p) = \theta_{s}(p) - \arccos \frac{P}{r_{G}(\theta_{s}(p))} - \arccos \frac{P}{r_{L}(\theta_{s}(p))}.$$ \hspace{1cm} (6)
This technique can be approximately generalized for processing realistic occultations with a noncircular geometry [Jensen et al., 2003]. Instead of Fourier transform, a more general Fourier Integral Operator (FIO) of the 2nd type can be used [Gorbunov and Lauritsen, 2002; Gorbunov, 2002a; Jensen et al., 2004; Gorbunov and Lauritsen, 2004]:

\[
\hat{\Phi}_2 u(p) = \sqrt{\frac{-ik}{2\pi}} \int a_2(p,y) \exp(ikS_2(p,y)) u(y) \, dy ,
\]

where \( a_2(p,y) \) is the amplitude function (defined using the energy conservation), and \( S_2(p,y) \) is the phase function. We can then write the following equation for the stationary phase point \( y_s(p) \):

\[
\frac{d\Psi_r(y)}{dy} + \frac{\partial S_2(p,y)}{\partial y} \bigg|_{y=y_s(p)} = 0 .
\]

This equation has a very simple physical sense. The derivative of the phase \( k S_2(p,y) \) of the oscillating kernel \( a_2(p,y) \exp(ikS_2(p,y)) \) of the integral operator (7) matches the frequency of the ray with a given impact parameter \( p \). Therefore a ray with impact parameter \( p \) is located by the stationary phase point. This fact, together with equation (5), allows for the derivation of the general expression for the phase function:

\[
\frac{\partial S_2(p,y)}{\partial y} \bigg|_{y=y_s(p)} = -\eta(y_s(p),p) ,
\]

where we used the fact that equation (9) must hold for an arbitrary realization of the wave field and, therefore, for an arbitrary function \( y_s(p) \). For nearly circular orbit, we can choose \( y = \theta \). A smooth model of impact parameter variation \( p_m(\theta) \) can be used to estimate the second and third terms in equation (5) [Jensen et al., 2003]. This allows for deriving the approximate expression for the phase function:

\[
S_2(p,\theta) = -p\theta - \int \left( \frac{\dot{r}_G(\theta)}{r_G(\theta)} \sqrt{\frac{r_G(\theta)}{r_s(\theta)} - p_m^2(\theta)} \right. \\
+ \left. \frac{\dot{r}_L(\theta)}{r_L(\theta)} \sqrt{\frac{r_L(\theta)}{r_s(\theta)} - p_m^2(\theta)} \right) d\theta \equiv \\
\equiv -p\theta - \int F(\theta) d\theta .
\]

This approximation reduces the integral operator (7) to the Fourier transform applied to the wave field multiplied with the reference signal \( u(\theta) \exp(-ik\int F(\theta)d\theta) \), where \( k \int F(\theta)d\theta \) can be viewed as a phase model. A generalization of the FSI technique, as well as the discussion of the amplitude functions can be found in the work of Jensen et al. [2004] and Gorbunov and Lauritsen [2004].

A different approach was investigated by Lauritsen and Lohmann [2002]. They used Fourier transform with respect to time \( t \) and replaced the phase of the wave field with expression \( k \Omega_0 \int \tilde{p}(t)dt \), where \( \Omega_0 = \langle \dot{\theta}(t) \rangle \) is a normalizing factor, \( \tilde{p}(t) = d\Psi/d\theta + \Delta \tilde{p}(t) \) is a model of impact parameter, and \( \Delta \tilde{p}(t) \) is a term that approximately corrects for the radial component of the satellite velocities, similar to \( F(\theta) \). This is a linear integral transform of the phase, which unifies frequencies of different rays, with respect to time. Lauritsen and Lohmann [2002] provide examples of good-quality retrievals of refraction angles by this method. However, this transform is nonlinear with respect to the complex field \( u(t) \), and therefore it does not conserve the energy of the field. Unlike this nonlinear transform, the FIOs used in the CT and FSI methods are linear and they conserve the energy. This may explain a worse quality of the amplitude in the transformed space presented in the work of Lauritsen and Lohmann [2002], as compared to CT and FSI amplitudes.

### 2.2. Canonical Transform Method

The CT method uses the concept of phase space with canonical coordinates. The observed wave field is first back-propagated to a vertical line; we use Cartesian coordinates \( x, y \) in the vertical occultation plane. Then, for a given position \( x \) of the back propagation plane, the back-propagated field \( u \) is a function of the vertical coordinate \( y \). The wave field in a single ray area can be represented in the form \( u(y) = A(y) \exp(ik\Psi(y)) \). We can introduce the momentum \( \eta = \partial\Psi/\partial y \) conjugated with the coordinate \( y \). The momentum \( \eta \) is equal to the vertical projection of the ray direction vector. The geometric optical rays are determined by the Hamilton system describing the dynamics of the ray coordinate \( y \) and momentum (direction) \( \eta \). We can consider another coordinate \( p \) and conjugated momentum \( \xi \) in the same phase space, where the new coordinate \( p \) is the impact parameter (Figure 1). The geometric optical ray equation can be rewritten in the new coordinates. If the transform from \( (y, \eta) \) to \( (p, \xi) \) is canonical, i.e., \( \det(\partial(p, \xi)/\partial(y, \eta)) = 1 \), then the ray equation will have the same Hamilton form also in the new coordinates. The wave field can also be asymptotically transformed to the representation of the new coordinates. This transform is given by the following FIO of the 1st type introduced by [Egorov, 1985]:

\[
\hat{\Phi}_1 u(p) = \sqrt{\frac{ik}{2\pi}} \int a_1(p, \eta) \exp(ikS_1(p, \eta)) \tilde{u}(\eta) d\eta,
\]

where \( a_1(p, \eta) \) is the amplitude function.
explained using the concept of stationary phase. The new momentum equals \( p' = \arcsin h(\eta) \). This relation allows for deriving the generating function:

\[
S_1(p, \eta) = \int y(p, \eta)d\eta = \int \frac{p + \sqrt{1 - \eta^2}}{1 - \eta^2} d\eta = \arcsin \eta - x\sqrt{1 - \eta^2},
\]  

(15)

The new momentum equals \( \xi = \partial S_1/\partial p = \arcsin \eta \). Operator (12) with phase function (15) transforms the wave field to the representation of impact parameter. If \( \Phi_1 u(p) = A'(p)\exp(ik\Psi'(p)) \), then the ray direction angle, or momentum, \( \xi(p) \) is equal to \( \partial \Psi'/\partial p \). The refraction angle \( \epsilon(p) \) is then found by using the formula of Bouger [Gorbunov, 2002a].

\[ \eta' = \eta, \quad \gamma' = -y. \]

(18)

(19)

This fact is used in the theory of asymptotic solutions of wave problems in the form of the Maslov operator [Maslov and Fedoriuk, 1981; Mishchenko et al., 1990]. In multipath areas we can use the old momentum \( \eta \) as the new coordinate \( y' \), with the new momentum \( \eta' \) being equal to \( -y \). This allows for finding a locally caustic-free projection of the ray manifold (cf. Figure 1). In the \( (\eta, -y) \) representation, the geometric optical solution can then be written and the inverse Fourier transform maps this solution back into the usual \((y, \eta)\) representation. The use of the Fourier transform for transfer to the momentum representation in quantum mechanics is also commonly known [Maslov and Fedoriuk, 1981].
We can summarize the relationship between the two methods. The CT and FSI methods are equivalent, and they both can be looked at from two view points, which complement each other and allow for deriving the same operators. Given a wave field \( u(y) \), we can think of it as a sum of components \( u_r(y) = A_r(y) \exp(i k \Psi_r(y)) \) corresponding to different rays \( u_r(y) \) is a signal composed of multiple tones). The wave field is subjected to an operator with an oscillating kernel. The \( y \) derivative of the phase of the kernel matches the frequency of the ray with given impact parameter \( p \), which is thus located by the stationary phase point. The refraction angle is a function of the derivative \( \xi \) of the phase of the transformed wave field. We can also introduce the phase space of geometric optical rays with coordinate \( y \) and momentum \( \eta \), where we consider a canonical transform to a new coordinate \( p \) and momentum \( \xi \), which are some functions of \( y \) and \( \eta \). The FIO associated with this transform is the operator with an oscillating kernel mentioned above. The momentum \( \xi \) in the new representation is equal to the derivative of the phase of the transformed wave field.

The main difference between the two approaches is that they are applied in different coordinate systems. The FSI method is formulated for a field measured along a satellite orbit, which is close to a circle, while the CT method is formulated for a back-propagated field given along a straight line. This implies that numerically FSI is more simple than the CT method, and it can also provide high resolution. Thus FSI can be very prospective for operational processing of radio occultations. An approach to the generalization of the FSI method using the principles of the theory of FIO applied directly to the field along a generic observation line was suggested by Gorbunov and Lauritsen [2002].

3. Resolution

Now we shall estimate the highest resolution that can be achieved in the reconstruction of refraction angle profiles. The resolution of the CT method has been discussed by Gorbunov [2002a]. It was shown that the operator (12) contains the combination \( -i k x / \sqrt{1 - \eta^2} \) \( \bar{u}_r(\eta) \), where \( \bar{u}_r(\eta) \) is the spatial spectrum of the wave field back-propagated to the horizontal coordinate \( x \). Because \( \exp(i k x / \sqrt{1 - \eta^2}) \) is the vacuum propagator from \( x = 0 \) to \( x \), this combination is equal to the spectrum \( \bar{u}_r(x) \) of the wave field back-propagated to \( x = 0 \). This means that the diffractive effects of vacuum propagation at a big distance are automatically corrected for by this operator. This is linked to the fact that the Fourier transform along a vertical line computes the plane wave expansion of a wave field, and the vacuum propagation of each plane wave can be described exactly using geometrical optics.

In the FSI method, we use polar coordinates \( r, \theta \) and Fourier transform the field \( u_r(\theta) \) with respect to the angle \( \theta \). We can introduce the complex spatial coordinate \( z = x + iy = r \exp(i \theta) \). Introducing the new coordinate \( z' = x' + iy' = \ln z/\rho_E = \ln(r/\rho_E) + i \theta_E \), where \( \rho_E \) is Earth’s radius, we can write the relationship between the Laplace operator \( \Delta_z \) in coordinates \( (x, y) \) and \( \Delta_{z'} \) in coordinates \( (x', y') \):

\[
\Delta_{z'} = \left| \frac{dz}{dx} \right|^2 \Delta_x = \frac{1}{r^2} \Delta_x.
\]

The wave equation in the coordinates \( (x', y') = (\ln r, \theta) \) can then be written as follows:

\[
\left[ \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + k^2 r^2(x') \right] u_{r(x')}(\theta) = 0,
\]

This equation has the same form as the wave equation in the \( (x, y) \) coordinates with effective refractive index \( n(x') = r = \rho_E \exp x' \). In the spectral representation (frequency being equal to impact parameter \( p \)) it can be rewritten as follows:

\[
\left( r \frac{\partial}{\partial r} \right)^2 + k^2 (r^2 - p^2) \bar{u}_r(p) = 0.
\]

This equation does not contain differential operators with respect to \( p \), which means that in the spectral representation there is no diffraction. This proves that in both FSI and CT methods the effects of diffraction due to a big propagation distance are reduced. It must be noticed that for the FSI method we used the symmetry of the wave equation with respect to rotations, and this requires that the field is measured along a circle. For a realistic observation line (which is close to a circle) the corresponding equation will contain small terms with derivatives with respect to \( \theta \). This will impose some restriction to the resolution. On the other hand, because satellite orbits are very smooth curves, we can expect that this effect will not play a leading role.

As discussed by Gorbunov and Gurvich [1998a], the synthesized aperture size \( A \) restricts the resolution. A wave diffracted by an inhomogeneity with vertical scale \( h \) has an angular spectrum width of \( \lambda/h \). For an observation distance \( L \), the size of a synthesized aperture which covers all the spatial spectrum must be equal to the following expression:

\[
A = \frac{L \lambda}{h}.
\]

Another limitation of the resolution is determined by diffraction inside the atmosphere. Given atmospheric inhomogeneities with characteristic vertical scale \( h \) and characteristic horizontal scale \( l \), diffraction on such
inhomogeneities is insignificant if the Fresnel zone size at a propagation distance of $l$ does not exceed $h$:

$$h > \sqrt{\lambda l}. \tag{24}$$

[28] If we assume that atmospheric inhomogeneities are nearly spherically layered then the characteristic length of the interaction between a ray and a layer is estimated as $l = \sqrt{2r_E h}$, where $r_E = 6370$ km is the mean Earth radius. This results in the following resolution estimation:

$$h \geq 3\sqrt{\lambda^2 r_E}, \tag{25}$$

where we neglected a factor of $2^{1/3} \approx 1.26$. For a wavelength of 0.2 m this results in a resolution of 60 m. For an observation distance of $L = 3000$ km, which is typical for radio occultations, in order to achieve a resolution of 60 m, the synthesized aperture size $A$ must be 10 km.

4. Numerical Simulations

[26] The inversion methods were compared using numerical simulations. We generated artificial radio occultation data by means of the latest version of the wave optics propagator used by Gorbunov [2002a]. The spherically symmetrical atmospheric refractivity field was modeled using high-resolution vertical profiles of refractivity from radiosonde data and simple analytical profiles. We modeled vertical radio occultations with an immovable GPS satellite and with the circular LEO orbit with a radius of $r_E + 720$ km. The same artificial data were then processed by the CT and FSI methods, and the results were compared. The radio occultation data have a sampling rate of 250 Hz, which corresponds to the spatial step 10 m between rays under weak refraction conditions. In the lower troposphere, due to strong regular refraction, the spatial step decreases to approximately 1 m.

[27] In both CT and FSI method, the simulated wave field was transformed into the impact parameter representation by the corresponding FIO, equations (12) and (7), respectively. For the calculation of refraction angle profiles, we applied a filter with a characteristic width of 10 m to the phase of the transformed wave field as a function of impact parameter.

[28] Figure 2 shows the results for a test profile (exponential model with a quasiperiodical perturbation):

$$n(z) = 1 + N_0 \exp\left(-\frac{z}{H}\right) \left[1 + \alpha \cos \left(\frac{2\pi z}{h}\right) \exp\left(-\frac{z^2}{L^2}\right)\right], \tag{26}$$
where \( z \) is the height above Earth’s surface, \( N_0 = 300 \times 10^{-6} \) is the characteristic refractivity at Earth’s surface (300 N units), \( H = 7.5 \) km is the characteristic vertical scale of refractivity field, \( \alpha = 0.002 \) is the relative magnitude of the perturbation, \( h = 0.05 \) km is the period of the perturbation, \( L = 3.0 \) km is the characteristic height of the perturbation area. According to the above estimate, this simulation is at the verge of the highest possible resolution. However, both methods prove capable of reproducing the small-scale structures of the refraction angle profiles. In this example, the accuracy of the CT method is slightly better.

Figure 3 shows the results of processing simulated radio occultation for tropical high resolution radiosonde data [Steurer, 1996]. Both methods reproduce the fine structures very well. In this example, FSI performs slightly better.

We also performed simulations with random uncorrelated noise. The vacuum amplitude of the wave field in the simulation was assumed to be 700. To each sample of the complex field we added uncorrelated noise with a magnitude of 10. This is stronger than typical noise that can be seen in real radio occultation. Typical value of noise for 50 Hz data, seen in the real occultation, is about 3. For 250 Hz it is estimated as \( \sqrt{250} / 50 \times 3 = 6 \). The results are presented in Figure 4. In order to improve the retrieval quality, we increased the smoothing filter width up to 30 m. We also applied prefiltering to the amplitude and phase of the simulated complex field before FSI and BP. The width of the additional filter was about 0.01 s, which corresponds to 30 m. The prefiltering procedure makes the FIO based methods more robust with respect to additive noise.

In processing this data set, both methods perform equally well. A general observation for both methods is that narrow spikes of the profile \( \epsilon(p) \) are not reproduced. This can be explained by the fact that the complex field corresponding to such spikes has a very low amplitude and the influence of the additive noise increases.

5. Conclusions

In this paper we have compared the canonical transform (CT) and full spectrum inversion (FSI) methods. We performed both theoretical analysis and numerical simulations. Both methods apply a Fourier integral
operator (FIO) to the observed wave field. The operator has an oscillating kernel which is chosen in such a way that the stationary phase point of the operator locates a single geometric optical ray with prescribed impact parameter \( p \). The transformed wave field is then a function of the impact parameter and refraction angles are calculated from the derivative of the phase of the transformed wave field. Both methods can also be looked at from the view point of the theory of Fourier integral operators associated with canonical transforms. Given a wave field \( u(y) \) measured along the observation trajectory parameterized with coordinate \( y \), we can introduce the momentum \( \eta \) conjugated with the coordinate \( y \). The momentum is equal to the derivative of the phase of the wave field. At each point \( y \) multiple rays can interfere. In order to disentangle multipath we introduce a canonical transform from old coordinates \((y, \eta)\) to new ones \((p, \xi)\). The refraction angle can be expressed as a function of \( p \) and new momentum \( \xi \). In the new coordinates there is no multipath if there is not more than one ray with given impact parameter. The transformation of the wave field is performed by the FIO whose phase function is equal to the generating function of the canonical transform. The momentum \( \xi \) is equal to the derivative of the transformed wave field with respect to the impact parameter \( p \).

[33] The main difference between the two methods is that they are applied in different coordinate systems. The CT method was formulated for processing the backpropagated wave field along a vertical line. The FSI method is formulated for wave field measurements along an orbit close to a circle. Both methods reduce diffraction effects due to propagation at a big observation distance in a vacuum because the wave field is transformed into \( p \) representation where there is no diffraction. The ultimate resolution is then defined by the effects of diffraction inside the atmosphere and for GPS wavelength it can be estimated to be about 60 m.

[34] We have shown that both methods are indeed different modifications of the same approach. Numerically, FSI can be implemented in a more effective way because it does not use a computationally expensive BP based on Fresnel integrals. CT uses BP as a preprocessing tool for reducing the observation geometry to a
vertical observation line. Elimination of BP reduces numerical inaccuracies. The FSI method can be approxi-
mately generalized for a realistic observation geometry [Jensen et al., 2003]. One more numerical advantage of the FSI method is that it can be applied directly to the data measured by a LEO satellite without stationarizing the GPS satellite, which is used for BP [Gorbunov et al., 1996; Kursinski et al., 2000]. This results in some advantages in processing LEO-LEO data. However, due to the movement of the GPS satellite the measured wave field is not stationary which reduces the effective synthetic aperture in both methods.

[35] FSI avoids the necessity of BP, which makes it computationally more efficient and accurate than BP+CT combination. This method maps the wave field into a representation where effects of diffraction due to the big propagation distance are reduced, which allows for a high resolution. This makes FSI or other CT-like meth-
ods applied directly to data measured along the orbit prospective for operative radio occultation processing.

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References


Lauritsen, K. B., and M. S. Lohmann (2002), Unfolding of radio occultation multipath behavior using phase models,


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