Electron impact ionization: A new parameterization for 100 eV to 1 MeV electrons

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Received 11 May 2008; revised 9 June 2008; accepted 18 July 2008; published 11 September 2008.

We present a new parameterization of the altitude profile of the ionization rate in the Earth’s atmosphere due to precipitating energetic electrons. Precipitating electrons are assumed to have a Maxwellian energy distribution and an isotropic pitch angle distribution above the atmosphere. In this study, two electron transport models (whose validity has been verified by observations) are employed to calculate the ionization rate, to which we have fit our new parameterization. To derive a new parameterization, we follow a similar scheme to that of Roble and Ridley (1987) but take into account further functional dependence on the incident electron energy. As a result, the new method presented in this paper provides a highly improved prediction for electron impact in a significantly extended energy range from 100 eV to 1 MeV, spanning 4 orders of magnitude. Note that we have neglected the contribution of bremsstrahlung X rays generated by energetic electrons, which are mostly important below 50 km altitude. The comparison of parameterization results with model calculations shows that the errors generally fall well within ±5% in both the altitude-integrated total ionization rate and the peak value. The altitude profile as a whole is also accurately predicted, with errors in the altitudes of the peak and e-folding ionization rates significantly less than 5 km. The proposed new parameterization method with high accuracy is thus ready to be implemented into global models to assess the electron impact on the ionosphere and the atmosphere.


1. Introduction

Energetic electrons of magnetospheric origin represent the major source of kinetic particle energy to the Earth’s upper and middle atmosphere. The resulting ionizing collisions with the neutral constituents significantly alter the ionosphere properties, and are responsible for sustaining the high-latitude ionosphere on the night side. Among precipitating electrons, auroral particles (approximately in the energy range of 1–30 keV) are considered the main particle energy source and have been intensively studied. For efficient calculation in the National Center for Atmospheric Research (NCAR) thermospheric general circulation model, Roble and Ridley [1987] (hereinafter referred to as RR87) proposed a parameterization scheme to evaluate auroral electron impact ionization. RR87 employed an empirical energy dissipation function derived by Lazarev [1967], which they extended to yield the ionization rate of a vertically incident electron beam with a Maxwellian energy distribution. In order to analyze the Upper Atmospheric Research Satellite (UARS) measurements, Lummerzheim [1992] adopted the RR87 algorithm and reported a new set of parameterization coefficients for precipitating electrons in an isotropic pitch angle distribution. These new coefficients have subsequently been adopted and widely used for auroral electron impact ionization in other NCAR models, like the thermosphere ionosphere mesosphere electrodynamics general circulation model (TIME-GCM) [Roble and Ridley, 1994] and the Whole Atmosphere Community Climate Model (WACCM) [cf. Garcia et al., 2007, and references therein].

In addition to auroral electrons, there are several other types of electron input of interest to the ionosphere and atmosphere community. Polar rain, characterized by weak drizzle of electron precipitation with an average energy between about 100 and 300 eV, is observed in the polar cap region [Newell et al., 1996]. Higher-energy electrons with an energy of over 30 keV (and up to a few MeV) also
impact the Earth’s atmosphere, originating either from the ring current or from the plasma sheet in the high-energy tail [Raben et al., 1995]. As both auroral and polar rain electrons have been routinely incorporated into the TIME-GCM and WACCM, it is of particular interest to extend the models to evaluate the effect of the medium-energy (~30–1000 keV) electrons on the ionosphere and atmosphere. A study of Codrescu et al. [1997] made an effort to construct statistical medium-energy electron precipitation patterns and assessed the effect of this extra particle source using the TIME-GCM. It was found that medium-energy electrons had a significant impact on the mesospheric composition of minor constituents, like NO and NO2. Recent satellite data analyses showed that dramatic ozone reduction of around 40% in the middle to upper polar stratosphere can be attributed to descending NOx (NO + NO2) that is originally produced by energetic particle precipitation at higher altitudes [Randall et al., 2007]. It is established that medium-energy particles play a certain important role in the Earth’s middle and low atmosphere. It is thus desired that the existing models like the TIME-GCM and WACCM be capable of modeling medium-energy particles and taking into account their effects.

[4] However, it is not straightforward to extend the RR87 parameterization to the medium-energy electrons. As noted by Lummerzheim [1992], the RR87 method is valid in the same electron energy range as the Lazarev [1967] derivation, that is, from a few hundred eV to around 32 keV. It was also found by Lummerzheim [1992] that the RR87 parameterization gave a fairly good estimate of the ionization rate over an extended energy range up to 100 keV. Therefore, to better allow for medium-energy electrons of >100 keV and the polar rain electrons with an energy as low as 100 eV, a new parameterization is required. This paper represents an effort to derive a new parameterization scheme that covers a significantly extended precipitating electron energy range from 100 eV to 1 MeV.

[5] An overview of this paper follows. In section 2, our parameterization technique is described and the limitation of the RR87 parameterization is discussed. In section 3, our new parameterization function is obtained from a least-squares fit to electron transport model results. In section 4, the validity of our formulas is confirmed through a systematic error analysis in the broad energy range from 100 eV to 1 MeV. Finally, the paper’s findings are summarized in section 5.

2. Numerical Method
2.1. Model Description
[6] In this paper, we study the geoeffectiveness of precipitating electrons in a Maxwellian energy distribution. The differential hemispherical number flux (keV^-1 cm^-2 s^-1) is specified by

\[ \phi(E) = \frac{Q_0}{2E_0} E \exp\left(-\frac{E}{E_0}\right), \]

where \( Q_0 \) is the total precipitating energy flux (keV cm^-2 s^-1), \( E_0 \) is the characteristic energy in keV, defined as the energy at which the spectral flux has a peak. That is, the characteristic energy in a Maxwellian distribution is half of the average energy. It is worth noting that in situ satellite measurements usually exhibit a non-Maxwellian feature with a high-energy tail in particle spectral fluxes [e.g., Frahm et al., 1997]. The Maxwellian approximation therefore imposes a limitation on atmospheric ionization rate specification at low altitudes. In addition, the pitch angle distribution at the top side boundary is assumed to be isotropic. In an attempt to accurately derive a parameterization function to fit the altitude profile of the electron impact ionization rate, we resort to two electron transport models.

[7] In this study, the transport of precipitating \( E_0 < 10 \) keV electrons and the resulting ionization rate are modeled using a multistream method [Lummerzheim et al., 1989; Lummerzheim and Lilensten, 1994]. The electron flux intensity in the atmosphere is calculated as a function of altitude, energy and pitch angle by solving steady state Boltzmann transport equations. The ionization rate can then be derived with the knowledge of the flux intensity and the corresponding cross sections. The validity of the model was justified in comparison with in situ measurements [Lummerzheim et al., 1989] and laboratory experiments [Lummerzheim and Lilensten, 1994]. The multistream method is suitable for modeling auroral and polar rain electrons as pitch angle scattering during the particle transport becomes more significant when incident electrons have a relatively low energy.

[8] However, owing to the limitation of the energy grid setting in the current multistream code, it is not appropriate to investigate electron precipitation with a characteristic energy of far above 10 keV without any extension of the energy grid. Considering that high-energy electrons penetrate fairly deep into the region of the dense atmosphere where the particles actually do not move any significant distance away, angular scattering becomes less important for high-energy electrons. Therefore, a two-stream electron transport model [Solomon et al., 1988; Solomon and Abreu, 1989] is adopted here for modeling incident \( E_0 \geq 10 \) keV electrons. Good agreement between the multistream and two-stream models for auroral electron energies ensures the continuity of electron impact ionization modeling in this study. In addition, as required by energy conservation, the altitude profile of the ionization rate calculated by the two-stream model is further multiplied by a small normalization factor so that the mean energy loss per ion pair production is about 35 eV, which is in accordance with laboratory measurements [e.g., Rees, 1989]. The scaling factor ranges from 1.06 to 1.15, depending on the precipitating electron energy. It should be pointed out that both the multistream and two-stream ion transport models are in agreement with Monte Carlo simulations [Solomon, 2001], confirming that they are reliable to be used for parameterization.

[9] In the ionization rate calculation, the contribution of bremsstrahlung X rays that are generated by precipitating energetic electrons is not considered. However, by neglecting ionization by bremsstrahlung X rays, calculation results at low altitudes may be considerably underestimated, as the ionization produced by the X rays peaks substantially lower in altitudes than does direct ionization by incident electrons. The importance of bremsstrahlung X rays in atmospheric
ionization was demonstrated by analyzing UARS measurements by Frahm et al. [1997].

2.2. Parameterization Scheme

To derive a fit to the modeled altitude profile of electron impact ionization, we adopt the RR87 scheme as the core of our parameterization but a further improvement is required for application to a much broader energy range of incident electrons. Here we first introduce the basic idea of the RR87 parameterization and discuss its limitations, followed by our improvement. Also considering that there were severe typos in the original formulation of the RR87 paper, it is appropriate to list the key equations here but with clarification provided.

In the RR87 work, the total electron impact ionization rate is calculated by scaling proportional to a normalized energy deposition function \( f \). That is,

\[
q_{\text{tot}} = \frac{Q_0}{2\Delta \varepsilon} \frac{1}{H^2},
\]

where \( Q_0 \) is the total incident electron energy flux (keV cm\(^{-2}\) s\(^{-1}\)), and \( \Delta \varepsilon = 35 \times 10^{-3} \) keV stands for the mean energy loss per ion pair production. \( H \) is the scale height in \( m \) defined by

\[
H = \frac{kT}{mg}.
\]

In the scale height calculation, \( k \) is the Boltzmann constant, \( T \) is the atmospheric temperature, \( m \) is the average molecular weight of the atmosphere, and \( g \) is the gravitational acceleration.

The energy deposition function \( f \) in equation (2) was found to depend on a normalized quantity \( y \), which is determined by a combination of the atmospheric density profile and the characteristic energy through

\[
y = \frac{1}{E_0} \left( \frac{\rho H}{4 \times 10^{-6}} \right)^{0.606},
\]

where \( E_0 \) is the characteristic energy of precipitating electrons in keV, and \( \rho \) is the atmospheric mass density in g cm\(^{-3}\). It is seen that \( y \) approximately has a power-law dependence on the column mass density. It is worth noting that the incident electron energy is taken into account by the inclusion of \( E_0 \) in the \( y \) definition.

In the RR87 study, the relationship between \( f \) and \( y \) is parameterized by the following equation:

\[
f(y) = C_1 y^{f_2} \exp(-C_3 y^{f_4}) + C_5 y^{f_6} \exp(-C_7 y^{f_8}),
\]

where the coefficients \( C_1 \) through \( C_8 \) are constants, independent of the atmospheric profile or the precipitating particle energy. The dependence of the energy deposition function on the characteristic energy of the precipitating electrons enters into the RR87 parameterization only through the parameter \( y \). The eight constant coefficients in equation (5) were established by employing the empirical relationship derived by Lazar [1967]. Therefore, the dependence of \( f(y) \) on \( y \) in equation (5) qualitatively illustrates how precipitating particles of a certain characteristic energy lose their energy through ionizing collisions as they penetrate the Earth’s atmospheric mass. The desired physical quantity of the ionization rate can finally be obtained by applying equation (2).

It should be emphasized that altitude does not explicitly play a role in the ionization rate calculation in the above formulation. In other words, it does not matter whether the position in the atmosphere is measured with respect to altitude or pressure. In fact, a pressure-based coordinate system is used in the TIME-GCM and WACCM.

Figure 1 compares the \( E_0 = 1 \) keV electron impact ionization derived from the multistream model [Lummerzheim et al., 1989; Lummerzheim and Lilensten, 1994] to the RR87 parameterization for an isotropic particle precipitation. The Mass Spectrometer Incoherent Scatter (MSIS-90) model atmosphere for a moderate solar and geomagnetic activity, \( F_{10.7} = 150 \) and \( A_P = 35 \), is used [Hedin, 1991]. Figure 1a shows the altitude profile of the atmospheric scale height and the \( y \) variable in the MSIS atmosphere, using equations (3) and (4), respectively. The model results of the ionization rate are presented as well. In Figure 1b, the modeled ionization rate is demonstrated in terms of the relationship between \( y \) and \( f \) using equation (2). The RR87 parameterized relationship (equation (5)) is also displayed for comparison.

It is shown in Figure 1b that in general the RR87 parameterization is in fairly good agreement with the model results in the case of \( E_0 = 1 \) keV electron precipitation. However, the ionization rate peaks at a lower \( y \) value and thus a higher altitude (see Figure 1a). Although the peak value of the parameterized energy deposition function \( f \) is a little larger, the ionization rate actually has a smaller peak than the modeled value (shown later in Figure 3). This paradoxical result comes from the fact that their peaks occur at different altitudes, where the scale height at the higher altitude is larger, resulting in a smaller \( q_{\text{tot}} \) (see equation (2)).

As limited by the working energy range of the Lazar [1967] approximation, the RR87 parameterization is expected to be valid for electron precipitation in a limited energy range. Figure 2 shows the model results in terms of the \( y \) and \( f \) relationship for a variety of incident electron energies, from \( E_0 = 100 \) eV to 1 MeV. The old parameterized relationship, which is independent of \( E_0 \), is superposed for comparison (equation (5)). As explained in section 2.1, the multistream model is used to calculate the ionization rate for \( E_0 < 10 \) keV, while the two-stream model is employed for \( E_0 \geq 10 \) keV with an appropriate scaling. To further examine the sensitivity to the background atmosphere, the MSIS-90 atmosphere for the ionization rate calculation is altered by changing both \( F_{10.7} \) and \( A_P \) index values. \( F_{10.7} \) is changed from 50 to 300 by a step of 50, while \( A_P \) is changed from 5 to 65 by a step of 10. As a consequence, the comparison in Figure 2 is made for 42 MSIS atmospheres in total (6 \( F_{10.7} \) by 7 \( A_P \) values). Because the \( C \) coefficients in equation (5) are independent of the atmosphere and the incident energy, it is seen that the parameterized dependence of \( f(y) \) on \( y \) is the same for varying electron precipitation in different atmospheres.

It is worth noting that although a wide range of atmospheric vertical distributions (that is, \( \rho(z) \) and \( H(z) \)) were used in Figure 2, the \( y \) versus \( f \) profiles closely cluster
with each other for any given value of \( E_0 \). This is particularly true for high-energy electron precipitation. However, this insensitivity may be partially due to the low sensitivity of the MSIS-90 model atmosphere to the \( F_{10.7} \) and \( A_p \) indices in the middle and low atmosphere [Hedin, 1991]. The fact that the relationship between \( y \) and \( f \) does not change much with the atmosphere partially justifies the parameterization using equation (5) by RR87. In fact, the energy deposition function \( f \) was originally designed to make the profiles independent of the atmosphere. This is the basic idea behind the Rees [1963] paper, which forms the basis of the RR87 parameterization. In this study, we adopt the same simplified assumption and consider only the average atmospheric effect in parameterizing the functional dependence of \( f \) on \( y \). The error due to neglecting the atmospheric effect will be systematically analyzed in section 4.

[19] It is evident in Figure 2 that the energy deposition function profile (\( f \) versus \( y \)) exhibits a high degree of variation when the precipitating electron energy is changed. A significant shift and deformation of the profile is observed. This is particularly prominent for both \( E_0 = 100 \text{ eV} \) and \( E_0 = 1 \text{ MeV} \), despite the fact that the RR87 parameterization matches the model results reasonably well in the characteristic energy range of 1–100 keV. The discrepancy between the parameterization and the detailed model results implies that an improvement is required to take into account a further effect of \( E_0 \), although the \( y \) definition in equation (4) partially allows for the precipitating electron energy.

**Figure 1.** Comparison between the Roble and Ridley [1987] parameterization and results from a multistream electron transport model. Shown here are (a) modeled ionization rates (without bremsstrahlung) for precipitating electrons of \( E_0 = 1 \text{ keV} \) and \( Q_0 = 1 \text{ erg cm}^{-2} \text{s}^{-1} \) (solid curve), calculated \( y \) values (described in the text) from the atmospheric profile but increased by 1000 times to fit the same bottom scale (dashed curve), and the vertical distribution of the scale height with the scale on the top axis (dotted curve), and (b) the relationships between \( y \) and a normalized energy deposition function \( f \) that are derived from the model results (solid) and the Roble and Ridley [1987] parameterization (dashed), respectively.

**Figure 2.** The relationship between \( y \) and \( f \) derived from the multistream and two-stream models for electron precipitation with varying characteristic energies (open circles). Results are shown for a set of 42 different MSIS model atmospheres (see text). The Roble and Ridley [1987] parameterized relationship is shown for comparison (solid curve).
In response to this requirement, we extend equation (5) by making each $C$ coefficient energy dependent. That is

$$f(y, E_0) = C_1(E_0)yC_1(E_0) + C_5(E_0)y^5C_1(E_0).$$

(6)

In equation (6), each coefficient $C_i(E_0)$ ($i = 1, \ldots, 8$) is approximated by a separate third-order polynomial in the following way,

$$C_i(E_0) = \exp\left(\sum_{j=0}^{3} P_{ij}(\ln(E_0))^j\right),$$

(7)

in which $E_0$ is in the unit of keV and $\ln(E_0)$ is the natural (base $e$) logarithm of $E_0$.

To evaluate the parameters of $P_{ij}$ ($i = 1, \ldots, 8; j = 0, \ldots, 3$), we performed a two-dimensional fitting of $f(y, E_0)$ over $y$ and $E_0$. In this work, hundreds of precipitating energies $E_0$ are selected to cover the whole energy range of 100 eV to 1 MeV. For each $E_0$, the multistream/two-stream model is employed to calculate the resulting altitude profile of the ionization rate in the MSIS atmospheres, as described in section 2.1. The two normalized quantities of $f$ and $y$ are then derived using equations (2) and (4), respectively. Considering the atmospheric effect (although obviously not as pronounced as $E_0$), we expect a marked improvement over the old one. As expected from the previous analysis, the old parameterization cannot be reasonably extended to energies as low as 100 eV or higher than 100 keV. This is particularly evident in comparisons at $E_0 = 100$ eV and 1 MeV. It is demonstrated in Figure 3 that at $E_0 = 100$ eV, the predicted ionization rate by the RR87 approach has a peak, which is shifted upward by around 30 km or more. In the case of $E_0 = 1$ MeV, the peak ionization rate is significantly underestimated and the altitude profile penetrates more than 10 km deeper. In contrast, after the correction by considering the additional precipitating electron energy effect through equation (6), our algorithm works well for $E_0$ from 100 eV up to 1 MeV, spanning 4 orders of magnitude in the energy range.

### 3. Parameterization Results

The parameterization coefficients of $P_{ij}$ ($i = 1, \ldots, 8; j = 0, \ldots, 3$) are evaluated by a two-dimensional least-squares fit to the model results of the multistream code [Lummerzheim et al., 1989; Lummerzheim and Lilensten, 1994] or the two-stream code [Solomon et al., 1988; Solomon and Abreu, 1989], depending on the precipitating characteristic energy (see section 2.1). The obtained $P_{ij}$ values are given in Table 1.

To calculate the desired expected quantity of $q_{tot}$ through our new parameterization, the following procedure is performed. The $P_{ij}$ coefficients in Table 1 are first entered into equation (7) to evaluate $C_i(E_0)$, which is then substituted into equation (6) to assess $f(y, E_0)$. The ionization rate is finally calculated by using equation (2).

Figure 3 presents the application results of our energy-dependent parameterization, comparing the altitude profiles of the ionization rates to those obtained from other methods: the multistream/two-stream model, and the RR87 parameterization. By “energy dependent,” we emphasize that the effect of precipitating electron energy $E_0$ is further taken into account in the $y$ versus $f$ relationship as shown in equation (6). In contrast, the relationship in RR87 is fixed and independent of $E_0$ (see equation (5)).

It is seen in Figure 3 that our parameterization method yields results in remarkably better agreement with model predictions than the RR87 method. This overall closer match with the model results confirms the validity of our parameterization, showing a marked improvement over the old one. As expected from the previous analysis, the old parameterization cannot be reasonably extended to energies as low as 100 eV or higher than 100 keV. This is particularly evident in comparisons at $E_0 = 100$ eV and 1 MeV. It is demonstrated in Figure 3 that at $E_0 = 100$ eV, the predicted ionization rate by the RR87 approach has a smaller peak, which is shifted upward by around 30 km or more. In the case of $E_0 = 1$ MeV, the peak ionization rate is significantly underestimated and the altitude profile penetrates more than 10 km deeper. In contrast, after the correction by considering the additional precipitating electron energy effect through equation (6), our algorithm works well for $E_0$ from 100 eV up to 1 MeV, spanning 4 orders of magnitude in the energy range.

### 4. Error Analysis

The validity of our improved parameterization is verified in Figure 3 through the comparison with the model results for electron precipitation with five representative characteristic energies in two sample atmospheres. In this section, a systematic error analysis is performed to extend the comparison for more incident electron energies and in a vast number of background atmospheric conditions.

For the derivation of the parameterization coefficients, 42 MSIS model atmospheres are employed to obtain an average effect by changing $P_{10,7}$ and $A_{10}$ indices while the rest of the MSIS model parameters are held constant. For a comprehensive error evaluation, we test the parameterization methods in many more atmospheric conditions by also varying days of year and locations in the MSIS-90 model. Four days of year are selected for the test purpose: spring, summer, equinox, and winter. The location is also varied in terms of geographic longitudes ($0^\circ$, $90^\circ$, $180^\circ$, and $270^\circ$) and geographic latitudes ($60^\circ$, $70^\circ$, and $80^\circ$). A total of 2016 model atmospheric conditions are used here. As an example of atmospheric variation, the temperature at 100 km altitude ranges from 170 K to 228 K, while the temperature at 300 km altitude ranges from 555 K to 1840 K.

Figure 4 shows the error analysis of the two parameterization methods in the 2016 MSIS model atmospheres for a varying precipitating electron characteristic energy from 100 eV to 1 MeV. For any given characteristic energy,

### Table 1. Parameterization Coefficients $P_{ij}$ for Equation (7), Giving Total Ionization Rate by Precipitating Electrons With a Maxwellian Energy and Isotropic Pitch Angle Distribution

<table>
<thead>
<tr>
<th>$P_{ij}$</th>
<th>$j = 0$</th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$</td>
<td>1.32171(−1)</td>
<td>−6.18200(−2)</td>
<td>−4.08124(−2)</td>
<td>1.6514(−2)</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>5.58425(−1)</td>
<td>−5.00739(−2)</td>
<td>5.69309(−2)</td>
<td>−4.0249(−3)</td>
</tr>
<tr>
<td>$i = 3$</td>
<td>1.69692(−1)</td>
<td>−2.58983(−2)</td>
<td>1.96822(−2)</td>
<td>1.20505(−3)</td>
</tr>
<tr>
<td>$i = 4$</td>
<td>−1.22271(−1)</td>
<td>−1.15532(−2)</td>
<td>5.37951(−6)</td>
<td>1.20189(−3)</td>
</tr>
<tr>
<td>$i = 5$</td>
<td>1.5701(0)</td>
<td>2.87986(−1)</td>
<td>−4.14857(−1)</td>
<td>5.18158(−2)</td>
</tr>
<tr>
<td>$i = 6$</td>
<td>8.83195(−1)</td>
<td>3.43102(−2)</td>
<td>−8.33599(−2)</td>
<td>1.02515(−2)</td>
</tr>
<tr>
<td>$i = 7$</td>
<td>1.90953(0)</td>
<td>−4.74704(−2)</td>
<td>−1.80200(−1)</td>
<td>2.46652(−2)</td>
</tr>
<tr>
<td>$i = 8$</td>
<td>−1.29566(0)</td>
<td>−2.10952(−1)</td>
<td>2.73106(−1)</td>
<td>−2.92752(−2)</td>
</tr>
</tbody>
</table>

*Numbers in parenthesis denote powers of 10. For example, 3.49979(−1) means 3.49979 × 10^{−1}.*
the ionization rates in a certain atmosphere that are derived from the two parameterization approaches are compared with the multistream/two-stream model results, giving relative errors for each method. By varying the background atmospheric conditions, the mean values of the errors are calculated and marked with open circles. The error bars in Figure 4 denote one standard deviation of the error distribution from the mean values. The error analysis here highlights the discrepancies in three aspects. First is the altitude-integrated ionization rate, which is reflective of the total energy deposition in the atmosphere through ionizing collisions. Second is the magnitude of peak ionization. Third is the characteristic altitude. In this third comparison, we focus on three altitude diagnostics. One is the altitude of the ionization peak. The two others are the e-folding altitudes above and below peak ionization. These three points together are qualitatively reflective of the ionization rate altitude profile.

It is clearly demonstrated that the peak ionization rate from the RR87 parameterization is generally underestimated, with larger errors at the low- and high-energy ends. The error is significant when the precipitating electron energy is lower than 1 keV or higher than 100 keV. It is seen that the relative errors in the peak ionization rate are about $-15\%$ at $E_0 = 100$ eV and $-30\%$ at $E_0 = 1$ MeV. This overall underestimation is understandable from Figure 2 with the caution that in addition to the normalized energy deposition function $f$, the ionization rate is still inversely proportional to the scale height $H$ through equation (2), which varies with altitude (and thus $y$). For the same reason, the variation with the atmospheric conditions manifests itself quite differently in the two parameterization results. For example, at $E_0 = 1$ MeV, the error bar of the old parameterization is considerably larger. This difference is
caused by the fact that the new method better predicts the peak ionization altitude, making the variation of the scale height at the peak significantly smaller and thus reducing the variation of the peak ionization rate.

The analysis of the altitude diagnostics (see definition above) shows that the altitude profile of the ionization rate given by the old parameterization significantly disagrees with the model results at the low- and high-energy ends. The whole profile shifts toward higher altitudes by up to around 40 km when $E_0 < 1$ keV and penetrates deeper by up to more than 10 km when $E_0 > 100$ keV. In contrast, the revised parameterization method proposed in this paper is in excellent agreement with model predictions for a broad energy range of electron precipitation from 100 eV to 1 MeV. The overall significant improvement indicates that our new parameterization method can provide an accurate prediction of the electron impact ionization rate and therefore is a reliable tool ready to be implemented into ionosphere and atmosphere models, like TIME-GCM and WACCM.

5. Summary and Conclusion

Recent model simulations and satellite observations clearly suggest that medium- and high-energy electron precipitation has a significant effect on the Earth’s middle atmosphere [Codrescu et al., 1997; Randall et al., 2007]. However, it is not straightforward to extend the existing Roble and Ridley [1987] parameterization for the energy far below 1 keV or higher than 100 keV. A new parameterization method is thus in an increased need to incorporate electron precipitation in a broad energy range. This need is particularly strong for the foreseeable future. The existing global ionosphere/atmosphere models are all extremely computational expensive even with modern supercomputers, requiring an accurate, easy, and fast parameterization method amenable for implementation.
In this paper, we present a new method to parameterize the altitude profile of the ionization rate generated by electron precipitation. Precipitating electrons are assumed to have a Maxwellian energy distribution and an isotropic pitch angle distribution. The resulting ionization rate in the atmosphere is obtained from two electron transport models, which our new parameterization is derived to fit. The validity of the electron transport models has been verified by comparisons with observations. In the ionization rate calculation, the contribution of bremsstrahlung X rays generated by energetic electrons is not considered, which are primarily important at low altitudes (below 50 km) [Frahm et al., 1997].

For the characteristic energy of \( E_0 < 10 \) keV, the multistream model [Lummerzheim et al., 1989; Lummerzheim and Lilensten, 1994] is employed, which is suitable to resolve angular scattering that is of particular interest when particle energies are low. Owing to the limitation of the current energy grid setting in the multistream code, we adopt the two-stream model [Solomon et al., 1988; Solomon and Abreu, 1989] to evaluate the ionization rate for \( E_0 \geq 10 \) keV. As required by the energy conservation, a small scaling factor is applied to the two-stream model results so that the mean energy loss per ion pair production is 35 eV, in accordance with laboratory measurements.

The RR87 scheme is adopted as the reference algorithm for this study. It is found that their application achieves a fairly good agreement with the model results only in a limited characteristic energy range of \( 1 - 100 \) keV. A significant error occurs when \( E_0 < 1 \) keV or \( E_0 > 100 \) keV. The error in the altitude-integrated ionization rate rises up to 10% when the characteristic energy decreases to 100 eV. The peak ionization rate is generally underestimated, reaching \(-15\%\) at \( E_0 = 100 \) eV and \(-30\%\) at \( E_0 = 1 \) MeV. The whole altitude profile shifts by up to around 40 km altitude higher when \( E_0 < 1 \) keV and penetrates deeper by up to more than 10 km when \( E_0 > 100 \) keV. These significant disagreements prevent the RR87 parameterization from being applied to study the effect of the polar rain low-energy electrons and high-energy electron precipitation from the ring current and the high-energy tail of the plasma sheet.

In this work, we propose an improvement to the RR87 parameterization. A two-dimensional least-squares fit to the electron transport model results is carried out to parameterize the dependence of the ionization rate on both the vertical location in the atmosphere and the precipitating electron energy. Although the electron energy was partially taken into account in the old method, a further consideration in our approach demonstrates a significant enhancement in the accuracy of calculations. The new parameterization is highly reliable for electron precipitation spanning 4 orders of magnitude in the energy range, from 100 eV up to 1 MeV. The comparison of the application of the new method with the model results shows that the errors generally fall within \( \pm 5\% \) in both the altitude-integrated and peak ionization rates. The altitude profile of the ionization rate as a whole is accurately predicted, with errors in the altitudes of the peak and e-folding ionization rates significantly less than 5 km.

It should be pointed out that the improved parameterization is computational efficient, with a slight increase in the computational load over the RR87 approach. Therefore, the proposed new parameterization method is ready to be implemented into global models to evaluate the electron impact on the ionosphere and the atmosphere.

Acknowledgments. The work at the University of Colorado was supported by NASA LWS grant NNX06AC05G. The work at the National Center for Atmospheric Research is sponsored by the National Science Foundation.

Wolfgang Baumjohann thanks Rudolf Treumann and Richard Link for their assistance in evaluating this paper.

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