Generalized VTD Retrieval of Atmospheric Vortex Kinematic Structure. Part I: Formulation and Error Analysis

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ABSTRACT

The primary circulation of atmospheric vortices, such as tropical cyclones and tornadoes, can be estimated from single-Doppler radar observations using the ground-based velocity track display (GBVTD) algorithm. The GBVTD algorithm has limitations in the following four areas: 1) distortion in the retrieved asymmetric wind fields, 2) a limited analysis domain, 3) the inability to resolve the cross-beam component of the mean wind, and 4) the inability to separate the asymmetric tangential and radial winds. This paper presents the generalized velocity track display (GVTD) algorithm, which eliminates the first two limitations inherent in the GBVTD technique and demonstrates the possibility of subjectively estimating the mean wind vector when its signature is visible beyond the influence of the vortex circulation.

In this new paradigm, the GVTD algorithm fits the atmospheric vortex circulation to a new variable $V_{dD}/R_T$ in a linear azimuth angle ($\theta'$), rather than the Doppler velocity $V_d$ in a nonlinear angle ($\psi$), which is used in GBVTD. Key vortex kinematic structures (e.g., mean wind, axisymmetric tangential wind, etc.) in the $V_{dD}/R_T$ space simplify the interpretation of the radar signature and eliminate the geometric distortion inherent in the $V_d$ display. This is a significant improvement in diagnosing vortex structures in both operations and research. The advantages of using $V_{dD}/R_T$ are illustrated using analytical atmospheric vortices, and the properties are compared with GBVTD. The characteristics of the $V_{dD}/R_T$ display of Typhoon Gladys (1994) can be approximated by a constant mean wind plus an axisymmetric vortex.

1. Introduction

Atmospheric vortices such as tropical cyclones and tornadoes possess a dipole Doppler velocity pattern when observed by a ground-based Doppler radar scanning in a plan-position indicator (PPI) mode (e.g., Donaldson 1970). The shape of the dipole Doppler velocity pattern of an axisymmetric vortex is a function of the distance between the “vortex circulation center” (hereafter, the center) and the radar, core diameter, and the ratio of peak tangential to peak radial wind. The dipole rotates clockwise (counterclockwise) when the radial wind is inflow (outflow), as shown in Wood and Brown (1992). When an axisymmetric vortex is located at infinite distance from the radar, its center can be determined as the midpoint of the line segment connecting the two peak dipole velocities (Wood and Brown 1992). As the vortex approaches the radar, the peak velocities of the dipole move toward the radar faster than the center. Hence, the dipole pattern is distorted and the center does not fall on the line segment connecting the two peak velocities of the dipole, which increases the complexity of accurately identifying the center in operational setting.

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Based on the rotational characteristics of a vortex, Lee et al. (1999) formulated a single-Doppler wind retrieval methodology, called the ground-based velocity track display (GBVTD), to retrieve the primary kinematic structures of atmospheric vortices (Lee et al. 2000; Roux et al. 2004; Bluestein et al. 2003; Harasti et al. 2004; Lee and Wurman 2005; Lee and Bell 2007; Tanamachi et al. 2007). The symbols and geometry of the GBVTD technique are illustrated in Fig. 1. For the convenience of discussion, we define the term “$\mathbf{R}_T$ vector” with a magnitude of $R_T$ and a direction pointing from the radar toward the center. Using a cylindrical coordinate system with the center as the origin, the GBVTD technique performs a Fourier decomposition of the Doppler velocity $V_d$ around each circle of radius $R$, and then estimates the three-dimensional (3D) tangential and radial circulations that cannot be deduced by existing single-Doppler wind retrieval methods (e.g., Browning and Wexler 1968; Donaldson 1991; Harasti 2003). Plausible axisymmetric 3D kinematic and dynamic quantities, such as the angular momentum, vertical vorticity, and perturbation pressure, can also be computed from the GBVTD-retrieved axisymmetric tangential and radial winds (Lee et al. 2000; Lee and Wurman 2005; Lee and Bell 2007).

The limitations of the GBVTD technique are as follows: 1) distortion in the retrieved asymmetric wind fields, 2) a limited analysis domain, 3) an inability to resolve the cross-beam component of the mean wind, and 4) an inability to separate the asymmetric tangential and radial winds. The first three limitations are caused by the sampling geometry, while the last is due to the intrinsic closure assumptions of the GBVTD technique. Hence, the GBVTD-derived vortex circulation is a proxy of the “true” circulation and may inherit large uncertainties resulting from the above limitations in certain situations (Lee et al. 1999).

This paper presents the generalized velocity track display (GVTD) technique and its applications to atmospheric vortices. The GVTD technique extends the foundation of GBVTD established in Lee et al. (1999) in an attempt to address the first three aforementioned limitations inherent in the GBVTD technique. Starting from the same radar observations, the GVTD technique introduces a new variable $V_d D/R_T$ by multiplying the distance of each gate ($D$) by $V_d$, and then scaling by the distance between the radar and the vortex center ($R_T$). Key vortex kinematic structures displayed in the $V_d D/R_T$ space simplify the interpretation of the radar signature and eliminate the geometric distortion inherited in the $V_d$ space (Jou et al. 1996). It will be shown that GVTD expands $V_d D/R_T$ into Fourier coefficients in a linear coordinate ($\theta'$) rather than expanding $V_d$ in a nonlinear coordinate ($\psi'$) in GBVTD. This results in a slightly complicated but mathematically exact representation, eliminating the required approximation of $\cos \alpha$ in GBVTD [Eq. (5) in Lee et al. (1999)]. GVTD is able to retrieve asymmetric vortex structures without distortion when the center is known accurately. The GVTD formulation can be applied to the extensions of the velocity track display (VTD) techniques (e.g., Roux et al. 2004; Liou et al. 2006) to improve their performance.

Section 2 describes the mathematical formulation of the GVTD technique. In section 3, characteristics of analytical wind patterns displayed in $V_d D/R_T$ space are given and compared with those displayed in $V_d$ space. Section 4 compares the center and radius of maximum...
wind (RMW) estimates between the $V_d/D_R$ and $V_d$ displays. In section 5, we illustrate and compare the wind fields retrieved from the GVTD and GBVTD techniques of several analytical vortices. Error analyses were conducted to investigate the differences between GVTD- and GBVTD-retrieved wind fields in the presence of asymmetry, a misplaced center, and uncertainty in the mean wind. Section 6 uses a simple flow model to simulate Typhoon Gladys’s observed characteristics. A summary and recommendations for future work are given in the final section.

\[ \hat{V}_d/cos\phi = V_M \cos(\theta_d - \theta_M) - V_T \sin(\theta - \theta_d) + V_R \cos(\theta - \theta_d) \]

\[ = V_M(\cos(\theta_d \cos \theta_M + \sin \theta_d \sin \theta_M)) - V_T(\sin \theta \cos(\theta_d - \cos \theta_d) - \cos \sin \theta_d) + V_R(\cos \theta \cos \theta_d + \sin \theta \sin \theta_d). \]  

For a Doppler velocity at point $E$ ($D$, $\theta_d$) (Fig. 1), one may deduce that

\[ D \cos \theta_d = R \cos \theta + R_T \cos \theta_T \]  

\[ D \sin \theta_d = R \sin \theta + R_T \sin \theta_T. \]  

Note that all angles are mathematical angles where positive is defined as being counterclockwise.

Substituting (3) and (4) into (2) and approximating $\hat{V}_d/cos\phi$ with $V_d$, we obtain

\[ V_d = (-V_T \sin \theta + V_R \cos \theta + V_M \cos \theta_M)(R \cos \theta \]

\[ + R_T \cos \theta_T)/D + (V_T \cos \theta + V_R \sin \theta \]

\[ + V_M \sin \theta_M)(R \sin \theta + R_T \sin \theta_T)/D. \]  

Rearranging (5) and applying trigonometry identities, we obtain

\[ V_d D R_T \]

\[ = -V_T \sin(\theta - \theta_T) + V_R \left[ \frac{R}{R_T} + \cos(\theta - \theta_T) \right] \]

\[ + V_M \cos(\theta_T - \theta_T) + \frac{R}{R_T} V_M \cos(\theta - \theta_M). \]  

Let $\theta' = \theta - \theta_T$ and use the relation $\theta - \theta_M = (\theta - \theta_T) + (\theta_T - \theta_M)$, and (6) becomes

\[ V_d D R_T = V_R R \frac{R}{R_T} \cos(\theta_T - \theta_M) \]

\[ - \left[ V_T + \left( \frac{R}{R_T} \right) V_M \sin(\theta_T - \theta_M) \right] \sin \theta' \]

\[ + \left[ V_R + \left( \frac{R}{R_T} \right) V_M \cos(\theta_T - \theta_M) \right] \cos \theta'. \]  

For a given $R$, the right-hand-side of (7) depends only on $\theta'$. Comparing Eq. (7) in Lee et al. (1999) with (7) herein, it can be seen that $V_d$, a function of nonlinear $\psi$ in GBVTD, corresponds to $V_d/D_R$, a function of linear $\theta'$ in GVTD. Note that Eq. (7) in Lee et al. (1999) required an approximation to link the unknown variable $\sin \alpha$ and the known constant $\sin \alpha_{max} = R/R_T$. When $R > R_T$, $\alpha_{max}$ is not defined\(^1\) (see Fig. 1). Explicitly moving $D$ to the left-hand side as part of the new variable makes (7) mathematically exact and valid for all radii beyond $R > R_T$.

Following Lee et al. (1999), we decompose $V_d/D_R$, $V_T$, and $V_R$ into Fourier components in the $\theta'$ coordinates:

\[ V_d D R_T (R, \theta') = A_0 + \sum_{n=1}^{N} A_n \cos n \theta' + \sum_{n=1}^{N} B_n \sin n \theta', \]  

\[ V_T (R, \theta') = V_T C_0 + \sum_{n=1}^{N} V_T C_n \cos n \theta' + \sum_{n=1}^{N} V_T S_n \sin n \theta', \]  

\[ V_R (R, \theta') = V_R C_0 + \sum_{n=1}^{N} V_R C_n \cos n \theta' + \sum_{n=1}^{N} V_R S_n \sin n \theta', \]  

in which $A_n$ ($V_T C_n$ and $V_R C_n$) and $B_n$ ($V_T S_n$ and $V_R S_n$) are the azimuthal wavenumber $n$ cosine and sine com-

\(^1\)This restriction does not really exist if $\sin \alpha_{max}$ is used in Eq. (B3) in Lee et al. (1999). However, it can be shown that when $R/R_T > 1$, $\psi$ spans an insufficient and highly nonlinear spaced subset of $0-2\pi$ for a meaningful GBVTD fit.
ponents of \( V_d D / R_T \) (\( V_T \) and \( V_R \)), as defined in Lee et al. (1999).

Substituting (8), (9), and (10) into (7), similar to Lee et al. (1999), we obtain the following:

\[
A_0 = \frac{R}{R_T} V_R C_0 + V_M \cos(\theta_T - \theta_M) - \frac{1}{2} V_T S_1 + \frac{1}{2} V_R C_1, \\
A_1 = \frac{R}{R_T} V_R C_1 + \frac{R}{R_T} V_M \cos(\theta_T - \theta_M) + V_R C_0 - \frac{1}{2} V_T S_2 + \frac{1}{2} V_R C_2, \\
B_1 = \frac{R}{R_T} V_R S_1 - \frac{R}{R_T} V_M \sin(\theta_T - \theta_M) - V_T C_0 + \frac{1}{2} V_T C_2 + \frac{1}{2} V_R S_2, \\
A_n(n \geq 2) = \frac{R}{R_T} V_R C_n + \frac{1}{2} (V_T S_{n-1} + V_R C_{n-1} - V_T S_{n+1} + V_R C_{n+1}), \text{ and} \\
B_n(n \geq 2) = -\frac{R}{R_T} V_R S_n + \frac{1}{2} (-V_T C_{n-1} + V_R S_{n-1} + V_T C_{n+1} + V_R S_{n+1}).
\]

Rearranging (11)–(15) to express each wave component of the vortex using these Fourier coefficients, we have

\[
V_M \cos(\theta_T - \theta_M) = A_0 - \frac{R}{R_T} V_R C_0 + \frac{1}{2} V_T S_1 - \frac{1}{2} V_R C_1, \\
V_T C_0 = -B_1 - B_3 + \frac{R}{R_T} [-V_M \sin(\theta_T - \theta_M) + V_R S_1 + V_R S_3] + V_R S_2, \\
V_R C_0 = \frac{A_0 + A_1 + A_2 + A_3 + A_4}{1 + \frac{R}{R_T}}, \\
V_T S_n = 2A_{n+1} - V_R C_n + V_T S_{n-2} - V_R C_{n+2} - \frac{2R}{R_T} V_R C_{n+1}, \text{ and} \\
V_T C_n = -2B_{n+1} + V_R S_n + V_T S_{n-2} + V_R C_{n+2} + \frac{2R}{R_T} V_R C_{n+1}.
\]

Equations (16)–(20) correspond to Eqs. (19)–(27) in Lee et al. (1999)\(^2\) for GBVTD with additional terms associated with \( R / R_T \). In the limit of \( R / R_T \sim 0 \), these two sets of equations are identical when they are truncated at the same wavenumber \( n \). It can be shown (appendix A) that (6) reduces to VTD [Eq. (3) in Lee et al. 1994] in the limit of \( R / R_T \sim 0 \) (i.e., \( D / R_T \sim 1 \)). In this situation, all radar beams of ground-based radar can be treated parallel with each other, similar to the sampling geometry in VTD. In addition, the most severe geometric constraint imposed in GBVTD (Lee et al. 1999), that is, the analysis domain of a storm is limited to \( R / R_T < 1 \), is no longer a constraint. The analysis domain in the GVTD extends over the entire domain wherever sufficient Doppler velocity data are available to yield reliable GVTD Fourier coefficient estimates. This point will be illustrated in section 5. Therefore, GVTD is a more general form of the VTD family of techniques.

GVTD faces similar problems encountered in Lee et al. (1999) where the numbers of unknown variables are greater than the number of equations. Hence, we assume the same closure assumptions as GBVTD, namely, that the asymmetric \( V_R \) is smaller than \( V_T \) and therefore can be ignored. Searching for dynamic closure assumptions for the VTD family of techniques remains a research topic.

3. Characteristics of the \( V_d D / R_T \) display

The characteristics of vortex signatures in \( V_d D / R_T \) space can be evaluated analytically from (7). Because (7) is similar to the VTD [Eq. (3) in Lee et al. 1994], characteristics of \( V_d D / R_T \) resemble those of \( V_d \) in VTD where radar beams are parallel to each other and there is no geometric distortion of the asymmetric structures. Regrouping (7) yields

\[
\frac{V_d D}{R_T} = -U_1 \sin(\theta' - \theta_0) + U_2,
\]

\(^2\) There is a typographical error in Eq. (20) of Lee et al. (1999). The correct equation is \( V_T C_0 = -B_1 - B_3 - V_M \sin(\theta_T - \theta_M) \sin\alpha_{max} + V_R S_2. \)
in which

\[
U_1 = \left[ V_T + \left( \frac{R}{R_T} \right) V_M \sin(\theta - \theta_M) \right]^2 + \left[ V_R + \left( \frac{R}{R_T} \right) V_M \cos(\theta - \theta_M) \right]^2 \right]^{1/2}, \tag{22}
\]

\[
U_2 = V_R \frac{R}{R_T} + V_M \cos(\theta - \theta_M), \quad \text{and} \tag{23}
\]

\[
\theta_0 = \tan^{-1} \frac{V_R + \frac{R}{R_T} V_M \cos(\theta - \theta_M)}{V_T + \frac{R}{R_T} V_M \sin(\theta - \theta_M)}. \tag{24}
\]

It can be concluded that for a fixed \( R \), (21) is a function of \( \theta' \) only as long as \( V_M, V_T, \) and \( V_R \) are functions of \( \theta' \). The existence of \( V_R \) and/or \( V_M \) in (23) raises or lowers the entire sine curve. Note that in VTD and GBVTD, \( V_M \) is the only factor that would shift the entire curve up and down for axisymmetric vortices. In (24), \( \theta_0 \) represents the phase shift of the sine curve (i.e., azimuthal rotation of the dipole). If there is no mean wind \( (V_M = 0) \), then \( \theta_0 \) reduces to \( \tan^{-1}(V_R/V_T) \), as in GBVTD (Fig. 2 in Lee et al. 1999). The effect of \( V_M \) on \( \theta_0 \) is further reduced by the factor \( R/R_T \) in the near-core region, but this effect may not be ignored at far radii of the vortex. If \( V_T \gg V_R \), then \( \theta_0 \approx 0 \). Thus, in a vortex without significant \( V_R, \theta_0 \) is generally small. Note that the phase shift of the dipole signature does not depend on \( V_M \) in \( V_d \) space, but does depend on \( V_M \) in \( V_dD/R_T \) space. This has the effect of complicating the estimation of the axisymmetric radial wind, as can be seen by comparing (18) with Eq. (21) of Lee et al. (1999). The mean wind vector can be estimated by using the hurricane volume velocity processing (HVVP) method (Harasti 2003) or using the unique signature of the mean wind in the \( V_dD/R_T \) display (shown below).

Following Brown and Wood (1991), an idealized vortex flow field is constructed to simulate the wind patterns in \( V_d \) and \( V_dD/R_T \). The complete flow fields include a uniform mean wind, an axisymmetric \( V_T \), and an axisymmetric \( V_R \). The mathematical expressions in natural coordinates are

\[
V_M = -V_M \sin(\theta - \theta_M) \mathbf{t} + V_M \cos(\theta - \theta_M) \mathbf{r},
\]

\[
V_T = V_{T_{\text{max}}} \left( \frac{R}{R_{\text{max}}} \right)^{M} \mathbf{t},
\]

\[
V_R = V_{R_{\text{max}}} \left( \frac{R}{R_{\text{max}}} \right)^{M} \mathbf{r}, \tag{25}
\]

in which \( \mathbf{t} \) is the unit vector in the tangential direction (positive counterclockwise) and \( \mathbf{r} \) is the unit vector in the radial direction (positive toward center); \( V_{T_{\text{max}}} \) and \( V_{R_{\text{max}}} \) are the maximum axisymmetric \( V_T \) and \( V_R \). Figure 2 shows a set of the flow fields, in which a plus sign marks the center at \((x, y) = (60\, \text{km}, 60\, \text{km})\), \( V_{T_{\text{max}}} = 40\, \text{m s}^{-1}, \) \( V_{R_{\text{max}}} = 10\, \text{m s}^{-1}, \), \( \theta_M = 180^\circ, \) and \( R_{\text{max}} = 20\, \text{km}. \) For a Rankine vortex, we have \( \lambda_r = 1 \) when \( R \leq R_{\text{max}} \), and \( \lambda_r = -1 \) when \( R > R_{\text{max}} \). The hypothetical Doppler radar is located at the origin.

A constant easterly mean wind and its corresponding \( V_d \) and \( V_dD/R_T \) displays are illustrated in Figs. 2a(1)–(3). The mean wind signature is a set of straight lines diverging from the radar in the \( V_d \) display [Fig. 2a(2)]. The wind direction is perpendicular to the zero Doppler velocity line pointing toward the positive contours and the wind speed is the maximum Doppler velocity in the domain. In the \( V_dD/R_T \) display, the easterly mean wind signature is a set of north–south-oriented parallel lines [Fig. 2a(3)]. It can be shown (appendix B) that the mean wind vector is the gradient of \( V_dD \). Note that \( R_T \) is a scale factor and is not needed to determine the mean wind vector. This parallel line signature can be identified by visually examining the \( V_dD/R_T \) contours not affected by the vortex circulation, usually in the quadrant opposite the center. An objective procedure to deduce the mean wind vector when mixed with a vortex will be presented in the forthcoming second part of this work. Hence, one of the unresolved quantities in the GBVTD formulation, the cross-beam mean wind, can be directly estimated in the \( V_dD/R_T \) display.

The flow fields \( V_d \) and \( V_dD/R_T \) displays of an axisymmetric vortex are portrayed in Figs. 2b(1)–(3). The striking differences between the \( V_d \) and \( V_dD/R_T \) displays [Figs. 2b(2),(3)] are in the shapes of the contours. The \( V_d \) pattern of an axisymmetric tangential vortex [Fig. 2b(2)] is distorted as a function proportional to \( R/R_T \). On the contrary, the \( V_dD/R_T \) contours are symmetric about the center [Fig. 2b(3)], independent of \( R/R_T \) with no distortion. Jou et al. (1996) first proposed using the midpoint of the line connecting the dipole in the \( V_dD/R_T \) display to estimate the center and the RMW [they called it the “velocity distance azimuth display” (“VDAD”) method].

Examples of the axisymmetric radial outflow are illustrated in Figs. 2c(1)–(3). When considering \( V_T = 0, \) \( V_M = 0 \) in (25), \( U_1 = V_R, U_2 = V_R(R/R_T), \) and \( \theta_0 = \pi/2 \) or \( 3\pi/2, \) (21) becomes \( V_dD/R_T = -V_R \sin(\theta - \pi/2) + V_R(R/R_T). \) This is the reason why the \( V_R \) signature in the \( V_dD/R_T \) display is not symmetric about the center and there is a \( \pi/2 \) phase difference between \( V_R \) and \( V_T \) in \( V_dD/R_T \) displays. Nevertheless, the contours are more symmetric in the \( V_dD/R_T \) display compared with the \( V_d \) display.
FIG. 2. (left) [a(1)] A constant easterly mean wind with a magnitude of 10 m s$^{-1}$. [b(1)] a Rankine-combined vortex, [c(1)] the axisymmetric radial wind, and [d(1)] the total wind [(a1) + (a2) + (a3)], and the corresponding (middle) observed Doppler velocity and (right) $V_d/D/R_T$ display of the simulated wind fields in the left-hand side. The Doppler radar is located at the lower-left corner.
Figures 2d(1)–(3) illustrate the flow field of a combination of $V_M$, $V_T$, and $V_R$, and the corresponding $V_d$ and $V_D/D/R_T$ displays. The combined flow field is asymmetric. However, the dipole is not significantly distorted in the $V_d/D/R_T$ display near the RMW, even with the addition of a constant $V_M$ and axisymmetric $V_R$, allowing the center and RMW to be estimated using the VDAD method [Fig. 2d(3)].

To further examine the characteristics of vortex signatures in the $V_d$ and $V_D/D/R_T$ displays as a function of $R_{max}$, two axisymmetric rotating vortices with $V_{T_{max}} = 50$ m s$^{-1}$ and an $R_{max}$ of 30 and 80 km are constructed, and their corresponding $V_d$ and $V_D/D/R_T$ displays are shown in Fig. 3. Figures 3a,c illustrate the $V_d$ and $V_D/D/R_T$ displays of the smaller vortex with an $R_{max}$ of 30 km where the radar is located outside the RMW, while Figs. 3b,d portray the corresponding profiles of $V_d$ and $V_D/D/R_T$ around two radii (at $R = 30$ and 60 km). As $R$ increases, the peak wind locations in the $V_d$ display (Fig. 3b) shift toward $\psi = \theta = \pi$, while the peak values of $V_d/D/R_T$ (Fig. 3d) remain at $\theta' = \pi/2$ and $\theta'' = 3\pi/2$. In the $V_d/D/R_T$ display, the center remains at the intersection between the zero Doppler velocity line and the line connecting the dipole, independent of the geometric factor $R/R_T$.

When the radar is inside the RMW of the larger vortex ($R_{max} = 80$ km, $R_T = 70.7$ km; hence $R_{max} > R_T$), the radar does not sample the full component of the $V_{T_{max}}$; therefore, the peak $V_d$ around the RMW (blue circle in Fig. 3e) is less than the $V_{T_{max}}$ (Fig. 3a). However, the corresponding $V_D/D/R_T$ profile at the $R = R_{max} = 80$ km and $R = 110$ km (blue line and red line in Figs. 3g,h) can recover the vortex intensity as in the $R_{max} < R_T$ case (Fig. 3c). The dipole structure can be fully recovered in the $V_d/D/R_T$ space, and even the radar does not sample the full component of $V_T$ at each radius. This property can be illustrated analytically by setting $V_R = 0$, $V_M = 0$, $U_1 = V_T$, $U_2 = 0$, and $\theta_0 = 0$ in (21); we will then have $V_d/D/R_T = -V_T \sin \theta'$. There is a clear advantage to displaying atmospheric vortices in $V_d/D/R_T$ space over the traditional $V_d$ space.

In summary, representing a vortex in $V_d/D/R_T$ space simplifies the vortex signatures and eliminates the dipole distortion as a function of $R_{max}/R_T$ in the traditional $V_d$ display. In particular, the signature of a constant mean wind is a set of parallel lines. The potential to separate the vortex and the mean wind in $V_d/D/R_T$ display provides a new paradigm to study the interaction between the vortex and the mean flow.

4. Center and RMW

It can be shown from (21) that the center is the midpoint of the line connecting the dipole in the $V_d/D/R_T$ display (i.e., the VDAD method) as long as $U_1$ and $U_2$ remain constant at the $R_{max}$ (i.e., any combination of axisymmetric $V_T$, axisymmetric $V_R$, and a constant $V_M$). The existence of axisymmetric $V_R$ and/or $V_M$ would add a constant magnitude and a constant phase shift to the sine curve at each radius that makes the dipole uneven in magnitude and rotates in azimuth. It is found that the VDAD method is especially useful for identifying the center of a near-axisymmetric vortex in a real-time operational environment. When significant asymmetric components exist, (21) is not valid and accurately estimating the center will require a more elaborate methodology, such as the "simplex" method (Lee and Marks 2000), which is beyond the scope of this paper.

We applied the Wood and Brown (1992) method to retrieve the center and $R_{max}$ [Fig. 2d(2)], where the estimated center is located at (60.33 km, 60.19 km) and $R_{max}$ is 19.65 km, compared with the true center located at (60 km, 60 km) and an $R_{max}$ of 20 km. These errors are quite small (the center error is 0.38 km and the $R_{max}$ error is 0.35 km). Next, we consider a more extreme case, for example, $R_{max}$ increases to 30 km, $V_{T_{max}}$ decreases to 25 m s$^{-1}$, $V_M$ increases to 20 m s$^{-1}$, and the direction of $V_M$ is from the southwest, parallel to the $R_T$ vector. Then, the retrieved center is (61.45 km, 60.34 km) and $R_{max}$ is 28.77 km. The errors increase to 1.49 and 1.23 km for the center and RMW, respectively. It is clear that the errors depend both on the assigned wind fields and the relative magnitude of the mean wind speed and direction. On the contrary, both centers estimated using the VDAD method are nearly perfect.

5. GVTD and GBVTD

A series of numerical experiments (using analytical vortices) were conducted to investigate the differences between GVTD- and GBVTD-retrieved wind fields in the presence of 1) asymmetry, 2) a misplaced center, and 3) uncertainty in the mean wind. The design of these experiments is listed in Table 1.

a. The asymmetry test (AS series)

In the asymmetry sensitivity test (AS series), the experimental design follows Lee et al. (1999), where the basic axisymmetric vortex is constructed as follows:

\[
V_T = V_{T_{max}} \frac{R}{R_{max}} \text{ for } R \leq R_{max}, \text{ or}
\]
\[
V_T = V_{T_{max}} \frac{R_{max}}{R} \text{ for } R > R_{max}; \text{ and}
\]
\[
V_R = \delta_1[(R_{max} - R)R]^{1/2} \text{ for } R \leq R_{max}, \text{ or}
\]
\[
V_R = \delta_2(R - R_{max})^{1/2} \left(\frac{R_{max}}{R}\right) \text{ for } R > R_{max};
\]
FIG. 3. A comparison of $V_d$ and $V_dD/R_T$ displays for two different radii (delineated as red and blue lines) for two vortices with different RMWs: (a) $V_d$ display for a pure rotating vortex with $R_{\text{max}} = 30$ km; (b) $V_d$ profiles at $R = 30$ and 60 km; (c) same as (a), but for $V_dD/R_T$ display; and (d) same as (b), but for $V_dD/R_T$ profiles. (e)–(h) Same as (a)–(d), but for $R_{\text{max}} = 80$ km > $R_T$ and the two $V_d$ profiles are at $R = 80$ and 110 km. The center “T” is located at $(x, y) = (200 \text{ km}, 200 \text{ km})$ and the hypothetical Doppler radar “O” is located at $(x, y) = (150 \text{ km}, 150 \text{ km})$, with $R_T = 50\sqrt{2} \text{ km}$. 
estimates, because of the zero GBVTD-retrieved winds but rather to no GBVTD (beyond two and three asymmetries). The white-colored area winds (right column), especially in the wavenumber nearly nonexistent in the GVTD-retrieved asymmetric tortions of the GBVTD-retrieved asymmetric winds three (fourth row) asymmetries. The pronounced dis-

tions following Lee et al. (1999).

where \( V_{T,\text{max}} = 50 \text{ m s}^{-1}, R_{\text{max}} = 30 \text{ km}, \delta_1 = 0.1 \text{ s}^{-1}, \) and \( \delta_2 = 3 \text{ m s}^{-1} \), respectively.

Four experiments were conducted, including the axisymmetric vortex (AS0), and wavenumber one, two, and three asymmetries (AS1, AS2, and AS3) embedded within the axisymmetric vortex. The asymmetric structures (wavenumbers \( n = 1, 2, \) and \( 3 \)) were constructed using the following equations and the parameters listed in Table 1:

\[
V_T = V_{T,\text{max}} \frac{R}{R_{\text{max}}} \left[ 1 + A_n \cos(n(\theta' - \theta_0)) \right] \quad \text{for} \quad R \leq R_{\text{max}}, \quad \text{and} \quad R > R_{\text{max}},
\]

where \( A_n = 0.2 \). Note that we will assume that there is no asymmetric radial component in the simulated vortex following Lee et al. (1999).

Table 1. Summary of sensitivity tests on GBVTD and GVTD. Three test series were conducted to quantify the asymmetry (AS), center displacement (C), and mean wind sensitivities (VM). The prefix G (M) represents results from GBVTD (GVTD). CxN (CyN) represents the response to a misplaced center toward east (north) for \( N \) km.

<table>
<thead>
<tr>
<th>Test series</th>
<th>Description</th>
<th>Parameter</th>
<th>GBVTD</th>
<th>GVTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS0</td>
<td>Wave 0</td>
<td>GAS0</td>
<td>MAS0</td>
<td>—</td>
</tr>
<tr>
<td>AS1</td>
<td>Wave 0 + 1</td>
<td>( \theta_0 = 90^\circ )</td>
<td>GAS1</td>
<td>MAS1</td>
</tr>
<tr>
<td>AS2</td>
<td>Wave 0 + 2</td>
<td>( \theta_0 = 90^\circ )</td>
<td>GAS2</td>
<td>MAS2</td>
</tr>
<tr>
<td>AS3</td>
<td>Wave 0 + 3</td>
<td>( \theta_0 = 0^\circ )</td>
<td>GAS3</td>
<td>MAS3</td>
</tr>
<tr>
<td>Cx</td>
<td>Center displacement in the x direction</td>
<td>1–10 km</td>
<td>GCxN</td>
<td>MCxN</td>
</tr>
<tr>
<td>Cy</td>
<td>Center displacement in the y direction</td>
<td>1–10 km</td>
<td>GCyN</td>
<td>MCyN</td>
</tr>
<tr>
<td>VM1</td>
<td>—</td>
<td>( \Delta \theta_M = 90^\circ )</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>VM2</td>
<td>—</td>
<td>( \Delta \theta_M = 30^\circ )</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

where \( \theta_M = \theta' \) and \( R_M = R_{\text{max}} \).

GBVTD; thus, the advantages of GVTD over GBVTD are clearly illustrated.

Figure 5 shows the percentage error distribution of the retrieved wind as a function of \( \theta' \) (x axis) and \( R \) (y axis) from wavenumber two and three asymmetries only (the errors in wavenumber zero and one cases are negligible, not shown). For GBVTD (Figs. 5a,b), the errors are positively correlated with \( R \) and are also highly dependent on the phase of the asymmetry. In general, the wavenumber two vortex was retrieved quite well by the GBVTD technique [Fig. 4c(2)]. The worst errors (>10%) occur along the \( \mathbf{R}_T \) vector beyond \( R = 40 \text{ km} \). For wavenumber three (Fig. 5b), GBVTD could not retrieve the peak wind along the \( \mathbf{R}_T \) vector beyond the center \([\theta = 0, \text{ see Fig. 4d(2)}]\), while significant phase and amplitude distortions occurred at large \( R \).

In contrast, these radius- and phase-dependent error distributions are not found in GVTD (Figs. 5c,d) and the errors are negligible (<1%), except for regions near the center. Hence, the GVTD analysis is quite robust and nearly eliminates the geometric distortions in the retrieved asymmetric wind fields.

b. The center displacement test (C series)

Figures 6a–c show the GBVTD- and GVTD-retrieved vortex structures when the center is displaced (a) 1, (b) 5, and (c) 10 km, along the \( \mathbf{R}_T \) vector (y axis) away from the true center. The original vortex contains only axisymmetric \( V_T \). It can be seen that both algorithms generate apparent wavenumber one components that occur in the opposite direction to the center displacement in all cases, while the amplitude increases as the center displacement increases. These errors are analogous to aliasing errors in signal processing. For a
FIG. 4. Comparison of GBVTD- and GVTD-retrieved vortex structure. [a(1)] The simulated axisymmetric wind field, [a(2)] the GBVTD-retrieved wind field, and [a(3)] the GVTD-retrieved wind field. Same as [a(1)]–[a(3)], but for [b(1)]–[b(3)] wavenumber one, [c(1)]–[c(3)] wavenumber two, and [d(1)]–[d(3)] wavenumber three cases. The center is located at (0, 0), while the hypothetical Doppler radar is located at (0, −80).
1-km center displacement, the error is small (not shown). Figure 7 shows that for 5- and 10-km center displacements, however, the errors near the RMW in the GBVTD- and GVTD-retrieved wind fields can be as large as 50% of the analytic axisymmetric vortex (Figs. 7c,d). These results strongly suggest that both methods are sensitive to the center uncertainties, but with similar error characteristics. To have a reasonably correct vortex wind retrieval (e.g., less than 20% of its axisymmetric tangential component), the uncertainty in the center cannot exceed 5 km.

To examine further, we calculated the root-mean-square error (RMSE) of the GBVTD- and GVTD-retrieved $V_{T\max}$ and $V_{R\max}$ for various center displacements. It can be seen that the RMSE of the retrieved $V_{T\max}$ as a function of center displacement in the $x$ and $y$ directions (Figs. 8a,b) is quasi-linearly proportional to the magnitude of the misplaced centers. The error in the GVTD-retrieved $V_{T\max}$ is about 40% less than the GBVTD-retrieved $V_{T\max}$. A 2-km center displacement produces about a 3% error ($1.5 \text{ m s}^{-1}$ error for $V_{T\max} = 50 \text{ m s}^{-1}$) in GBVTD and a 2% error in GVTD. The errors are symmetric when the center is displaced perpendicular to the $R_T$ vector. When the center is misplaced along the $R_T$ vector, the errors are larger (smaller) around the near (far) side of the center. The error distributions of $V_{R\max}$ are very different between the two methods (Figs. 8c,d). The $V_{R\max}$ errors in GBVTD are more symmetric to the center while the $V_{R\max}$ errors in GVTD are more sensitive to the center
Fig. 6. Comparison of GBVTD- and GVTD-retrieved pure rotational vortex structure with prescribed center displacement along the $R_t$ vector (y axis). Results are for the case of [a(1)] GBVTD- and [a(2)] GVTD-retrieved structure for a center displacement of 1 km. Same as [a(1)] and [a(2)], but for a center displacement of [b(1)], [b(2)] 5 and [c(1)], [c(2)] 10 km, respectively.
displacement perpendicular to the $\text{R}_T$ vector. In a typical situation where the misplaced center is ~2 km (Lee and Marks 2000), both methods perform very well.

c. The mean wind sensitivity (VM series)

The sensitivity of the GVTD-retrieved axisymmetric vortex on the uncertainty of the mean wind in the direction perpendicular to the $\text{R}_T$ vector is illustrated in Fig. 9. The error distributions are quite different between the retrieved $V_{\text{T}_{\text{max}}}$ and $V_{\text{R}_{\text{max}}}$. It is clear that the retrieved $V_{\text{T}_{\text{max}}}$ is sensitive to the error in the mean wind speed. A 50% error in the mean wind speed results in ~10% error in the retrieved $V_{\text{T}_{\text{max}}}$. The error of $V_{\text{R}_{\text{max}}}$ increases proportionally as the assigned error in the mean wind speed. However, the error of $V_{\text{R}_{\text{max}}}$ is more sensitive to the mean wind direction instead. The situation is reversed while the mean wind direction is along the $\text{R}_T$ vector (VM2 test, not shown); the retrieved $V_{\text{T}_{\text{max}}}$ is more sensitive to the mean wind direction and the retrieved $V_{\text{R}_{\text{max}}}$ is more sensitive to the mean wind speed.

6. Typhoon Gladys

In this section, we use Typhoon Gladys (1994) to gain understanding of the mean wind and vortex signatures in the $V_d/R_T$ display. According to the Joint Typhoon Warning Center (JTWC), Gladys was a relatively small typhoon with moderate intensity. The $V_d$ constant-altitude PPI (CAPPI) display of Gladys at 4-km height (Fig. 10a) shows that Gladys’ inner-core diameter is about 35 km, indicated by the circle in the lower-right-hand corner of the display. The approaching Doppler velocity exceeded 50 m s$^{-1}$ and the receding compo-
The pronounced asymmetric structure indicates a possible combination of a strong mean flow and/or an asymmetric vortex. Figure 10b shows the corresponding $V_{d}/D/R_T$ display. It is clear that the vortex circulation was mostly confined to lower-right corner of the display, where the near-parallel straight lines aligned in a north–south direction to the left of the radar (opposite side of the center) suggested a likely east–west-oriented mean wind at this level.

The flow field of a Gladys-sized Rankine vortex with a RMW of 16.5 km and $V_{T_{\text{max}}}$ of 35 m s$^{-1}$ embedded in a 20 m s$^{-1}$ easterly mean wind is simulated, and the corresponding $V_{d}$ and $V_{d}/D/R_T$ displays are shown in Figs. 10c,d. Even with no asymmetric $V_T$ and $V_R$ in the
simulation, the similarity between the observed and simulated $V_d$ (Figs. 10a,c) and $V_{d/R_T}$ (Figs. 10b,d) is very encouraging. With the $V_{d/R_T}$ display, the gross features of the vortex and its accompanied mean flow characteristics can be estimated with a reasonable accuracy, while the mean wind is not straightforward enough for identification in the $V_d$ display (Fig. 10a). Note that an east–west-oriented convective line ~70 km north of the radar forces the $V_{d/R_T}$ contours to be oriented in the east–west direction in Fig. 10b instead of north–south, as in Fig. 10d. Differences in the actual and simulated $V_{d/R_T}$ are also apparent in the rainbands northeast of the radar where asymmetric vortex components are likely.

7. Summary and future work

This paper introduces the GVTD technique with a new variable $V_{d/R_T}$ as the new paradigm to display, interpret, and retrieve kinematic structures of atmospheric vortices. Using analytical vortices, the properties of the $V_{d/R_T}$ display of atmospheric vortices and the GVTD technique are examined and compared with the radar signatures in the $V_d$ display and the GBVTD technique. It is evident that the $V_{d/R_T}$ display simplifies the vortex interpretation and eliminates the geometric distortion of the dipole signature displayed in $V_d$. It is shown that GVTD is a more general form for the VTD family of techniques. The advantages of the GVTD technique over the GBVTD technique are as follows:

1) Negligible geometric distortion: The $V_{d/R_T}$ variable relates the vortex circulation in a linear coordinate system. Hence, the pronounced distortion of retrieved asymmetric winds in GBVTD has been nearly eliminated, especially when high-wave-number asymmetries are involved and/or $R/R_T \sim 1$.

2) Expanded analysis domain: In GBVTD, the analysis domain is limited by $R/R_T \leq 1$, where the distortion of the retrieved wind fields worsens as $R/R_T$ approaches unity. In GVTD, the analysis can be extended to cover the entire domain of the Doppler radar whenever there are enough data for meaningful GVTD analysis, as portrayed in Fig. 4. The ability to recover the dipole structure for $R > R_T$ is particularly striking. This characteristic is especially important for assimilating GVTD-retrieved winds into a numerical model in the future.

3) Relatively straightforward: The subjective estimation of the mean wind is from the $V_{d/R_T}$ display when the vortex circulation is not dominating the Doppler velocities. In this situation, a constant mean wind appears as parallel lines and can be easily recognized subjectively. The possibility to separate the vortex signature from the mean wind signature provides a useful tool for studying the vortex mean flow interactions in the future.

When estimating the center location and RMW in the $V_{d/R_T}$ space, the VDAD method has advantages over the Wood and Brown (1992) method in the $V_d$ space, especially for a near-axisymmetric vortex. The
VDAD method is particularly useful in an operational environment for quick determination of the gross features of the vortex. A more quantitative and robust algorithm to estimate the center location and RMW is to combine GVTD and a simplex method that will be presented in a future paper.

Beyond all of these advantages, there are limitations in the GVTD technique worth noting. First, from the center displacement tests (C series), it can be seen that the retrieved $V_{\text{max}}$ from GVTD is 3 times more sensitive to the accuracy of the center than $V_{\text{max}}$ derived from the GBVTD technique (Figs. 8c,d). However, in a typical uncertainty of ~2 km around the center, the errors are not significant. Second, the subjective determination of the mean wind vector will have difficulty when the vortex circulation is large [e.g., Hurricane Katrina (2005)]. Third, the distance weighting of $V_{\text{max}}/R_T$ rescales Doppler velocities. It is possible that the missing data at a large range resulting from noise or limited unambiguous range may affect the least squares fit of the GVTD coefficients differently than in the GBVTD technique.

Several research activities are currently underway to evaluate and address these limitations. Taking advan-
tage of the constant mean wind signature in the \( V_dD/R_T \) display, an automated method has been developed to estimate the best mean wind vector and center location simultaneously. We will apply GVTD to real atmospheric vortices to examine the limitations related to real observations (such as missing data, distance weighting, etc.). These topics will be reported in future papers.

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\[ V_d^{\text{GVTD}} = V_M \cos \left( \frac{\pi}{2} - \theta_M^{\text{GVTD}} \right) - V_T \sin \left( \theta^{\text{GVTD}} - \frac{\pi}{2} \right) + V_R \cos \left( \theta^{\text{GVTD}} - \frac{\pi}{2} \right), \]
\[ = V_M \cos \left( \frac{\pi}{2} - \theta_M^{\text{VTD}} - \pi \right) - V_T \sin \left( \theta^{\text{VTD}} + \pi - \frac{\pi}{2} \right) + V_R \cos \left( \theta^{\text{VTD}} + \pi - \frac{\pi}{2} \right), \]
\[ = V_M \cos \left( -\frac{\pi}{2} - \theta_M^{\text{VTD}} \right) - V_T \sin \left( \theta^{\text{VTD}} + \frac{\pi}{2} \right) + V_R \cos \left( \theta^{\text{VTD}} + \frac{\pi}{2} \right), \]
\[ = -V_M \sin \theta_M^{\text{VTD}} - V_T \cos \theta^{\text{VTD}} - V_R \sin \theta^{\text{VTD}}, \text{ and} \]
\[ = -V_d^{\text{VTD}}, \]

which proves GVTD reduces to VTD in the limit of \( R/R_T \sim 0 \).

APPENDIX B

Derivation of the Mean Wind Vector

Starting from (5), moving \( D \) to the left-hand side, and considering a uniform mean wind only, we have

\[ V_dD = V_M \cos \theta_M (R \cos \theta + R_T \cos \theta_T) \]
\[ + V_M \sin \theta_M (R \sin \theta + R_T \sin \theta_T) \]
\[ = V_M \cos \theta_M (x' + x_T) + V_M \sin \theta_M (y' + y_T) \]
\[ = V_M(x \cos \theta_M + y \sin \theta_M), \quad \text{(B1)} \]

where \( x_T = R_T \cos \theta_T, \ y_T = R_T \sin \theta_T, \ x' = R \cos \theta, \text{ and } y' = R \sin \theta \) (Fig. 1). The origin of the Cartesian coordinate \((x, y)\) is located at the radar. This equation is in the form of a straight line, \( ax + by = c \), because \( V_M \text{ and } \theta_T \) are constant for a uniform mean wind.

By taking the gradient of (B1), we have

\[ \nabla (V_dD) = \left[ \frac{\partial}{\partial x} (V_dD) + \frac{\partial}{\partial y} (V_dD) \right] \]
\[ = (V_M \cos \theta_M, V_M \sin \theta_M). \quad \text{(B2)} \]

Therefore, the direction of the gradient vector is \( \theta_M \) while the magnitude of the gradient vector is \( V_M \).

Note that (B2) is independent of \( R_T \) and is expressed in a Cartesian coordinate system. As a result, estimating the mean wind vector using \( V_dD \) can be applied to any flow field, and is not limited to atmospheric vortices.

APPENDIX A

VTD and GVTD

This appendix intends to show GVTD is a general form of VTD by proving that (6) is identical to VTD [Eq. (3) in Lee et al. 1994] in the limit of \( R/R_T \sim 0 \). Note that the geometry and symbols between airborne Doppler radar (e.g., Fig. 3 in Lee et al. 1994) and ground-based radar in this paper (Fig. 1) are different. Assuming the flight track in VTD is oriented in the east–west direction (Fig. 3 in Lee et al. 1994), it is equivalent to have a \( \theta_T = \pi/2 \) in the GVTD geometry (Fig. 1). The azimuth angle \( \theta \) in VTD \( (\theta^{\text{VTD}}) \) (\( \phi \) in Lee et al. 1994) and \( \theta \) in GVTD \( (\theta^{\text{GVTD}}) \) result in \( \theta^{\text{VTD}} = \theta^{\text{GVTD}} - \pi \). In addition, positive \( V_d \) in VTD \( (V_d^{\text{VTD}}) \) corresponds to positive \( V_T \) (cyclonic rotation) and \( V_R \) (radial outflow) of a vortex, while the opposite is true for \( V_d \) in GVTD \( (V_d^{\text{GVTD}}) \). In the limit of \( R/R_T \sim 0 \), replacing \( \theta \) with \( \theta^{\text{GVTD}} \) and \( \theta_T - \theta_M \) with \( \pi/2 - \theta_M^{\text{GVTD}} \), (6) becomes
REFERENCES


