Angular and range interferometry to measure wind

Guifu Zhang, 1 Richard J. Doviak, 2 J. Vivekanandan, 1 and Tian-You Yu 3

Received 27 June 2003; revised 8 October 2003; accepted 20 October 2003; published 18 December 2003.

[1] Radial wind is routinely measured by Doppler method, whereas winds transverse to the radar beam are measured using an interferometric technique in which three or more spaced antennas are used (i.e., the Spaced Antenna (SA) technique). In this paper, an interferometric technique is examined whereby a single antenna is used to measure both radial and transverse winds. Angular Interferometry (AI) determines transverse wind, and Range Interferometry (RI) determines radial wind. The cross-correlation of signals, received from different angles and from different ranges by a single antenna, is derived based on wave scattering from random fluctuations of refractive index. The radial and transverse wind components are estimated from the cross-correlation of signals received from different ranges and different directions. The theoretical standard deviation of the estimated wind is derived, and its dependence on spatial resolution, observation time, and turbulence is presented. The theory shows that AI requires small beam size to measure transverse wind accurately, contrary to the SA technique, whereas RI requires fine range resolution to perform well.

INDEX TERMS: 6969 Radio Science: Remote Sensing; 0699 Electromagnetics: General or miscellaneous; 6952 Radio Science: Radar atmospheric physics; 6974 Radio Science: Signal processing; KEYWORDS: single antenna interferometry, wind measurement, cross-correlation ratio, weather radar, MST radar, wind profiler


1. Introduction

[2] Components of the wind can be measured by the Doppler method, interferometric techniques, or by Tracking Reflectivity Echoes by Correlation (TREC) [Doviak and Zrnic, 1993; Briggs et al., 1950; Rinehart, 1979]. The Doppler method measures radial velocity, and interferometric techniques measure the wind component transverse to the beam axis. Both the Doppler and the interferometric techniques directly measure wind components (i.e., the velocity of randomly distributed scatterers advected by wind). The vector wind can be determined using the Doppler method (e.g., through a VAD type analysis) if the wind is uniform over a large area, but the interferometric technique can measure the vector wind within the resolution volume of the radar, thus providing finer resolution. Both of these techniques can determine the wind even if the reflectivity field is statistically homogenous. On the other hand, the TREC method tracks features in the reflectivity field to determine feature motions [Crane, 1979; Rinehart, 1979; Tuttle and Foote, 1990]. Thus, if reflectivity is not conserved, reflectivity motion is not necessarily the same as the wind velocity; furthermore the TREC method does not work if the reflectivity field is homogenous.

[3] The Doppler method uses the phase of the auto-correlation function of received signals to estimate radial wind. Because the phase can only be measured within an interval of 2π, the Doppler method is prone to aliasing which causes the estimated wind velocity to alias with a period of λ/2Ts, where Ts is the pulse repetition time. Current dealiasing techniques are based on spatial/time continuity of the wind field. We develop an alternative method, Range Interferometry (RI), that is immune to aliasing, and which could supplement Doppler wind measurements. Interferometric techniques use the magnitude of the cross-correlation function, and thus the problem of phase aliasing does not exist.

[4] Interferometry had been successfully developed and applied to measure transverse wind [Briggs et al., 1950; Briggs, 1984; Vincent and Rottger, 1980; May et al., 1989; Doviak et al., 1996]. Interferometry for wind measurement is based on the spaced antenna (SA) technique, in which scattered signals received at separate

1Research Application Program, National Center for Atmospheric Research, Boulder, Colorado, USA.
2National Severe Storms Laboratory, Norman, Oklahoma, USA.
3School of Electrical and Computer Engineering, University of Oklahoma, Norman, Oklahoma, USA.

Copyright 2003 by the American Geophysical Union. 0048-6604/03/2003RS002927
antennas are cross-correlated to estimate the transverse wind. A single antenna interferometer was proposed to measure ocean surface features using air-borne synthetic aperture radars (SAR) [Fitch, 1991]. To our knowledge, however, single antenna interferometry has not been used in the field of atmospheric remote sensing with ground-based radars. Radar interferometry with a single antenna/receiver for wind measurement has not been investigated or demonstrated.

[5] In this paper, we introduce single-antenna interferometry to measure the wind vector, and study its feasibility for practical applications. We propose Angular Interferometry (AI) to determine the transverse wind components from the angular cross-correlation function, and Range Interferometry (RI) to measure the radial wind component from the range cross-correlation function. It is noted that neither AI nor RI are TREC methods because both AI and RI apply even if the reflectivity field is uniform.

[6] This paper is organized as follows: In section 2 a conceptual description of single antenna interferometry for wind measurement is presented; In section 3 the problem is formulated, and the angular and range cross-correlation functions are derived for signals received by a single-antenna/receiver; In section 4 the cross-correlation ratio (CCR) method [Zhang et al., 2003] is used to determine the transverse and radial wind components, and to analyze the standard errors of the wind estimates. Finally, the feasibility of the method for practical applications is discussed.

2. A Conceptual Description of Single-Antenna Interferometry for Wind Measurements

[7] The accepted explanation for radar interferometric measurement of crossbeam wind is that the echo diffraction pattern advects across an array of receiving antennas at twice the speed that scatterers are advected across the beam [Briggs, 1984]. Doviak et al. [1996], however, consider pairs of scatterers and receivers to prove that diffraction patterns do not necessarily advect at twice the speed of wind. Considering a pair of receivers, symmetrically placed about a transmitting antenna to form a pair of side-by-side bistatic scattering volumes, and wind along the baseline of the receivers, they showed by applying the reciprocity theorem that echoes in the two receivers are perfectly correlated when signals from one receiver is lagged a time difference equal to the time it takes the scatterers to advect from one scattering volume to the next.

[8] This later interpretation (i.e., spaced resolution volumes, rather than spaced antennas), leads us to an alternative explanation of interferometry applied to the measurement of wind advecting either discrete scatterers (e.g., rain drops) or Bragg scatterers [Doviak and Zrnic, 1993]. Ignoring the effects of turbulence, wind simply advects scatterers without changing their relative displacements. If the radar’s resolution volume $V_6$ [Doviak and Zrnic, 1993, section 4.4.4] can be strictly translated the same vector distance that the scatterers have moved, the phase path (i.e., from the transmitter to each of the scatterers and back to the receivers) differences would be the same for all the scatterers in each of the $V_6$s. For example, if scatterers are horizontally advected, $V_6$ must also be strictly translated horizontally (i.e., without rotation as would occur if the center of $V_6$ is translated horizontally and the radar is at a fixed location). If $V_6$ is to be strictly translated horizontally, both the transmitter and receiver must be translated horizontally the same vector distance as the scatterers or, if the transmitter is fixed, the receiver must be symmetrically and horizontally translated across the transmitting antenna twice the distance that the scatterers have moved (i.e., a bistatic radar configuration is required). This explanation also applies if the scatterers and sample volumes advect radially. If the resolution volume strictly follows the motion of the scatterers such that the phases to all scatterers are fixed (i.e., ignoring turbulence), the magnitude of cross-correlation function is maximal. Otherwise, the cross-correlation of the received signals decreases.

[9] Based on the above interpretation of SA interferometry, we propose a single-antenna/receiver interferometric technique to measure wind. As shown in Figure 1a, AI locates the angular displacement of the scatterers to measure transverse wind. An angular cross-correlation function of the signals from different directions is constructed to estimate transverse wind. Similarly, RI locates the radial displacement of the scatterers along range (Figure 1b), and estimates the radial wind from the cross-correlation of signals from different ranges.

[10] The radar that uses AI can have a phased array antenna or single dish reflector provided the signals from two adjacent directions are acquired rapidly to maintain signal correlation, and repeatedly to have many independent samples. The phased array weather radar being assembled by National Severe Storms Laboratory (NSSL) has beam agility and is a good candidate to test AI for wind measurement. The angular correlation functions at positive and negative angles about the transmitting beam can also be obtained with a single reflector of a monopulse radar. If the beam shifts in the direction that follows the displacement of the scatterers, the correlation of the signals is high. Beam shifts in the opposite direction leads to a lower correlation. Hence, the ratio of the correlation functions of signals from $V_6$s displaced in the direction of the wind and against it should give transverse wind information.

[11] RI can be realized with a pulsed Doppler radar with a wide band receiver that samples signals from two
refractive index fluctuations be obtained by integrating the spatial distribution of spherical coordinate system, and a resolution volume \( V \) located at range \( r \) and at an angular displacement \( \theta \) from a reference axis (Figure 2; two \( V_6 \)s are shown for a later reference). That is, \( \sigma_0 \) is related to that of one-way angular weighting function \( \sigma_{01} \) as \( \sigma_0 = \sigma_{01} / \sqrt{2} \), and the standard deviation of the one-way radiation pattern, \( \sigma_{01} = \gamma \lambda / D \), is related to the commonly used one-way 3dB beam width as \( \theta_1 = 2.36 \sigma_{01} \). Parameters, \( \lambda \), \( D \), and \( \gamma \) are wavelength, antenna size and antenna efficiency factor, respectively.

The cross-correlation function is the ensemble average of the product, \( V(\vec{r}_1, t_1)V^*(\vec{r}_2, t_2) \), in which \( V(\vec{r}_1, t) \) and \( V(\vec{r}_2, t) \) are the received signals from the resolution volumes centered at \( \vec{r}_1 \) and \( \vec{r}_2 \). The cross-correlation function of the received signals is

\[
C(\vec{r}_1, t_1; \vec{r}_2, t_2) = A^2 \int \int \frac{\Delta n(\vec{r}_1, t_1) \Delta n(\vec{r}_2, t_2)}{r_1^2 r_2^2} \exp \left( - \frac{|\vec{s}_1 - \vec{s}_2|^2}{4 \sigma_0^2} - \frac{(r_1 - r_2)^2}{4 \sigma_r^2} - 2jk(r_1 - r_2) \right) \, d\vec{r}_1 \, d\vec{r}_2. \tag{2}
\]

To simplify the integrations in (2):

3. Formulation for Single Antenna Radar Interferometry

Consider a radar located at the origin \((0, 0, 0)\) of a spherical coordinate system, and a resolution volume \( V_6 \) located at range \( \vec{r} \) and at an angular displacement \( \vec{s} \) from a reference axis (Figure 2; two \( V_6 \)s are shown for a later reference). The received complex signal \( V(\vec{r}, t) \), back-scattered from a resolution volume \( V_6 \) centered at \( \vec{r} \), can be obtained by integrating the spatial distribution of refractive index fluctuations \( \Delta n(\vec{r}', t) \) weighted with angular and range weighting functions [Doviak and Zrnic, 1993, section 11.5]. That is,

\[
V(\vec{r}, t) = A \int \frac{\Delta n(\vec{r}', t)}{r'^2} \exp \left( - \frac{|\vec{s}' - \vec{s}|^2}{4 \sigma_0^2} - \frac{(r' - r)^2}{4 \sigma_r^2} - 2jk(r' - r) \right) \, d\vec{r}',
\tag{1}
\]

where \( A \) is a constant dependent on radar parameters, \( \vec{s} \) is the angular distance vector from the reference axis located between two spaced resolution volumes, \( r \) is the range to the center of \( V_6 \), and a prime defines the location of \( \Delta n(\vec{r}', t) \). The first term in the exponent is the angular weighting function, the second term is the range weighting function, and \( 2k(r' - r) \) is the phase path difference between the location of \( \Delta n(\vec{r}', t) \) and the center of \( V_6 \). The range resolution, \( \sigma_r \), (Figure 2), is defined as the square root of the second moment of the range weighting function [Doviak and Zrnic, 1993, section 11.5]. The square root of the second moment of the two-way angular weighting function \( \sigma_0 \) is related to that of one-way angular weighting function \( \sigma_{01} \) as \( \sigma_0 = \sigma_{01} / \sqrt{2} \), and the standard deviation of the one-way radiation pattern, \( \sigma_{01} = \gamma \lambda / D \), is related to the commonly used one-way 3dB beam width as \( \theta_1 = 2.36 \sigma_{01} \). Parameters, \( \lambda \), \( D \), and \( \gamma \) are wavelength, antenna size and antenna efficiency factor, respectively.

The cross-correlation function is the ensemble average of the product, \( V(\vec{r}_1, t_1)V^*(\vec{r}_2, t_2) \), in which \( V(\vec{r}_1, t) \) and \( V(\vec{r}_2, t) \) are the received signals from the resolution volumes centered at \( \vec{r}_1 \) and \( \vec{r}_2 \). The cross-correlation function of the received signals is

\[
C(\vec{r}_1, t_1; \vec{r}_2, t_2) = A^2 \int \int \frac{\Delta n(\vec{r}_1, t_1) \Delta n(\vec{r}_2, t_2)}{r_1^2 r_2^2} \exp \left( - \frac{|\vec{s}_1 - \vec{s}_2|^2}{4 \sigma_0^2} - \frac{(r_1 - r_2)^2}{4 \sigma_r^2} - 2jk(r_1 - r_2) \right) \, d\vec{r}_1 \, d\vec{r}_2. \tag{2}
\]
1. Assume that the field of Δn(\(r', t\)) is statistically homogeneous and stationary so that its correlation function is only a function of the distance \(r''_0 = r''_1 - r''_1\) between the two locations of Δn(\(r', t\)), and the time lag difference \(\tau = t_2 - t_1\). Thus this correlation function can be written as,

\[ B_n(r'', \tau) = \langle Δn(r'_1, t_1)Δn(r'_2, t_2) \rangle. \] (3)

Because we only have samples of the signal spaced \(T_s\) (i.e., the Pulse Repetition Time), \(\tau\) changes incrementally in \(T_s\) steps along sample-time \(\tau = mT_s\) [Doviak and Zrnic, 1993, section 4.1].

2. Assume a Taylor series expansion for the phase term up to the second order [Doviak and Zrnic, 1993, section 11.5], i.e.,

\[ r' - r = \ell' - \ell + \frac{\rho^2 - \rho^2_0}{2r}. \] (4)

3. Assume a narrow radar beam and a small angular difference (i.e., \(\sigma_0 \ll 1\) and \(|\vec{s}_1 - \vec{s}_2| \ll 1\)), so that the angular displacement vector, \(\vec{s}\), can be represented as,

\[ |\vec{s}' - \vec{s}| \approx \frac{|\vec{\rho}' - \vec{\rho}|}{r}, \] (5)

where \(\vec{\rho}\) is the rectilinear displacement transverse to the reference axis.

4. Assume a fine range resolution, so that the approximations, \(\rho'_1 \approx \rho'_2 \approx \rho\) and \(r' - r = \ell' - \ell\) can be used in equation (2), where \(\ell' - \ell\) is the longitudinal displacement parallel to the reference axis.

Using the assumptions 1–4 in (2) and noting that \(\rho'\) and \(\ell'\) are independent variables, we separate the integrals and have

\[ C(\vec{r}_1, t_1; \vec{r}_2, t_2) \approx \frac{A^2}{r^4} \int \int \int B_n(\vec{r}', \tau) \cdot \exp \left( -\frac{|\vec{\rho}'_1 - \vec{\rho}_1|^2 + |\vec{\rho}'_2 - \vec{\rho}_2|^2}{4r^2\sigma_0^2} - jk \left( \frac{\rho'^2_1 - \rho^2_1 - \rho'^2_2 + \rho^2_2}{r} \right) \right) \cdot \exp \left( -\frac{(\ell'_1 - \ell_1)^2 + (\ell'_2 - \ell_2)^2}{4\sigma_r^2} \right) \cdot d\vec{\rho}'_1 d\ell'_1 d\vec{\rho}'_2 d\ell'_2. \] (6)

Equation (6) constitutes the general formulation for the cross-correlation function of the signals received by a single antenna receiver from two spaced resolution volumes.

[19] Making the following coordinate transformation to center and difference coordinates,

\[ q_c = \frac{1}{2}(q_1 + q_2), \quad q_d = q_1 - q_2, \] (7)

for the variables \(\rho', \vec{\rho}, \ell', \) and \(\ell,\) and performing the integrations over the center variables \(q'_c\) and \(q'_d\) in (6), we obtain,

\[ C(\vec{r}_d, \tau) \approx \frac{A^2}{r^4} \sqrt{2\pi\sigma_0^2} \sqrt{\frac{\sigma_r^2}{8}} \int \int B_n(\vec{r}', \tau) \cdot \exp \left( -\frac{|\vec{\rho}_d - \vec{\rho}_d|^2}{8\sigma_0^2} - 2k^2\sigma_0^2\rho_d^2 \right) \cdot \exp \left( -\frac{(\ell'_d - \ell_d)^2}{8\sigma_r^2} - 2jk(\ell'_d - \ell_d) \right) d\vec{\rho}_d d\ell'_d, \] (8)

where the first exponential contains both the angular resolution volume weighting term and the Fresnel term. It can be shown that (8) reduces to equation (11.122) in Doviak and Zrnic [1993] when \(\rho_d = \ell_d = \tau = 0\). In this case the cross-correlation function becomes the autocorrelation function at zero lag, and is simply the backscattered power.

[20] We use the following Gaussian correlation function for the refractive index fluctuation field, \(\Delta n\) [Doviak et al., 1996],

\[ B_n(\vec{r'}, \tau) = \frac{\rho^2_{c\perp} \rho_{c\parallel}}{\beta^2_T \beta^2_\ell} \exp \left( -\frac{|\vec{\rho}_d - \vec{\nu}_T\tau|^2}{2\beta^2_T} - \frac{(\ell'_d - \nu_T\tau)^2}{2\beta^2_\ell} \right), \] (9)

where \(\beta_T^2 = \rho^2_{c\parallel} + \sigma^2_{\nu_T} \sigma^2_{\nu_T}, \) and \(\beta_\ell^2 = \rho^2_{c\parallel} + \sigma^2_{\nu_T} \sigma^2_{\nu_T}, \sigma_{\nu_T}\) is the standard deviation of the radial component of wind (i.e., turbulence), and \(\rho_{c\perp}, \rho_{c\parallel}, \sigma_{\nu_T}\) are the correlation lengths of \(\Delta n\) in directions transverse and longitudinal to the reference axis (Figure 2). \(\vec{\nu}_T = v_{0x}\hat{x} + v_{0y}\hat{y}\) and \(\nu_T\) are wind components transverse and along the reference axis, respectively. Substituting (9) into (8), and integrating over \(\vec{\rho}_d\) and \(\ell'_d,\) yields the following cross-correlation function,

\[ C(\vec{r}_d, \tau) = \frac{8\pi^3 A^2 \sigma_0^2 \sigma_r^2 \rho^2_{c\perp} \rho_{c\parallel}}{r^4} \times \frac{1}{1 + \beta^2_T/(4r^2\sigma_0^2) + 4k^2\sigma_0^2\sigma_r^2} \cdot \exp \left( -\frac{|\vec{\rho}_d - \vec{\nu}_T\tau|^2}{8\sigma_0^2} + \frac{k^2\rho^2_{c\perp} \rho^2_{c\parallel}/(2r^2)}{1 + \beta^2_T/(4r^2\sigma_0^2) + 4k^2\sigma_0^2\sigma_r^2} + 2k^2\sigma_0^2\nu_T^2 \right) \] \times \frac{1}{\sqrt{1 + \beta^2_\ell/(4\sigma_r^2)}} \cdot \exp \left( -\frac{(\ell'_d - \nu_T\tau)^2}{8\sigma_r^2} + \frac{2k^2\beta^2_\ell - 2jk(\ell'_d - \nu_T\tau)}{1 + \beta^2_\ell/(4\sigma_r^2)} \right). \] (10)
the displacement of the resolution volumes. The third term, the Fresnel term, accounts for the decorrelation due to the scatterers’ motion in a transverse rectilinear direction rather than along arcs at constant range, known as range defocusing. The last term shows the de-correlation due to turbulence. Turbulence and Fresnel terms usually cause faster de-correlation of received signals than the displacement of resolution volumes.

[22] By letting \( r_d = 0 \) in (11) we obtain the magnitude of the angular cross-correlation function of signals from sample volumes at the same range. This is given by

\[
|C_A(\theta_d, \tau)| \approx S \exp \left( -\frac{|r_{\theta_d} - \bar{v}_\tau\tau|^2}{8r^2\sigma_0^2} - \frac{|r_{v\tau}|^2}{8r^2\sigma_r^2} - 2k^2\sigma_0^2\sigma_r^2\tau^2 - 2k^2\sigma_0^2\sigma_r^2\tau^2 \right),
\]

(12)

It can be seen that if the scatterers’ displacement in sample-time \( \tau \) matches the separation of the resolution volume center, the correlation reaches a maximum. Figure 3 shows the normalized cross-correlation coefficient as a function of the angular separation \( \theta_d \) and sample-time lag \( \tau \). The other parameters used for the calculation are \( v_{0x} = 50 \) m/s, \( v_{0y} = 0 \) m/s, \( \sigma_{0t} = 0.42 \) degree, \( r = 1 \) km, \( \sigma_r = 0.1 \) m/s, and \( \lambda = 0.1 \) m. The correlation ellipse width perpendicular to the major axis (i.e., the line along \( \theta_d = v_{0x}/r \)) is roughly \( 2\sigma_0 \), and its length along the major axis depends principally on the turbulence and Fresnel terms in equation (12); the smaller are the coefficients of \( \tau^2 \), the longer is the major axis. For laminar flow (i.e., \( \sigma_r = 0 \)), the length is limited to \( 1/(2k\sigma_0v_r) \) by the Fresnel term. Thus, the smaller is \( v_r \) the longer is the major axis, and thus a longer lag time can be used to estimate the tilt or slope \( v_{\tau/r} \) of the major axis. A longer major axis also makes easier the estimation of the transverse wind from the tilt \( v_{\tau/r} \). Likewise, the smaller is \( \sigma_0 \) the narrower and longer is the correlation ellipse; thus narrow beams are favored for accurate transverse wind measurement using the AI approach.

[23] Similarly, letting \( s_{\theta_d} = 0 \) in (11), we obtain the cross-correlation magnitude as a function of range separation and the radial wind component,

\[
|C_R(r_d, \tau)| \approx S \exp \left( -\frac{(r_d - v_r\tau)^2}{8\sigma_r^2} - \frac{v_r^2\tau^2}{8r^2\sigma_0^2} - 2k^2\sigma_0^2\sigma_r^2\tau^2 - 2k^2\sigma_0^2\sigma_r^2\tau^2 \right).
\]

(13)

The normalized cross-correlation coefficient is calculated and shown in Figure 4 as a function of range spacing and time lag. The parameters used for the calculation are \( v_r = 0 \) m/s, \( v_{0x} = 50 \) m/s, \( \sigma_{0t} = 0.42 \) degree, \( \sigma_r = 10 \) m, \( \sigma_t = 0.1 \) m/s and \( \lambda = 0.1 \) m. Again we have an ellipse with an
axis tilt that depends on the range spacing and the radial wind component.

4. Wind Estimate Errors

4.1. Wind Estimation From Correlation Ratios

To measure the wind vector, we calculate the wind components from estimates of the angular and range cross-correlation functions. We use the cross-correlation ratio (CCR) method developed to estimate transverse cross-correlation functions. We use the cross-correlation components from estimates of the angular and range cross-correlation functions. The logarithm of the ratio of angular cross-correlation function, and the de-correlation parameter, the coherence time, the time lag to the peak of the cross-correlation function, are determined using (15) and (17). In practice, however, the angular and range cross-correlation functions are not precisely known; only estimates can be obtained from measurements.

Because the wind components are linearly proportional to the logarithm of the cross-correlation ratios, the standard deviation of the wind estimates \( \hat{v}_T \) and \( \hat{v}_r \) are simply related to the SD of \( \hat{L}_A (\theta_d, \tau) \) and \( \hat{L}_R (r_d, \tau) \) estimates, respectively. That is,

\[
SD(\hat{v}_0) = \frac{2\sigma^2}{\theta_d \tau} SD(\hat{L}_A (\theta_d, \tau)) \tag{18}
\]

and

\[
SD(\hat{v}_r) = \frac{2\sigma^2}{r_d \tau} SD(\hat{L}_R (r_d, \tau)) \tag{19}
\]

The cross-correlation function magnitude can be written in the form,

\[
|C(\tau)| = S \exp \left( -\frac{(\tau - \tau_p)^2}{2\tau^2_c} - \eta \right) \tag{20}
\]

where the Gaussian parameters \( \tau_c, \tau_p \) and \( \eta \) are the coherence time, the time lag to the peak of the cross-correlation function, and the de-correlation parameter. The SD of the estimated cross-correlation ratio \( \hat{L}(\tau) \) has been derived in Zhang et al. [2003] and is

\[
SD(\hat{L}) = \frac{\tau}{\tau_c \sqrt{M^2_{\rho_0}}} \left( 1 + \frac{2\tau^2_p}{\tau^2_c} - \rho^2_0 \right)^{1/2} \tag{21}
\]
where \( M_f = MT_s/\sqrt{\pi \tau_c} \) is the number of independent samples, \( M \) is the total number of correlated samples, and \( \rho_0 \) is the cross-correlation coefficient at zero lag. Because the angular and range cross-correlation functions can be written in the form (20), the standard deviations of the estimates for the transverse and radial wind components can be derived as shown in the next section.

### 4.2.1. Standard Deviation of Transverse Wind Estimates

Rewriting the angular correlation function (11) in the form of (20) for \( r_d = 0 \) leads to the Gaussian parameters \( \tau_c, \tau_p \) and \( \eta \) having the following expressions:

\[
\tau_c^{(A)} = \left( \frac{\nu_T^2}{4r^2\sigma_0^2} + \frac{\nu_r^2}{4r^2} + 4k^2\sigma_0^2\nu_T^2 + 4k^2\sigma_r^2 \right)^{-1/2}
\]

where \( \eta \) is the number of independent samples, \( M_f \) is the total number of correlated samples, and \( \rho_0 \) is the cross-correlation coefficient at zero lag. Because the angular and range cross-correlation functions can be written in the form (20), the standard deviations of the estimates for the transverse and radial wind components can be derived as shown in the next section.

\[
\tau_p^{(A)} = \frac{\theta_d\nu_r\nu_T^2}{4r\sigma_0^2}
\]

Under far field conditions (i.e., \( r \geq \frac{\nu_T^2}{\nu_r^2} \) assumed in the derivations, the Fresnel term (i.e., the third term in (22) is much larger than the first term. Thus the expression for the correlation time can be reduced to,

\[
\tau_c^{(A)} = \left( \frac{\nu_T^2}{4\sigma_T^2} + 4k^2\left(\sigma_0^2\nu_T^2 + \sigma_r^2\right) \right)^{-1/2}
\]

Because \( \sigma_0 \) is much smaller than 1, it is easily seen that unless turbulence is extremely small compared to the transverse wind, the correlation time is principally controlled by turbulence. If \( \sigma_r \ll \sigma_0\nu_T \) then the correlation time is controlled by wind; otherwise it becomes \((2k\sigma_r)^{-1}\).

\[\text{[28]}\] The transverse cross-correlation coefficient at zero lag, \( \rho_0^{(A)} \), can be obtained from (12) by setting \( \tau = 0 \). Assuming resolution volume separation is small \( (\theta_d \ll \sigma_0) \), the approximate expression

\[
\rho_0^{(A)} = \exp\left( -\frac{\theta_d^2}{8\sigma_0^2} \right) \approx 1 - \frac{\theta_d^2}{8\sigma_0^2}
\]

can be used in (21). Substituting (26) into (21), along with the expressions for \( \tau_c^{(A)} \) and \( \tau_p^{(A)} \), and then using these in (18),

\[
SD(\dot{v}_0) = \frac{\tau_c^2 \rho_0^{(A)} (\rho_0^{(A)})}{M_f} \left( 1 + \frac{\nu_T^2 \tau_c^{(A)} \rho_0^{(A)}}{2r^2\sigma_0^2} \right)^{1/2}
\]

If \( \tau_c^{(A)} \) is controlled by the transverse wind, then \( \tau_c^{(A)} \approx (2k\sigma_0\nu_T)^{-1} \). For this relation to be valid, the radial wind component and turbulence must satisfy the conditions, \( \nu_r \ll 4\pi\gamma\left(\frac{D}{r}\right)\nu_T \) and \( \sigma_r \ll \gamma\left(\frac{D}{r}\right)\nu_T \). In this case, it can be shown that, because the solutions are valid for the far field (i.e., \( r > 2D^2/\lambda = r_f \)), the second term in the radical of (27) is small compared to the first, and the standard error of the transverse wind estimates \( SD(\dot{v}_0) \) reduces to

\[\text{[29]}\]

\[
SD(\dot{v}_0) \approx \frac{4\pi\gamma^2 r}{\rho_0^{(A)} \sqrt{M_f}} \frac{r}{r_f} \nu_T, \quad r > r_f.
\]

Thus, under the specified conditions, the transverse wind is estimated more accurately at closer ranges. But satisfying the condition on turbulence is unlikely unless the flow is nearly laminar (i.e., turbulence is much weaker than the mean flow).

\[\text{[30]}\] The standard deviation of the transverse wind estimates as a function of turbulence at various ranges calculated from (27) is plotted in Figure 5. This figure shows that the standard deviation increases as the turbulence becomes stronger, and as range increases.

\[\text{[31]}\] For comparison, the standard deviation of the wind estimates obtained with Spaced Antenna (SA) techniques is also shown in Figure 5. The radar characteristics are the same as that used in the calculation for the AI approach (i.e., the transmitter antenna size \( D_T = 6.0 \) m) except the two receiving antennas have a smaller size (i.e., \( D_R = 3.0 \) m) and are separated by 3 m (i.e., the same aperture is used for transmitting and receiving). The accuracy of the SA technique is independent of range and, as shown, provides more accurate wind measurement than the AI approach. This is due to the difference in the transverse correlation lengths as explained in the following paragraph.
length allows a short time for scatterers to advect to maintain signal coherence, and thus an easier measurement of wind. The beam size for a collimated beam in the far field is much larger than the antenna size. Therefore, the wind measurement with the AI technique is less accurate than with the SA technique in the far field. In near field, however, SA and AI techniques should provide comparable accuracy of wind measurements when the receiving SA antennas are highly overlapped. Less (or no) overlapping in SA antennas degrades its performance when the CCR method is used for wind estimation. AI can outperform the SA technique for a focused beam when the beam size (i.e., at the focal point) is smaller than antenna size.

4.2.2. Standard Deviation of Radial Wind Estimates

[32] There are two approaches in using equation (13) to obtain the radial wind from RI: (1) the range-time series from a pair, or multiple pairs, of transmitted pulses could be lagged with respect to one another, or (2) a sample-time series at a pair of gates, or multiple pairs, of transmitted pulses to obtain the radial wind from RI: (1) the range-time of (20), it can be shown that (13) is written in the Gaussian correlation form another. In the second approach, if the range correlation function (13) is given by (22), but the time-lag to correlation peak \( \tau_{p}^{(R)} \) and peak magnitude \( \eta^{(R)} \) are given by expressions,

\[
\tau_{p}^{(R)} = \frac{r_{d}v_{r}}{4\sigma_{r}^{2}}
\]

\[
\eta^{(R)} = \frac{r_{d}^{2}}{8\sigma_{r}^{2}} \frac{\sigma_{r}^{2}v_{r}^{2}}{32\sigma_{r}^{4}}
\]

The range correlation coefficient \( \rho_{0}^{(R)} \) can be obtained from (13) by setting \( \tau = 0 \). Thus

\[
\rho_{0}^{(R)} = \exp \left( -\frac{r_{d}^{2}}{8\sigma_{r}^{2}} \right) \approx 1 - \frac{r_{d}^{2}}{8\sigma_{r}^{2}}.
\]

Substituting (30), (31), and (32) into (21), and the result into (19) yields:

\[
SD(\bar{v}_r) = \frac{\sigma_r}{\tau_\rho_0^{(R)} \sqrt{M}} \left( 1 + \frac{v_r^2 \tau_{cr}^2}{2\tau_{cr}^2} \right)^{1/2}
\]

If the flow is laminar (i.e., \( \sigma_r = 0 \)), it can be shown that

\[
SD(\bar{v}_r) = \frac{1}{2\rho_0^{(R)} \sqrt{M}} \sqrt{\frac{3}{\tau_\rho_0^{(R)}}} v_T, \text{ if } v_r \gg 4\pi \gamma \left( \frac{\sigma_r \lambda}{D} \right) v_T
\]

and

\[
SD(\bar{v}_r) = \frac{4\pi \gamma}{\rho_0^{(R)} \sqrt{M}} \left( \frac{\sigma_r \lambda}{D} \right) v_T, \text{ if } v_r \ll 4\pi \gamma \left( \frac{\sigma_r \lambda}{D} \right) v_T.
\]

If turbulence controls the correlation time, then

\[
SD(\bar{v}_r) = \frac{4\pi}{\rho_0^{(R)} \sqrt{M}} \left( \frac{\sigma_r \lambda}{\lambda} \right) v_T.
\]

If a pair of range-time series are lagged with respect to one another (i.e., approach 1), the above equations also hold, but \( M \) must be replaced with \( M_{BR} = M_{BR}v_T\sqrt{\tau_{cr}} \), where \( \delta \tau_{cr} \ll 2\sigma_r/c \) is the gate spacing, and \( \tau_{cr} = 4\sigma/c \) is the correlation time along range-time. The standard deviation calculated from (33) is shown in Figure 6 as a function of turbulence for various range resolutions of 20, 40, 80 and 160 m. The other parameters are \( v_T = 0 \text{ m/s}, v_r = 50 \text{ m/s}, r_d = 10 \text{ m}, \text{ and } \lambda = 0.1 \text{ m} \). As expected, error in wind estimate increases as \( \sigma_r \) and turbulence \( \lambda \), increase.

[33] For typical Doppler radar measurements of wind, RI gives radial wind estimates with accuracies much worse than that obtained from Doppler measurements. But RI is not subject to aliasing errors that often plague Doppler measurements. Thus RI could be an alternative approach to resolving velocity ambiguities. In this case, accuracy requirements are relaxed and we only need to estimate \( v_r \) sufficiently well to resolve the ambiguity. A valid radial wind measurement would require the estimation error to be smaller than the unambiguous velocity interval \( 2v_{fa} = \lambda/2T_c \), (i.e., \( SD(\bar{v}_r) \leq \lambda/2T_c \)). If flow is laminar (i.e., \( \sigma_r = 0 \)), (35) shows, for typical weather and radar parameters (i.e., \( \sigma_r = 100 \text{ m} \); \( v_T \approx 10 \text{ m s}^{-1}; T_c \approx 10^{-3} \text{ s}; D \approx 10 \text{ m}; \lambda = 0.1 \text{ m}; \gamma \approx 0.5 \)), that at most a few hundred independent samples would have to be averaged. However, if correlation time is dominated by

![Figure 5. Standard deviation of transverse wind estimates as a function of turbulence at ranges 0.72, 2, 10, and 50 km. Other parameters are \( \sigma_0 = 0.42 \text{ degree}, v_{0x} = 50 \text{ m/s}, v_{0y} = 0 \text{ m/s}, v_r = 0 \text{ m/s}, T_d = 60 \text{ s}. \) See color version of this figure in the HTML.](image-url)
5. Summary and Conclusions

We develop and discuss single antenna interferometry to measure three-dimensional wind. We provide an alternative explanation of interferometry for wind measurement from that commonly used (i.e., detecting the diffraction pattern displacement across pairs of receiving antennas) to explain Spaced Antenna (SA) measurements. We extend the SA interferometric technique that uses multiple antennas to single antenna radars. We propose Angular Interferometry (AI) to determine transverse wind and Range Interferometry (RI) to determine radial wind using time series data collected from different beam positions and from different range gates. Angular and range cross-correlation functions of signals received by a single antenna radar are derived based on wave scattering from random fluctuations of refraction index. The wind vector is estimated from the ratio of the correlation function at positive and negative lags to estimate the transverse wind from the angular cross-correlation function, and the radial wind from the range cross-correlation function. The feasibility of the method was studied through error analysis. The standard deviations of the estimated wind velocities derived and their sensitivity to resolution, dwell time, and turbulence are analyzed.

In the case of turbulence, \( \tau_c = \lambda/(4\pi \sigma_r) \), and then the following condition on \( M_l \) is obtained:

\[
\sqrt{M_l} > 8\pi \frac{\sigma_r T_s}{\lambda} \frac{\sigma_r}{\lambda}.
\]

For the assumed radar parameters, and \( \sigma_r = 1 \text{ m/s} \), the number of independent samples required is almost 10,000!

Although this might be achievable for some research radars, it is not practical for operational weather radars. On the other hand, if one is allowed to decrease the pulse width by a factor of 10 (and increase the bandwidth by ten to about 10 MHz, a reasonable value) the number of independent samples required reduces to about 630. Considering that we can obtain about 6 independent samples during a typical dwell time to estimate moments in a resolution volume, we need only 100 more independent samples. However, we can average about 100 estimates in range, which would reduce range resolution to about 1 km. That is, assuming the conditions hold over this range interval, we should be able to estimate the high-speed radial wind within the typical dwell time of weather radars. Worsening the range resolution to 1 km should be acceptable because the angular resolution of operational weather radars (e.g., the WSR-88D) is more than 1 km for ranges larger than 60 km, and at present, the range resolution of reflectivity estimates is also 1 km.

References


---

R. J. Doviak, National Severe Storms Laboratory, 1313 Halley Circle, Norman, OK 73069, USA.

J. Vivekanandan and G. Zhang, Research Application Program, National Center for Atmospheric Research, P.O. Box 300, Boulder, CO 80516, USA. (guzhang@ucar.edu)

T.-Y. Yu, School of Electrical and Computer Engineering, University of Oklahoma, Norman, OK 73019, USA.