Modeling of tropospheric RO signals

Acquisition of the tropospheric RO signals

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Problems of radio occultations in lower troposphere had been noticed since GPS/MET experiment 1995-1997:

(i) large errors;
(ii) negative N-bias;

These problems are especially serious in moist tropical and sub-tropical troposphere.

At first, it was thought that the problems are related to incorrect solving inverse problem, i.e., calculation of bending angles from Doppler, under multi-path propagation typical for the moist troposphere.
Development of advanced radio-holographic methods (CT, FSI) solved the problem of interpretation (inversion) of multi-tone RO signals. But large errors and negative N-bias in lower troposphere remained.

Statistics of comparisons of RO refractivity with ECMWF (August 2002)

At present, it is clear that there are two main error sources:

(i) receiver tracking errors (induced by low SNR and complicated signal dynamics due to multi-path);
(ii) errors of Abel inversion in the presence of super-refraction on top of PBL.
Modeling of tropospheric RO signals

In order to develop and validate the signal tracking technique that converts the EM field, received by antenna, into digital signal with minimal corruption, it is necessary to have realistic models of the RO signals propagated through lower troposphere.

Ray-tracing is not applicable:
(i) the size of N irregularities is often smaller than the Fresnel’s zone;
(ii) ray-tracing is not stable for small-scale N irregularities;
(iii) finding multiple rays arriving at one point and their summation with individual phases and amplitudes is very complicated.

However, it is possible to accurately model (simulate) RO signals by solving the wave propagation problem (Helmholtz equation) by multiple phase screens technique.
The multiple phase screen method

The atmosphere is represented by a large number of infinitely thin phase screens normal to the direction of initial propagation.

The phase on each screen is: \[ s(z) = \int_{-\Delta l/2}^{+\Delta l/2} [n(y, z) - 1] dy \]

Helmholtz equation is solved in a vacuum, between the screens, by expansion of the solution into the series of plane waves and by satisfying boundary conditions on each screen (a plane wave is a fundamental solution of the Helmholtz equation).
Complex EM field on input to a screen: \( u_{in}(z) \)
(for incident plane wave normal to 1\textsuperscript{st} screen: \( u_{in}(z) = 1 \))

EM field on output of the screen, expanded in Fourier series:
\[
u_{out}(z) = u_{in}(z) \exp[iks(z)] = \sum_{-M/2}^{+M/2} c_m \exp(2\pi imz / H)
\]

where: \( H \) is the vertical dimension of the screen, \( M \) is the number of data

Each harmonic on output of the screen is associated with the plane wave in space:
\[
k_z = 2\pi m / H, \quad k_y = \sqrt{k^2 - 4\pi^2 m^2 / H^2}, \quad k^2 = k_y^2 + k_z^2
\]

EM field in space after the screen:
\[
u(y, z) = \sum_{-M/2}^{+M/2} \tilde{c}_m(y) \exp(2\pi imz / H) \quad \text{where:} \quad \tilde{c}_m(y) = c_m \exp[ik_y(y - y_s)]
\]
\[
u(y_s + \Delta l, z) \quad \text{is used as the boundary condition on input to the next screen}
\]
\[
u(y, z) \quad \text{after last screen is propagated to observation trajectory}
\]

**Important:** propagation of EM field from screen to screen is based on the forward and inverse Fourier transforms and is thus computationally efficient with the use of the FFT
The refractivity profiles used for RO signal modeling

1,2,3 – high resolution tropical radiosonde profiles (Pacific Ocean). A,B – models. All $N$ refractivities are treated as spherically-symmetric. This is not realistic for the small-scale $N$ irregularities. This results in “worst-case” (most complicated) signals given $N$ vertical structure. Thus is useful for testing acquisition (tracking) and inversion techniques.

![Graph showing refractivity profiles with labels 1, 2, 3, A, and B, and units for altitude (km) and refractivity (N units).]
The modeled RO signals (amplitude)

“Observation altitude” is the height of straight line between transmitter and receiver (has negative values due to bending).

Sampling frequency = 50 Hz.

Receiver velocity = 3.2 km/s.

Small-scale N irregularities result in propagation of RO signals down to lower observation altitudes than for smooth N profiles.

High frequency of amplitude fluctuation indicates the multi-path propagation.
The modeled RO signals (Doppler)

Sampling frequency = 50 Hz. Receiver velocity = 3.2 km/s. Strong fluctuation of Doppler indicates the multi-path propagation.
Spectra of the modeled RO signals

Sampling frequency = 50 Hz. Receiver velocity = 3.2 km/s. Time window = 1.28s. The signal is frequency-detrended:  \[ u(t) = a(t) \exp[i \phi(t) - i < \phi(t) >] \]
where:  \(< \phi(t) >\) is the smoothed RO signal phase
(obtained by cubic spline regression with 1.28s window)

Despite the complicated structure of RO signals (Doppler spikes of several hundred Hz magnitude) the width of their spectra does not exceed \(~50\) Hz.

Height of straight line transmitter-receiver:
\(-180\) km - +20 km

A – single-path propagation

B-F – multi-path propagation
Visualization of RO signals by sliding spectrograms

A useful tool for understanding structure of RO signals, estimation of their mean frequency and for the inversions. Each horizontal cross-section is the Fourier spectrum obtained in sliding window (the center of the window assigned to either time or position of receiver).
Sliding spectrograms of RO signals simulated with surface boundary condition which produces reflected signals.
Random phase acceleration of the modeled RO signals

The phase acceleration is the characteristic of a signal important for the closed-loop tracking. Large phase acceleration results in large errors of the extrapolation of phase. Not important for the open-loop tracking.

\[
\frac{\phi_{t+1} - 2\phi_t + \phi_{t-1}}{2\pi \Delta t}
\]

Dotted lines show the phase acceleration +/-6g (+/-60m/s^2) ~ 300 Hz/s (the max. acceleration guaranteed to be processed by a generic GPS receiver)
Why the fluctuation of Doppler frequency has peak magnitude of hundreds of Hz (phase acc. >1000Hz/s) while the width of the spectrum does not exceed ~50 Hz?

Let consider a signal which consists of two sub-signals with close frequencies and close amplitudes.

\[
\begin{align*}
\omega_1 &< \omega < \omega_2 \\
u_1 &= a_1 \exp(i\omega_1 t) \\
u_2 &= a_2 \exp(i\omega_2 t) \\
u &= u_1 + u_2
\end{align*}
\]

When \((\omega_2 - \omega_1)t \sim \pi\) (the sub-signals are out of phase) the phase of the sum changes rapidly by \(\sim \pi\) (the spike in Doppler) while the amplitude is close to zero.
An example of Doppler and amplitude for the modeled RO signal

Large spikes in frequency are always accompanied by dips in amplitude (a problem for signal acquisition by closed-loop tracking)
The spread of frequencies of received RO signal is directly related to the spread of arrival angles of rays:

$$\omega_{dop} = k_z v_{rec}$$

$$\sin \alpha = \frac{k_z}{k} = \frac{\omega_{dop}}{kv_{rec}}$$

for coarse estimates: $\alpha << 1$

Spread of ray arrival angle: $\Delta \alpha \sim \frac{c \Delta f_{dop}}{v_{rec} f}$

For: $\Delta f_{dop} \sim 50Hz, \ v_{rec} \sim 3.2km/s, \ \Delta \alpha \sim 3 \cdot 10^{-3} \text{rad}$

Rays are arriving from height range: $\Delta z \sim l \Delta \alpha$

For: $l \sim 3000km, \ \Delta z \sim 10km$

Due to limited vertical size of the atmosphere, the spread of arrival angles (and the spread of spectrum, for a given receiver velocity) decreases with the increase of the distance from receiver to Earth’s limb.
What is the variance of mean frequency of RO signal, related to large-scale weather variations of refractivity (how well the mean frequency can be predicted)?

This can be estimated by ray-tracing, by use of global atmospheric models.

NCEP T62 NWP model.

An ensemble for each latitude consists of 192 N profiles.

Excess Doppler frequency shift is calculated for GPS and LEO (7150km orbit).

The corresponding variance of arrival angles is about 10 times smaller than the variance of the bending at a given height.

The ensemble for each latitude is shifted in vertical by 50 Hz for display purposes.
Acquisition (tracking) of the tropospheric RO signals
Phase and amplitude of acquired RO signal

How to define the phase and amplitude (two functions) from acquired real RO signal (one function)?

Formal analytical continuation of a real function in complex plane, in practice, is an ill-conditioned problem.

In practice, the definition of the phase and amplitude is possible for narrow-band signals:

\[ u(t) = a(t) \sin(\omega_0 t + \phi(t)) \quad \text{where: } |d\phi/dt| \ll |\omega_0| \]

In-phase signal:

\[ I(t) = u(t) \sin(\omega_0 t) = \frac{a(t)}{2} \cos \phi(t) - \frac{a(t)}{2} \cos[2\omega_0 t + \phi(t)] \]

Quadrature signal:

\[ Q(t) = u(t) \cos(\omega_0 t) = \frac{a(t)}{2} \sin \phi(t) + \frac{a(t)}{2} \sin[2\omega_0 t + \phi(t)] \]

The phase: \( \phi = \arctan(Q/I) \)    The amplitude: \( a = 2\sqrt{I^2 + Q^2} \)

I and Q can be thought as the real and imaginary parts of complex signal.
Lay-out of the spectrum of RO signal

\[ f_0 = 1.57542 \text{GHz} \quad \text{L1 GPS carrier frequency} \]
\[ f_{dop\ vac} \sim 20 \div 30 \text{kHz} \quad \text{Doppler frequency shift in a vacuum} \]
\[ f_{dop\ atm} \sim 0 \div 1 \text{kHz} \quad \text{Excess atmospheric Doppler frequency shift} \]
\[ \Delta f_{atm} < 50 \text{Hz} \quad \text{atmosphere-induced spread of spectrum} \]
Principle of digital closed loop tracking

The main goal of the phase-locked loop is reduction of the frequency of signal as close to zero as possible by modeling its phase based on extrapolation of the previously extracted phase. Then the residual phase is determined $|\tilde{\phi}| < \pi$

Large random phase acceleration results in the large errors of the projected (extrapolated) phase model. A generic GPS receiver is capable of tracking signals with phase acceleration $6g \sim 60m/s^2 \sim 300Hz/s$. 

Due to the in-real-time feedback, the phase-locked loop is an optimal tracking technique for single-tone signals.

But, the feedback makes this technique unstable under the conditions:

(i) low signal-to-noise ratio (SNR)

(ii) complicated structure of the phase (e.g., large phase acceleration), typical for multi-tone signals, which results in large errors of predicted (extrapolated) phase

Both (i) and (ii) result in large errors of extraction of the residual phase.

Since the residual phase is used for updating the phase model, the errors can accumulate.

Extracting the residual phase and updating the phase model allow different algorithms. The results of tracking multi-tone signals under low SNR may significantly depend on the implementation of PLL (on tuning of the loop parameters). This was confirmed in GPS/MET.
An example of PLL tracking errors in the troposphere

Question: How to make tracking of RO signals in the troposphere (multi-path, low SNR) stable?
Answer: To not use the feedback for updating the phase model (open loop).
Principles of the open-loop tracking

Digital signal processing is based on the sampling theorem:

A continuous signal can be fully reconstructed from its discrete samples when the sampling frequency is not smaller than the (double-sided) spectral bandwidth of complex signal.

A) Sampling frequency **100Hz**.
The spectrum is preserved. The signal is fully reconstructed.

B) Sampling frequency **50Hz**.
The spectrum is aliased, but without overlapping of harmonics. The signal can be reconstructed after additional downconversion.

C) Sampling frequency **25Hz**.
The spectrum is aliased with overlapping of harmonics. The signal may not be recovered.
Some basic concepts

Aliasing is the apparent shift of harmonic in the spectrum when the frequency of the harmonic is larger than half of the sampling frequency. The frequency shift $= \pm n \cdot f_{samp}$

Down-conversion (up-conversion) is the multiplication of complex signal by complex harmonic function, that reduces (increases) mean frequency of the signal, by shifting its spectrum.

$$u(t) = a(t) \exp[i\phi(t)], \quad u'(t) = a(t) \exp[i\phi(t) \pm 2\pi if_0 t]$$
When aliasing does not result in overlapping of harmonics in the spectrum, i.e., the sampling frequency is larger than the spread part of the spectrum, the spectrum can be reconstructed by additional down-conversion

\[
\text{The down-conversion that corrects the shape of the spectrum:}
\]

\[
u_{\text{correct}} = u_{\text{samp}} \exp(2\pi i \Delta f t)
\]
The low-sampled signal with corrected spectrum (mean frequency reduced to ~0) can be up-sampled at higher rate.

This can be done by:
(i) calculating its spectrum at low sampling frequency;
(ii) filling zeroes in the spectrum at higher frequencies;
(iii) reconstructing the signal from the extended spectrum, at higher sampling rate. This can be treated as the Fourier interpolation.

Why do we need the up-sampling?
1) To reduce the phase lapse between samples when connecting the extracted phase between the samples (resolving cycle ambiguities)
2) Before the up-conversion for radio-holographic inversions (such as the FSI...)

\[ f \]

- original spectrum
- extended spectrum (filled zeroes)
- low sampling frequency band
- high sampling frequency band
Thus, for reconstruction of a signal from discrete samples it is sufficient that the sampling frequency is not smaller than $\geq$ the spread part of the spectrum + + the uncertainty of mean frequency.

In fact, in RO, by assuming that it is possible to estimate the “center” of the spectrum from its shape, it is sufficient that the sampling frequency is not smaller than the spread part of the spectrum.

Uncertainty of GPS carrier frequency $< 1 \text{Hz}$

Uncertainty of vacuum Doppler (based on orbit determination) $\sim 1-2 \text{Hz}$

Weather-related uncertainty of mean atmospheric Doppler $\sim 10-15 \text{Hz}$

Spread of RO signal spectrum in the troposphere $< 50 \text{Hz}$

**Question:** why to not directly sample raw RO signal at 100 Hz frequency? The spectrum will be aliased, but it could be corrected by down-conversion in post-processing (?)
Answer: because this will result in aliasing of noise from outside to inside the sampling frequency band and significant reduction of SNR.

Thus, if one wants to sample at low rate, the noise must be filtered before the sampling.

For the noise (low-pass) filtering the signal must be down-converted to as close to zero mean frequency as possible. This is done optimally by PLL, but PLL does not perform stable for multi-tone signals and under low SNR.
Open-loop tracking of RO signal consists of:

Prior to an occultation:
1) calculation of the frequency model of RO signal with account for predicted GPS and LEO orbit and refraction of radio waves in the atmosphere

During an occultation:
2) down-conversion (complex multiplication) of RO signal with the pre-calculated frequency model (without a feedback from received signal!) in order to reduce the mean frequency
3) low-pass filtering (integration) of the down-converted RO signal
4) sampling and transmitting complex RO signal (I and Q) for post-processing

In post-processing:
5) estimation of the residual mean frequency shift from the sliding spectrogram
6) additional down-conversion for minimization of the residual mean frequency shift
7) up-sampling by Fourier interpolation
8) extraction of the residual phase and amplitude
9) up-conversion of RO signal with account for all models used for the down-conversions (by adjusting the real-time model for solved LEO clock)
Calculation of the frequency model of RO signal

The frequency model is based on predicted GPS and LEO orbits and accounts for refraction of radio-waves in the atmosphere:

\[ \omega_{\text{mod}}^{dop} = -k \left( \frac{dr_1}{dt} \sqrt{1 - \frac{a^2}{r_1^2}} + \frac{dr_2}{dt} \sqrt{1 - \frac{a^2}{r_2^2}} + a \frac{d\theta}{dt} \right) \]

\[ \alpha_{\text{mod}} (a - r_c) = \arcsin\left(\frac{a}{r_1}\right) + \arcsin\left(\frac{a}{r_2}\right) + \theta - \pi \]

These equations must be solved concurrently

\[ \frac{dr_1}{dt}, \frac{dr_2}{dt}, \frac{d\theta}{dt} \] are known from predicted orbits

\[ \alpha_{\text{mod}} (h) \] is the model which accounts for refraction of radio-waves

where: \( \alpha \) is the bending angle, \( h \) is the height of ray asymptote

Accuracy of the frequency model based on orbits and refractivity (bending angle) climatology is 10-15 Hz for LEO height 500-1000 km (the accuracy increases with increasing orbit height).
The difference between L1 excess Doppler and its model estimated from GPS and LEO orbits and CIRA+Q refractivity climatology.

A – RO signal modeled from hires radiosonde.
B,C – GPS/MET observation RO signals.
Down-conversion of RO signal

\[ f_{\text{mod}} = f_{0} + f_{\text{dop}} \]

\[ u'(t) = u(t) \exp(-2\pi i f_{\text{mod}} t) \]

As the result, the signal frequency becomes close to zero. Miss-modeling is about 10-15 Hz. This is larger than for stable operating PLL, but smaller than for the unstable operating PLL under multi-path conditions and low SNR.
Low-pass filtering of down-converted RO signal

The down-converted, high-rate sampled RO signal contains wide-band noise. Must be low-pass filtered to prevent aliasing of this noise into sampling band. For the filtering, they commonly use the integration of the complex signal:

\[ u'(t) = \int_{t-\Delta t/2}^{t+\Delta t/2} u(t') dt' \]

The frequency response of such filter is equal: \( \left( \pi f \right)^{-1} \sin(\pi f \Delta t) \)

3.2kHz white noise after passing through 20ms sliding integration filter
Post-processing of sampled complex RO signal

Sliding spectrograms of 50 Hz sampled simulated RO signal after down-conversion with two frequency models:

- **perfect frequency model**
- **maximal frequency miss-modeling 15 Hz**
Estimation of the mean frequency miss-modeling from the sliding spectrogram

The spectrum in each window is cross-correlated with a model of the spectrum (a simple model, the sine wave, works OK). The shift of the maximum of the cross-correlation function gives an estimate of the shift of the center of the spectrum.

The estimated frequency shift as the function of time is used as the model for an additional down-conversion of RO signal, for further reduction of its mean frequency.
Effect of mean frequency miss-modeling on RH inversion

Two error sources:
1) spectral aliasing (more significant)
2) damping of the aliased spectrum due to integration (less significant)

Inversion of sampled RO signal with 15 Hz mean frequency mismodeling

true spectrum
aliased spectrum (50 Hz sampling, 0 ms int.)
aliased spectrum (50 Hz sampling, 20 ms int.)

50 Hz sampling
50 Hz sampling + frequency correction before the inversion
100 Hz sampling
Extraction of the residual phase and amplitude and connection of the phase

Amplitude: \( a = 2\sqrt{I^2 + Q^2} \) \hspace{1cm} Raw phase: \( \phi = \arctan_2(Q, I) \)

Accumulated (continuous, connected) phase is calculated successively, by adding \( 2\pi, 0, -2\pi \) to \( \phi_i \) whatever minimizes \( |\phi_i - \phi_{i-1}| \)

Up-sampling to higher rate prior to extraction of the accumulated phase allows to reduce the probability of the cycle slips
1 MHz C/A code demodulation

C/A code replica in receiver can be controlled by phase model generated similar to the Doppler model.
Accuracy of the neutral atmospheric model +/-15 m
Ionospheric group delay at 1.5 GHz can be as large as ~300 ns (~100 m)
Miss-phasing of the signal and replica is +/-65 m (20% of C/A code chip)
This will result in 20% power loss (equivalent ~1dB loss of antenna gain).

The excess phase delay for 768 refractivity profiles produced by NCEP T62 NWP model at 0-75deg N.
Summary

PLL is an optimal signal acquisition technique for single-tone RO signals (above the moist troposphere).

Tropospheric multi-tone RO signals may not be reliably acquired by PLL.

Must be acquired in open-loop mode:

1) down-convert with the frequency model which takes into account predicted orbits and refraction in standard atmosphere (NO feedback from acquired RO signal!), low-pass filtering and transmitting I and Q for post-processing.

2) Determining of frequency miss-modeling from the sliding spectrogram; additional down-conversion, (up-sampling), extraction of the phase and amplitude.

Important: open-loop allows tracking both setting and rising occultations (PLL allows only setting).

Currently, open-loop tracking is possible for only L1 GPS signal (C/A).