Reconsidering the role of Rossby waves in the Madden-Julian Oscillation

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ABSTRACT: Many studies attempt to gain insight into atmospheric and oceanic phenomena in the tropics using the Gill (1980) model because it simplifies the equatorial wave spectrum by making the “longwave approximation.” How can the equatorial wave spectrum be simplified to reproduce the waves important for the Madden-Julian Oscillation (MJO)? The MJO is the main intraseasonal fluctuation in tropical weather, modulating precipitation, pressure, and winds all year. Global Climate Models (GCMs) do not simulate the MJO well due in part to a lack of physical understanding. In order to improve physical understanding, the dynamical aspects of the MJO were investigated in a simple model framework. Steady state anomalies of winds and geopotential heights, as well as momentum fluxes in two simplified shallow-water models on the equatorial β-plane, were compared to those in a “complete” shallow-water model on the equatorial β-plane, which represented all waves. The simplified models consisted of a “filtered” model, which produced all Rossby and Kelvin waves, and a “truncated” model, which produced all Kelvin waves and only long Rossby waves. Three case scenarios of diabatic heating were analyzed. Winds, geopotential heights, and momentum fluxes were generally weaker, and maximum values were less concentrated in the “filtered” and “truncated” models. Therefore, short Rossby waves must be well-represented in idealized models of the MJO, and inertia-gravity waves may play an important role as well. Idealized models such as the one used here can help improve physical understanding of the MJO, and GCMs can be interpreted and revised so that the MJO is simulated with more accuracy.
1. Introduction

Imagine if meteorologists could predict weather more than three weeks in advance. An accurate forecast this far in advance would help many industries, such as agriculture, water resource management, and transportation (Barlow et al. 2005). Many scientists are trying to improve weather predictions on timescales longer than the typical 7-10 day timescale, especially intraseasonal phenomena (30-90 days) such as the Madden-Julian Oscillation (MJO).

The MJO is the main intraseasonal fluctuation that determines weather variations in the tropics (Madden and Julian 1994), developing at the equator in the form of atmospheric equatorial waves. Models often incorporate equatorial wave processes to gain insight into climate variability forming near the equator, e.g., the MJO and the El Niño Southern Oscillation (ENSO) (Schubert et al. 2009). Equatorial waves modulate heat, moisture, and momentum within and beyond the tropics. Equatorial waves are coupled with convective clouds and precipitation in the MJO. What makes a phenomenon like the MJO unique is that these convectively coupled waves travel at relatively slow speeds, approximately 5-10 m s⁻¹.

In order to better understand longer-range weather and climate predictions, more research must be performed on intraseasonal phenomena, such as the MJO, using data and model analysis (Madden and Julian 1994). This study compares two well-established analytical models, which both use equatorial wave theory, to learn more about the dynamical characteristics of the MJO.

1.1. Background

1.1.1. The Madden-Julian Oscillation (MJO)

The MJO, also known as the tropical intraseasonal oscillation, is a weather event that involves convective clouds, precipitation, and momentum processes, which travel across much of the tropical belt (approximately 20S-20N). The MJO forms in the Indian Ocean and dissipates in the eastern Pacific Ocean over a time period of 30-60 days (Madden and Julian 1971) (Fig. 1). The tropical intraseasonal oscillation is of great importance to communities in southeastern Asia because its activity determines whether an area will experience relatively dry conditions or monsoon conditions. Since a large number of communities depend on climate conditions for agricultural and energy production, precise weather forecasts are of great importance (Barlow et al. 2005).

Much of the current research on the MJO suggests that although observational and data analysis studies are improving at a satisfactory pace, the majority of modeling studies are not (improving at the same pace). Global Climate Models (GCMs) often are unsatisfactory when simulating the dynamics and structure of convectively-coupled waves (Frierson 2008; Majda and Khouider 2008). In some of the new models the MJO is simulated well, but understanding why tests our knowledge of the dynamical and physical processes. In order to improve understanding of the dynamics and structure of the MJO, Majda and Biello (2005) suggest that the simple physics and dynamics of convectively-coupled waves must be analyzed further using theoretical studies of atmospheric equatorial waves and tropical convection.

1.1.2. Atmospheric equatorial waves

Atmospheric equatorial waves are excited by localized atmospheric forcings. The main atmospheric forcing in the tropics is diabatic heating produced by latent heat release. Latent heat release occurs as a result of the sun heating the upper ocean and evaporating into water vapor in the lower atmosphere, which condenses into liquid, producing deep convective clouds. Cloud formation in the tropics is often accompanied by atmospheric waves. These waves transport geophysical features inside and outside of the tropics. Waves in the atmosphere are anisotropic, i.e., their response is not the same in all directions, producing different types of wave structures. There are four types of equatorially trapped waves in the tropics: Kelvin waves, inertia-gravity waves, Rossby waves, and mixed Rossby-gravity waves (Fig. 2). The first scientist to present comprehensive mathematical solutions for all of these equatorial waves was Matsuno (1966).

1.1.3. Theoretical models of equatorial waves and the MJO

Matsuno (1966) derived and solved the linearized shallow-water equations of motion on the equatorial β-plane, often referred to as the primitive equations model. The equatorial β-plane is frequently used to study phenomena
near the equator because it simplifies the equations of motion. One assumes that both the earth is a flat plane and that the Coriolis parameter varies linearly with latitude in the $\beta$-plane approximation. After Matsuno (1966) many other studies, such as Gill (1980), used the linearized shallow-water equations on the equatorial $\beta$-plane. Gill (1980) evaluated the steady state response of equatorial waves to idealized heat sources by using an analytical model based on the linearized shallow-water equations. The model is often referred to as the “longwave approximation” because even though inertia-gravity waves are filtered out, Kelvin and long Rossby waves are retained (Schubert et al. 2009). The Gill (1980) model has been used to simulate numerous atmospheric and oceanic phenomena in tropics, including the monsoon circulation and the Walker circulation. Gill (1980) has been used primarily to
gain simple understanding into modeling climate phenomena. However, a possible deficiency of the longwave approximation is its distortion of the dispersion relation for short Rossby waves (Stevens et al. 1990), illustrated in Figure 3. An improvement of the Rossby wave spectrum is needed when studying circulations that involve higher resolution (multiscale) features, such as those in the MJO.

1.2. The current study

Two steady state analytical models are presented and compared to the steady state primitive equation model ("complete" model) in this study: a “filtered” model and a “truncated” model. The “complete” model is similar to Matsuno (1966) in that it is solved analytically and it does not filter out any equatorial waves. The “filtered” model resolves all Rossby and Kelvin waves, and neglects inertia-gravity and mixed Rossby-gravity waves. The “truncated” model is the same as the “filtered” model that only resolves long Rossby waves. It only resolves Rossby waves up to wavenumber 3, while the “filtered” and “complete” models resolve wavenumbers up to 200. All of these models include forced waves that arise from latent heating. The goal of this study is to see how dynamical variables important for a strong MJO signal change as inertia-gravity waves, mixed Rossby-gravity waves, and short Rossby waves are excluded from the equatorial wave spectrum.
A number of studies have utilized the longwave approximation (Gill (1980)) for simulating the MJO (e.g., Chao (1987)), however an accurate representation requires both precise Rossby wave dynamics west of the region of interest, and Kelvin wave dynamics east of the region of interest. It has been noted that inertia-gravity waves can be filtered out when studying the MJO because they occur on a different timescale than intraseasonal weather (Fig. 4). The ultimate goal is to create a model to study the MJO that simplifies the equatorial wave spectrum, which involves filtering out inertia-gravity waves while accurately representing Rossby and Kelvin waves for all wavelengths. We would like to reevaluate the complete equatorial wave spectrum in more detail to better understand filtering. A new filtered model that reproduces a more accurate idealized MJO may help explain other dynamical phenomena in tropical climate (Schubert et al. 2009).

This paper is organized as follows. The methods section is organized into a section which presents the primitive shallow-water equations (“complete” model) and discusses the “filtered” and “truncated” models, as well as a section which describes the three case studies with tables of variable values and the overall relevance of the methodology. The results and discussion section is presented in three sections, one for each case. All figures within each case are compared to each other as well as the overall differences between each case. The conclusion section summarizes the whole study, its far-reaching significance, and what could be improved for future studies.
Figure 4. A presentation of the wavenumber \( (s) \) vs. frequency (day\(^{-1}\)) spectral peaks of a long record of satellite-observed outgoing longwave radiation for the symmetric component of the Outgoing Longwave Radiation (OLR) with respect to the equator. To represent the peaks, the contours show a ratio of the actual power with an estimate of the red-noise background power. A ratio of greater than 1.1 is a statistically significant spectrum (95\% level. The dispersion curves for Kelvin waves, \( n = 1 \) mode Rossby waves (ER), westward inertia-gravity waves (WIG), and eastward inertia-gravity waves (EIG) for equivalent depths of 12, 25, and 50 m are shown. Also, TD-type and the MJO are shown. TD-type stands for tropical depression type disturbances. From Wheeler and Kiladis (1999).
2. Methods

2.1. Primitive Equations

We will begin by introducing a set of primitive shallow-water equations linearized about a mean resting state on the equatorial $\beta$-plane. In a shallow-water model the atmosphere is simulated as an incompressible fluid (e.g., water), and the fluid depth is much smaller than the horizontal scale of the flow. The primitive equations are:

\[
\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \phi}{\partial x} = -\alpha u, \tag{2.1}
\]

\[
\frac{\partial v}{\partial t} + \beta y u + \frac{\partial \phi}{\partial y} = -\alpha v, \tag{2.2}
\]

\[
\frac{\partial \phi}{\partial z} = R T, \tag{2.3}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = w, \tag{2.4}
\]

and

\[
\frac{\partial T}{\partial t} + \Gamma w = -\alpha T + \frac{Q}{c_p}. \tag{2.5}
\]

We define all variables in Table I.

We solve this set of equations making the following assumptions. We seek solutions on a domain that is infinite in $y$ and periodic over $-\pi a \leq x \leq \pi a$. The dependent variables $u, v, \phi, w, T$ are assumed to approach zero as $y \to \pm \infty$. The vertical direction is confined to $z = 0$ and $z = z_T = \ln(1010/200) = 1.619$, with boundary conditions $w = 0$ at $z = 0$ and $z = z_T$.

A shallow-water model describes only horizontal variations in a fluid. Therefore, we will separate the horizontal and vertical structure by performing a vertical normal mode transform on equations (2.1) - (2.5) (Fulton and Schubert (1985)):

\[
\begin{pmatrix}
  u(x, y, z, t) \\
  v(x, y, z, t) \\
  \phi(x, y, z, t)
\end{pmatrix} = \sum_{l=1}^{\infty} \begin{pmatrix}
  u_l(x, y, t) \\
  v_l(x, y, t) \\
  \phi_l(x, y, t)
\end{pmatrix} Z_l(z),
\]

\[
\begin{pmatrix}
  T(x, y, z, t) \\
  w(x, y, z, t) \\
  Q(x, y, z, t)
\end{pmatrix} = \sum_{l=1}^{\infty} \begin{pmatrix}
  T_l(x, y, t) \\
  w_l(x, y, t) \\
  Q_l(x, y, t)
\end{pmatrix} Z_l'(z),
\]

where $l$ is an integer that denotes the vertical mode, and the orthogonal functions $Z_l(z)$ and their derivatives compose the vertical structure. In this study, only the first baroclinic mode, $l = 1$, is represented because it compares well to observations (Fig. 5). (The specific form of the vertical structure including its derivation is explained in more detail in Fulton and Schubert (1985)).

The diabatic heating function is defined as:

\[
Q(x, y, t) = \frac{Q_0}{2} e^{-\left(\frac{y-y_0}{b_0}\right)^2} \begin{cases}
  1 + \cos \left(\frac{z(x-ct)}{a_0}\right) & \text{if } x - ct \leq a_0, \\
  0 & \text{if } x - ct \geq a_0,
\end{cases}
\]

where $c$ is the propagation speed of the heating, $y_0$ is the center of the heating, $a_0$ its half width in $x$, $b_0$ its e-folding width in $y$, and $Q_0$ its peak amplitude. The values for these variables are listed in Table II. This forcing function can mimic an eastward propagating MJO event when $c > 0$ ms$^{-1}$. A limitation to this type of prescribed diabatic forcing is that the wind circulations that arise from the heating cannot communicate with the forcing. When dealing with simple theoretical models there are limitations in order to be able to solve the equations analytically.

Another limitation to the primitive equations model is that we can only analyze the steady state solutions (i.e., once all transient effects are negligible). In nature, a steady state is seldom reached, but it has been proven to be a relatively accurate approximation for the initial quasi-equilibrium state of Global Climate Models (GCMs).
Table I. Variable definitions for equations (2.1) - (2.5).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>eastward velocity component</td>
<td>$(\text{ms}^{-1})$</td>
</tr>
<tr>
<td>$v$</td>
<td>northward velocity component</td>
<td>$(\text{ms}^{-1})$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>perturbation geopotential height</td>
<td>$(\text{m})$</td>
</tr>
<tr>
<td>$w = \frac{Dz}{Dt}$</td>
<td>“vertical log-pressure velocity”</td>
<td>$(\text{hPa day}^{-1})$</td>
</tr>
<tr>
<td>$T$</td>
<td>the perturbation temperature</td>
<td>$(\text{K})$</td>
</tr>
<tr>
<td>$\beta = \frac{2\Omega}{a}$</td>
<td>equatorial value of northward gradient of Coriolis parameter</td>
<td>$2.29 \times 10^{-11} \text{ (m s}^{-1})$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Earth’s rotation rate</td>
<td>$7.29 \times 10^{-5} \text{ (s}^{-1})$</td>
</tr>
<tr>
<td>$a$</td>
<td>Earth’s radius</td>
<td>6371 (km)</td>
</tr>
<tr>
<td>$R$</td>
<td>dry air gas constant</td>
<td>287 (Jkg$^{-1}$K$^{-1}$)</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat capacity at constant pressure</td>
<td>1004 (Jkg$^{-1}$K$^{-1}$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>dissipation coefficient (Rayleigh friction, Newtonian cooling)</td>
<td>0.25 (day$^{-1}$)</td>
</tr>
<tr>
<td>$\Gamma = \frac{dT}{dz} + \kappa \bar{T}$</td>
<td>basic state static stability</td>
<td>23.79 (K)</td>
</tr>
<tr>
<td>$\kappa = \frac{R}{c_p}$</td>
<td>dimensionless ratio between dry air and specific heat capacity</td>
<td>0.2859</td>
</tr>
<tr>
<td>$Q$</td>
<td>diabatic heating/forcing</td>
<td>$(\text{K day}^{-1})$</td>
</tr>
</tbody>
</table>

Figure 5. The curves labeled $Z(z)$ (blue,dashed) and $Z'(z)$ (red, solid) - interpreted using the lower scale are the vertical structure functions. The curve labeled $Q = c_p$ (orange, dotted) - interpreted using the upper scale - is the 120-day mean vertical profile of heating rate for the western Pacific warm pool, as determined by Johnson and Ciesielski (2000). Note that $Z'(z)$ reaches its maximum at $p = 395$ hPa. From Schubert and Masarik (2006).

In order to continue the derivation of (2.1)-(2.5) we must remove the time dependent terms, applying the steady state assumption. We then combine (2.1)-(2.5) so that we are left with five equations for five unknown variables:

$$
\begin{pmatrix}
  u(x, y, z) \\
  v(x, y, z) \\
  \phi(x, y, z)
\end{pmatrix}
= Z(z) \sum_{m=-\infty}^{\infty} \sum_{n=-1}^{\infty} \sum_{r=0}^{2} \eta_{mnr} \begin{pmatrix} U_{mnr}(\hat{y}) \\ V_{mnr}(\hat{y}) \\ \Phi_{mnr}(\hat{y}) \end{pmatrix} e^{imx/a},
$$

\(2.8\)

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Table II. Outline of case scenarios.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>SS</th>
<th>PS</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>(km)</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$b_0$</td>
<td>(km)</td>
<td>555</td>
<td>555</td>
<td>555</td>
</tr>
<tr>
<td>$y_0$</td>
<td>(km)</td>
<td>0</td>
<td>0</td>
<td>555</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>(K day$^{-1}$)</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$c$</td>
<td>(ms$^{-1}$)</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
\left( \begin{array}{c}
T(x, y, z) \\
w(x, y, z)
\end{array} \right) = \frac{Z'(z)}{R} \sum_{m=-\infty}^{\infty} \sum_{n=-1}^{1} \sum_{r=0}^{2} \left( \frac{1}{i \nu_{mnr} r^{-1}} \right) \hat{\eta}_{mnr} \Phi_{mnr}(\hat{y}) e^{imx/a}.
\] (2.9)

We arrived at the solution (2.8) - (2.9) first by Fourier transforming in $x$, then Hermite transforming in $y$. In doing this we have converted the primitive equations from physical space (i.e., $x$ and $y$) to spectral or wave space (i.e., $m$, and $n$), where $m$ is zonal wavenumber and $n$ is meridional mode. Fourier transforms are convenient because they allow us to represent any function using the superposition multiple sine and cosine functions. Hermite transforms are similar in that one can superpose multiple polynomial functions to represent any function. (For more details on the mathematical derivation, refer to Schubert and Masarik (2006).)

2.2. The “filtered and “truncated” models

The “filtered” model filters out inertia-gravity waves and mixed Rossby-gravity waves and represents all Rossby and Kelvin waves. This is done by running the “complete” model (2.8) - (2.9) only for $n = -1, 1-200$, $r = 0$ waves. We use this model because previous research suggests that Rossby and Kelvin waves are the most important equatorial wave types for the MJO. We would like to investigate dynamical variables important for a strong MJO signal that neglect the contribution of inertia-gravity waves.

The “truncated” model is a simplification of the “filtered” model in that it only produces long Rossby waves greater than wavenumber 3 and all Kelvin waves. This would change (2.8) - (2.9) by summing $m = (-3,200)$ instead of $m = (-200,200)$. As in the “filtered” model $n = -1, 1-200$, $r = 0$. Our approach is not to recreate the Gill model, but to interpret the effect of neglecting short Rossby waves in the “filtered” model.

2.3. Three Case Scenarios

We run the “complete”, “filtered”, and “truncated” models under three case scenarios of diabatic heating. The first case scenario involves a stationary heat source that is symmetric about the equator, called the “stationary symmetric” (SS) case. The second case scenario involves a 5 ms$^{-1}$ eastward moving heat source that is also symmetric about the equator, called the “propagating symmetric” (PS) case. The third case scenario involves a 5 ms$^{-1}$ eastward moving heat source that is displaced about 5$^\circ$N (555 km), called the “propagating antisymmetric” (PA) case. Each case has the same heating structure, which is zonally-elongated and agrees relatively well with NASA’s Modern Era Retrospective-analysis for Research and Applications (MERRA) reanalysis precipitation composite of MJO anomalies. Each diabatic heating case scenario is plotted over the equatorial region of approximately 24$^\circ$S-24$^\circ$N ($\pm$ 2700 km) and over all longitudes. The resulting figures concentrate on a longitudinal area close to the heat source, which is approximately 80$^\circ$ wide (8880 km). The details of each case scenario are outlined in Table II, and a sample domain and heating is shown in Figure 6. For practicality we assume the heating is located in the Indian Ocean for all of the cases, since the MJO can be either stationary or moving eastward during its lifetime in the Indian Ocean (Zhang 2005; Li et al. 2009). These results can also be displayed over the West Pacific Ocean because the MJO can be either stationary or moving eastward over the West Pacific warm pool (Rui and Wang 1990; Weickan and Khalsa 1990; Li et al. 2009). For each case scenario we analyze variables important for the initiation and progression of a MJO event: wind anomalies, geopotential height anomalies, the meridional flux of the zonal momentum, and the vertical flux of the zonal momentum (using vertical-pressure velocity - refer to the Appendix). These variables are defined in Table III. We display the solutions of these variables for the
Table III. Variables analyzed in all case scenarios.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>(Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v'$</td>
<td>wind anomaly</td>
<td>($\text{ms}^{-1}$)</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>geopotential anomaly</td>
<td>(m)</td>
</tr>
<tr>
<td>$u'v'$</td>
<td>meridional flux of zonal momentum</td>
<td>($\text{m}^2\text{s}^{-2}$)</td>
</tr>
<tr>
<td>$u'\omega'$</td>
<td>vertical flux of zonal momentum</td>
<td>($\text{ms}^{-1}\text{hPa day}^{-1}$)</td>
</tr>
</tbody>
</table>

“complete”, “filtered”, and “truncated” models and the difference between the three in the next section. We also present the plots and highlight their main features.

Figure 6. The diabatic heating structure contoured for each case scenario: SS (top), PS (middle), and PA (bottom). Refer to Table II for the specific variables and their respective values in each case scenario.
3. Results and Discussion

In this section we present the model results for three shallow-water models on the equatorial $\beta$-plane to investigate what dynamical roles inertia-gravity waves, mixed Rossby-gravity waves, and short Rossby waves play in an idealized MJO event. Simple idealized studies may improve our physical understanding of dynamical processes that occur during the MJO. The solutions of the three models used here involve a superposition of zonal wavenumbers (sum over $m$), meridional wavenumbers (sum over $m$), and wave types (sum over $r$) in (2.8) - (2.9).

3.1. Stationary symmetric (SS) case scenario

We present the steady state solutions of a stationary diabatic heating, which is centered on the equator. First we analyze the wind and geopotential height anomalies (Fig. 7) of all three models. The “complete” model produces three wave types in this case: inertia-gravity, Rossby, and Kelvin waves. The mixed Rossby-gravity wave solution is exactly zero when the heating is centered on the equator. The “filtered” model produces all Rossby and Kelvin waves, and the “truncated” model produces long Rossby waves and all Kelvin waves.

Figure 7. The wind and geopotential height anomalies for each model in the SS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). Winds are in vector form, and the geopotential heights are shaded.
The major differences between the “filtered” model and the “complete” model include more divergent winds close to the heating because the “filtered” model does not resolve inertia-gravity waves (Fig. 8, top). Winds are also weaker near the heating — the “filtered” model maximum winds are 9.15 m/s while the “complete” model maximum winds are 10.01 m/s. These maximum winds are located just west of the heating in both cases and can be thought of as westerly wind bursts (WWBs). WWBs are winds greater than 5 m/s over a distance of at least 10° (Harten 1996). Also, “filtered” model geopotential heights are too weak just west of the heating, and too strong just east of the heating (Fig. 8, top). The maximum difference between the “filtered” model and the “complete” is 18 m, located just west of the heating. There are even larger differences in the winds and geopotential heights between the “truncated” and “complete” model (Fig. 8, bottom). As in the “filtered” model, most of the differences occur close to the heating. Winds are up to 5.01 m/s too weak just west of the heating, and the geopotential heights are 43.95 m too weak just west of the heating. It also seems as if the Rossby waves “break-off” from the rest of the flow (Fig. 7, top, middle). As a result there are multiple relative maximums in geopotential height in the “truncated” model, while there is only one relative maximum in the “complete” and “filtered” models.

Figure 8. The differences in winds and geopotential height anomalies between each model in the SS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom). Wind differences are in vector form, and the geopotential height differences are shaded.
Taking a closer look into the Rossby waves, specifically, illustrates features lost when short Rossby waves are excluded in the “truncated” model (Fig. 9). Since we are only going to investigate the Rossby wave structure, both the “complete” and “filtered” are the same. We will refer to the full Rossby wave solutions as the “complete” model. The wind field is much weaker in the “truncated” model compared to the “complete” model, especially near the heating (Fig. 9, top, middle). The “truncated” model is up 4.24 m/s too weak just west of the heating, and winds prevail farther east of the heating when compared the “complete” model (Fig. 9, bottom). The wind circulations northwest and southwest of the equator are also weaker in the “truncated” model. Geopotential heights are generally weaker and less concentrated, especially near the heat source (Fig. 9, bottom). Geopotential heights are up to 27.80 m too weak just west of the heating. Farther west of the heating, geopotential heights are too strong, but these differences are less significant than those closer to the heating. These differences in geopotential heights help explain why the geopotential height field is less concentrated in the “truncated” model when compared to the “filtered” model. Also, the shape of the geopotential height differences resemble relatively small Rossby waves (Fig. 9, bottom). This may be a manifestation of not resolving enough Rossby waves — when all modes \((m,n)\) are summed up in the “truncated” model, multiple Rossby wave pairs are located around the zonal domain. In the “filtered” model all of the Rossby waves that show up in the zonal domain sum to zero except for the pair of Rossby waves near the heat source.

Figure 9. Wind and geopotential height anomalies for Rossby waves in the SS case scenario: “complete” model (top), “truncated” model (middle), and “truncated” - “complete” difference (bottom). Winds are in vector form, and the geopotential heights are shaded.

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Fluxes of zonal momentum are also important for a strong MJO signal. First, we analyze the meridional flux of zonal momentum, $u'v'$, which is the transport of the east-west wind component by the north-south wind component.

Figure 10 illustrates the horizontal distribution of $u'v'$ between the three models. In general, both the “filtered” and “truncated” model have major differences when compared to the “complete” model (Fig. 11). The maximum in $u'v'$ for the “complete” model is 9 m$^2$s$^{-2}$, for the “filtered” model 5.35 m$^2$s$^{-2}$, and for the truncated model 1.53 m$^2$s$^{-2}$. The “filtered” model has momentum flux just east of the heating from the interaction between Rossby and Kelvin waves, while the “complete” model has negligible $u'v'$ just east of the heating due to the added contribution of inertia-gravity waves. Winds are exclusively zonal in the “complete” model just east of the heating while the “filtered” model has both a zonal and meridional wind component (Fig. 8). This result would explain why the $u'v'$ would be very small in the “complete” model, but not in the “filtered” model.

Near the north and south of the heating and just east of the heating, the “filtered” has a much larger $u'v'$ than the “complete” model (Fig. 10). The “truncated” model may seem to be more accurate spatially than the “filtered” model when looking at Figure 22 (top, middle), but the $u'v'$ in the “truncated” model is far too weak. For completeness, the difference in $u'v'$ between the “filtered” model and the “truncated” model are shown in Figure 22 (bottom), but these results are not surprising. A nonlinear variable such as $u'v'$ yields larger differences between the models when compared to the wind and geopotential height anomalies.

![Figure 10. The momentum flux $u'v'$ for each model in the SS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'v'$ is shaded.](image)
Next we take a look at the vertical flux of zonal momentum, $u'\omega'$, which is basically the transport of the east-west wind component by the vertical wind component. Figure 11 displays the horizontal distribution of $u'\omega'$ between the three models. Both the “filtered” and “truncated” models are inaccurate when compared to the “complete” model, with the “filtered” model being 1775 ms$^{-1}$hPa day$^{-1}$ too weak, and the “truncated” model being 1958 ms$^{-1}$hPa day$^{-1}$ too weak (Fig. 23). Spatially, both the “filtered” and “truncated” models are not as inaccurate when compared to other variables analyzed, with large values of $u'\omega'$ located in the area of diabatic heating (Fig. 11). Figure 23 (bottom) illustrates that the “filtered” model and the “truncated” model do not differ as much in $u'\omega'$ when compared to each other.

![Figure 11](image_url)

**Figure 11.** The momentum flux $u'\omega'$ for each model in the SS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'\omega'$ is shaded.

In conclusion, a stationary symmetric MJO-like heating produces winds, geopotential heights, the meridional flux of zonal momentum, and the vertical flux of zonal momentum that are influenced greatly by both short Rossby and inertia-gravity waves. The variables are weaker and less concentrated when short Rossby and inertia-gravity waves are excluded, in general. Momentum fluxes are influenced the most, since they are nonlinear variables. Next we investigate a propagating symmetric MJO-like heating.
3.2. Propagating symmetric (PS) case scenario

In this section, the steady state solutions of a diabatic heating propagating eastward at 5 ms$^{-1}$ centered on the equator are investigated for all three models.

![Diagram showing wind and geopotential height anomalies for each model in the PS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). Winds are in vector form, and the geopotential heights are shaded.]

Figure 12. The wind and geopotential height anomalies for each model in the PS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). Winds are in vector form, and the geopotential heights are shaded.

In general, the wind and geopotential anomalies in both the “filtered” and “truncated” models are weaker and less concentrated, with the largest difference located close the heating (Fig. 12). Without inertia-gravity waves, the geopotential heights and winds in the “filtered” and “truncated” models create a “lump” north and south of the heating, due to a lack of wind convergence. In this case scenario the “filtered” and “truncated” model winds anomalies and geopotential heights anomalies improve compared to the SS case scenario. The winds are weaker and the geopotential heights are stronger overall in this case scenario. Specifically, west of the heating, winds and geopotential heights are weaker in this case scenario. Also, east of the heating, winds and geopotential heights are stronger in this case scenario. This causes the Rossby waves to “break-off” the Kelvin waves in both the “filtered” and “truncated” models more than the first scenario (Fig. 7). An eastward moving heating excites the Kelvin wave more in this case scenario than the first case scenario since Kelvin waves are located to the east of the heating. The maximum winds in the “complete” geopotential height in the “complete” model are 7.49 m/s, in the “filtered”
model 6.69 m/s, and in the “truncated” model 4.93 m/s. The criteria for WWBs is satisfied for the “complete” and “filtered” models, but not the “truncated” model.

As stated above, the Kelvin waves are excited more in this case scenario; therefore, we expect smaller differences between all three models when compared to the previous case scenario. The maximum wind difference between the “complete” model and the “filtered” model is 0.87 ms\(^{-1}\), while the maximum wind difference between the “complete” model and the “truncated” model is 3.11 ms\(^{-1}\); both are smaller than those values in the previous case scenario. The maximum geopotential height difference between the “complete” model and the “filtered” model is 14.83 m, while the maximum geopotential height difference between the “complete” model and the “truncated” model is 31.14 m (Fig. 13).

Figure 13. The differences in winds and geopotential height anomalies between each model in the PS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom). Wind differences are in vector form, and the geopotential height differences are shaded.
When comparing the Rossby waves between the models, we still see there are differences between the “complete” and the “truncated” models. However, as previously stated, these differences are smaller in this case scenario. The Rossby wave structure is more concentrated and stronger close to the heating in the “complete” model, whereas the Rossby wave structure is spread out more in the “truncated” model (Fig. 14, top, middle). The winds and geopotential heights are weaker in the “truncated” model compared to the “complete” model with a maximum wind of 5.92 m/s and a maximum geopotential height of -61.17 m. When compared to a stationary symmetric heating, the differences between the Rossby waves in the “complete” and “truncated” models in a propagating symmetric heating are smaller and located closer to the heating (Fig. 14, bottom). The shape of the geopotential height differences resembles relatively small Rossby waves, just as in the stationary symmetric heating case scenario, but they are weaker in magnitude and more concentrated.

Figure 14. Wind and geopotential height anomalies for Rossby waves in the PS case scenario: “complete” model (top), “truncated” model (middle), and “truncated” - “complete” difference (bottom). Winds are in vector form, and the geopotential heights are shaded.
Fluxes of zonal momentum are important for a strong MJO signal, as stated earlier. First, we analyze the meridional flux of zonal momentum, $u'v'$. Even though the “filtered” and “truncated” models perform better in the wind and geopotential height fields in this case scenario, these two models still are not sufficiently accurate in the $u'v'$ field (Fig. 15). The differences between all three models are either to the west of the heating, where the Rossby waves are located, or in the location of the heating, where the inertia-gravity waves are present. The maximum $u'v'$ in the “complete” model is located to the west of the heating whereas the maximum $u'v'$ for both the “filtered” and “truncated” models are located to the east of the heating.

Figure 15. The momentum flux $u'v'$ for each model in the PS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'v'$ is shaded.
We now analyze the vertical flux of zonal momentum, $u'\omega'$, Even though the “filtered” and “truncated” models perform better in the wind and geopotential height fields in this case scenario, these two models still are not sufficiently accurate in the $u'\omega'$ field (Fig. 16). Both the “filtered” and “truncated” models are just far too weak, similar to a stationary symmetric heating. The maximum $u'\omega'$ in the “filtered” model is $1100.4 \text{ ms}^{-1}\text{hPa day}^{-1}$ weaker than the “complete” model, while the maximum $u'\omega'$ in the “truncated” model is $1179.4 \text{ ms}^{-1}\text{hPa day}^{-1}$ weaker than the “complete” model (Fig. 25).

Figure 16. The momentum flux $u'\omega'$ for each model in the PS case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'\omega'$ is shaded.
3.3. Propagating antisymmetric (PA) case scenario

We also evaluate a heating that is centered at 5°N and propagating at 5 m/s since the heating during a MJO event is never actually exactly centered on the equator. We know that when the heating is centered off the equator that mixed Rossby-gravity waves will play a role in our variables of interest, however previous studies have suggested that their contributions are negligible when the heating is only slightly centered off the equator (Schubert and Masarik 2006). Therefore, we will not discuss their role in as much depth. Also, we will only state the major differences qualitatively in this case scenario because the results here are quite similar to the previous case scenarios.

Figure 17. The wind and geopotential height anomalies for each model in the PA case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). Winds are in vector form, and the geopotential heights are shaded.

As seen in Figure 17, the winds and geopotential heights are weaker in the “filtered” and “truncated” models compared to the “complete” model. Spatially, the differences in horizontal structure in the “filtered” and “truncated” models compared to the “complete” model are similar to the previous case scenarios. There are differences in the winds and geopotential for the “filtered” and “truncated” models north and south of the equator due to the contribution of inertia-gravity waves (Fig. 18, top) and short Rossby waves (Fig. 18, bottom).

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Figure 18. The differences in winds and geopotential height anomalies between each model in the PS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom). Wind differences are in vector form, and the geopotential height differences are shaded.
The differences in short Rossby waves are located mostly in the Rossby wave north of the equator (Fig. 19). This is not promising since in nature, the Rossby wave located in the hemisphere of the heating will dominate, and the other Rossby wave will dissipate. Therefore these discrepancies between the “complete” and “truncated” models will grow even larger over time.

Figure 19. Wind and geopotential height anomalies for Rossby waves in the PA case scenario: “complete” model (top), “truncated” model (middle), and “truncated” - “complete” difference (bottom). Winds are in vector form, and the geopotential heights are shaded.
Momentum fluxes $u'v'$ and $u'\omega'$ in the “filtered” and “truncated models (Fig. 20, 21) illustrate even larger differences (when compared to the “complete” model) compared to the other case scenarios, spatially, and in magnitude. A propagating antisymmetric heating case scenario is the most realistic of the three because heating in the tropics is usually displaced off the equator. The results from these figures suggest that inertia-gravity waves and short Rossby waves are both important in the MJO.

Figure 20. The momentum flux $u'v'$ for each model in the PA case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'v'$ is shaded.
Figure 21. The momentum flux $u'\omega'$ for each model in the PA case scenario: “complete” model (top), “filtered” model (middle), and “truncated” (bottom). The flux $u'\omega'$ is shaded.
4. Conclusion and Future Work

In this study, we analyze idealized models of the dynamical aspects that simulate the Madden-Julian Oscillation (MJO). Global Climate Models (GCMs) often do an unsatisfactory job of simulating many dynamical properties of the MJO, due in part to a lack of physical understanding. Using simpler models that focus on more specific dynamical aspects of this phenomenon will help improve our understanding of processes misrepresented in GCMs. We analyze steady state wind and geopotential height anomalies, as well as steady state momentum fluxes in two simplified shallow-water models on the equatorial $\beta$-plane. The two models are simplified in that they filter or neglect certain equatorial waves in the wave spectrum. These models, given the names the “filtered” model and “truncated” model, are compared to a “complete” model that represents all the equatorial waves in the wave spectrum.

The results from three case scenarios illustrate that both the “filtered” model and the “truncated” model generally have weaker and less concentrated winds, geopotential heights, and momentum fluxes. These models perform best when a heat source is centered on the equator and moves eastward at 5 ms$^{-1}$ and worst when a heat source is stationary. This is mainly due to an enhanced excitation of the Kelvin wave when the heating moves eastward, since the Kelvin wave response occurs east of the heating. Despite this, all of the case scenarios show large differences between the “truncated” model and the “complete” model.

We suggest that short Rossby waves must not be neglected in an idealized model because longwaves cannot reproduce strong enough features in winds or geopotential heights. Inertia-gravity waves play an important role as well in providing wind convergence that would enhance momentum fluxes of the zonal wind, which is a key element to a strong MJO signal. These inertia-gravity waves could be approximated in future models and may not need to be fully represented over the whole wave spectrum since they approach high wave frequencies (small time periods) in the wave spectrum.

Future studies should investigate the Gill (1980) approximation in more depth, because it is much simpler than most idealized models. However, it only approximates short Rossby waves and neglects inertia-gravity waves. It does a better job of simulating the “complete” model than the “truncated” model, and it may even do better than the “filtered” model. We need to learn more about what waves need to be represented well, or even at all, in the equatorial wave spectrum. This would help simplify the dynamics of an idealized model of the MJO, and then more moisture variables, or thermodynamic variables, can be represented in such a model. We know that there is more to the MJO than just the dynamics, and so it would be useful to have a model that represents more features of the MJO. Most GCMs do not resolve the whole wave spectrum due to their relatively low resolutions and numerical approximations. This may be an issue to also address in future studies. Then the statement arises, “Maybe the MJO is too complex a phenomenon to simulate in an idealized model,” but we still have to address more issues that arise in the theory. Other complex phenomena, such as the El Niño Southern Oscillation (ENSO), are understood well, including the theoretical aspects.

Scientists need to investigate problems with GCM simulations and the MJO from multiple directions, using multiple methods. This includes using simpler theory of dynamics, such as idealized models, to gain more insight into the MJO.
5. References


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6. Appendix

We calculate the vertical log-pressure velocity in our plots using the relation \( \omega = \frac{Dp}{Dt} = p_0 e^{-z} w \). We calculate this form of the vertical velocity because the primitive equations are converted to vertical log-pressure coordinates.

There are a number of figures which were computed and discussed, but not plotted in the results section in order to conserve space. In Figures 22-27 we present the differences in the \( u'v' \) and \( u'\omega' \) fields between the three models for each case scenario.

Figure 22. The differences in \( u'v' \) between each model in the SS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).
Figure 23. The differences in $u'/\omega'$ between each model in the SS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).
Figure 24. The differences in $u'v'$ between each model in the PS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).
Figure 25. The differences in $u'/\omega'$ between each model in the PS case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).
Figure 26. The differences in $u'/v'$ between each model in the PA case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).
Figure 27. The differences in $u'\omega'$ between each model in the PA case scenario: “filtered” model - “complete” model (top), and “truncated” model - “complete” model (bottom).