Path-Length Sensitivity of
the Lyman-Alpha Humidiometer

Carl A. Friehe
1. Introduction

The Lyman-alpha humidiometer is often used for the measurement of humidity fluctuations in the atmosphere. The principle of the device is discussed by Tillman (1965), and a description of an operational unit is given by Randall et al. (1965). The device is based on the absorption of a specific wavelength of light in the vacuum ultraviolet (the Lyman-alpha line, 1215 Å) by molecular hydrogen in water vapor in the air. Lyman-alpha radiation is usually generated by glow discharge or radio-frequency-excited hydrogen tubes, and detected by nitrous oxide or photomultiplier tubes. Lithium or magnesium fluoride tube windows are used to pass the Lyman-alpha radiation to the atmospheric path.

The absorption is governed (for ideal performance of a pure Lyman-alpha source tube, see Tillman, 1965) by Beer's Law:

\[
\frac{I}{I_0} = e^{-\frac{k \rho_x}{\rho_0}}
\]  

where \( I \) = received intensity

\( I_0 \) = source intensity

\( x \) = path length

\( k \) = absorption coefficient for \( H_2O = 387 \, \text{cm}^{-1} \)

\( \frac{\rho}{\rho_0} \) = dimensionless concentration, referred to STP \( \rho_0 \) conditions (0.806 kg/m\(^3\) at 1013 mb, 273 K)

Some analysis considering departures from the ideal Beer's Law is given by Buck (1975).

2. Analysis

We are interested in optimizing the sensitivity of the humidiometer to water vapor density changes. The sensitivity, \( S \), from (1), is

\[
S = \frac{dI/I_0}{d\rho/\rho_0} = -k \frac{\rho_x}{\rho_0}
\]  

(2)
As can be seen from (2), the sensitivity, for a given $p/p_o$ to be measured, varies from 0 at $x = 0$, to $S + 0$ as $x \to \infty$ because of the exponential term. The dependence of $S$ on $x$ is

$$\frac{dS}{dx} = k^2 \frac{p}{p_o} x e^{-\frac{k \cdot p}{p_o} x} - k \cdot \frac{p}{p_o} e^{-\frac{k \cdot p}{p_o} x}. \quad (3)$$

The maximum value of $|S|$ is obtained by setting (3) equal to zero and solving for the optimum value $x = x^*$; the result is

$$x^* = \left( \frac{k p_o}{p} \right)^{-1}. \quad (4)$$

Table 1 lists value of $x^*$ and $S(x^*)$ for different values of $p/p_o$, assuming saturated conditions. The intensity ratio, $I/I_0$, is $1/e = 0.368$ at $x^*$.

<table>
<thead>
<tr>
<th>Dew Point</th>
<th>$p$</th>
<th>$p_o/p$</th>
<th>$x^*$</th>
<th>$S(x^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>gm m$^{-3}$</td>
<td>-</td>
<td>cm</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>4.85</td>
<td>166.1</td>
<td>0.429</td>
<td>-161.10</td>
</tr>
<tr>
<td>5</td>
<td>6.80</td>
<td>118.5</td>
<td>0.306</td>
<td>-114.56</td>
</tr>
<tr>
<td>10</td>
<td>9.40</td>
<td>85.62</td>
<td>0.221</td>
<td>-83.50</td>
</tr>
<tr>
<td>15</td>
<td>12.83</td>
<td>62.82</td>
<td>0.162</td>
<td>-23.11</td>
</tr>
<tr>
<td>20</td>
<td>17.30</td>
<td>46.51</td>
<td>0.121</td>
<td>-17.22</td>
</tr>
<tr>
<td>25</td>
<td>23.0</td>
<td>35.04</td>
<td>0.091</td>
<td>-12.89</td>
</tr>
<tr>
<td>30</td>
<td>30.4</td>
<td>26.51</td>
<td>0.069</td>
<td>-9.75</td>
</tr>
</tbody>
</table>

The function $S$ is plotted in Figure 1 as $S' = \frac{p}{p_o} S\left(\frac{x}{x^*}\right) = -\frac{x}{x^*} e^{-\frac{x}{x^*}}$ versus $x/x^*$.

The optimum path length, $x^*$, is shown, in Figure 2 as a function of water vapor density, $p$, and the maximum sensitivity, $S$, as a function of $p$ in Figure 3.

These figures give guidelines for choosing the optimum path length for a particular measurement situation. The sensitivity curve (Figure 1) is reasonably broad around $x/x^* = 1$, so that reasonable performance may be expected if $x$ is within a factor of 2 of $x^*$. 
3. Frequency Response

The frequency response of a commercial (ERC Company) Model B Lyman-alpha is given in Figure 4. The response is flat to within ±0.2 dB to 250 Hz. An amplifier circuit constructed at the University of California at San Diego has improved response. The response was determined by modulating the source tube voltage with a signal generator. The source tube was assumed to respond ideally; apparently the photochemical reaction in the detector is limiting the upper frequency response. The response is not simple (i.e., not that of a simple R-C circuit), and appears to improve slightly with higher-intensity received light levels. Further work is necessary to completely understand and improve the frequency response of the Lyman-alpha humidiometer, although it is already suited for present aircraft needs.

Recently Priestley and Cartwright (1982) have used the source modulation technique to test the response of several Lyman-alpha circuits. They were able to achieve response to 10 kHz (+ ld/3) with their detector amplifier design, and indicate that the reaction time in the detector tube is not a limiting factor.

The optimum sensitivity analysis presented here was used in the design specification of the NCAR/RAF Lyman-alpha 3 aircraft humidiometer. In this instrument (Buck, 1979) the path length is manually set at 0.25, 0.5, 1.0, 1.5, or 2 cm depending on expected flight conditions. This provides optimum sensitivity for conditions ranging from the moist tropical ocean boundary layer to dry, high altitude conditions.

References


Acknowledgments

This work was done at the University of California, San Diego. The author would like to thank Mr. Jon B. Haughdal for devising the frequency response technique.
Figure 1. Normalized sensitivity and received intensity as a function of the ratio of actual path length to optimum path length.
Figure 2. Optimum path length as a function of average absolute humidity.
Figure 3. Optimum sensitivity as a function of average absolute humidity.
Figure 4. Frequency response of commercial and prototype circuit.