Experiments with
Buoyancy-Driven Ocean Circulation

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ABSTRACT

Seven numerical experiments are described, in which both wind and sources of buoyancy drive a quasigeostrophic model of the ocean circulation. The 'thermal' gyres develop upon the potential vorticity field, $Q$, set by the wind-gyres, and the thermal- and wind-gyres interact significantly.

The model is quasigeostrophic with three layers in the vertical and 200 x 200 horizontal resolution, representing a 4000 x 4000 km square, mid-latitude ocean. Free slip and 'hyper'-slip are applied at the vertical coasts, with high-order viscosity in the interior. Typical duration of the experiments is 10 to 20 model years, which allows the average flow fields and energies to equilibrate.

The mean $Q$-field develops 'islands' (that is, maxima and minima) in the vicinity of the buoyancy sources, which contrast the 'plateaus' and 'escarpments' that dominate purely wind-driven simulations. Although the buoyancy-driven circulations have the same qualitative nature as that predicted by steady, nonlinear theory (Rhines, 1985), there are many differences: eddies induce a non-Sverdrup component of the mean depth-integrated circulation; the intensity of the flow is reduced, relative to laminar theory (due to the vertical eddy-transport of the horizontal momentum of the counter-rotating thermal gyres); the pattern of mean vertical velocity is greatly altered with respect to that of steady eddy-free theory; with strong forcing, the flow 'collapses' into meridional inertial boundary currents and zonal jets, rather than following smooth, broad interior currents predicted by Sverdrup theory.
Meridional flow in the interior ocean, far from boundaries, is difficult to achieve in the simple steady theories. With thermal forcing but in absence of wind forcing, meridional flow is confined to boundary currents and regions of direct external forcing. Here either (i) interaction with the wind-driven gyres, or (ii) eddy induction, make possible broad regions of north-south flow.

As diagnostics we show: spin-up histories of both linear and non-linear cases; mean fields after spin-up; time-averaged vorticity balances along selected latitudes and longitudes; time-averaged buoyancy balances; 'Sverdrup' balance diagnostics, in which $f\bar{w}/\partial z$ and $\bar{v}$ are compared; scatter plots of mean $\bar{Q}$ and mean $\bar{\psi}$, which show the transition from weak, non-functional dependence to strong, functional dependence $\bar{Q} = \bar{Q}(\bar{\psi})$, as the strength of the forcing is increased, and energy-histories of the spin-up of the circulations.

Owing to the great strength of the eastern boundary currents in the strongly driven cases, we include one run with enhanced grid resolution near both eastern and western boundaries (a total of 280 gridpoints east-west, rather than 200).

**TACTICAL NOTE**

We have put together this record of a set of experiments to transmit more or less the full experience of the simulations to the reader. Because research is pulse-like rather than steady it may be that investigations in this area will one day cease, and it seems important to record more than can be accommodated by the journals. One day, perhaps, there will be computer archives of this kind of work, in which the results and the model itself will be preserved.
ACKNOWLEDGEMENTS

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INTRODUCTION

Study of thermohaline circulation of the oceans has been hampered by the enormous complexity of the nonlinear, diffusive system. To predict from boundary conditions both the circulation, the associated density perturbations and also the underlying stratification, is a formidable problem. Diffusive adjustment takes thousands of years, even with unrealistically large diffusion constants. The compound nonlinearity includes both horizontal and vertical advection of unknown field variables. Numerical simulations of the Bryan-Cox type use primitive equations, and their long adjustment toward steady state takes so many iterations that spatial resolution is minimal.

In view of this difficulty it seems particularly valuable to examine a 'partial' thermohaline system: the synoptic quasigeostrophic (QG) equations with source terms in the layer-height (continuity) equation. The advantage is that we have longer time-steps than the primitive equation (PE) model, and far more rapid adjustment (decades rather than millenia). Of course we gain this advantage by sacrificing vertical resolution (three layers here rather than, typically, 15 levels in a modern Bryan-Cox simulation), giving up any notion of predicting the basic stratification (the three layers have prescribed density), omitting Kelvin-wave dynamics from the boundary regions, and forbidding the constant-potential density surfaces from rising to contact the sea surface.

Nevertheless, the QG system contains many aspects of the observed thermohaline system. By prescribing interior buoyancy sources we can track the mode waters and thermal gyres of the mid-depth ocean. All of
the lateral advection effects, and a perturbation form of vertical advection, are included in the QG system. It is as if we 'unwarped' the true isopycnal surfaces, and applied sea surface heating and cooling, or evaporation-precipitation forcing, at interior levels rather than at the geometric sea surface. Even more than in the PE simulations, we are uncertain about the physics implied by the forcing terms; the mesoscale deep convection and mixed layer model will one day be constructed, which will describe these source terms as functions of the flow and stability themselves. As we shall see, this is a rich system by itself, and we cannot help but think that it will aid us in understanding the full thermohaline system.

Structure of internal modes of circulation. A key question to keep in mind in idealized studies like this is the nature of large-scale internal modes of circulation. With just $\beta$ and vortex stretching contributing to the potential vorticity, we have a low-order mathematical problem, albeit nonlinear. We might ask the simple question, "can a weakly diffusive ocean interior have meridional flow occurring in broad, baroclinic circulation modes?" The problem with such north-south flows is that they tend to act like baroclinic Rossby waves (which are robustly nonlinear solutions) and hence propagate westward. Nature abhors an east-west density gradient unless it is held in place by Ekman pumping or eastward advection.

We observe, for example in the Atlantic, identifiable water masses with several apparent reversals of meridional flow in the water column. In some instances these are mode waters sweeping into and under the subtropical gyre. In other cases the buoyancy driven modes lie in the
western half of the basin, and travel great distances passing beneath many different regimes of wind-driven flow, and often crossing the equator: this describes the 'parfait' of water masses found in the western Atlantic. The distribution of tracers is so broad in longitude that it is difficult to believe that these are classical western boundary currents.

In accompanying theory (Rhines, 1985) we show that simple few-layer QG models require either a mean barotropic wind-driven circulation or a very special eastward zonal barotropic flow if non-zonal baroclinic circulation is to occur. Below we show (i) how the wind-gyres can set the pathways for these added components of flow, allowing meridional velocities to occur, and (ii) how eddy effects can drive meridional flow in the interior, if the flow is sufficiently inertial.
THE MODEL

The numerical model was constructed by Holland (1978), and the present form is unchanged but for the increased horizontal and vertical resolution. The stratification is set to represent the subtropical oceans with three constant density layers, with 20 km resolution, making the size of the square 200 x 200 grid 4000 km. The increased vertical resolution gives two Rossby radii of deformation, 45km and 25km, corresponding to the two baroclinic Rossby wave modes. Increasing the vertical resolution puts added demands on horizontal resolution, by allowing smaller energetic eddies to occur. The increase of the domain size over that treated by Holland (1978) also has its limits: the uniformity of the basic stratification becomes less realistic, and the amplitude restrictions on the flow become greater (the tilted interfaces would like to rise and break the restriction that their vertical displacement be far smaller than the mean layer thickness).

The boundary conditions are that

\[ \frac{\partial \psi}{\partial s} = 0 = \nabla^2 \psi \]

along the boundary, where \( s \) is a horizontal coordinate parallel to the boundary. The latter condition is the extra one required by the high-order viscosity, \( \nabla^6 \psi \), in the vorticity equation. There is weak bottom friction \( -R \nabla^2 \psi \) in the vorticity equation as well. For the parameter values chosen, the lateral friction is weak compared with the resolved eddy-flux of momentum and potential vorticity.

The interface equations, which play the role of the density equation in the QG system, are
\[ h_L + J(\tilde{\psi}, h) = w + S \]

where \( h \) is the interfacial negative (downward) vertical displacement, proportional to the difference between \( \psi \) above and below, \( w \) is the vertical velocity, \( S \) is the prescribed source "heating" function, and \( \tilde{\psi} \) is the \( H \)-weighted average of the streamfunctions above and below the interface (\( H \) is the mean layer thickness). In the potential vorticity equation the source term thus appears differenced between layers in the vertical. The corresponding form of the buoyancy forcing of the potential vorticity equation for continuous stratification is

\[ f \left( \frac{\rho}{\rho}^{-1} S_z \right) \]

which depends upon the vertical derivative of \( S_z \), the convergence of buoyancy.
THE EXPERIMENTS

The buoyancy forcing is distributed such as to favor the second internal vertical mode. It operates in the eastern portion of the basin, Figs. 1b and lc. The form of the forcing is

\[ S' = A \exp\left(\frac{x-B}{C}\right) \sin Gy \quad \text{(Fig. lc)} \]

\[ = A \exp\left(-\frac{(x-D)^2}{E^2}\right) \sin Gy \quad \text{(Fig. 1b)} \]

and

\[ S = \frac{f_0S'/g'_i}{g'_i} \]

where \( A \) is given in Table 2, \( B = 4000 \) km, \( C = 500 \) km, \( D = 750 \) km, \( E = 250 \) km, \( G = 2\pi/4000 \) km, and \( g'_i \) is the reduced gravity.

It is configured with an effective cooling at the upper (300 m) interface in the northern half of the basin, and a warming immediately below, at the 1000 m interface. Exactly the opposite pattern is applied in the southern half-basin. One can think of these source terms as representing patterns of internal mixing (e.g., mixing in the middle layer producing cooling above heating, tending to spread apart the isopycnal surfaces in the subpolar gyre; conversely, mixing most intense in the top and bottom layers, tending to pinch together the layer interfaces in the subtropical gyre). Alternatively, the cause could be patterns of deep convection, or finally, a crude representation of outcrop forcing, where the actual encounter of the isopycnals with the upper mixed layer is represented by internal sources.\(^1\) At each interface the

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\(^1\) Hendershott (private communication) has recently derived a QG version of the ventilated thermocline model by imposing the outcrop forcing on the interior interfaces; the QG model looks like a staircase with Ekman pumping prescribed on ever deeper interfaces, as one moves poleward.
basin-integrated heating or cooling vanishes, the source patterns being antisymmetric with respect to the middle latitude. The two configurations, Figs. 1b and 1c, are chosen simply to explore the circulation generated by relatively compact source regions, ones which are spatially removed from the center of activity of the wind-gyres. In fact, these eastern regions of buoyancy forcing do indeed have geophysical relevance, as we shall see below.

The wind forcing is the familiar sinusoidally varying wind-stress curl, Fig. 1a. This configuration was chosen such that the S source terms were nonzero outside of the mean wind-gyres found in the second and third layers of the numerical experiments. When S overlaps the closed contour region of the wind gyre, theory suggests some dramatically intense circulation. More realistic configurations of buoyancy forcing will be a goal of succeeding experiments.

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<tr>
<td>29</td>
<td>strong thermal + wind</td>
<td>Fig. 1b</td>
</tr>
<tr>
<td>31S</td>
<td>strong thermal + wind + stretched grid</td>
<td>Fig. 1b</td>
</tr>
</tbody>
</table>
The wind configuration is in all cases the same. The amplitude of the thermal forcing is as follows. Suppose that $S$ were entirely balanced by vertical motion; then the maximum value of the vertical velocity would be:

<table>
<thead>
<tr>
<th></th>
<th>Upper Interface</th>
<th>Lower Interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong cases</td>
<td>$5.2 \times 10^{-6} \text{ m sec}^{-1}$</td>
<td>$1.15 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>(164 m yr$^{-1}$)</td>
<td>(363 m yr$^{-1}$)</td>
</tr>
<tr>
<td>weak cases</td>
<td>$1.0 \times 10^{-6} \text{ m sec}^{-1}$</td>
<td>$2.3 \times 10^{-6} \text{ m sec}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>(33 m yr$^{-1}$)</td>
<td>(72.5 m yr$^{-1}$)</td>
</tr>
</tbody>
</table>

The strong forcing corresponds to a forcing amplitude, $S'$, of $2 \times 10^{-3}$ and weak cases correspond to $S'$ of $4 \times 10^{-4}$ m$^2$sec$^{-2}$. Other parameters are given in Table 2. In fact, the experiments will show that $w$ falls short of this linear density balance (comparisons with 'reality' should be made with the observed $w$ values rather than these linear estimates). Lateral density advection is significant, both in the contributions from mean and from eddy transports.
THERMAL FORCING, ALONE

SPIN-UP; LINEAR CASE (#30). Figure 12.1 shows the switch-on response of the linear problem, with no pre-existing wind-driven circulation. The second internal Rossby mode is dominant, yet one sees a weak first mode propagate more quickly westward. The observed first mode speed is about 4.4 km per day, and the second mode speed 1.1 km per day, in agreement with theory. The steady Sverdrup balance is established by day 3600, with meridional flow in the directly forced regions connected via zonal currents to western boundary currents. The counterrotating gyres have respective transports very close to the theoretically predicted values,

\[
\psi_1 = 2.4 \text{ Sverdrups} \\
\psi_2 = 7.7 \text{ Sverdrups} \\
\psi_3 = 5.3 \text{ Sverdrups}
\]

Here we have used the weaker of the two forcing amplitudes to set the amplitude; with the stronger forcing the linear theory would predict transports just five times these values.

The final time-averaged fields are shown in Fig. 2.7. Note that even at this weak forcing amplitude the linear theory predicts major distortion of the potential vorticity field, which would invalidate the theory. But a nonlinear steady theory (Rhines, 1985) shows very similar behavior for this problem.

SPIN-UP; NONLINEAR CASE, THERMAL ONLY (#22). The corresponding nonlinear, eddy-containing simulation, Figure 12.2, shows the thermal
gyres developing baroclinic instability after just one year of development. \( \partial Q/\partial y \) is negative near the middle latitude, and this is the site of the instability. The waves develop downstream along the eastern boundary and soon fill the thermal gyres. This occurs before the baroclinic Rossby waves have filled out the steady circulation, and established the broad region of meridional flow in the region of non-zero forcing, \( S \). The instability arrests this development. Late in the run, day 7080, the field consists of interior eddies, zonal jets of the type observed by Rhines (1977) and Williams (1978), and intense inertial boundary currents on both eastern and western boundaries. The sense of the thermally driven gyres is the same as in linear theory. Yet, an extra pair of small gyres appears in the central latitude band of \( \phi_1 \), with others near the north and south boundaries. The eddy activity is so strong that the expected \( \bar{w} \)-patterns due to the forcing (essentially \( \bar{w} = -S \)) are not realized. Instead, the developing thermal gyres 'rub' on one another so intensively (via the form-drag associated with baroclinic instability) that \( \bar{w} \) is reversed at the upper interface. For example, in the northern gyre the imposed cooling at the upper interface would normally cause sinking in the region of forcing (Fig. 1c). Yet, this sinking occurs entirely near the eastern boundary, and rising motion occurs elsewhere. The rising is a dynamical response to the strong vertical shear, and tends to oppose it. These events are connected with the collapse of the 'normal' meridional circulation from the broad region of the forcing down to an infinitesimal inertial eastern boundary current.
Accompanying the eddy activity is a strong generation of barotropic motion (also emphasized in our earlier 'process' models). This mode is particularly visible in the lowest layer, $\psi_5$, Fig. 12.3. Notice how quickly the barotropic induced signal reaches westward to the boundary, where short reflected waves appear by day 660 (Fig. 12.3c). This is a non-Sverdrup depth-averaged circulation, whose eventual time average form is shown in Fig. 6b. The induced circulation has rather sharp zonal jets, closed with boundary currents in both east and west. The sense of the barotropic flow is the same as that of $\psi_1$, and its transport amplitude (typically 20 Sverdrups) is greater than that of the upper layer (typically 10 Sverdrups).

During spin-up, the baroclinic field is limited in the speed at which it can extend westward (Fig. 12.4). However baroclinic dipole eddies develop due to the artificial symmetry of the developing flow. These modon-like features propagate westward far faster than linear waves. Round-off error soon breaks this symmetry, however, and by day 840 the modons have themselves destabilized and broken apart. Subsequently the only striking modon events occur at boundaries, where eddies and their images form modon pairs. This boundary propagation is striking, and it strongly affects the time-average flow near the northern and southern boundaries (look at Figs. 2.3a, 2.8, 5).

The mean fields for Case 22 are shown in Fig. 2.4. In Table 1 below we give the amplitude of the observed circulation in each of the layers, and compare it with the linear prediction. Although all of the transport passes through the western boundary currents in the linear case, it does not do so here. The thermal gyres are strongly inertial,
and much of the transport closes meridionally in mid-ocean. Paradoxi-
cally, almost all of the transport does pass through the eastern bound-
ary current.

**INTERACTION BETWEEN THERMAL GYRES AND WIND-GYRES.**

**WIND + WEAK THERMAL FORCING (#25, 28).** The remaining runs were
begun from the purely wind-driven case, #4, with buoyancy driving
switched on after the wind-gyres were in equilibrium. The growth of
basin integrated energies for various runs is shown in Fig. 11. The
mean two-gyre structure of the wind-forced experiment is familiar: the
circulation extends downward through the form-drag supplied by eddies.
There are three principal sites for eddy generation: the region of jet
separation, the westward flowing inertial recirculation a few hundred km
away from the separatrix, and the distant westward flow of the Sverdrup
interior (e.g., in the 'North Equatorial Current'). Two notable
features of the wind-driven case have been described elsewhere (e.g.,
Rhines and Young, 1982; Holland, Keffer and Rhines, 1984). First, the
'hydrographic' picture of the mean circulation, Figs. 2.1g-h, shows
broad gyres while the 'velocity' picture, Figs. 2.1a, c, e, is dominated
by far smaller gyres, in the deep water. This difference parallels the
western North Atlantic flow, where the barotropic mode seems to have far
smaller inertial recirculation than the density field exhibits. Second,
the potential vorticity field is thoroughly mixed by the flow, such that
in layers remote from direct sources of Q, homogenization occurs within
the gyres. This seems a robust result, under a variety of boundary
current regimes. In the top layer, where the wind-curl acts directly,
there are residual variations in Q within the gyres, and a strong
escarpment of $Q$ between subtropical and subpolar gyres; these events are very much parallel to the observed state of the North Pacific (Holland et al., 1984).

In run 28, a weak thermal source-sink pattern is switched on. As the energy history (Fig. 11c) shows, this is a small perturbation to the pre-existing flow. The thermal circulation propagates along the mean potential vorticity field set by the wind gyres. The effect is most visible in the middle layer ($\psi_3$, Fig. 2.2c), where the wind-gyre is weaker than it is above. The potential vorticity shows that the homogenized plateau is actually extended by the thermal forcing (compare Figs. 2.1d, 2.2d), and in addition, weak maxima now appear in $Q_3$. Young (personal communication) and Rhines (1984) discuss the way in which the source terms, $S$, determine the $Q(\psi)$ relation.

**Transport.** The transports are summarized in Table 1. They are close to linearized predictions, yet the vorticity analysis below shows that even at these weak forcing levels there are eddy-induced changes in the circulation.

**Vertical velocity.** The thermal gyres agree qualitatively with steady theory in this case. However, the vertical velocity, $\bar{w}_2$, is reversed from its expected sense at the upper interface. Fig. 9.1a shows a cut across the latitude $j = 60$ (60/200 of the way from the southern boundary toward the northern). Rather than the upward $\bar{w}$ expected from a heating region we find $\bar{w} < 0$ in the vicinity of the forcing. The thermal balance involves eddy heat convergence (Fig. 9.1b), invalidating the linear relation $\bar{w} = -S$. $\bar{w}_4$, the mean vertical velocity at the lower interface (Fig. 9.2a) is downward in the southern
half-basin as expected in a cooling region. Fig. 9.2d is the residual
time dependence after the ten-year averaging (also in this case using
the mirror symmetry with respect to the middle latitude to average
j = 60 with j = 144). The residual is small.

**Vorticity balance.** Fig. 9.4 shows the same result in the averaged
potential vorticity balance. The eddy term $J(\eta_3', \tilde{\psi}_3')$ is not small with
respect to the laminar terms [a steady large-scale nonlinear balance
would have

$$J(\tilde{\psi}_3, \eta_3 + \beta y) = SS_2 - SS_4$$

where $\eta_3 = (f^2_0/H_3)((\psi_1 - \psi_3)/g_2') - (\psi_3 - \psi_5)/g_4'$, and

$SS_1 = f^2_0 S./H_3$.

Although $\bar{w}_2$ is reversed from its expected sense, we nevertheless
have the thermal contribution to the gyre in the upper layer, $\psi_1$, in the
expected (linear) sense. Although the middle layer gyre 'drags' the
upper layer along with it, the upper layer gyre shifts eastward due to
inertial effects, such that the meridional flow near the forcing region
is reversed in sign.

**Sverdrup balance.** Fig. 9.6 shows cuts of time averaged $f_0 \partial \omega/\partial z$,
$-\beta v$, and the linear expectation based on the available forcing, S (the
smooth curves). $\partial \bar{w}/\partial z$ may be identified by its noisy character. This
test is somewhat different from that of the above vorticity balance; $\bar{w}$
may involve nonlinear contributions (convergence of eddy heat flux), yet
$\beta v = f\bar{w}_z$ may still hold. It is the convergence of relative vorticity
flux, $u'\tilde{\gamma}'$, that can upset the 'turbulent' Sverdrup balance.
One sees that much of the flow obeys the Sverdrup balance (vide the mirror-imaging, for example in 9.6d). However the literal agreement with the linear $w$-prediction is close only in the bottom layer (Fig. 9.7a). There it is still noisy. In the middle layer, Fig. 9.6d, the eddy relative vorticity flux terms are clearly active, although the sense of the theory is correct. In the upper layer, Fig. 9.6b, the sense of the thermally driven vertical velocity is to shift eastward the wind gyre in the eastern region (from mid-basin almost to the eastern boundary), thus reversing the meridional flow near the forcing.

If one were setting out to make equivalent vorticity tests at sea, he should note the noise level of the various second-order quantities. Here the $I = 160$ cuts (160/200 of the way from western boundary to eastern) have ten years' averaging, while the $j = 60$ cuts have effectively 20 years' averaging.

Case 25 is another realization of wind plus weak thermal circulation, but with the $\exp(x) \sin y$ source function, Fig. 1c.

**WIND + STRONG THERMAL FORCING (23, 29, 31S).** Runs 23 and 29 have five times the strength of thermal forcing found in runs 25 and 28, or just the same strength as in run 22. Forcing of this magnitude may occur in deep convection regions like the northeast Atlantic or the 18° water formation region of the Sargasso Sea. As we shall see, it is an extremely strong driving effect. The following important point about scaling emerges: the weaker the basic stratification, the stronger will be the mechanical effect of a given heat flux to the atmosphere. Under the current configuration of the model, the density jumps between layers are large, and correspond to the subtropical gyre. If we reinterpret
the model in terms of the subpolar gyre, the same heat flux to the atmosphere will yield far larger w-velocities and far stronger circulation. Thus, although the magnitude of $S$ for runs 22, 23 and 29 are excessive for realistic subtropical situations, they are not so in subpolar latitudes.

This run, with the same thermal-forcing amplitude as run 22, discussed above, is still farther from linear theory. The five-times stronger forcing yields transports that are about 4.3 times greater (as measured by $\psi_3$): see Table 1. The thermal gyres are clearly visible in all layers, and their eastward shift has become extreme. Comparing Figs. 2.3c and 2.3d, we see that $\psi_3$ and $Q_3$ have a very similar appearance. They develop an inertial nature; rather than finding meridional flow only in the vicinity of external forcing, $S$, we find great gyres that reshape the $Q$-field, creating meridional pathways in a much larger domain. The circulations tend to close in eastern boundary currents, while only a fraction of their transport reaches the western boundary (just opposite to linear predictions! Table 1).

There is a threshold amplitude that must be exceeded for this kind of inertial gyre to exist. In order that vortex stretching compensate $\beta$ across the span of latitudes of the gyre, we need $U > O(\beta L^2_\rho)$, where $U$ is the flow speed and $L_\rho$ the Rossby Radius.

The potential vorticity fields for weak and strong thermal forcing (+wind) are shown in perspective in Fig. 4. The growth of the 'thermal islands' in $Q$ contrasts the wind-driven plateau lying to the west.

Vorticity balance. As in earlier runs we find strong eddy form drag working between the counter rotating gyres, reducing their net
transport. Meanwhile, inertial effects shift the gyres eastward, leaving a turbulent Sverdrup balance in the mean (Fig. 10.6), yet one in which the external forcing (and the linearized balance) is barely visible. Only in the lowest layer (layer 5) is the balance somewhat more like linear theory (Fig. 10.7a-c).

THE $\Theta$-SPIRAL. We apply Bryden's (1975) derivation of the relation between the veering of horizontal current with depth and the lateral density advection (essentially the result of taking $uX$ the time-mean thermal-wind equation); the overbar is a time-average. Stommel (1979) has noted, at station Juliette in the northeast Atlantic, the tendency for velocity spirals to develop where there is intensive cooling. The relation is

$$
\left(\frac{f_0}{g}\right) \left|\overline{u} \right|^2 \Phi \Bigg|_z = \overline{u^*} \nabla_h z = - (\overline{w_\rho} + \nabla_h \overline{u^*_h \rho'}) + S .
$$

where the subscript $(_h)$ denotes horizontal components. $\Phi$ is the bearing of the horizontal current with respect to East. The spiraling $\Phi_z$ is due to a mixture of heating/cooling and vertical advection of density, and lateral eddy flux of density [mixing terms and external forcing are subsumed in $S$]. Thus one sees $\beta$-spirals ($w$-induced) in adiabatic regions, and "cooling spirals" or "$\Theta$-spirals" where $S$ or eddy heat flux dominates the RHS of the above relation. As we remarked earlier, the linear buoyancy balance $S = \overline{w_\rho}$, is not accurate in any but the linearized experiments. The term $\overline{w^*_\rho}$ has been left off the RHS on the basis of geostrophic scaling.
In a steady flow the sense of the result is that vertical variation of the flow orientation $\Phi$ requires thermal wind velocity normal to $y$, and hence a tilt in the isopycnal surfaces along the direction of $y$. Steady flow in this direction must thus either rise or sink, or alternatively be heated or cooled sufficiently to allow $w$ to vanish. The eddy contribution to $\bar{w}$ may be significant. In dynamical terms it corresponds to the form drag of one layer on the next, which is found below (vorticity balance) to be strong. In the heat equation it represents the lateral eddy heat and mass-flux occurring when there is systematic correlation between vertical displacement and horizontal velocity.

In experiment 22 (thermal driving only), Fig. 2.4, one can see the $\theta$-spiral in the eastern half of the basin. Compare the $\Phi_1 - \Phi_3$-fields. In the southeast quadrant, say just west of the gyre center in the upper layer, the mean velocity is northward in the upper layer and northeastward in the layer below. This veering is consistent with interface contours lying parallel with the vector difference of the two velocities, or a downslope to the north (Fig. 2.4g). In turn, this orientation of the interface would match a downward directed $\bar{w}$ in a steady balance. But here the contribution of eddy-heat flux is also 'downward', while $S$, the external buoyancy forcing is a warming, hence itself driving a rising motion. The veering with depth leads to a north-south displacement between the gyre centers in the respective layers. This 'phase shift' with depth contrasts steady theory, in which the gyre centers lie one above another.

In experiment 29 (wind + strong thermal forcing) we can make this graphical study looking along latitude $J = 60$ ($J$ is the $y$-gridpoint), in
Figs. 2.3a,c,g. The sense of veering is clockwise-downward, to the west of the southeastern thermal gyre center, and counterclockwise-downward to its east. Sketching the vector velocity difference, this would correspond to a downward \( \vec{w} \) to the left of the gyre center, and upward \( \vec{w} \) to the right. Now look at the actual mean balance of the interface equation, Figs. 10.1, 10.2. To the accuracy of the mean thermal wind equation, the veering corresponds to \( \bar{u}_h \cdot \bar{v}_h = J(\bar{h}, \bar{\psi}) \), which as expected is positive (west of gyre center) and negative (east of gyre center). Now, how does the partition of this mean advection work out among \( \vec{w}, S \), and eddy flux (note Fig. 10.1c = Fig. 10.1a - Fig. 10.1b - \( S + \) residual time dependence)? We see downward \( \vec{w}_2 \) along \( J = 60 \), everywhere except in the boundary currents. The eddy heat- or thickness flux, \( \bar{v} \cdot \bar{u}' h' = -J(\bar{h}', \bar{\psi}') \) is dominantly positive in the vicinity of the thermal gyre, and there it is about twice as large as \( \vec{w} \). Thus with this strong thermal driving, the thermal balance in the core of the gyre (which is heated at the upper interface and cooled at the lower interface) is roughly

\[
\downarrow \text{\( \beta \)-spiral} \downarrow \\
\bar{u} \cdot \bar{v}_h + \bar{v} \cdot \bar{u}' h' + \vec{w} = S \\
\downarrow \text{\( \theta \)-spiral} \downarrow
\]

\( I = 170, J = 60: [-3 +8 -2 = 3] \times 10^{-6} \)

at the upper interface, where the first term alone relates to the veering. At this point the mean advection is changing sign and hence is small. The balance is dominated not by the 'thermal' effect of \( S \), but by the eddy heat flux that follows as a consequence of the mechanical
effect of $S$. The Eulerian mean $\overline{w}_2$ is reversed in sign from the expected result (that heating leads to rising motion). The expected rising motion is pinched into the intense eastern boundary current.

Now consider a much simpler locale, at the middle longitude of the section $J = 60$ (same figures). There $S$ vanishes, and eddy form drag is small. The dominant balance is

$I = 100, J = 60: [2 + 0 - 2 = 0] \times 10^{-6}$.

which is simple flow along geostrophic contours; mean advection and vertical velocity. Still farther west, we enter the more active parts of the wind-driven gyre, and find

$I = 35, J = 60: [1.5 - 2.5 + 1 = 0] \times 10^{-6}$.

Here the eddy form drag is active, and is balanced by $\overline{w}$ and, more strongly, by mean advection. The corresponding effects may be seen in the vorticity balances as well.

$Q-\psi$ scatter plots. The inertial character of the flow suggests looking for a functional relation between the mean $\overline{Q}$ and $\overline{\psi}$ fields. The plots, Fig. 7, are given both for the entire basin, at a given depth, and also for selected latitudes. In a sense these sum up the mean dynamics of the model. In the wind + weak thermal case (#28, Fig. 7.3) the points form a cloud, which represents a broadening of the figure for the purely wind-driven case (Fig. 7.1). In the middle layer, for example, the linear theory would have $Q$ increasing systematically with latitude, and the motionally induced perturbations riding on top of that change (Fig. 7.3e,f,g,h). Note how constrained these regression plots are in the neighboring problem of a model with three equal-depth layers; there, we have in the middle layer
\[ Q_3 = F(\psi_1 - 2\psi_3 + \psi_5) + \beta y \]
\[ = F(\psi_B - 3\psi_3) + \beta y \quad (\psi_B = \Sigma \psi_i) . \]

This says that a given latitude,

\[ \frac{\partial Q_3}{\partial \psi_3} = -3F + \frac{F\partial \psi_B}{B\psi_3} . \]

If \( \psi_B \) is small, this suggests a constant slope, with mean ordinate \( \beta y \) just as we find in Fig. 7.3j,k,l and 7.4j,k,l.

In the wind + strong thermal case, Fig. 7.4, by contrast, \( Q_3 \) and \( \psi_3 \) develop much more of a simple functional relation. The great thermal gyres lie on a fuzzy line of negative slope; the rectangular cross of the pure-wind case, Fig. 7.1e, bends over to the left. The cuts at specific latitudes, Figs. 7.4f,g,h show, (a) that the same functional relation holds over the entire thermal gyre, and (b) that at a given latitude there is relatively small hysteresis. That is

\[ \int Q d\psi = \int Q \left( \frac{\partial \psi}{\partial x} \right) dx = \int Q v \ dx \]

is small. The northward- and southward-flowing circulation fall nearly upon the same \( Q(\psi) \), suggesting relatively little dissipation of \( Q \) as fluid proceeds about the circuit.

There is something paradoxical about this, for it would suggest that the local vorticity balance should nearly be a balance between mean-flow stretching, \( J(\psi', \eta) \) and \( \beta v \). But we have seen above that the eddy form drag, \( J(\psi', \eta') \) is at least as large a term. It appears that the external forcing, \( S \), is nearly balanced by form drag while the mean advective balance is separately obeyed (as shows so clearly in the \( Q-\psi \) scatter plots).
**Transport comparisons.** Figs. 3 show comparative cuts across the mean fields of many of the experiments. In Fig. 3.1 we see the clear growth of the thermal gyres as the intensity of S is increased. Their eastward shift and formation of eastern boundary currents with stronger forcing is evident. The north-south cuts (Figs. 3.2, 3.4) show in addition the development of concentrated east-west jets, and boundary currents at the northern and southern boundaries, compared with the sinusoidal y-dependence of the forcing and linear response.

**Boundary currents.** Because of the intensity and importance of the novel eastern boundary currents in these experiments, we made a three-year run with coordinate stretching on both eastern and western boundaries. The formulation of the model is due to Haidvogel, who kindly made it available to us. There is a total of 280 points from west to east, with exponential stretching; the grid interval is as small as 1 km near the boundary. No-slip boundary conditions were applied, with Navier-Stokes lateral friction rather than hyperviscosity. Otherwise the parameters were identical to run 29, which served as initial conditions for the simulation. While three years is not time enough for full adjustment, the eastern boundary region seems to have settled down during this interval. The fully resolved boundary currents are visible in the transects, Figs. 3.1, 3.2. The time-mean fields are not greatly changed from those of run 29 (Fig. 2.8).

The advantage of the extra resolution shows particularly clearly in the Q-Ψ scatter plots, Fig. 7.5. There are now many points in the boundary layers, which show up as being relatively widely spaced in the
figures. There is some extra hysteresis (enclosed area) due to dissipation in the boundary currents. Some of this change, relative to Fig. 7.4, case 29, is probably due to the presence of Navier-Stokes friction.

We note in passing that the density height-fields set up by the thermal gyres result in a pattern of layer thickness which is propitious to eastern boundary current development. For example, in the subtropical gyre the thickness pattern \(- (h_2 - h_4)\) (the \(h\)-fields are negative interface deflections, Figs. 2.8g-h) represents a pinching together of the middle layer in the eastern subtropical gyre. Now, the cyclonic circulation there enters the boundary current, where this thickness increases. The attendant vortex-stretching is of the correct sign to produce strong positive relative vorticity, \(\zeta\). This is just the sense of \(\zeta\) required by the northward-flowing boundary current. Thus the interfacial slopes play the role of \(\beta\) in one-layer inertial gyres, and in doing so the classical rules (for instance that the interior zonal flow must be westward in order to match with inertial boundary layers) are broken. It appears that some of the same stretching dynamics is working in the western boundary currents. We are led to wonder whether there is not a simple stratified inertial boundary current theory that has, as yet, eluded the theoreticians.

Deep circulation. The lowest layer (spanning 1000 m to 5000 m depth) has direct thermal driving, and also eddy form drag acting from above. The dynamics of this layer appear to be quite inertial, in the strongly driven cases (e.g., Fig. 2.3e). It appears, however, that this layer does not feed back dynamically on the layers above. Comparing Figs. 3.1c and 3.1h or Figs. 2.3c and 2.3h we see that the mean
streamfunction in the middle layer is nearly identical to the lower interface profile (inverted). This suggests that very little vortex stretching is provided by the mean lower interface. With $\psi_3 = h_4$, the model is responding as a 2 1/2 layer model in which the lower interface acts as a free surface, and its height contours are streamlines (just as in the classic deep-lower-layer-at-rest-models). The lowest layer is driven strongly from above, and perhaps participates weakly in the baroclinic instability process, but otherwise might be neglected in calculating the flows overhead. This might lead to considerable savings of computing time in future 'stripped down' explorations of the thermal gyres.
NOTES ON OBSERVATIONS

In subtropical regions we note that beneath the levels of intense wind-gyre activity the mid-depth circulation proceeds along seemingly independent paths. Maps of tracers and potential vorticity reveal these flows, and we suggest that in many cases they involve buoyancy forcing and response that senses the wind gyres overhead. Examples involving the Mediterranean water and various mode waters will be discussed in the published version of this work.

In subpolar regions the potential vorticity may on occasion take the form of the 'thermal islands' found here. In the neighborhood of $\sigma_0 = 26.5$ to 27.0 in the northern North Pacific, there is a halocline driven by precipitation at the surface. This buoyancy driven feature appears as a maximum in Q-maps, and may be traced as it sweeps into the subtropical gyre to the south (Holland et al, 1984).

In the subpolar North Atlantic, the intense cooling may act to bring the Sverdrup transport upward to a shallower level (as linear theory would predict). But, in addition the eddy intensity observed there (say, at 55°N) is fully strong enough to produce the level of non-Sverdrup induction found in our largest-amplitude simulations. These simulations show increased transport relative to the Sverdrup prediction at higher latitudes, and decreased transport at lower latitudes within the subpolar gyres.

In the North Pacific we note that some studies of synoptic hydrography show the circulation to form rather sharp eastern boundary currents on occasion (see the maps of Tabata, 1982). These features occur in the model only with higher amplitude buoyancy forcing, and they form or die quickly, under changes in the amplitude of the driving. In view
of the eddy activity in observed eastern boundary currents (e.g., Ikeda and Emery, 1984), we wonder whether the inertial sharpening observed here may not be common. Even though local forcing is known to be an important component of those currents, their occurrence with distant forcing suggests that they may in part be features of the basin-scale circulation.
CONCLUSION

Mesoscale eddies have played a strong role in shaping these mixed thermal- and wind-driven flows. Although some years ago eddies were associated principally with western boundary currents, it is now clear that there are many important sources of eddies elsewhere in the oceans. If one looks in regions of strong buoyancy forcing like the subpolar gyre of the North Atlantic, hydrographic sections (Worthington and Wright, 1970), show eddies of perhaps 400 m vertical displacement amplitude to be commonplace. Examining Dickson's (1982) survey of current meter data, one sees weak eddy kinetic energy in the eastern North Atlantic at 30°N, but the intensity rises abruptly north of 50°N to levels in excess of those in the (vigorous) Local Dynamics Experiment of Polymode (32°N, 70°W). At 59°N, 11°40'W, time-averaged eddy energies of 180 cm² sec⁻² occur (0-800 m depth) compared with an average of about 110 cm² sec⁻² in the LDE.

For comparison, the present model simulations have about 100 cm² sec⁻² average (0-1000 m depth) eddy kinetic energy in the eastern region of the strongly forced runs (not including the intense boundary current). The isopycnal height field perturbations at 300 m depth are typically 250 m, peak to trough. In the weakly forced runs (#25, #28) the eastern eddy k.e. is roughly 50 cm² sec⁻¹ (0-1000 m depth) while the height-field perturbations are typically 50 to 100 m. One has the impression that the model, when strongly forced, is producing eddy activity comparable with that observed in the subpolar gyres.
REFERENCES


TABLE 1. Transports in the thermal gyres, in Sverdrups, \(10^6 \text{ m}^3 \text{ sec}^{-1}\), velocities in m sec\(^{-1}\). The sign indicates that the gyres reverse in sense from one layer to the next. The three layers are labeled 1, 3, and 5.

\begin{tabular}{|c|c|c|c|c|}
\hline
Layer & Maximum Transport & Transport Reaching W.B.L. & Velocity (interior) & Velocity (jets) \\
\hline
1 & 12.5 & 12.5 & & \\
3 & -38.5 & -38.5 & & \\
5 & 26.5 & 26.5 & & \\
\hline
\end{tabular}

#30 (Linearized thermal forcing) (amplitude set to correspond with the strong thermal case, #22)

\begin{tabular}{|c|c|c|c|c|}
\hline
Layer & Maximum Transport & Transport Reaching W.B.L. & Velocity (interior) & Velocity (jets) \\
\hline
1 & 13.5 & 3 & .015 & .06 \\
3 & -24.5 & -14 & -.06 & -.08 \\
5 & 25.5 & 20 & .005 & .01 \\
\hline
\end{tabular}

#22 (Strong thermal forcing)

\begin{tabular}{|c|c|c|c|c|}
\hline
Layer & Maximum Transport & Transport Reaching W.B.L. & Velocity (interior) & Velocity (jets) \\
\hline
1 & 2.5 & 2.5 & & \\
3 & -6 & 0 & & \\
5 & 5 & 0 & & \\
\hline
\end{tabular}

#28 (Wind + weak thermal)

\begin{tabular}{|c|c|c|c|c|}
\hline
Layer & Maximum Transport & Transport Reaching W.B.L. & Velocity (interior) & Velocity (jets) \\
\hline
1 & 18.9 & 15 & & \\
3 & -25.9 & -8.5 & & \\
5 & 24 & 17 & & \\
\hline
\end{tabular}

#29 (Wind + strong thermal)
<table>
<thead>
<tr>
<th>Experiment</th>
<th>4</th>
<th>22</th>
<th>23</th>
<th>25</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>wind</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(pascal m⁻²)</td>
<td>1x10⁻⁴</td>
<td>0</td>
<td>1x10⁻⁴</td>
<td>1x10⁻⁴</td>
<td>1x10⁻⁴</td>
<td>1x10⁻⁴</td>
<td>0</td>
<td>1x10⁻⁴</td>
</tr>
<tr>
<td>thermal forcing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x10⁻⁴ m² sec⁻²)</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>4</td>
<td>4</td>
<td>20</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>Navier-Stokes friction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m² sec⁻¹)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>biharmonic friction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m² sec⁻¹)</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
<td>8x10⁹</td>
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</tr>
<tr>
<td>bottom friction</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(sec⁻¹)</td>
<td>1x10⁻⁷</td>
<td>1x10⁻⁷</td>
<td>1x10⁻⁷</td>
<td>1x10⁻⁷</td>
<td>1x10⁻⁷</td>
<td>1x10⁻⁷</td>
<td>0</td>
<td>1x10⁻⁷</td>
</tr>
<tr>
<td>timesteps per day</td>
<td>15</td>
<td>6</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>4</td>
<td>48</td>
</tr>
<tr>
<td>length of run (days)</td>
<td>3600</td>
<td>7200</td>
<td>2520</td>
<td>2340</td>
<td>6480</td>
<td>4680</td>
<td>5040</td>
<td>1080</td>
</tr>
<tr>
<td>averaging time (years)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>initial conditions</td>
<td>rest</td>
<td>run 4</td>
<td>run 23</td>
<td>run 4</td>
<td>run 28</td>
<td>rest</td>
<td>run 29</td>
<td></td>
</tr>
<tr>
<td>Experiment</td>
<td>4</td>
<td>22</td>
<td>23</td>
<td>25</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>------------</td>
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<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>$P_2$ (mean PE of upper interface, m sec$^{-2}$)</td>
<td>280</td>
<td>113</td>
<td>300</td>
<td>260</td>
<td>260</td>
<td>330</td>
<td>18</td>
<td>330</td>
</tr>
<tr>
<td>$P_4$ (mean PE of lower interface, m sec$^{-2}$)</td>
<td>40</td>
<td>140</td>
<td>100</td>
<td>50</td>
<td>30</td>
<td>90</td>
<td>17</td>
<td>70</td>
</tr>
<tr>
<td>$K$ (mean total KE m sec$^{-2}$)</td>
<td>100</td>
<td>64</td>
<td>140</td>
<td>90</td>
<td>95</td>
<td>135</td>
<td>1.4</td>
<td>63</td>
</tr>
<tr>
<td>$\phi_1$-max (max transport of wind-gyre) (x10$^3$ m$^3$ sec$^{-1}$ = 1 Sverdrup)</td>
<td>46</td>
<td>0</td>
<td>30</td>
<td>35</td>
<td>45</td>
<td>32</td>
<td>0</td>
<td>26</td>
</tr>
<tr>
<td>$\psi_1$-max (thermal gyre only)</td>
<td>0</td>
<td>14</td>
<td>17</td>
<td>2</td>
<td>2</td>
<td>18</td>
<td>2.4</td>
<td>20</td>
</tr>
<tr>
<td>$\psi_3$-max (as above, for wind-gyre of middle layer)</td>
<td>39</td>
<td>0</td>
<td>21</td>
<td>29</td>
<td>36</td>
<td>26</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>Experiment</td>
<td>4</td>
<td>22</td>
<td>23</td>
<td>25</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>------------</td>
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<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>
| $\psi_3$-max  
(thermal gyre only) | 0   | 25  | 27  | 14  | 5.8 | 26  | 7.7 | 31  |
| $\psi_5$-max  
(as above, for wind-gyre of lower layer) | 80  | 0   | 30  | 65  | 72  | 64  | 0   | 20  |
| $\psi_5$-max  
(thermal gyre only) | 0   | 26  | 24  | 8   | 5.5 | 24  | 5   | 32  |
1a Wind stress curl (abscissa = y (km))

1b Pattern of θ forcing for runs 28, 29, 31S

1c Pattern of θ forcing for runs 22, 23, 25, 30
Contour intervals $\Psi_1, 10^4$, $Q_1, 1 \times 10^{-5}$, $\Psi_3, 7 \times 10^3$, $Q_3, 5 \times 10^{-6}$ (run 4)
Contour intervals \( 65, 2 \times 10^3; \) \( Q_5, 5 \times 10^{-6}; \) \( h_2, 30; \) \( h_u, 20 \) (run 4)
2.2 a-d  Contour intervals $\Psi_1$, $10^4$; $Q_1$, $1 \times 10^{-5}$; $\Psi_3$, $6 \times 10^3$; $Q_3$, $5 \times 10^{-6}$ (run 28)
2.2 e-h  Contour intervals \( v_5, 2 \times 10^3 \); \( q_5, 5 \times 10^{-6} \); \( h_2, 30 \); \( h_4, 20 \) (run 28)
2.3 a-d Contour intervals $V_1, 10^4$; $Q_1, 1 \times 10^{-5}$; $V_3, 4 \times 10^3$; $Q_3, 5 \times 10^{-6}$ (run 29)
2.3 e-h

Contour intervals \( \Psi_5, 10^3; Q_5, 5 \times 10^{-6}; h_2, 20; h_4, 20 \) (run 29)
Contour intervals $y_1, 5 \times 10^3; \quad Q_1, 9 \times 10^{-5}; \quad y_3, 4 \times 10^3; \quad Q_3, 5 \times 10^{-6}$ (run 22)
Contour intervals $v_5, 8 \times 10^2; \quad Q_5, 5 \times 10^{-6}; \quad h_2, 10; \quad h_4, 20$ (run 22)
Contour intervals $\Psi_1, 10^4$; $Q_1, 1 \times 10^{-5}$; $\Psi_3, 4 \times 10^3$; $Q_3, 5 \times 10^{-6}$ (run 23)
Contour intervals \( y_5, 10^3; \) \( Q_5, 5 \times 10^{-6}; \) \( h_2, 20; \) \( h_4, 20 \) (run 23)
Contour intervals \( \psi_1, 10^4 \); \( \phi_1, 1 \times 10^{-5} \); \( \psi_3, 5 \times 10^3 \); \( \phi_3, 5 \times 10^{-6} \) (run 25)
2.6 e-h

Contour intervals \( \Psi_s, 2 \times 10^3 \); \( Q_s, 5 \times 10^{-5} \); \( h_2, 20 \);
\( h_u, 10 \) (run 25)
2.7 a-d  
Contour intervals $\Psi_1$, $10^3$; $Q_1$, $9 \times 10^{-6}$; $\Psi_3$, $10^3$; $Q_3$, $5 \times 10^{-6}$ (run 30)
Contour intervals $v_5$, $2 \times 10^2$; $Q$, $5 \times 10^{-6}$; $h_2$, 8; $h_4$, 10 (run 30)
2.8 a-d  Contour intervals $\psi_1$, $10^4$; $Q_1$, $1 \times 10^{-5}$; $\psi_3$, $5 \times 10^3$; $Q_3$, $5 \times 10^{-6}$; (run 31S)
Contour intervals $T_5$, 10; $Q_5$, $5 \times 10^{-6}$; $_{h2}$, 20; $_{h4}$, 30 (run 31S)
3.1 a-d solid, run 4; long dash, run 28; dot, run 29; short dash, run 31S; \( \psi_1, Q_1, \psi_3, Q_3 \), \( j=52 \)
3.1 e-h solid, run 4; long dash, run 28; dot, run 29; short dash, run 31S; \( \Psi_5 \), \( Q_5 \), \( h_2 \), \( h_3 \), \( J=52 \)
3.1 i-j

solid, run 4; long dash, run 28; dot, run 29;
short dash, run 31S; \( W_2, W_4, J=52 \)
3.2 a-d  solid, run 4; long dash, run 28; dot, run 29; short dash, run 31S; $\Psi_1$, $Q_1$, $\Psi_3$, $Q_3$, I=160
3.2 e-h  solid, run 4; long dash, run 28; dot, run 29; short dash, run 31S; $\Psi_5$, $Q_5$, $h_2$, $h_4$, $i=160$
3.2 i-j  
*solid*, run 4; *long dash*, run 28; *dot*, run 29; 
*short dash*, run 31S; \( W_2 \), \( W_4 \), \( I=160 \)
3.3 a-d solid, run 4; long dash, run 22; dot, run 23; short dash, run 25; $\Psi_1$, $Q_1$, $\Psi_3$, $Q_3$, J=52
3.3 e-h  solid, run 4; long dash, run 22; dot, run 23; short dash, run 25; $\psi_5$, $Q_5$, $h_2$, $h_4$, $J=52$
3.3 i-j solid, run 4; long dash, run 22; dot, run 23; short dash, run 25; $W_2$, $W_4$, J=52
3.4 a-d solid, run 4; long dash, run 22; dot, run 23; short dash, run 25; $\Psi_1$, $Q_1$, $\Psi_3$, $Q_3$, I=160
3.4 e-h  solid, run 4; long dash, run 22; dot, run 23;
short dash, run 25; $\Psi_5$, $Q_5$, $h_2$, $h_4$, $I=160$
CASE 4  22 23 25 W-2  I=160

3.4 i-j solid, run 4; long dash, run 22; dot, run 23;
short dash, run 25;  W_2, W_4, I=160
3.5 a-c Run 30, $\Psi_1$, $\Psi_3$, $\Psi_5$, $J=52$
3.5 d-f Run 30, $\bar{V}_1$, $\bar{V}_3$, $\bar{V}_5$, I=160
3.5 g-h  Run 30, \( \bar{h_2}, \bar{h_4}, \) I=160
4. a-b  
Run 28, $Q_3$; Run 29, $Q_3$
5. a-d  Run 31S, instantaneous $\Psi_1$, CI=$10^4$, day 1060, 1080; $\Psi_3$, CI=$9 \times 10^5$, day 1060, 1080
5. e-h  
Run 31S, instantaneous $y_5$, CI=$4 \times 10^3$, day 1060, 1080; 
h_2, CI=$10^3$, day 1060;  
h_4, CI=$8 \times 10^3$, day 1060
6. a-d  Barotropic $\psi$, run 4, CI=2 x $10^7$; run 22, CI=0.6 x $10^7$; run 28, CI=1 x $10^7$; run 29, CI=1 x $10^7$
WHOLE- I QG CASE 4

Q - T scatter plot, run 4, 0 < Q < 2 \times 10^{-4},
-1.5 \times 10^5 < \psi < 1.5 \times 10^5; layer 1, J=40, J=60, J=80
7.1 e-h $\bar{Q} - \bar{\psi}$ scatter plot, run 4, $5 \times 10^{-5} < Q < 1.5 \times 10^{-4}$, $-4.5 \times 10^{4} < \psi < 4.5 \times 10^{4}$; layer 3, J=40, J=60, J=80
7.1 i-1 \( \bar{Q} - \bar{\Psi} \) scatter plot, run 4, \( 5 \times 10^{-5} < \bar{Q} < 1.5 \times 10^{-4} \), \(-2 \times 10^{0} < \bar{\Psi} < 2 \times 10^{4}\); layer 5, \( J=40 \), \( J=60 \), \( J=80 \).
7.2 a-d $\bar{Q} - \bar{\Psi}$ scatter plot, run 22, $6 \times 10^{-6} < Q < 1.8 \times 10^{-4}$, $-5 \times 10^{5} < \Psi < 5 \times 10^{5}$; layer 1, $J=40$, $J=60$, $J=80$
$Q - \Psi$ scatter plot, run 22, $5 \times 10^{-5} < Q < 1.5 \times 10^{-4}$, $-4 \times 10^{-4} < \Psi < 4 \times 10^{-4}$; layer 3, J=40, J=60, J=80
WHILE 6 GGE CASE 22

LAYER— 5 CASE 22 J= 40

Q - scatter plot, run 22, 5 x 10^{-5} < Q < 1.5 x 10^{-4},
-7 x 10^{3} < \psi < 7 x 10^{3}; layer 5, J=40, J=60, J=80
7.3 a-d \( \bar{Q} - \bar{\psi} \) scatter plot, run 28, \( 0 < Q < 1.8 \times 10^{-4} \), 
\( -2 \times 10^5 < \psi < 2 \times 10^5 \); layer 1, \( J=40, J=60, J=80 \)
7.3 e-h \( Q - \bar{\Psi} \) scatter plot, run 28, \( 5 \times 10^{-5} < Q < 1.5 \times 10^{-4}, \)
\(-5 \times 10^6 < \Psi < 5 \times 10^6 \); layer 3, \( J=40, J=60, J=80 \)
7.3 i-1 

$Q - \overline{\Psi}$ scatter plot, run 28, $5 \times 10^{-5} < Q < 1.5 \times 10^{-4}$, $-2 \times 10^4 < \Psi < 2 \times 10^4$; layer 5, $J=40$, $J=60$, $J=80$
7.3 e-h \[ \bar{Q} - \bar{\psi} \] scatter plot, run 28, \( 5 \times 10^{-5} < Q < 1.5 \times 10^{-4} \), \(-5 \times 10^{-6} < \psi < 5 \times 10^{-4} \); layer 3, \( J=40, J=60, J=80 \)
7.3 i-1 \( \bar{Q} - \bar{\Psi} \) scatter plot, run 28, \( 5 \times 10^{-5} < Q < 1.5 \times 10^{-4} \), \( -2 \times 10^{-6} < \Psi < 2 \times 10^{6} \); layer 5, \( J=40 \), \( J=60 \), \( J=80 \)
7.4 a-d  \( Q - \Psi \) scatter plot, run 29, \( 0 < Q < 2 \times 10^{-4}, -1.5 \times 10^5 < \Psi < 1.5 \times 10^5 \); layer 1, \( J=40, J=60, J=80 \)
\[ Q - \Psi \text{ scatter plot, run 29, } 5 \times 10^{-5} < Q < 1.5 \times 10^{-4}, \]
\[ -4.5 \times 10^4 < \Psi < 4.5 \times 10^4; \] layer 3, $J=40$, $J=60$, $J=80$
7.4 i-1
\( \bar{Q} - \bar{W} \) scatter plot, run 29, \( 5 \times 10^{-5} < Q < 1.5 \times 10^{-4} \),
\( -2 \times 10^4 < \psi < 2 \times 10^4 \); layer 5, \( J=40, J=60, J=80 \)
7.5 a-d $\bar{Q} - \bar{\Psi}$ scatter plot, run 31S, $0 < Q < 2 \times 10^{-4}$, $-9 \times 10^4 < \Psi < 9 \times 10^4$; layer 1, $J=40$, $J=60$, $J=80$
7.5 e-h $\bar{Q} - \bar{\psi}$ scatter plot, run 31S, $5 \times 10^{-5} < Q < 1.5 \times 10^{-4}$, $-5 \times 10^4 < \psi < 5 \times 10^4$; layer 3, J=40, J=60, J=80
$Q - \bar{V}$ scatter plot, run 31S, $5 \times 10^{-5} < Q < 1.5 \times 10^{-4}$, $-9 \times 10^{3} < \bar{V} < 9 \times 10^{7}$; layer 5, $J=40, J=60, J=80$
8.1 Run 29 (a) $\overline{Q_u}$ west half basin, max vector = $2.2 \times 10^{-5}$
(b) $\overline{Q_u}$ east half basin, max vector = $0.29 \times 10^{-5}$;
(c) $\overline{Q_{u_t}}$ west half basin, max vector = $1.9 \times 10^{-7}$;
(d) $\overline{Q_{u_s}}$ east half basin, max vector = $0.94 \times 10^{-7}$
8.2 Run 29, (a) $\mathbf{\nabla} \cdot \mathbf{u}_3$ east half basin, max vector = $8.7 \times 10^{-7}$;
(b) $\mathbf{u}_3$ east half basin, max vector = $7 \times 10^{-7}$
9.1 a-d Run 28. density balance, $\bar{w}_2$, $\bar{J}(h'_2, \psi'_2)$, $J(h_2, \psi_2)$, $\partial h_2/\partial t$, $J=60$
9.2 a-d Run 28, density balance, $\bar{W}_4, J(h_4', \psi_4'), J(h_4, \bar{v}_4), \partial h_4 / \partial t, J=60$
9.3 a-c Run 28, vorticity balance, $\beta \overline{v}$, $\overline{J(\eta_1, \Psi_1)}$, $\overline{J(\eta_1, \Psi_1)}$, $J=60$
9.4 a-c  Run 28, vorticity balance, $\beta \overline{\psi_3}$, $\overline{\mathcal{J}(\eta_3, \psi_3)}$, $\mathcal{J}(\overline{\eta_3}, \overline{\psi_3})$, $J=60$
Run 28, vorticity balance, $\beta \psi_5$, $\overline{J(\eta_5^*, \psi_5^*)}$, $J(\eta_5, \psi_5)$, J=60

9.5 a-c
CASE 28 DWDZ-1

CASE 28 DWDZ-3

Run 28, $-\beta v$, $f w_{z'} (\beta v)$, layer 1, $I=160$; layer 1, $J=60$; layer 3, $I=160$; layer 3, $J=60$
9.7 a-b  Run 28, $-\beta v$, $fw_z$, $(\beta v)_\text{linear}$, layer 5, $I=160$; layer 5, $J=60$
10.1 a-d Run 29, density balance, $\bar{W}_2$, $\bar{J}(h_2', \psi_2')$, $J(h_2, \psi_2)$, $\partial h_2/\partial t$, $J=60$
10.2 a-d Run 29, density balance, $\bar{w}_u$, $\overline{h'_4}$, $\Psi'$,
$J(h'_4, \Psi'_4)$, $\delta h'_4/\delta t$, $J=60$
10.3 a-c Run 29, vorticity balance, $\overline{\partial v_1}$, $J(\eta_1', \psi_1')$, $J(\eta_1, \psi_1)$, $J=60$
10.4 a-c  Run 29, vorticity balance, $\beta \bar{v}_3$, $J(\eta_3', \Psi_3')$, $J(\eta_3, \Psi_3)$, $J=60$
10.5 a-c Run 29, vorticity balance, $\beta \vec{v}_5$, $\bar{J}(n_5, \Psi_5)$, $\bar{J}(n_5, \Psi_5)$, J=60
10.6 a-d Run 29, \(-\delta v, f_{wz}, (\delta v)_{linear}\) layer 1, \(I=160\); layer 1, \(J=60\); layer 3, \(I=160\); layer 3, \(J=60\)
10.7 a-c  Run 29, $-\beta v$, $fw$, $(\beta v)_{\text{linear}}$, layer 5, $I=160$, $J=50$, $J=60$
11. a-d  Time dependent energetics,
(a) Run 30, P_2, P_4, K_3, K_1, K_5;
(b) Run 22, P_4, P_2, K_5, K_3, K_1;
(c) Run 28, P_2, K_1, K_5, P_4, K_3;
(d) Run 29, P_2, P_4, K_5, K_1, K_3
12.1 a-d Run 30, instantaneous \( \psi_1 \), day 60, CI=70; day 240, CI=300; day 840, CI=800; day 3600, CI=10
12.2 a-d  Run 22, instantaneous $V_1$, day 60, CI=300; day 260, CI=10^3; day 360, CI=2 x 10^7; day 400, CI=2 x 10^9
12.2 e-h Run 22, instantaneous $Y_1$, day 460, CI=4 x $10^3$; 
day 660, CI=6 x $10^3$; day 1020, CI=9 x $10^3$; 
day 7080, CI=8 x $10^3$
12.3 a-d  Run 22, instantaneous $\Psi_5$, day 360, CI=400; day 500, CI=$10^3$; day 660, CI=$10^3$; day 7080, CI=$4 \times 10^3$
Run 22, instantaneous $h_4$, day 720, CI=$5 \times 10^3$; day 840, CI=$5 \times 10^3$