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HIGH ALTITUDE OBSERVATORY
of the
University of Colorado

Solar Research Memorandum No. 44

From: Gerard Wlérick.

Subject: Photographic Photometry of the Coronal Emission Lines.

INTRODUCTION.

It is known that to avoid too much scattering of the photospheric light, the first objective of a coronagraph is a simple lens. As this lens is not achromatic, some difficulties appear when one wants to measure simultaneously the intensities of different coronal emission lines. (See Solar Research Memorandum No. 34 of D. E. Billings.) These troubles arise as it is not possible to focus at the same time the occulting disk and the slit of the spectrograph for different wavelengths.

I shall now study this problem from the following point of view: what information may we draw from one single photographic plate recording the coronal spectrum? This study brings me also to consider the different factors which play a role in coronal photometry: vignetting, spill, and scattered light.

The optics of the coronagraph consist essentially of two parts: the first objective B, and the secondary optics S. I shall consider (Table 1) the different settings used at Climax and Sacramento Peak. I shall study also the setting used at Pic du Midi, as the final small Climax coronagraph will have similar properties.

Table 1

Setting Number	Optical System B + S	Optical System S	Occulting Disk Focused on:	Image of Sun Focused on Spectrograph for:	Example
1	achromatic	"hyper-chromatic"	$H\alpha$	every λ	Pic du Midi
2	nonachromatic	achromatic	$H\alpha$	$H\alpha$	Sacramento Peak
3	nonachromatic	nonachromatic	$H\alpha$	$H\alpha$	Climax 1
4	nonachromatic	nonachromatic	$H\alpha$	$\sim$6000 A°	Climax 2

Before reviewing these four settings, I shall indicate the optical properties of the first objective, by taking the concrete case of the Climax "5-inch" coronagraph as it was used at the beginning of 1955.

Properties of the First Objective.

1. Glass: Borosilicate crown.

2. Lens diameter: 12.5 cm (5 inches).

3. Diameter used: $D = 7.6$ cm (3 inches).

4. Focal length for $H\alpha$: $f = 248$ cm (98 inches).

5. Numerical aperture: $A = D/f = 0.031$.

6. Chromatic properties: Let n_r be the refractive index for $H\alpha$ and n the index for another wavelength. The corresponding change Δf in focal length is:

$$\Delta f = f\ (n-n_r)/n-1 \qquad (1)$$

7. Table 2 gives information concerning the primary images of the sun corresponding to different colors:

Table 2

λ in A°	4861	5100	5303	5980	6374	6563
Color of the Image	$H\beta$	blue green	green	yellow	red	$H\alpha$
Notation for Center		b	g	y	r'	r
Notation for a Diameter		b_1b_2	g_1g_2	y_1y_2	$r'_1r'_2$	r_1r_2
$\Delta f/f$	0.0153	0.0122	0.01	0.0041	0.00126	0
Δf in cm	3.8	3.04	2.48	1.02	0.31	0
Mean Diameter of Solar Image in cm	2.270	2.278	2.283	2.298	2.305	2.308

8. Notations.

1 center of the first objective B. (Figure 1)

l_1, l_2 edges of the useful part of one diameter of B.

d_1 , d_2 edges of the occulting disk.

α angular height of the part of the inner corona masked by the occulting disk.

$$\alpha = r_2 d_2 / f$$

1" and 1' of arc represent, respectively, on the sun 720 km and 43,500 km.

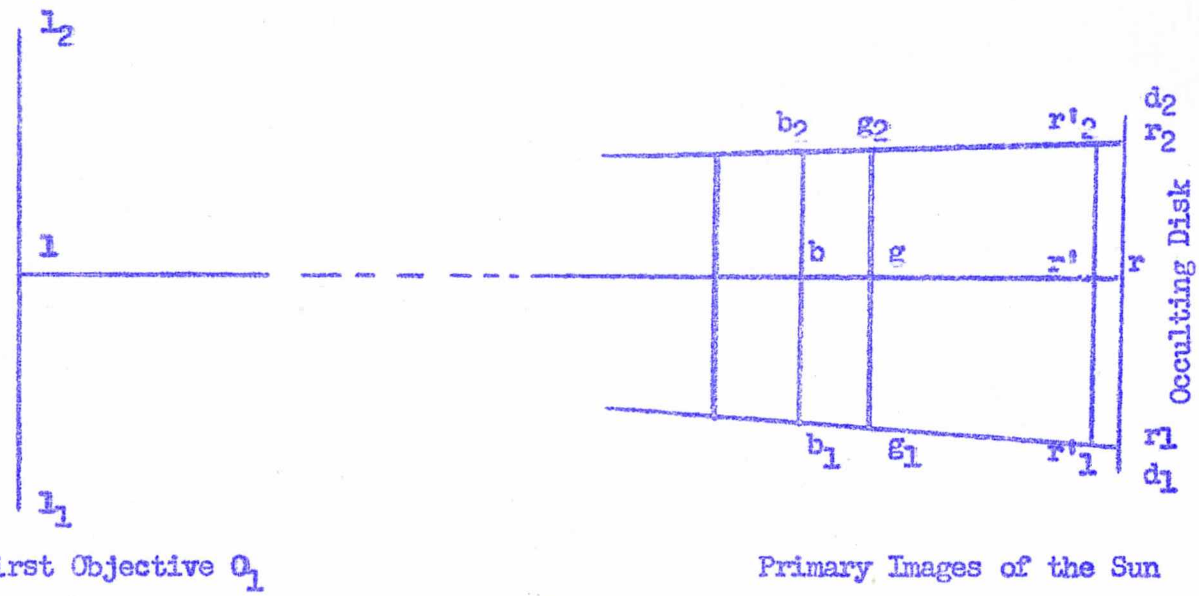


Figure 1.

I. First Setting.

Optical System B+S Achromatic as at Pic du Midi.

The advantage of this system is the following: the final images corresponding to different wavelengths are formed in the plane of the slit of the spectrograph simultaneously and with the same size. Thus one photographic spectrum reveals fine details corresponding to different prominence or coronal lines. Indeed this setting works quite perfectly for spectra taken at some distance from the limb. However, we shall see that for observations very close to the limb (~10,000 to 30,000 km), it can not give simultaneously the true values of the intensities of the different lines. That comes from the fact that the occulting disk is in the right position for one wavelength only. I shall compute as example the case where the disk is focused for $H\alpha$, a wavelength very convenient for prominence studies. In such a case the trouble is small for the red coronal line 6374 A°, but important for the green line 5303A°.

A. Final Image of the Corona Corresponding to the Green Emission Line.

If we look at some point of the corona rather close to the edge of the occulting disk, the brightness of the green image of such a point is attenuated because some green light rays are stopped by the occulting disk.

Let g_3 and g_4 be the two points where the green image cuts the lines ld_2 and l_2d_2 (Figure 2). These points have the following properties:

1. The brightness of the final image given by the point g_3 is only 0.5 times the true brightness (Figure 3a) because all of the rays coming from the upper half of the lens are stopped.

2. The brightness of the final image, for all points farther from the axis than g_4, is not attenuated.

3. The results for the Hα line are given for comparison in Figure 3b.

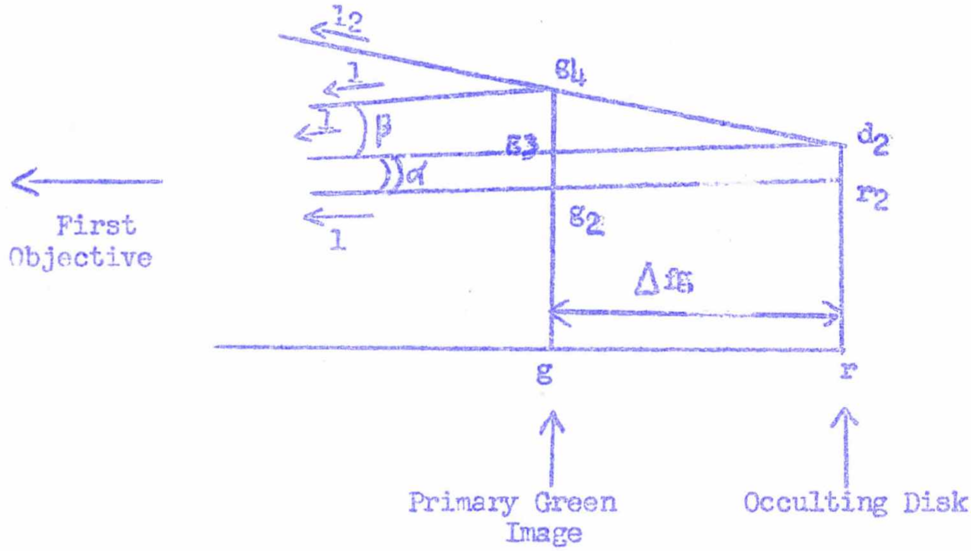


Figure 2.

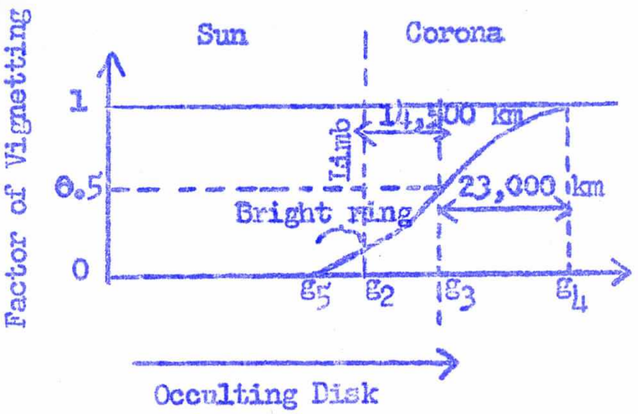
We have:

$$g_3g_4 = ll_2 \Delta f_g / f$$

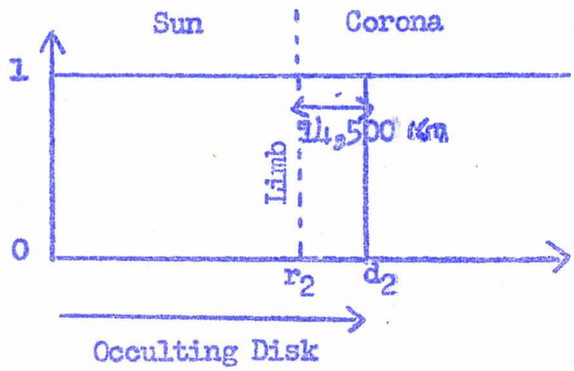
and the angular distance corresponding to g_3g_4 is:

$$\boxed{\beta = 1/2 \, A \, \Delta f_g / f} \qquad (2)$$

The numerical computation gives $\beta = 32''$ corresponding to 23,000 km. The figures 3a and 3b correspond to the case where $\alpha = 20'' = 14,500$ km.



a. 5303 A°



b. H α

Figure 3.

B. Scattered Light.

The photospheric light of wavelength $\lambda = 5303$ A° is not completely stopped by the occulting disk which is placed at the Hα focus. As a consequence, a bright green ring appears on the plane of the slit of the spectrograph. It corresponds to the segment $g_5\ g_2$ of Figure 3a, which has an angular height equal to $\beta - \alpha$. Taking into account the limb darkening the relative amount a of photospheric light not stopped by the disk has the value:

$$a = 1/960 \int_{960 - \beta + \alpha}^{960} v\, d\xi$$

960 = solar radius expressed in seconds of arc.

v = factor of vignetting.

With the values of α and β used, we found very roughly:

$$a \sim 10^{-3}$$

A part of this photospheric light is scattered by the edge of the occulting disk and the secondary optics and increases the background of the green image due to the sky and to the light scattered by the first objective. In fact, the corresponding increase is small; however, if one wants to avoid this effect he has only to take $\alpha \geqslant \beta$.

C. Conclusion.

In this setting, when the occulting disk is focused on Hα, there is, for the green line, some vignetting for the points of the corona which are imaged close to the edge of the occulting disk. The angular height β of the vignetted area is proportional to the numerical aperture A. The angular height above the limb at which we may observe without vignetting is equal to $\alpha + \beta$, where α is the height above the limb corresponding to the edge of the occulting disk. To have a very pure spectrum it is reasonable to take $\alpha \simeq \beta$. Above an angular height 2β the system works very well without correction. Especially it gives a precise image of the 5303 A° coronal emission.

II. Second Setting.

System S Achromatic, Occulting Disk and Slit Focused in H α.

Let us consider the image of the slit of the spectrograph given by the two lenses F and D: this image is an arc of circumference and is located in the plane of the occulting disk. Let P be a point of the slit and p its image. Every ray which enters the slit at P has passed through the point p which acts as a pupil.

A. Position of the Slit of the Spectrograph.

We determine the closest position of the slit with respect to the limb, by specifying that the photospheric light (spill) enters the pupil p only for wavelengths smaller than a given wavelength λ_1. This corresponds to a compromise between two objectives: to observe as close as possible to the limb, to use the maximum part of the spectrum and in any case to have no trouble in the spectral region 5300-6700 A. I shall compute, as example, the case $\lambda_1 = 5100$ A° which about corresponds to the routine at Sacramento Peak. If the spill arises for $\lambda \leq 5100$ A, the lines l_1p and lr_2 cross at the point b_2 (Figure 4).

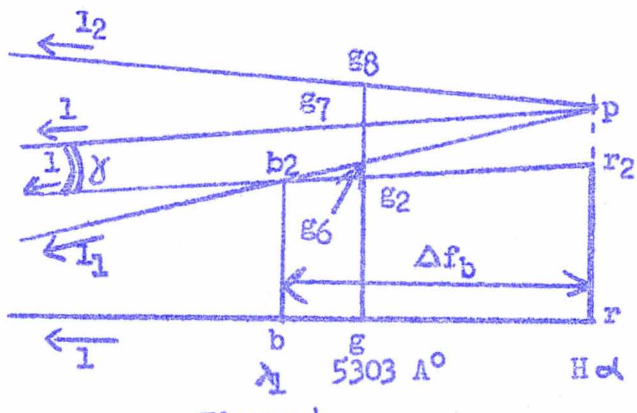


Figure 4.

We have: $r_2p = ll_1 \Delta f_b/f$

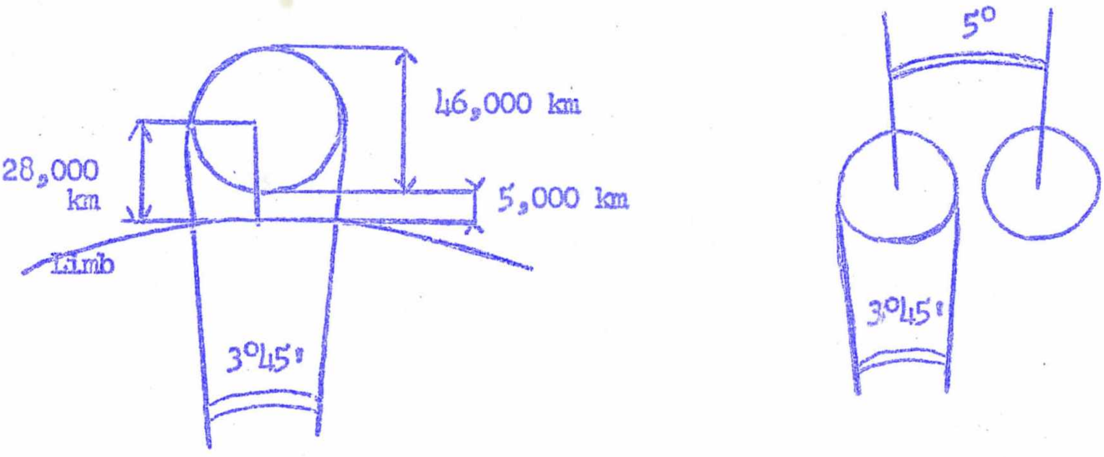
The point p belongs to the red image and corresponds to an angular distance γ from the sun.

$$\boxed{\gamma = 1/2\ A\ \Delta f_b/f} \tag{3}$$

The computation indicates: $\gamma = 1.89 \; 10^{-4}$ radian = 39". The observation is thus made at 28,000 km above the limb.

B. <u>How the Green Line is Observed.</u>

Let g_6, g_7, and g_8 be the crossing points of the line g_1g_2 with the lines pl_1, pl, pl_2. To enter the spectrograph at P, every green line passes through p and so has cut the green image in the interior of a circle which has $g_6 \, g_8$ as diameter and g_7 as center. Thus a point in the green line of the photographic plate of the spectrograph corresponds to rays of light emitted by a small circle of the corona (Figure 5a) that we may call the circle of confusion.



a. b.

Figure 5.

The diameter of the circle is given by: $g_6g_8 = l_1l_2 \; \Delta f_g/f$

It corresponds to an angular diameter δ in the corona:

$$\boxed{\delta = A \; \Delta f_g/f} \tag{4}$$

Thus,

$$\delta = 2\beta \tag{5}$$

Table 3 gives the characteristic values related to the green circle of confusion.

Table 3

Characteristics of the Circle of Confusion for $\lambda = 5303A^{o}$.

Unit	Seconds of Arc	Km	Heliographic Angle
Distance of the Center from the Limb.	$\gamma = 39''$	28,000	
Diameter	$\delta = 64''$	46,000	$3^{o}\ 45'$

If we make measurements at points 5^{o} apart in heliographic latitude, we see that there is no overlapping between the corresponding circles (Figure 5b).

C. Coronal Photometry.

If the corona was a halo of uniform brightness, the ratio G/R of the green line intensity to the red line intensity, which is measured by this system, would be the true one for the following reason: everything happens as if we were using an out of focus image of the green corona in the plane of the red image; this pseudo-image has the same dimension as the red image and the pupil p is also the same.

As the brightness of the 5303 A^{o} corona has very heterogeneous structure, the ratio G/R may be either smaller or greater than the true ratio.

D. Light Scattered by the Edge of the Occulting Disk.

As the occulting disk is focused for every wavelength, the light scattered by its edge does not cause trouble, as it does not enter the slit. It is an advantage of this system. For a similar reason, the value of the diameter of the occulting disk is not critical, but the best is that the disk overoccults appreciably.

E. Conclusions.

1. With this setting, we must observe rather far from the limb ($D > 28,000$ km) to avoid the spill for the wavelengths greater than 5100 A.

2. We lose definition for the green image and the G/R ratio is not the true one.

3. On the other side, we have no trouble coming from the light scattered by the edge of the occulting disk.

III. Third Setting.

This setting differs only from the second one in that S is no more achromatic. Again let P be a point of the slit of the spectrograph. P has now different images, for different colors, in the S system. Let p_r, p_g, and p_b be these images for the red, green and blue green light. p_r, p_g, and p_b act as pupils for their respective wavelength. The Figure 6 shows qualitatively the setting corresponding to the Climax coronagraph, when its slit is focused for H α. The pupils are on the principal axis and the occulting disk is outside of the axis.

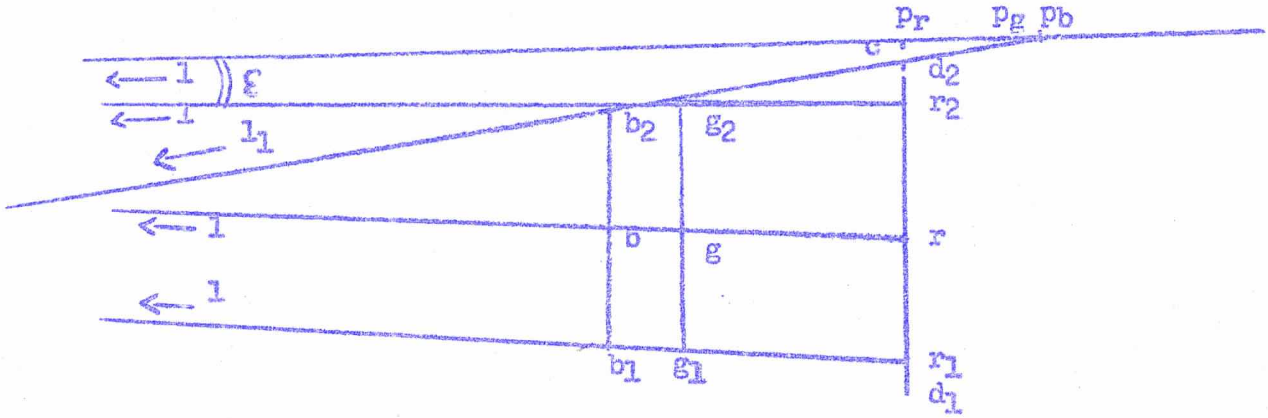


Figure 6.

A. Position of the Slit of the Spectrograph.

I assume first that the occulting disk overoccults just a little bit the H α image of the photosphere, and I specify again that the photospheric light (spill) enters the slit of the spectrograph only for the wavelengths smaller than 5100 A°. This is realized when the lines l_1p_b and lr_2 cross at the edge b_2 of the blue green image (Figure 6).

The important parameter is the distance p_rp_b. I call it $\Delta' f_b$. The angular distance ε at which we observe from the limb is equal to the angle between the lines lp_r and lr_2. We have:

$$\boxed{\varepsilon = A/2(\Delta f_b + \Delta' f_b)/f} \qquad (6)$$

This angular distance ε is bigger than the distance γ corresponding to the second setting. I put:

$$R = (\Delta f_b + \Delta' f_b)/\Delta f_b \qquad (7)$$

It follows:

$$\boxed{\varepsilon = R\gamma} \qquad (8)$$

Dick Hansen has performed a special experiment to determine the value of R. His result is:

$$R \sim 1.7$$

We thus obtain:

$$\varepsilon \sim 66''$$

corresponding to a distance of observation of about 48,000 km above the limb.

B. How the Green Emission Line Is Observed.

As in the second setting, we have again for the green line a circle of confusion $g_g g_n$ (Figure 7), but now the circle is bigger, in the ratio:

$$(\Delta f_g + \Delta' f_g)/\Delta f_g$$

One may show very simply that:

$$(\Delta f_g + \Delta' f_g)/\Delta f_g = (\Delta f_b + \Delta' f_b)/\Delta f_b = R$$

if the dispersive properties of the first objective B and the system D + F are either the same or proportional one to the other, the three lines $p_r r_2$, $p_g g_2$, $p_b b_2$ must cross practically at the same point, c, as the lines $p_r p_b$, and $b_2 r_2$ are nearly parallel (Figure 6).

I call ζ the angular distance corresponding to the diameter of the circle of confusion. We have:

$$\zeta = R\,\delta \qquad (9)$$

Table 4 gives the characteristic values related to the green circle of confusion.

Table 4

Characteristics of the Circle of Confusion for $\lambda = 5303$ A°.

Unit	Second of Arc	Km	Heliographic Angle
Distance of the Center from the Limb	$\varepsilon = 66''$	47,500	
Diameter	$\zeta = 109''$	80,000	6° 30'

If we measure at points 5° apart in heliographic latitude, there is a small overlapping between the corresponding circle of confusion.

C. Coronal Photometry.

The photometry is affected by two factors:

1. As in setting 2, the G/R ratio tends to be different from the true one because, in the circle of confusion, the brightness of the corona is not uniform.

2. We have also to take into account the dimensions of the pupils and of the images used. In the red ($H\alpha$), we use the true image and the pupil p_r. In the green, everything happens as if we were using the pupil p_g and a pseudo image in the plane of this pupil p_g. The pupil p_g is smaller than the pupil p_r and the pseudo image in the plane of p_g is bigger than the true red image. These two facts tend to decrease the G/R ratio.

D. Importance of the Size of the Occulting Disk.

We have assumed that the disk overoccults just a little bit. It is interesting to see what happens when the size of the disk increases, without changing the angular distance ε from the limb at which we observe the corona in the red (r_2p_r = constant).

I consider the points a, c and e where the lines l_1g_2, l_1p_b and l_1p_g cut the line r_2p_r (See Figure 7).

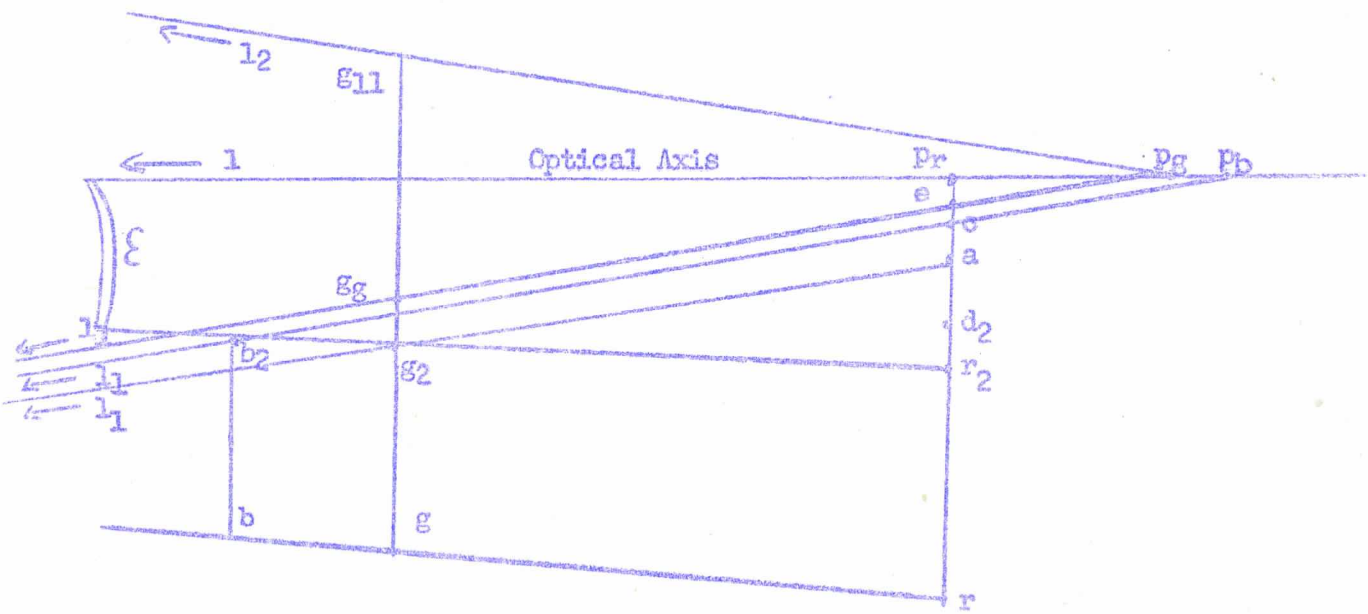


Figure 7.

We may consider four different cases:

a. d_2 between r_2 and a: the edge d_2 receives and scatters some photospheric green light, a part of which goes through the green pupil p_g.

b. d_2 between a and c: the edge d_2 doesn't receive any given photospheric light.

c. d_2 between c and e: the spill disappears for every wavelength.

d. d_2 between e and p_r: some "vignetting" of the green image appears as the lower part of the green circle of confusion $g_g g_{11}$ is no more useful.

It is advisable to keep the spill, as it is a simple means to know at which distance we observe from the limb. The best dimension of the disk seems thus to be obtained when d_2 is between a and c, (ac $\sim$ 0.1 mm). The point a corresponds to α = 32". Thus the disk would overoccult appreciably.

E. Conclusion.

1. The last paragraph shows that this third setting may be made similar to the first one (vignetting, no spill), or to the second one (spill, no vignetting).

2. If we want the spill to appear only for $\lambda \leq 5100$ A°, we must observe at 47,500 km above the limb and the diameter of the circle of confusion for $\lambda = 5303$ A° will then be of the order of 80,000 km. Thus this third setting is not so good as the first or the second one.

IV. Fourth Setting.

Very often, in routine observation, the Climax "5-inch" coronagraph has been working in the following conditions: the occulting disk in focus for Hα, the spectrograph in such a position that the "green image of the occulting disk" is focused on the slit. In that setting the spectrograph is in focus for an image of the sun of wavelength λ_2 such that 5303 A° $< \lambda_2 < 6563$ A°. That has the advantage of decreasing the very great asymmetry of observation between green and red coronal lines, such as it exists in setting 3.

A. Computation of λ_2.

The pupil p_g must be in the plane of the occulting disk (Figure 8). The pupil p_y corresponding to λ_2 is at a distance Δf_y of the occulting disk such that:

$$\boxed{\Delta f_y = \Delta f_g (R-1)/R} \qquad (10)$$

We obtain:

$$\Delta f_y = 1.02 \text{ cm}$$

and

$$\lambda_2 = 5980 \text{ A}^\circ$$

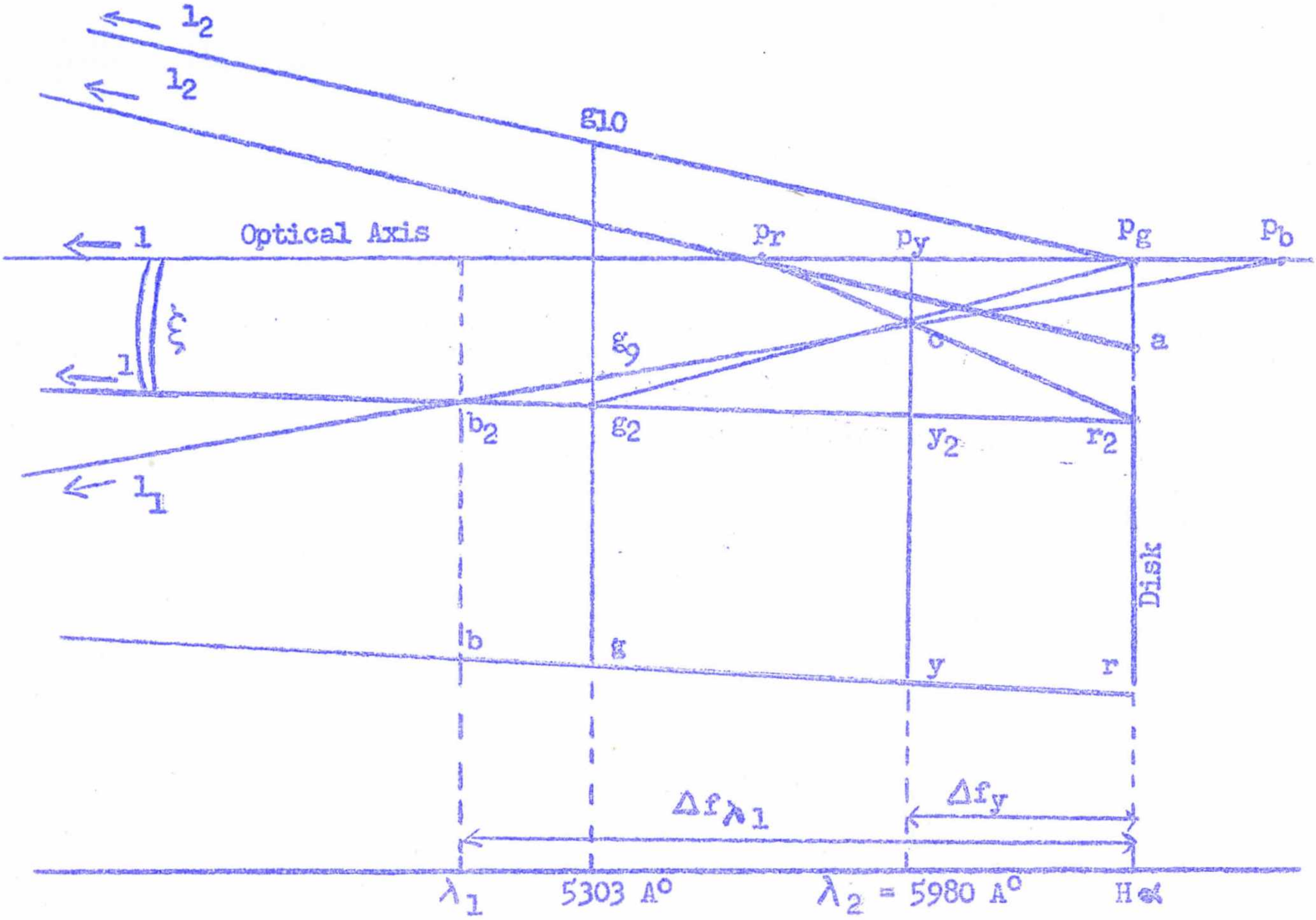


Figure 8.

B. Position of the Slit of the Spectrograph.

I assume that the spill arises for wavelengths smaller than a given value λ_1. I call ξ the angular distance at which we observe from the limb. We have: (Figure 8)

$$\xi = y_2p_y/f = R\ y_2c/f$$

$$y_2c = A/2\ (\Delta f_{\lambda 1} - \Delta f_y)$$

Thus:

$$\boxed{\xi = A/2\ R\ (\Delta f_{\lambda 1} - \Delta f_y)/f} \qquad (11)$$

I give the value of ξ for two different values of λ_1:

λ_1 = 5100 A°	ξ = 44"	Corresponding to	32,000 km
λ_1 = 4861 A°	ξ = 61"	Corresponding to	44,000 km

C. How the Green Emission Line is Observed.

For $\lambda = 5303$ A°, we have the circle of confusion $g_9\ g_{10}$. I call K its angular **diameter:**

$$K = A\ \Delta f_g/f = \delta \qquad (12)$$

The circle of confusion has the same diameter as in setting 2. We thus have $K = 64''$ corresponding to 46,000 km.

Remark: K is independent of the value chosen for λ_1. Thus when λ_1 decreases, we observe farther from the limb but the diameter of the green circle of confusion remains the same.

D. Coronal Photometry.

The remarks made for the third setting are valid for the fourth one.

E. Importance of the Size of the Occulting Disk.

One may make a discussion similar to that used for setting 3. When the size of the disk increases, the vignetting appears. It affects first H α (point a of Figure 8) and then the smaller wavelengths.

F. Conclusion.

For the simultaneous observations in the wavelengths interval 5303-6563 A°, this setting has properties which are not very different from those of setting 2.

General Conclusions.

1. The minimum distance of observation above the limb, as well as the diameter of the circle of confusion at a given wavelength, is proportional to:

   a. The numerical aperture A.
   b. The dispersive properties of the first objective B.

2. The importance of the diameter of the circle of confusion for $\lambda = 5303$ A° (46,000 km or more) explains very well the different values obtained for the intensities of the green line when one takes spectra of an active region which are in focus or out of focus for the green. That agrees completely with the results reported by D. E. Billings in Solar Research Memorandum No. 34. Fortunately, with the new "5-inch" coronagraph, now in installation at Climax, it will be easy to take successively in-focus spectra of the corona in the green and in the red line.

3. The present work will permit connecting the observations made with the old coronagraph to these of the new one.

Acknowledgement.

I thank Dr. D. E. Billings and Mr. Richard Hansen for the very helpful discussions that I had with them. I thank also G. W. Curtis who computed the shape of the spill curve appearing in Appendix 2. We will very much appreciate critical comments on this report from the staff at Sacramento Peak, and others who may be interested in these results.

Appendix I.

Distance of Observation Deduced from Spill for in-Focus Spectra.

As the technique to take in-focus spectra in the red and in the green is developing now at Climax and Sacramento Peak, it is useful to have tables giving the relation between the wavelength at which the spill appears and the distance of observation above the limb.

The angular distance η is given by a formula identical to formulae (3) and (6). We have:

$$\boxed{\eta_{\text{radian}} = A/2\ R\ \Delta f_{\lambda_1}/f} \tag{13}$$

or

$$\boxed{\eta_{\text{seconds of arc}} = 2.06 \times 10^5\ A/2\ R\ \Delta f_{\lambda_1}/f}$$

I shall again make the computation with the following values: A = 0.031 (D = 3 inches), R = 1 (Secondary optics achromatic) and R = 1.7 (Climax secondary optics until June 1955). The results appear in two tables.

Table A1.

Disk and Spectrograph Focused on Hα.

Wavelength λ_1 Where spill Appears		5893	6100	6300	6563
$\Delta f_{\lambda_1}/f$		4.64×10^{-3}	3.08×10^{-3}	1.69×10^{-3}	0
R = 1	η in seconds of arc	14.8"	9.8"	5.4"	
	Distance in km	10,700	7,100	3,900	
R = 1.7	η in seconds of arc	25.1"	16.7"	9.15"	
	Distance in km	18,000	12,000	6,600	

Table A2.
Disk and Spectrograph Focused on 5303 A°.

Wavelength λ_1 Where spill Appears		4861	5000	5100	5200	5303	5450	5600	5750	5893
$\Delta f_{\lambda_1}/f$		5.4×10^{-3}	3.6×10^{-3}	2.3×10^{-3}	1.08×10^{-3}	0	1.38×10^{-3}	2.88×10^{-3}	4.16×10^{-3}	5.4×10^{-3}
R=1	ζ in sec. of arc	17.3"	11.5"	7.35"	3.45"		4.4"	9.2"	13.3"	17.3"
	distance above limb in km	12,500	8,300	5,300	2,500		3,200	6,600	9,600	12,500
R=1.7	ζ in sec. of arc	29.25"	195"	12.45"	5.85"		7.5"	15.6"	22.5"	29.25"
	distance above limb in km	21,000	14,000	9,000	4,200		5,400	11,200	16,200	21,000

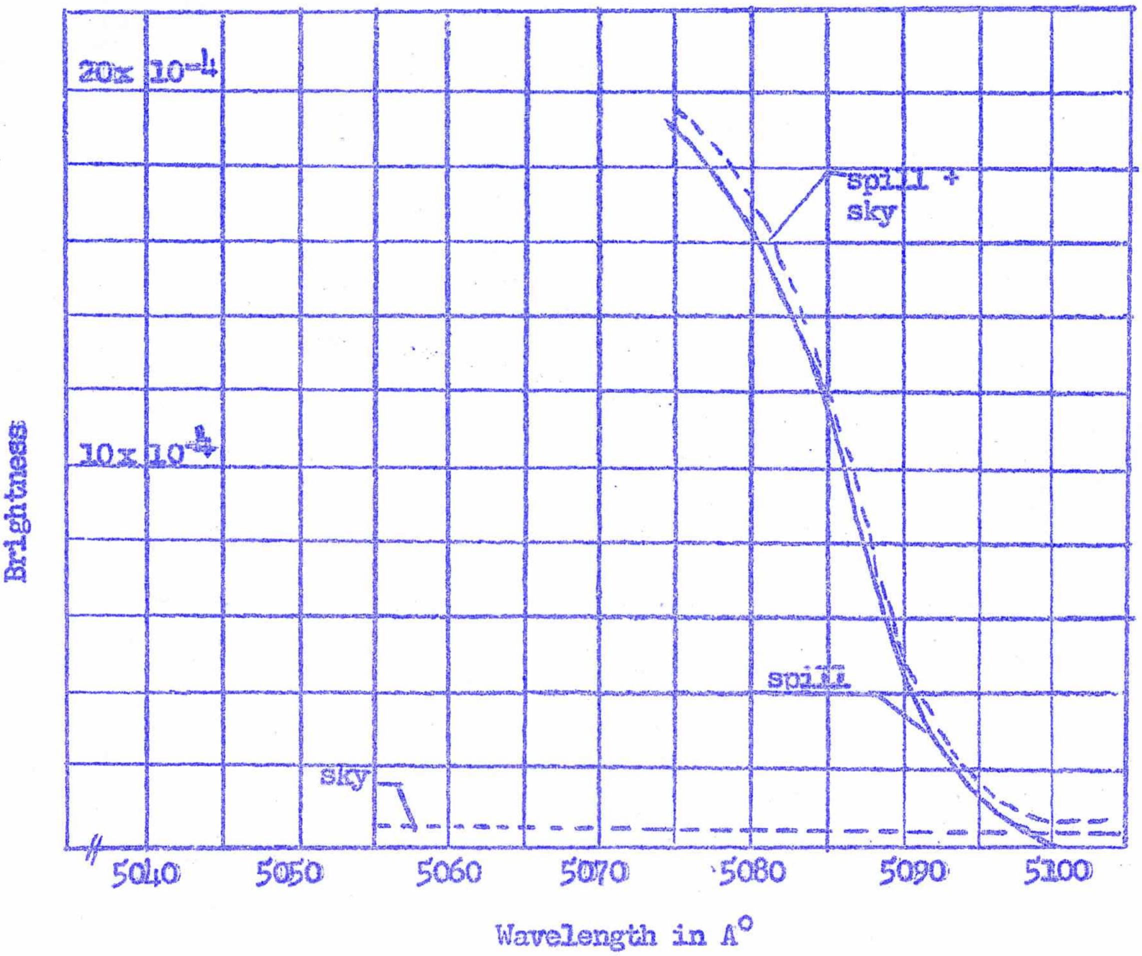


Figure 9.

Brightness Due to Spill

(The brightness of the center of the sun is used as unit)

Appendix II.

G. William Curtis has computed the brightness due to the spill as a function of wavelength. The results appear in Figure 9. One sees that the spill brightness increases very quickly. In a wavelength interval of 25 A°, the brightness is already of the order of 2000×10^{-6} B. Thus it can be concluded that the spreading of the spill observed on the plates is mostly due to seeing.

End of Memorandum