Heterogeneous Convective-Scale Background Error Covariances with the Inclusion of Hydrometeor Variables

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ABSTRACT
Convective-scale models used in NWP nowadays include detailed realistic parameterization for the representation of cloud and precipitation processes. Yet they still lack advanced data assimilation schemes able to efficiently use observations to initialize hydrometeor fields. This challenging task may benefit from a better understanding of the statistical structure of background errors in precipitating areas for both traditional and hydrometeor variables, which is the goal of this study. A special binning has been devised to compute separate background error covariance matrices for precipitating and nonprecipitating areas. This binning is based on bidimensional geographical masks defined by the vertical averaged rain content of the background error perturbations. The sample for computing the covariances is taken from an ensemble of short range forecasts run at 3-km resolution for the prediction of two specific cases of convective storms over the United States. The covariance matrices and associated diagnostics are built on the control variable transform formulation typical of variational data assimilation. The comparison especially highlights the strong coupling of specific humidity, cloud, and rain content with divergence. Shorter horizontal correlations have been obtained in precipitating areas. Vertical correlations mostly reflect the cloud vertical extension due to the convective processes. The statistics for hydrometeor variables show physically meaningful autocovariances and statistical couplings with other variables. Issues for data assimilation of radar reflectivity or more generally of observations linked to cloud and rain content with this kind of background error matrix formulation are thereon briefly discussed.

1. Introduction
A long-standing problem in NWP has been to provide initial conditions to convective-scale¹ models for the prediction of convective systems and precipitations, given the available observations (Errico et al. 2007). The assimilation of cloud- or precipitation-related observations raises several important issues, such as the choice of observable, the depiction of observation error statistics, the characterization of model error in cloud and precipitation parameterization schemes, and the nonlinearity of the model forecasts and observation operators at those spatiotemporal scales (Lopez 2007). Current operational data assimilation systems use some form of Kalman filtering suitable for Gaussian errors. Following Auligne et al. (2010), cloud processes are highly nonlinear, giving non-Gaussian errors, but quasi-linear data assimilation techniques can be kept as a starting point and progress can be made on their respective advantages by hybridizing them. Simpler and physically consistent schemes such as latent heat nudging may also be of interest, through they have shortcomings such as neglecting precipitation advection, time lag, and a strong sensitivity to tuning (Leuenberger and Rossa 2007).

¹ Also called the meso-γ scale (Orlanski 1975).

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Even in the “Gaussian” framework, the depiction and representation of background error covariances at the convective-scale is not straightforward. In a variational data assimilation scheme, the background error penalty term is constructed from a control variable transform (CVT), which is a sequence of operators that approximate the square root of the true term as reviewed by Bannister (2008). The calibration of these operators is carried out over a sample of background errors built from either differences of forecasts valid at the same time (Parrish and Derber 1992) or from ensemble forecasts (Pereira and Berre 2006). In the ensemble Kalman filter (EnKF; Evensen 2003), the background error covariance matrix $\mathbf{B}$ is computed as the sample ensemble covariance matrix around the ensemble mean. This covariance matrix suffers from erroneous large-scale correlations because of sampling noise. Using a Schur product with a correlation function of compact support often improves the estimate of sampling noise. Recently, Caron and Fillion (2010) and Montmerle and Berre (2010; hereafter MB2010) compared background error statistics in precipitating and nonprecipitating areas. MB2010 used geographical masks based on vertically averaged rain content to diagnose background error statistics with a convective-scale nonhydrostatic NWP model (Brousseau et al. 2008). Strong differences were found in the error variances, in the correlation length scales, and in the cross covariances between humidity and divergence errors between precipitating and nonprecipitating areas. MB2010 also suggested a methodology, named as a “heterogeneous” variational scheme, to build a piecewise analysis based on separate background error covariances for precipitating and nonprecipitating areas. Albeit the hybrid method will naturally represent flow-dependent covariances, the heterogeneous scheme may also prove useful to impose some covariances that are typical of a mesoscale structure such as a convective cell or as fog (Montmerle et al. 2010) when they are not present or displaced in the background.

After these approaches, this paper advances a synthetic description of the background error covariances, including for hydrometeor variables, that have been obtained through 3-km resolution ensemble forecasts from the Weather Research and Forecasting (WRF) model (Skamarock et al. 2008; Dowell and Stensrud 2008). For that purpose, the technique of the geographical masks based on rain content introduced by Caron and Fillion (2010) and MB2010 is used because hydrometeor errors are likely to be highly dependent on cloud physics. Andersson et al. (2005) pointed out that forecast differences for specific humidity are easier to approximate with a Gaussian for a limited geographical region and similar values of the background field. Here, it is also shown that it is meaningful to compute hydrometeor background error covariances only for similar values of precipitations.

The paper is organized as follows. Section 2 describes the convective-scale ensemble over which the diagnostics are performed. Section 3 introduces the updated formulation of the CVT in WRF variational data assimilation (Var) devised for this study. It includes a change of control variables with respect to previous formulations and a binning based on the rain content of the perturbations. Section 4 describes and compares the background error covariances in precipitating and nonprecipitating areas. The conclusion summarizes the results and discusses some issues about the implementation of a hydrometeor control variable in variational data assimilation.
with the heterogeneous formulation and on the possible use of these diagnostics for EnKF localization.

2. The dataset

a. The ensemble

Surface observations are known to be of primary importance for the description of mesoscale structures (Ducrocq et al. 2000), and in particular of cold pools and inflow jets that are frequently associated with mesoscale convective systems. As part of the National Oceanic and Atmospheric Administration (NOAA) Hazardous Weather Testbed 2007 Spring Forecasting Experiment, a mesoscale short-range ensemble was designed with the WRF model to explore the value of these surface observations for the forecasts of severe events (Stensrud et al. 2009). For this purpose, hourly analyses were produced at 30-km resolution on an extended domain covering the contiguous 48 states of the United States. Routine hourly surface observations of 2-m potential temperature, 2-m dewpoint temperature, and 10-m winds were assimilated within a 30-member ensemble with the Data Assimilation Research Testbed (DART; Anderson et al. 2009) square root EnKF. The ensemble is started by creating initial and boundary perturbations consistent with the static covariance information from WRF 3D-Var, following Torn et al. (2006) and Stensrud et al. (2000) for the humidity perturbations. In addition to the initial dispersion of the ensemble and to the perturbations deduced from the EnKF, physics perturbations were introduced following the methodology of Stensrud et al. (2000), with variations of land surface, planetary boundary layer, radiation, convection, and microphysical perturbations. This ensemble was shown to well represent both initial condition and model physics uncertainties and results on case studies are examined in Stensrud et al. (2009).

This mesoscale ensemble analyses and forecasts were used by Dowell and Stensrud (2008) to provide initial and boundary conditions for an ensemble of forecasts that utilized a 3-km horizontal grid spacing with explicit convection, for several severe weather cases during spring 2007. This convective-scale ensemble was shown to provide valuable guidance about convective storm timing, location, and mode and is further exploited herein to estimate background error statistics with a focus on hydrometeor variables.

b. Meteorological situations

Two different meteorological situations are here considered: a case of high plains tornado outbreak at 0300 UTC 29 March 2007 and a multicellular storm at 0000 UTC 6 May 2007. Note that the sizes and locations of the domains are a different, but the forecast lead time is the same (6 h). Both ensembles have been run using initial and boundary conditions from the largest-scale EnKF.

Some features of these two cases are given in Fig. 1: the location where the ensemble produces strong precipitation is indicated by the ensemble-estimated probability of strong reflectivity occurrence at the lowest model level (within a radius of 15 km from grid point). Following Dowell and Stensrud (2008), the cyclonic supercells in the ensemble members are detected as locations where strong value of vertical velocity and vorticity arise ($\omega > 0, \zeta > 0, \text{and } \omega \times \zeta > 0.1 \text{ m s}^{-1}$ at 600 hPa). As shown by the ensemble-averaged 10-m wind, the storms are located in an environment of strong low-level convergence. Both cases, albeit different, look reasonably similar in terms of number of supercells and structure of the low-level flow. The situation at 0300 UTC 29 March 2007 looks weaker compared to the case at 0000 UTC 6 May 2007, which is associated with a strong probability of high reflectivities.

3. The heterogeneous 3D-Var with a rain-dependent binning and hydrometeor variables

a. Introduction to 3D-Var

The 3D-Var scheme is framed to provide an analysis $\mathbf{x}_a$ that minimizes a cost function $J(x)$ given a background $\mathbf{x}_b$:

$$\mathbf{x}_a = \text{Arg min} J,$$

$$J(x) = J_b(x) + J_o(x)$$
$$= \frac{1}{2} (x - x_b)^T B^{-1} (x - x_b)$$
$$+ \frac{1}{2} [y - \mathcal{H}(x)] \mathcal{R}^{-1} [y - \mathcal{H}(x)],$$

where, denoting by $n$ the dimension of $\mathbf{x}$ and by $p$ the dimension of the observations $\mathbf{y}$,

- $\mathcal{H}$ is the nonlinear observation operator;
- $B$ of dimensions $n \times n$ is the background error covariance matrix;
- $\mathcal{R}$ of dimensions $p \times p$ is the observation error covariance matrix.

The background term $J_b$ measures the distance in model space between any state $\mathbf{x}$ and the background $\mathbf{x}_b$, weighted by the inverse of the background error covariance matrix. The observation term $J_o$ measures the distance in observation space between any state $\mathbf{x}$ and the observation $\mathbf{y}$, weighted by the inverse of the observation error covariance matrix.

The innovation vector is further defined as the departure between the observation and the background in observation space: $d = y - \mathcal{H}(x_b)$. The incremental
approach (Courtier et al. 1994) replaces the problem Eqs. (1)–(2) by linearization with respect to the increment \( v = x - x_0 \):

\[
J(v) = \frac{1}{2} v^T B^{-1} v + \frac{1}{2} (d - Hv)^T R^{-1} (d - Hv),
\]

where \( H \) of dimensions \( p \times n \) is the linearized observation operator. The minimization is easier to accomplish as the cost function becomes quadratic.

b. The formulation of the control variable transform

The use of the variable \( v = B^{1/2} u \), where \( B^{1/2} \) is a square root of \( B \), reduces the cost function to be

\[
J(u) = \frac{1}{2} u^T u + \frac{1}{2} (d - HB^{1/2} u)^T R^{-1} (d - HB^{1/2} u).
\]

This change of variable generally improves the conditioning of the minimization, which results in faster convergence (Lorenc 1997; Gauthier et al. 1999). Because of its size, \( B^{1/2} \) is modeled as a sequence of operators. Different constructions of \( B^{1/2} \) used by several 3D-Var formulations are reviewed in Bannister (2008).

The WRF CVT (Barker et al. 2004) was adapted from the Met Office CVT as defined by Lorenc et al. (2000), but the horizontal correlations were prescribed with homogeneous recursive filters for the regional models, based on Hayden and Purser (1995). Michel and Auligne (2010) updated this formulation with the introduction of a new CVT:

\[
v = u^{th} U^v U^{ih} S u.
\]

where \( S \) is a gridpoint variance scaling factor; \( u^{th} \) is the application of high order, homogeneous (Purser et al. 2003a) or inhomogeneous (Purser et al. 2003b) recursive filters to impose horizontal correlations; \( U^v \) is the application of vertical correlations through homogeneous empirical orthogonal functions (EOF) and \( U^{ih} \) changes the control variables to model variables through a physical transform. The gridpoint variance scaling factor is not used in this study because background error covariances are taken to be piecewise constant over the geographical masks.

The calibration of operators defining \( B^{1/2} \) is performed apart. Samples of background error are used to successively estimate the parameters in the balance (or cross covariances), in the vertical covariances, in the variance and the horizontal length scales. More information on how those operators are computed and calibrated may be found in Wu et al. (2002), Barker et al. (2004), and Michel and Auligne (2010).

c. The rain-dependent binning

The variable \( B^{1/2} \) in WRF-Var has been devised using a gridpoint approach. To allow for geographical variations

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**FIG. 1.** Probability (%) that the radar reflectivity exceeds the level 50 dBZ within 15 km of grid point over the ensemble for (a) the 0300 UTC 29 Mar 2007 case and (b) the 0000 UTC 6 May 2007 case (shadings). Supercells locations defined as grid point of joint strong and positive values of vertical velocity and vorticity \((\omega > 0, \xi > 0, \text{and } \omega \times \xi > 0.1 \text{ m s}^{-1})\) are shown with dots. The 10-m ensemble mean wind is plotted with barbs.
of the covariances, the regression coefficients and the EOF can be computed over restricted bins of the domain (Wu et al. 2002). The binning system can be broadened to cases where there is dependence on the rain content of the background error samples. Similarly to MB2010 but using a gridpoint space approach, we define bidimensional geographical masks as areas where the values of the vertically averaged rain content in the background exceed (or not) a certain threshold.

Specifically, statistics are averaged into the bin of “heavy precipitation” $P_h$, when at points where both background fields used in the background error sample have vertically averaged rain content $\int q_r \geq 0.1$ g kg$^{-1}$, into the bin of “light precipitation” $P_l$, where both backgrounds have vertically averaged rain content $\int q_r < 0.1$ g kg$^{-1}$ and $\int q_r \geq 0.01$ g kg$^{-1}$, and into the bin of “no precipitation” $N_p$ were both backgrounds have vertically averaged rain content $\int q_r < 0.01$ g kg$^{-1}$. When the two background fields differ in class, their statistics are however kept in a special “mixed” bin. This bin can be seen as corresponding to the situation where the background is precipitating but the observation is not, for instance because of a displacement error. The bin operators are the corresponding to the situation where the background is precipitating but the observation is not, for instance because of a displacement error. The bin operators are projections; for example, any background error $\epsilon_b$ may be orthogonally decomposed as a sum of heavy, light, and no-rain one:

$$\epsilon_b = P_h \epsilon_b + P_l \epsilon_b + N_p \epsilon_b.$$  \hfill (6)

The background error covariance matrix can thus be decomposed into

$$B = \langle \epsilon_b \epsilon_b^T \rangle = \langle P_h \epsilon_b \epsilon_b^T P_h^T \rangle + \langle P_l \epsilon_b \epsilon_b^T P_l^T \rangle + \langle N_p \epsilon_b \epsilon_b^T N_p^T \rangle,$$  \hfill (7)

where “$\langle \rangle$” stands for a mathematical expectation, and where the cross covariances between the masks have been neglected to keep the number of covariances reasonable (cross covariances will however be introduced into the formulation in section 2f by spatial smoothing of the masks). In practice, the mathematical expectation is approximated by an average over the ensemble members and an average over the bins, thus leading to

$$B = P_h B_h P_h^T + P_l B_l P_l^T + N_p B_p N_p^T.$$  \hfill (8)

In this latter equation, the geographical masks that were depending on each ensemble member are now set up from the precipitating or nonprecipitating areas in a background from 3D-Var or with the observations. The matrices $B_{P_h}$, $B_{P_l}$, and $B_{N_p}$ are deduced from the square root form [Eq. (5)] written for each bin:

$$B = P_h U_{h}^T B_{h} U_{h}^T + P_l U_{l}^T B_{l} U_{l}^T + N_p U_{n}^T B_{n} U_{n}^T,$$  \hfill (9)

$$\text{if } U_{h}^T B_{h} U_{h}^T = U_{l}^T B_{l} U_{l}^T = U_{n}^T B_{n} U_{n}^T,$$  \hfill (10)

$$\text{if } U_{h}^T B_{h} U_{h}^T = U_{l}^T B_{l} U_{l}^T = U_{n}^T B_{n} U_{n}^T.$$  \hfill (11)

d. The physical transform

The state vector has been extended and includes the vorticity $\zeta$, the divergence $\eta$, the temperature $T$, the surface pressure $P_s$, the specific humidity $q$, the cloud content $q_o$, and the rain content $q_r$. Alternatively, the state vector for hydrometeors could instead admit the logarithm of cloud content $\log_{10}(q_o/q_r^s)$ and of rain content $\log_{10}(q_r/q_r^s)$ using the hybrid multivariate normal and lognormal approach of Fletcher and Zupanski (2006), where errors are taken to be multiplicative (Fletcher and Zupanski 2007) and $q_r^s = q_o^s = 1$ g kg$^{-1}$ are normalization constants. This logarithmic transform has been suggested to bring the probability density function of background errors for some variables closer to a Gaussian (Fletcher and Zupanski 2007). The quantitative comparison between normal and hybrid normal lognormal approaches requires updates to WRF 3D-Var and is beyond the scope of this paper.

Otherwise, the general formalism of Derber and Bouttier (1999) is used: full variables are transformed into approximately uncorrelated variables through the physical transform (sometimes also called the balance operator), which may be written as

$$\begin{pmatrix} \zeta \\ \eta \\ T \\ P_s \\ q \\ [\log q_o] \\ [\log q_r] \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_3 & 1 & 0 & 0 & 0 & 0 & 0 \\ M_3 & 1 & 0 & 0 & 0 & 0 & 0 \\ M_5 & 1 & 0 & 0 & 0 & 0 & 0 \\ M_7 & M_8 & M_9 & 1 & 0 & 0 & 0 \\ M_{10} & M_{11} & M_{12} & M_{13} & M_{14} & 1 & 0 \\ M_{15} & M_{16} & M_{17} & M_{18} & M_{19} & M_{20} & 1 \end{pmatrix},$$  \hfill (12)
where \([\log_{10}]\) denotes the optional logarithm transform from the hybrid normal lognormal approach (Fletcher and Zupanski 2007). The inversion of the Laplacian operator \(\Delta^{-1}\) transforms the vorticity into a streamfunction that is proportional to the geopotential height in the geostrophic framework on an \(f\) plane. Using the vorticity as a control variable instead of the streamfunction is costly in a gridpoint assimilation system as it requires the linear balance equation to be solved at every iteration of the minimization. The first four lines of this matrix transform are quite standard in the design of \(\mathbf{B}^{1/2}\); for example, see among others Parrish et al. (1997), Derber and Bouttier (1999), and Wu et al. (2002). The fifth line for \(q\) is similar to the transform designed by Berre (2000) and tested in an operational context by Fischer et al. (2005). Here, the difference lies in the fact that these correlations depend on the rain binning as in MB2010.

The last two lines statistically couple hydrometeors with the other variables, with the noticeable difference that the geostrophic transform has been discarded as there is few coupling between variables that have such different scales and because the errors probably have very different sources (e.g., microphysics and clouds for hydrometeors).

All the operators \(\mathbf{M}_i\), where \(i = 1, 2, \ldots, 20\), have the same structure (except for surface pressure), depending on the binning and on the vertical level. They are based on ordinary linear least squares as, for example,

\[
\mathbf{M}_{14} = (\mathbf{Q}^T \mathbf{Q})^{-1} \mathbf{Q}^T \mathbf{q}_c.
\]

where \(\mathbf{Q}\) and \(\mathbf{q}_c\), respectively, denote vectors of samples of humidity \(q\) and cloud content \(q_c\) in some rain-dependent binning therefore the matrices \(\mathbf{Q}^T \mathbf{Q}\) and \(\mathbf{Q}^T \mathbf{q}_c\), after appropriate normalization are, respectively, the bin-averaged vertical covariance matrix of \(q\) and the bin-averaged vertical cross-covariance matrix between \(q\) and \(q_c\). This is meaningful only if the matrix \(\mathbf{Q}^T \mathbf{Q}\) is well conditioned; otherwise regularization techniques can alleviate the problem (Neumaier 1998). This gridpoint framework prohibits a scale-dependence of the regression does in spectral or wavelet spaces.

\section*{e. Design of correlations}

The design of the vertical correlations is based on the decomposition of EOF. The vertical covariances are computed and a diagonalization is performed for each bin, resulting in vertical correlations that depend on the geographical masks.

In the variational scheme horizontal correlations are produced by recursive filters, which produce Gaussian-like shapes. A larger class of shapes can be obtained by positive linear combinations (Purser et al. 2003a). For variables such as vorticity and divergence, negative lobes of the correlation function can be reproduced if a scaled Laplacian operator is applied (Purser et al. 2003b) or with carefully designed nonpositive linear combinations (Mironze and Weaver 2010) that preserve the positive definite character of the covariances.

The model for the horizontal correlations decidedly depends on the reduction of the number of tunable parameters, such as, for instance, one or more length scales with associated weighting coefficients. A general statistical framework to estimate these coefficients is described by Dee and da Silva (1999). A widespread and simpler approach consists in using a diagnostic as an input for the model of the horizontal correlations. In Wu et al. (2002), a single length scale parameter was computed from the ratio of the variance of a field over the variance of the Laplacian of this field as an input for the recursive filters. This has the benefit of relying on a simple computation of variances that can easily be made bin dependent.

In this study, the single length scale parameter \(L\) is first diagnosed following Wu et al. (2002) and Michel and Auligné (2010). To broaden the range of correlation functions and especially that of fat tails, the correlation is written as a positive linear combination of three different Gaussian-shaped functions of scales \(L/2, L, \text{and } 2L\). The nonnegative least squares algorithm designed by Lawson and Hanson (1974) is used to fit the weighting coefficients of the combination to the locally diagnosed correlation function.

\section*{f. Illustration}

So far, the discussion has focused on the definition and calibration of square root operators defining the background error covariance matrices. The complete background error covariance matrix is the sum of the different background error covariance matrices weighted by the geographical masks \([\text{Eq. (8)}]\). If the masks are strictly taken as piecewise constant (for instance, with 0 in heavy precipitating areas and 1 otherwise for \(P_h\)), then the resulting increments will exactly vanish on the border between geographical masks. This situation is illustrated in Fig. 2a, where single observation results for temperature are shown to have shapes strongly driven by the geographical mask. This is likely to produce spurious numerical artifacts. Correlations between the precipitating and nonprecipitating regions need to be introduced by spatially smoothing the geographical masks, as suggested by MB2010.

Sometimes the mislocation of clouds in the background will cause deficiency in the analyses (Auligné et al. 2010), because a displacement error may cause a non-Gaussian distribution for background errors (Lawson and Hansen 2005; Ravela et al. 2007). Given reliable observations of reflectivity (for instance from a dense radar network), it
is possible to use the mixed bin at places where the background is not precipitating but where the observations show precipitations. This feature is quite unique to the heterogeneous 3D-Var and may help producing clouds and rain where necessary.

We shall now describe some elementary operations on geographical masks that may be of interest when using the heterogeneous 3D-Var in a realistic context. The first one, from the preceding discussion, is the ensemble union of observed and forecast precipitating areas to produce clouds and rain where necessary. Enlarging the mask with the help of observations is unlikely to be sufficient to provide good analysis. In particular, one may want to get rid of precipitating areas that are much smaller than the typical background length scale to ensure that the analysis increment do not follow exactly the geographical masks. In short, it may be desirable to smooth out the contour of the geographical masks shown in Fig. 2a. One solution for this is the use of algorithms from mathematical morphology [see, for instance, appendix A of Michel and Bouttier (2006) and references therein]. In particular, the opening of an image is obtained as successive erosion and dilatations. Intuitively, dilation expands an image object and erosion shrinks it. Opening smoothes a contour in an image, breaking narrow isthmuses and eliminating thin protrusions. The result is shown in Fig. 2b. The contours of the geographical mask have been smoothed and the tiniest cells have been removed by the algorithm. Together with convolution of the mask (as in MB2010), this ensures smooth transition for analysis increments between the different zones, as shown in Fig. 2b. When the precipitating area is small, the analysis increment however exhibits some fat tails that are a contribution from the nonprecipitating area. The amount of smoothing, both for the opening procedure and for the convolution, results from a trade-off between smoothness and representation of precipitating areas. It will need to be set with an objective criterion and is the topic of current study by Menétrier and Montmerle (2011).

**g. Gaussianity**

Auligne et al. (2010) have suggested that focusing first on the covariances of hydrometeor errors may be a valuable strategy, if the displacement error is taken into account as a first step. Recent progress has been made in non-Gaussian data assimilation [see Bocquet et al. (2010) for a recent review], such as the use of Gaussian anamorphosis prior to the analysis update, but it is still not clear how to handle multivariate aspects. Section 4 will show in particular that using a lognormal transform significantly alters the multivariate aspects, and in particular the coupling with divergence of hydrometeors found in these convective situations.

Figure 3 shows the histograms of background errors projected onto EOF modes, for different bins (based on the geographical masks previously defined, not to be confused with the bins of the histogram itself). Results from Fig. 3 should be interpreted while keeping in mind that
Fig. 3. Normalized histograms (adjacent rectangles in solid black lines) displaying relative frequencies of the occurrence of background errors normalized by their standard deviation and projected on the first EOF for different variables and geographical masks: (a) temperature in heavy rain bin, (b) specific humidity in heavy rain bin, (c) cloud content in the mixed bin, (d) rain content in the mixed bin, (e) cloud content in the heavy rain bin, and (f) rain content in the heavy rain bin. Also shown the Gaussian (solid gray) and exponential (dashed gray) continuous distributions of same variance.
the total distributions shown are spatial averages over different grid points, each of which could have their own characteristic errors (Holm et al. 2002). Because of this spatial averaging, it is not directly possible to relate (non-)Gaussianity of histogram to the (non)Gaussianity of the underlying probability density functions. Rather, this should be understood as a consistency check of whether the covariances are representative of the (averaged) probability density functions.

The background errors for wind and temperature variables appear to have near-normal distributions in all bins (see Fig. 3a). For humidity, a near-normal distribution is observed in the rainy bins (Fig. 3b) but not in the non-rainy bin (not shown), which may be a result of the inhomogeneity of the humidity field (Holm et al. 2002). The background errors for hydrometeor variables look nearly Gaussian in the rainy bins (Figs. 3e,f) but not in the mixed bin (Figs. 3c,f) or the nonprecipitating one (not shown). This support the idea that the probability density functions of hydrometeor background errors may be reasonably approximated by Gaussian in cases where there is no displacement error of rainy areas. We turn now to the description of the covariances obtained in this case.

4. Diagnostics of background error covariances

a. Background error standard deviations

The vertical profiles of bin-averaged background error standard deviations $\sigma_b$ are shown in Fig. 4 for the traditional control variables (vorticity, divergence, temperature, and specific humidity) for the March case. Overall, as in MB2010, they are bigger by a large factor for wind variables in precipitating areas with respect to non-precipitating ones. Moreover these standard deviations depend on the amount of rain chosen as a threshold to define the geographical masks (the standard deviations in “light rain” conditions are smaller than in “heavy rain” conditions). This probably reflects an important small-scale dynamical circulation associated with convective precipitating clouds; in particular the divergence profile (Fig. 4b) shows two extremes on the vertical that could be linked to the uncertainty in cloud top and bottoms. Temperature errors (Fig. 4c) are slightly bigger in precipitating areas than in nonprecipitating ones within the clouds but of smaller amplitude at lowest levels, which may reflect some isothermal effects caused by a strong convection. The overshoot of convective towers over the tropopause could explain the large $\sigma_b(T)$ at 200 hPa. Humidity errors also show two extrema on the vertical that could be linked with the uncertainty in cloud top and bottoms. Background error standard deviations for vorticity and divergence were similar in the May case. Figure 5 shows the vertical profiles obtained for temperature and specific humidity. The contrast between precipitating and nonprecipitating areas is even more apparent, with larger errors in precipitating areas. Contrary to the March case, the standard deviation for temperature is larger in precipitating areas than in nonprecipitating ones at all altitudes.

The vertical profiles of background error standard deviations for the March case for hydrometeors (Fig. 6) share some similarities with the mean distribution of those hydrometeors themselves (not shown). As for dynamical variables, standard deviations are increasing at larger precipitation thresholds. Small $\sigma_b$ for $q_c$ are visible in nonprecipitating areas, which shows that some non-precipitating clouds are present in the ensemble. The standard deviation of cloud content error is monopolar shaped with a maximum at 700 hPa. The standard deviation of rain error has two local extremes, one inside the cloud (toward 450 hPa) that is probably linked to the occurrence of rain at higher altitudes in strong convective towers with broad vertical extension and one below the cloud base (toward 800 hPa). The evaporation of precipitations may induce a decrease of the variance at lower levels. Similar results have been obtained for the May case (Fig. 7), but the variance of cloud content is broader. The quantity of rain error is also larger (it reaches 1.35 g kg$^{-1}$ rather than 0.95 g kg$^{-1}$). The highest value for rain error is located at higher altitude than for the March case, which indicates the strong dependence of rain covariances on the environmental conditions, for example, the altitude of the freezing level or the vertical extension of convective towers. These findings are consistent with the stronger intensity mentioned in section 2.

The logarithmic transform stresses relative, rather than absolute, variations: a small amount change in $\log q_c$ may be written $d[\log q_c] = dq_c/q_c$. Figures 6c,d show the standard deviations of background errors when the logarithmic transform is used in the March case. The vertical structures are noticeably broader. The maximum amplitude is reached at cloud bottom and top, where the relative variations of $q_c$ and $q_r$ are maximum.

b. Physical transform

Figure 8 shows the scaled eigenvalues of the vertical background error covariances matrices that need to be inverted to form the regression coefficients in Eq. (13). Generally, good conditioning ($\sim 10^3$) can be obtained for most variables except for humidity $q_r$ ($\sim 10^5$) and hydrometeors. This problem appears for both total and unbalanced hydrometeors. Using the log transform only alleviate the problem for $q_r$ in the May case (Fig. 8). The bin-averaged vertical covariance matrices for hydrometeors in rainy areas are of reduced rank, with three
null eigenvalues. This may be caused by the limited sampling or lack of statistical diversity in the ensemble. Regularization is therefore required at least for the $\mathbf{M}_{14}$, $\mathbf{M}_{19}$, and $\mathbf{M}_{20}$ regression coefficients. Following Neumaier (1998), the covariance matrix $(\mathbf{Q}^T \mathbf{Q})$ may be approximated by some well-defined invertible regularized matrix, which can then be used in Eq. (13). Difficulties may arise from the choice of the specific form of the approximate inverse (which usually follows from an assumption about the smoothness of the regression coefficients $\mathbf{M}$) and from the choice for the (small) regularization parameter $\lambda$. In practice, the ensemble is split into two independent samples of 15 members. Covariance matrices are estimated over the first ensemble; then the second ensemble is used to find the best truncated eigenvalue decomposition of $(\mathbf{Q}^T \mathbf{Q})$ that minimizes the

Fig. 4. Vertical profiles of background error standard deviations for (a) vorticity, (b) divergence, (c) temperature, and (d) specific humidity computed over rain-dependent binnings: heavy precipitating areas (dotted–dashed line), light precipitating areas (dotted gray line), mixed situations (dashed line), and nonprecipitating areas (solid line). Case at 0300 UTC 29 Mar 2007.
unexplained variance. The regression coefficients are then recomputed over the whole ensemble or set to 0 if they do not reduce the variance. The procedure has been found effective to solve the conditioning issue.

It is common to assess the goodness-of-fit of the linear least squares by comparing how much of the variation of the total variables can be reduced, or explained, by the regression onto the predictors (Boultier et al. 1997). This is achieved through partial ratios of explained regressed variance to the total variance. Figure 9 shows the results from nonprecipitating areas for temperature (Fig. 9a) and specific humidity (Fig. 9b). The vorticity is the main contributor and explains about 50% of the variance in temperature. This value is smaller than the ones for larger-scale global models (Derber and Boultier 1999; Ingleby 2001; Wu et al. 2002) because of the loss of geostrophy at smaller scales (Berre 2000), but still larger than those in MB2010 and also larger than the ones for the May case (not shown). Thus around 30% of the variance of $q$ is explained; a result comparable with the description in Berre (2000), except that there is no coupling at all with divergence in this case.

The situation is very different in precipitating areas, with the divergence as the predominant statistical coupling for humidity in the midtroposphere (Fig. 9d), and the divergence and the vorticity-derived balanced mass for temperature (Fig. 9c). This confirms the interest for considering multivariate relationships for the specific humidity, especially within precipitating areas. Such a relationship may help sustain convection while assimilating humidity-related observations (Caumont et al. 2010) and Doppler winds (Montmerle and Faccani 2009).

For hydrometeors (Fig. 10), the prevalent coupling is with the unbalanced divergence, which is directly linked to the vertical motion through the continuity equation. This does not mean that couplings with temperature or humidity are small, as a significant part of temperature and humidity is explained by the unbalanced divergence. Furthermore, there is very good agreement between the sum of the explained variance ratios (“total” in Fig. 10) and one minus the unexplained variance ratio (“total 2” in Fig. 10), thus corroborating the fact that the regularization does not significantly affect the orthogonality of the successive regressions. When using the log transform, the relative order of the predictor has been found to differ dramatically: Fig. 11 shows that the prevalent coupling of the variance of the logarithm of rain content are unbalanced humidity and temperature below 800 hPa. Temperature and humidity are better predictants of the relative variations of rain content than of the absolute variations. This highlights that the use of the log transform may significantly alter the multivariate coupling with divergence in the low troposphere. It would be necessary to apply the log transform to unbalanced variables to avoid this.

The use of $q_c$ and $g_r$ variables will cause analysis increments to exhibit a complex structure in all other wind
and mass variables. Additional diagnostics are required to infer physical meaning, and ascertaining of the vertical cross covariances can provide clues to the underlying structure of errors (Berre 2000; Michel and Auligné 2010). Cross covariances of $q$ and $q_c$ with the unbalanced divergence are shown in Fig. 12. The same structure as in MB2010 is observed, with a positive humidity error linked to the low-level convergence and a higher-level divergence in the convective clouds (Fig. 12a). A similar structure exists for cloud content but is vertically displaced toward midlevels (Fig. 12b). The cross covariance of $q_r$ background error with unbalanced divergence is more complex, with a tripolar structure (Fig. 12c). A positive rain increment at the lowest levels is linked with the convergence and the divergence associated with downdrafts below 900 hPa. The cross covariance with unbalanced
Fig. 7. Vertical profiles of background error standard deviations for (a) cloud content and (b) rain content. Same legend as in Fig. 6. Case at 0000 UTC 6 May 2007.

Temperature highlights some impacts of microphysics: the influence of evaporation is noticeable by the occurrence of a negative cross covariance below 850 hPa (the "cold pool" effect, cf. Fig. 12d), and the effect of latent heat release is noticeable by the occurrence of positive cross covariances in the cloud. Similar structure is found when computing the cross covariance between $q$ and $q_r$, with the effect of evaporation below the cloud and the effect of saturation in the cloud (not shown).

At convective scale, the notion of balance has been recently revisited by Caron and Fillion (2010). Their main...
finding is that the vertical motion should be in balance with physical tendencies. This result is believed to be consistent with the large coupling of hydrometeors fields with divergence found here. However, it is likely that the rain increments that will be produced by the heterogeneous scheme will be washed out at the very first steps by the physics package. If the rain increment is big, this may yield unbalance. One possibility would be to use some form of incremental analysis update (Bloom et al. 1996). The question of the balanced character of the analysis increments will be addressed with the first assimilation experiments when the full heterogeneous control variable
c. Autocorrelations

The structure of vertical correlations has also been found to depend strongly on the rain-dependent binning for both total and unbalanced variables. Figure 13 highlights how the total covariances differ for temperature and humidity variables. In particular, the negative correlations that appear in nonprecipitating air between the stratosphere and the troposphere (Fig. 13a) do not exist in precipitating areas (Fig. 13b). Correlations for humidity are broader in precipitating areas and above 850 hPa, reflecting the mixing in clouds. Similar results have been found for the May case (not shown).

Horizontal length scales have been found to be smaller by a factor of 2 in precipitating areas (Fig. 14), which is consistent with the results of MB2010. The mixed bin, which is the cloud/no-cloud situation, has an intermediate length scale, which is likely to be dominated by the displacement error and by the size of convective cells. Temperature and humidity horizontal...
correlations in precipitating areas are tightened toward 600 hPa, which may mean a smaller horizontal mixing.

In practice, the horizontal length scales are diagnosed on unbalanced variables decomposed on EOF modes. This impedes the full representation of the variation of length scales with height (Ingleby 2001) but allows for a weak nonseparability of the tridimensional covariances. The EOF modes, when ranked with their variances, are more and more oscillating on the vertical (Bouttier et al. 1997; Barker et al. 2004; Michel and Auligné 2010) and thus of smaller vertical scale. As shown in Fig. 15, the horizontal length scales of unbalanced temperature (Fig. 15a) and unbalanced humidity (Fig. 15b) in non-precipitating areas decrease with the index of the EOF mode. It ensues that there is a geostrophic scaling between horizontal and vertical scales. This is less pronounced for heavy precipitating areas, which is consistent with the findings of Caron and Fillion (2010) as precipitating areas are “less geostrophic,” because a significant part of the background error is linked to microphysical processes.

The structure of vertical correlations for hydrometeors (shown in Figs. 16a,b) shares some similarities with the results of Amerault and Zou (2006), despite using different models, methodologies and weather phenomena.
The broad vertical correlations for $q_c$ in the midtroposphere denotes the vertical mixing within precipitating clouds. Rain is vertically highly correlated under 800 hPa, which is the direct consequence of rain falling freely under this level. A similar shape is found when using the log transform, but the vertical extent of the correlations is reduced (Figs. 16c,d).

The horizontal length scales of cloud and rainwater contents (Fig. 17) reach about 6 km, which is 2 grid spacings and is smaller than for other variables. Similar results have been obtained using the log transform (not shown). For cloud content, the maximum displayed at 950 hPa below the cloud is probably an artifact due to poor sampling at those vertical levels, as it does not appear in the diagnosis of the May ensemble. This is not a problem in practice as length scales are prescribed for unbalanced variables on truncated EOF modes. Errors in rain content tend to be correlated over larger distances at the lowest levels (6 km at 1000 hPa but 5 km at 800 hPa), which may be due to a coalescence by collision. This result shows that hydrometeor errors tend to be less extended on the horizontal than other variables, and this discrepancy should be taken into account in localization procedures used to filter sampling noise in the EnKF. Finer diagnosis based on sample correlations (not shown) indicate that negative correlations occasionally occur at longer range in the midtroposphere, in agreement with the large statistical coupling found with divergence.

**FIG. 13.** Average vertical autocorrelations for temperature in (a) nonprecipitating areas and (c) heavy precipitating areas, and total specific humidity in (b) nonprecipitating areas and (d) heavy precipitating areas as a function of pressure level (contour interval 0.2).
Fig. 14. The $e$-folding horizontal length scale (km) as a function of pressure level for (a) temperature and (b) specific humidity for different rain-dependent binnings. Case at 0300 UTC 29 Mar 2007.

Fig. 15. The $e$-folding horizontal length scale (km) as a function of EOF mode for (a) unbalanced temperature and (b) unbalanced specific humidity for different rain-dependent binnings. Case at 0300 UTC 29 Mar 2007.
5. Conclusions and discussion

Variational assimilation heavily relies on the design of the background error covariance matrix to filter and spread spatially the information brought by the observations. Because of technical and mathematical difficulties in the construction of three-dimensional covariance operators, this matrix is generally simplified and represents only some spatial and temporal average of the true matrix. In contrast, the EnKF uses a sampled background error covariance matrix that is affected by sampling noise but dependent on the meteorological context and on the quality and density of observations.

In this study, a sample from an EnKF is used to build up a background error covariance matrix for 3D-Var in which the computation of statistics considers precipitating and nonprecipitating areas separately. For traditional control variables, the two regions are found to be characterized by different statistics, including bigger variance, larger coupling of humidity, and shorter length scales in precipitating versus nonprecipitating areas. This has two implications, for both the EnKF and 3D-Var: localization can be made more restrictive in precipitating areas for the EnKF, and 3D-Var formulations at cloud scales should be able to represent these major inhomogeneities. In this way, these two algorithms could assimilate more observations in a physically consistent way in precipitating areas where the background uncertainty is known to be large.

Another caveat of background error control variable transforms is that they generally ignore hydrometeors.
In this paper, using the geographical masks, the full covariance structure of background errors for liquid hydrometeors was computed, including the balance and the spatial correlations that are generally neglected. The inclusion of cloud and rainwater contents required the use of regularized regression; otherwise the same framework as for standard variables was used. Significant coupling was found with divergence, and realistic spatial correlations exist, which may help spread of the information from observations to related hydrometeors and then to other variables. It was found that the direct use of the lognormal approach was having a strong impact on the structure of the physical transform, in particular reducing the coupling with divergence.

The computation of the covariances was accomplished for similar values of the rain amount in the background. This is necessary to keep the probability density functions of hydrometeor variables close to the Gaussian case. This may be seen as a (simple) spatial dividing–localization strategy in the terminology of Bocquet et al. (2010). In practice, however, it is not clear yet if the heterogeneous scheme can handle the difficult situations of precipitating background and nonprecipitating observation, as this can happen because of displacement problems. It is possible to produce hydrometeor increment in arbitrary regions by extending and smoothing the geographical masks, but the scheme neglects the non-Gaussianity induced by this situation. This will be explored in future studies, following the ideas developed in Auligne et al. (2010).

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