Inhomogeneous Background Error Modeling for WRF-Var Using the NMC Method

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ABSTRACT
Background error modeling plays a key role in a variational data assimilation system. The National Meteorological Center (NMC) method has been widely used in variational data assimilation systems to generate a forecast error ensemble from which the climatological background error covariance can be modeled. In this paper, the characteristics of the background error modeling via the NMC method are investigated for the variational data assimilation system of the Weather Research and Forecasting (WRF-Var) Model. The background error statistics are extracted from short-term 3-km-resolution forecasts in June, July, and August 2012 over a limited-area domain. It is found 1) that background error variances vary from month to month and also have a feature of diurnal variations in the low-level atmosphere and 2) that u- and v-wind variances are underestimated and their autocorrelation length scales are overestimated when the default control variable option in WRF-Var is used. A new approach of control variable transform (CVT) is proposed to model the background error statistics based on the NMC method. The new approach is capable of extracting inhomogeneous and anisotropic climatological information from the forecast error ensemble obtained via the NMC method. Single observation assimilation experiments show that the proposed method not only has the merit of incorporating geographically dependent covariance information, but also is able to produce a multivariate analysis. The results from the data assimilation and forecast study of a real convective case show that the use of the new CVT improves synoptic weather system and precipitation forecasts for up to 12 h.

1. Introduction
The background error (BE) covariance matrix plays a key role in a variational data assimilation system by weighing the importance of the a priori state by smoothing and spreading information from observation points and by imposing balance between the analysis variables (Daley 1991; Bannister 2008a,b). However, the BE covariance matrix must be estimated because the true state of the atmosphere is not known. Furthermore, the estimated
covariance matrix cannot be directly applied to data assimilation systems because it is numerically unfeasible. The alternative is therefore to derive BE statistics, such as variance and length scale, from the estimated error covariance based on a BE model that employs various assumptions.

Two methods are commonly used to obtain the estimated error covariance from which the BE statistics can be derived. The so-called National Meteorological Center (NMC; now named the National Centers for Environmental Prediction) method (Parrish and Derber 1992) is one approach that is widely employed to estimate the BE covariance. This method uses the differences between forecasts of different lengths, but valid at the same time, to evaluate short-range forecast errors. The forecasts are usually obtained by running a numerical weather prediction (NWP) model for a month or longer. Hence, the estimated error covariance using this method is intended to represent the climatology of the forecast uncertainty. An alternative analysis ensemble method was proposed by Houtekamer et al. (1996) and has been implemented operationally at the European Centre for Medium-Range Weather Forecasts (ECMWF; Fisher 2003). This method is based on an ensemble of assimilation experiments in which the observations are randomly perturbed, as well as the physical parameterizations, to generate an ensemble of short-term forecasts. With this approach, ensembles from different times are applied in order to get a good variability in the “super ensemble.” From each participating ensemble, differences between ensemble members, along with the length of the background forecast, are used to contribute to the covariance calculation.

Since the BE matrix is of high dimensions, assumptions are necessary to model the BE covariance such that it becomes manageable in data assimilation systems. The present numerical weather prediction model typically has a dimension of $10^7$ or more, so the BE matrix has $10^{14}$ elements, which cannot be explicitly used in data assimilation systems. In practice, it is usually assumed that the BE covariance is homogeneous and isotropic with a Gaussian distribution. These assumptions can have a significant impact on extracting BE information from forecast ensembles using either the NMC or the ensemble method.

Another component in BE covariance modeling that can have an impact on the BE statistics is the so-called control variable transform (CVT) that converts NWP model variables to control variables of a data assimilation system and introduces approximate balance relations (Bannister 2008a,b). Two kinds of CVTs in BE modeling for wind are widely used in operational variational data assimilation. Vorticity and (unbalanced) divergence are used in the data assimilation systems at ECMWF (Courtier et al. 1998) and Météo France (Fischer et al. 2005). Stream function and unbalanced velocity potential are widely used as control variables in global data assimilation systems and some regional data assimilation systems (e.g., Ingleby 2001; Barker et al. 2004, 2012; Zupanski et al. 2005; Rawlins et al. 2007; Huang et al. 2009; Wang et al. 2013a).

The NMC method has been employed to specify BE statistics for the variational data assimilation system for the Weather Research and Forecasting (WRF-Var; Barker et al. 2012) Model. WRF-Var assumes that the BE covariance is homogeneous and isotropic in its BE modeling, and it also applies a CVT that transforms $\mu$ and $\nu$ to streamfunction and velocity potential that are used as the momentum control variables. The CVT includes an operation that computes and removes the statistical correlations between streamfunction and velocity potential and between temperature and surface pressure. Apparently, all of these operations in BE modeling can affect the quality of data assimilation results. For example, several studies have found an overestimation of the length scale in WRF-Var via the NMC method, which has a negative impact on WRF-Var analysis. Barker et al. (2004) suggested applying empirical tuning factors to the length scales (ranging between 0.5 and 1). Previous studies (e.g., Xiao and Sun 2007; Sugimoto et al. 2009; Ha and Lee 2012; Li et al. 2012; Sun et al. 2012; Wang et al. 2013b) showed that radar radial velocity data assimilation using the WRF-Var system with reduced length scales improved analyses and forecasts. However, none of these studies have examined the behavior of the BE and the analyses resulting from the tuning. Therefore, one of the objectives of this study is to investigate and understand the characteristics of the modeled BE using the NMC method. Another objective is to present and test a new CVT scheme that has the ability to extract the inhomogeneous and anisotropic climatological information from the estimated error covariance obtained by the NMC method. As far as the authors know, this is the first work using the new proposed CVT to study the climatological BE modeling in the context of the WRF-Var system. We will describe this new scheme and compare its BE features with those from the existing scheme in WRF-Var using single observation experiments and real data experiments.

The WRF-Var system has been extensively used in the research community and operational centers (Barker et al. 2012; Huang et al. 2013). For examples, WRF-Var was adopted in the Rapid Update Cycling Data Assimilation and Forecasting System at the Beijing Meteorological Bureau (BJ-RUC; Chen et al. 2009), which has been in operation since June 2008. The WRF-Var system with radar data assimilation has shown a consistently
positive impact on precipitation prediction (Wang et al. 2013b). The operational application of the WRF-Var system at Taiwan’s Central Weather Bureau has significantly reduced typhoon track forecast errors (Hsiao et al. 2012). Our study using this community system will not only benefit the numerous users of WRF-Var throughout the world, but it is also applicable to other operational variational data assimilation systems.

This paper is organized as follows. Section 2 provides a description of the NMC method and features of the BE statistics over the Beijing region. The new CVT is introduced in section 3. Single observation data assimilation experiments using two CVTs are presented in section 4. The impacts of the new CVT on real data assimilation and forecast for a convective case that occurred in Beijing is presented in section 5. A summary and discussion are given in the final section.

2. Background error modeling

a. The NMC method

A common method to model the BE covariance matrix is to take the difference between pairs of forecasts of different lead times, but each valid at the same time (Parrish and Derber 1992). The ensemble of the forecast differences is usually obtained from forecasts over a reasonably long period of time (e.g., a month). This makes the NMC method suitable for climatological forecast error statistics. In WRF-Var, the background error covariance matrix may be considered by the following expression:

$$
\mathbf{B} = (\mathbf{x}^{24} - \mathbf{x}^{12})(\mathbf{x}^{24} - \mathbf{x}^{12})^T ,
$$

(1)

where \( \mathbf{x}^{24} \) and \( \mathbf{x}^{12} \) are 24- and 12-h forecasts, respectively, valid at the same time. The overbar denotes an average over time and/or space. The two forecasts can be written in terms of “truth” and their errors:

$$
\mathbf{x}^{24} = \mathbf{x}^{\text{truth}} + \mathbf{e}^{24} + \mathbf{b}^{24} \quad \text{and} \quad \mathbf{x}^{12} = \mathbf{x}^{\text{truth}} + \mathbf{e}^{12} + \mathbf{b}^{12} .
$$

(2)

Here, \( \mathbf{x}^{\text{truth}} \) is the true atmospheric state at the valid time, \( \mathbf{e}^{24} \) and \( \mathbf{e}^{12} \) are the random errors, and \( \mathbf{b}^{24} \) and \( \mathbf{b}^{12} \) are the biases in each forecast. Assuming there is no bias or the bias is constant in time, \( \mathbf{b}^{24} = \mathbf{b}^{12} \), the forecast difference is

$$
\mathbf{x}^{\text{diff}} = \mathbf{x}^{24} - \mathbf{x}^{12} = \mathbf{e}^{24} - \mathbf{e}^{12} .
$$

(3)

The BE covariance matrix is written as

$$
\mathbf{B} = (\mathbf{e}^{24} - \mathbf{e}^{12})(\mathbf{e}^{24} - \mathbf{e}^{12})^T = (\mathbf{e}^{24} - \mathbf{e}^{12})(\mathbf{e}^{24} - \mathbf{e}^{12})^T = (\mathbf{e}^{24})(\mathbf{e}^{24})^T + (\mathbf{e}^{12})(\mathbf{e}^{12})^T - (\mathbf{e}^{24})(\mathbf{e}^{12})^T - (\mathbf{e}^{12})(\mathbf{e}^{24})^T .
$$

(5)

It is seen that the BE modeling using Eq. (1) includes three parts: 24-h BE, 12-h BE, and their correlations. In a real data assimilation and forecast system, analyses are usually updated every 6 or 3 h. It suggests that the background error covariance generated by forecast differences with longer lead times needs to be tuned in real applications.

b. Error variance estimation using the BJ-RUC operational forecasts

Before discussing the BE modeling, it is crucial to check features of error variance in a raw dataset of error estimates using Eq. (5). The BE modeling should reproduce those features as much as possible. The ensemble of error estimates, referred to as the NMC ensemble, consists of the differences between 24-h and 12-h forecasts valid at the same times. It is obtained from operational forecasts produced by the BJ-RUC system (Chen et al. 2009) during the period between 1 June and 31 August 2012. All the short-term BE statistics in this study are derived from this NMC ensemble. The statistics are over the inner domain (its coverage and size can be seen in Fig. 1) with 3-km grid spacing.

Horizontal distributions of standard deviation of the error at three model levels are shown in Fig. 1. It is obvious that even over the inside small domain (about 1300 \( \times \) 1600 km\(^2\)), the variance distributions show geographically dependent structures. These features are significant for wind around the 30th model level (approximately 250 hPa), and for temperature around the 11th model level (approximately 850 hPa). It is noted that the WRF-Var uses a domain-averaged error variance; thus, it does not account for these location-dependent features. In the next section, an effective yet efficient scheme is proposed for the location-dependent error variance modeling for WRF-Var. In general, the value of the standard deviation of the wind error is about 3 m s\(^{-1}\) at the 11th (approximately 850 hPa) and 25th (approximately 500 hPa) model levels. The value increases to be about 5 m s\(^{-1}\) at the extratropical jet (33rd) level. For temperature \( T \), the value is about 1 K in most of the domain. For relative humidity, the values of the error are between 10% and 20%. The vertical structures of the standard deviation of the error variances are depicted in Fig. 2. It is seen that the error of the wind and temperature (Figs. 2a–c)
exhibit two maximum regions: one below the 11th model level (approximately 850 hPa) and the other around the 30th model level (approximately 250 hPa). Figures 1 and 2 also reveal that the errors for $u$, $v$, and $T$ in the north region of the model grid are larger than in the south region below the 11th model level. This is related to the surface observation density (Fig. 13a, below) in BJ-RUC, as will be discussed in section 5.
The monthly variations of background errors are found to be rather significant. Figure 3 depicts the BE variances for $u$, $v$, $T$, and RH. It is seen that the BE variances for wind and temperature in June and July are relatively larger than in August. The diurnal variations of forecast error near surface are clearly shown in Fig. 4. The error variances for wind and temperature in the evening low atmosphere, 1200 UTC (2000 local time), are larger than those in the morning, 0000 UTC (0800 local time). The above results indicate that even with the climatological BE statistics, the seasonally and diurnally dependent variances can be achieved. The large temperature error near the surface suggests that surface observations could be important data sources for a regional rapid update cycle data assimilation system. BE covariance accounting for the diurnal variation and geographically dependent error variance may benefit the surface data assimilation.

c. B modeling in WRF-Var

The BE covariance estimated by Eq. (5) is not directly used in a variational data assimilation system. A CVT and some assumptions are made to model BE covariance in an efficient and affordable way. In the WRF-Var system, a control variable transform $\delta x = Uv$ is applied to model background errors, where $v$ represents the control variable vector and $\delta x$ stands for the analysis increment vector. The $U$ transform maps control variables from control variable space to analysis space. The CVT $\delta x = Uv$ is implemented through a series of operations ($U = U_p U_y U_h$; Barker et al. 2004). The default control variables [CV option 5 (CV5)] in WRF-Var include the streamfunction $\psi$, the unbalanced part of velocity potential $\chi$, the unbalanced part of temperature $T$, the unbalanced part of surface pressure $P_s$, and pseudo–relative humidity RH. The term “unbalanced” refers to the residual from the balance with the streamfunction.

The operators $U_p$, $U_y$, and $U_h$ are described in detail by Barker et al. (2004). The horizontal transform $U_h$ models the autocorrelation of a control variable using recursive filters. The horizontal correlations are assumed to be homogeneous (i.e., not dependent on geographic position) and isotropic for each control variable. The vertical transform $U_h$ is performed via an empirical orthogonal function (EOF) decomposition of the vertical component of BE on model levels. The variances and vertical correlations of each control variable are modeled during this stage. In the default CV5, the time- and...
domain-averaged vertical component of the BE is used, assuming that the variances and vertical correlations are constant on each model level and do not depend on geographic positions. In addition, the horizontal correlation for each control variable is not modeled in the physical space, but in the EOF spaces. The physical variable transform \( U_p \) involves the balance transform and conversion of control variables to analysis variable increments. The statistical balance transform is applied in this stage, defined by

\[
\begin{pmatrix}
\psi \\
\chi \\
T \\
Ps \\
RH
\end{pmatrix}
= \begin{pmatrix}
I & 0 & 0 & 0 \\
C_{\chi,\psi} & I & 0 & 0 \\
C_{T,\psi} & 0 & I & 0 \\
C_{Ps,\psi} & 0 & 0 & I \\
C_{RH,\psi} & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\psi \\
\chi \\
T \\
Ps \\
RH
\end{pmatrix},
\]

where \( I \) is the identity matrix, and \( C_{\chi,\psi}, C_{T,\psi}, \) and \( C_{Ps,\psi} \) stand for statistical regression matrices between \( \chi, T, \) and \( \psi \), respectively. The analysis variables of \( u \)-wind (\( u \)), \( v \)-wind (\( v \)), and specific humidity (\( q \)) can be obtained by a transform as follows:

\[
\begin{pmatrix}
u \\
v \\
\chi \\
P_s \\
RH
\end{pmatrix} = \begin{pmatrix}
C_{u,\psi} & C_{u,\chi} & 0 & 0 & 0 \\
C_{v,\psi} & C_{v,\chi} & 0 & 0 & 0 \\
I & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & C_{q,\chi,\psi}
\end{pmatrix}
\begin{pmatrix}
\psi \\
\chi \\
T \\
P_s \\
RH
\end{pmatrix}.
\]

\[
(7)
\]

Fig. 3. Profiles of forecast error in terms of std dev for (a) \( u \), (b) \( v \), (c) \( T \), and (d) RH estimated by the NMC method for June, July, and August 2012.
d. Features of variance and correlation modeling

With the CVT and additional assumptions previously described, a natural question to ask is, what information is lost/filtered out by using CV5 in WRF-Var? To answer this question, we compare the error standard deviation directly computed from the NMC ensemble with that modeled by CV5 in WRF-Var.

The standard deviation in the 3-month forecasts directly estimated by Eq. (1) without CVT is named STD_NMC. The standard deviation derived from the CV5 BE statistics is named STD_CV5. This is achieved by the following steps: 1) A Gaussian resampling with 200 samples of the control vector is performed using the variance and length scales in the CV5 control variable space; 2) the control variables are transformed back to the analysis variables using $\delta x = \mathbf{U} \delta v$; and 3) the STD_CV5 is computed in the analysis variable space.

The vertical profiles of STD_NMC and STD_CV5 are plotted in Fig. 5. It is seen that standard deviations (STDs) of all the variables are underestimated, especially for $u$ and $v$, suggesting that the CVT that converts the wind components to streamfunction and velocity potential may have contributed to their significant underestimates. As mentioned in the introduction of this paper, a common remedy for the problem used in the past was to reduce the length scales. It is shown in Fig. 5 that the $u$ and $v$ variances (STD_CV5_L05) are comparable to STD_NMC when the length scales are cut by half. This is not surprising, because the velocity components are related to the gradients of streamfunction and velocity potential. Reducing the length scales of the

![Fig. 4. Profiles of forecast error in terms of std dev for (a) $u$, (b) $v$, (c) $T$, and (d) RH estimated by the NMC method at 0000 UTC (black curve) and 1200 UTC (red curve).](image-url)
streamfunction and velocity potential increases their gradients and, hence, results in higher values of velocity increments. However, as will be seen next, the artificial tuning can cause other problems.

Horizontal correlations for $u$ and $v$ from the NMC ensemble and from the CV5 BE covariance matrix (CV5 B) at the 25th model level (approximately the 500-hPa surface) along the east–west and north–south directions are shown in Fig. 6. The correlations using the NMC ensemble are directly calculated so that they can be used as reference for the time- and domain-averaged statistics from the CV5 B. It is seen that the correlations from CV5 B decrease more slowly, compared with those calculated from the NMC ensemble in a shorter range, resulting in a significantly longer correlation length for both $u$ and $v$ at correlation $= 0.5$. It is noted that the correlations for $v$ in the east–west (north–south) direction and for $u$ in the north–south (east–west) direction are almost the same. Moreover, the correlations for $v$ in the east–west direction and for $u$ in the north–south direction display negative values with substantial magnitude. Xie and MacDonald (2012) pointed out that the long-distance negative correlation is an inevitable numerical problem when using the streamfunction–velocity potential as momentum control variables. The problem of negative correlations is even magnified as the length scales are tuned to half of their computed values.

The previous comparisons of the CV5-modeled BE statistics with the NMC statistics reveal rather significant differences between them. Besides the issue associated with the CVT, other possible reasons could include the facts that CV5 BE modeling is unable to account for the spatial variation and anisotropic nature of the background error, the horizontal correlation modeling is carried out on EOF (decomposed from time-averaged vertical covariance) spaces, and only one Gaussian fitting is used to model the pattern of the horizontal correlations.
3. New CVT for BE modeling: Alpha control variable

In this section, a new CVT is proposed to account for the inhomogeneous and anisotropic background error statistics in the NMC ensemble. This is done by a control variable transform that assumes the analysis increment is a linear combination of the estimated errors in the NMC ensemble. Or, in other words, the analysis increment is expressed in a subspace expanded by the NMC ensemble:

\[
x' = \sum_{k=1}^{K} (\alpha_k \cdot \mathbf{x}_k^d),
\]

where \( K \) is the total number of ensemble members, and the vector \( \mathbf{x}_k^d(k = 1, K) \) is the \( k \)th unbiased element in the NMC ensemble normalized by \( K^{1/2} \),

\[
\mathbf{x}_k^d = (\mathbf{x}_k^\text{diff} - \bar{\mathbf{x}}) / \sqrt{K},
\]

where \( \mathbf{x}_k^\text{diff} \) is the forecast difference defined by Eq. (4). In practice, the time-averaged bias \( \bar{\mathbf{x}} \) is removed from

FIG. 6. Horizontal correlations for \( u \) (filled squares) and \( v \) (open circles) in the NMC ensembles (black curves), by BE modeling using CV5 (red curves) and CV5 with length scales reduced by 0.5 (blue curves) at the 25th model level (approximately 500-hPa surface) along the (top) east–west and (bottom) north–south directions.
the forecast differences. Note that the vector $x_d^k$ includes the model prognostic variables as its elements. The vector $a_k$ stands for the control variables for the $k$th forecast difference. The symbol $\circ$ denotes the Schur product of the vectors $a_k$ and $x_d^k$.

A similar transform to Eq. (8) was used by Wang et al. (2008) in the development of a WRF-based hybrid ensemble-3DVar data assimilation system. However, in their scheme, an ensemble of forecast perturbations was used to incorporate the flow-dependent error covariance of the day. Here we use the similar transform, but on the NMC ensemble that represents climatological errors. Note that the hybrid ensemble-3DVar scheme proposed by Wang et al. (2008) still depends on the climatological BE that we aim at improving through this study.

The analysis increment $x^0 = \sum_{k=1}^K (a_k \circ x_d^k)$ is achieved by minimizing the following cost function:

$$J(a) = \frac{1}{2} (a)^T A^{-1} (a) + \frac{1}{2} (d - H'x')^T R^{-1} (d - H'x'),$$

where $d$ denotes innovation, $H'$ is the linearized observation operator, and $a$ is a vector formed by concatenating $K$ vectors $a_k$. The vector $a$ will be called the alpha control variable (CVA) in this paper. The quantity $A$ defines the horizontal and vertical correlation localizations of $a$. Specifically, the horizontal and vertical correlation localizations are implemented through recursive filters (Wang et al. 2008) and a vertical correlation matrix, respectively.

A general formulation to form the vertical covariance matrix can be written as

$$\text{Cov}(l_1, l_2) = \exp \left( -\frac{d(l_1, l_2)^2}{D(l_1)^2} \right),$$

where $\text{Cov}(l_1, l_2)$ represents the correlation between model levels $l_1$ and $l_2$, $d$ is the distance in a specified coordinate between model level $l_1$ and $l_2$, and $D$ stands for the level-dependent vertical localization radius.

The default vertical correlation matrix in WRF-Var is defined in model level space: specifically, $d(l_1, l_2) = l_2 - l_1$, $D(l_1) = 10(l_1/N)$, and $N$ is the total number of model levels.

$$\text{Cov}(l_1, l_2) = \exp \left( -\frac{(l_2 - l_1)^2}{10(l_1/N)^2} \right),$$

It is seen that the level-dependent localization radius $D(l_i)$ only depends on the number index of the model level. This indicates that an observation at the model level with a large number index will be widely spread in the vertical direction, since WRF has a pressure-based vertical coordinate. This may reduce the impact of observations that are located in low model levels.

In addition to the above formulation [Eq. (12)], a specific application of Eq. (11) in the height coordinate is also tested:

$$\text{Cov}(l_1, l_2) = \exp \left( -\frac{[Z(l_2) - Z(l_1)]^2}{[\Delta Z(l_1)]^2} \right),$$

where $Z$ is domain-averaged height at a model level. For simplicity, a constant vertical localization radius $\Delta Z(l_i)$ is used, as in Li et al. (2012). Figure 7 compares the

![Figure 7](image-url)

**FIG. 7.** The correlation matrices for the alpha control variable localization using (a) Eq. (12) and (b) Eq. (13).
patterns of the correlation matrices resulting from these two localization schemes. In the following single observation experiments, the above two vertical localization specifications (Fig. 7) will be tested.

4. Single observation data assimilation experiments

The B matrix weights the background state and spreads out observation information in horizontal and vertical directions. Increments from single observation experiments can be used to estimate BE variance and demonstrate how the BE covariance spreads the observation information spatially, providing a graphic representation of the BE structure function (Huang et al. 2009; Gustafsson et al. 2012). The increment $\delta x^u$ at the single observation location can be expressed in a scaled form:

$$\delta x^u = (\sigma^2d)/(\sigma^2 + e^2), \quad (14)$$

where the scale $d$ is innovation, $e^2$ is the observation error variance, and $\sigma^2$ is the BE variance in observation space. If $d$, $e^2$, and $\delta x^u$ are known, the BE variance $\sigma^2$ can be easily derived from Eq. (14), as follows:

$$\sigma^2 = \frac{e^2\delta x^u}{d - \delta x^u}. \quad (15)$$

a. Experimental design

Six single observation data assimilation experiments are conducted to compare the BE covariance structures between CVA and CV5. A summary of these experiments is provided in Table 1. The single observation is set either at location (271, 212, 25) or at location (364, 150, 15) in the model grid with the intent of evaluating the geographical dependence of the BE. The two locations are named P1 and P2 (see white dots in Figs. 8a,b). The experiments CV5-P1 and CV5-L05-P2 are designed to see the impact of CV5 BE and its tuning on analysis increments. The scaling factor of 0.5 is adopted from BJ-RUC to reduce the length scales (Wang et al. 2013b) for CV5-L05-P2. Four experiments using that CVA are conducted. The first two experiments (CVA-L300-P1 and CVA-L300-P2) are without vertical localization, and the other two (CVA-L100-P1-VL1 and CVA-L100-P2-VL1) are with vertical localization to examine the influence of the vertical localization scheme. The numbers 300 and 100 in the names represent the horizontal length scales. The suffixes “VL1” and “VL2” represent the vertical localization schemes represented by Eq. (12) and Eq. (13), respectively.

b. Analysis increments

In this section, first the experiments assimilating a single $u$ observation are presented. The innovation is $1.0 \text{ m s}^{-1}$, and the observation error is $1.0 \text{ m s}^{-1}$. The variance scaling factor for each control variable is 1.0, which is the default value in WRF-Var. We will examine the horizontal and vertical structures of $u$ analysis increments. Then the multivariate features of analysis increments in selected experiments will be described.

1) Structures of $u$ increments

The horizontal $u$ increments on the 25th model level (approximately the 500-hPa pressure level) are shown in Fig. 8. The vertical south–north cross sections of the $u$ increments across the single observation location are shown in Fig. 9. The maximum value of the $u$ analysis increments in CV5-P1 is 0.66 (Figs. 8a, 9a). The corresponding BE standard deviation derived using Eq. (15) is 1.39, indicating again that CV5 underestimates the wind variance. This result is consistent with the BE standard deviation estimation presented in section 2b (Fig. 5a). In contrast, the four experiments using CVA produce the maximum analysis increment of 0.88 or 0.89 (Figs. 8c–f and 9c–f). The corresponding BE standard deviation $\sigma$ derived using Eq. (15) is about 2.8, which is obviously a better estimate.

Comparing the horizontal spread of the single observation between Figs. 8a and 8c, and between Figs. 9a and 9c, it is found that CV5 produces a wider spread that is consistent with the correlation in Fig. 6. By reducing the length scale by half, the maximum value of the $u$ increment in CV5-P2-L05 (Figs. 8b, 9b) is increased to $0.88 \text{ m s}^{-1}$, which is almost the same as in the experiment using CVA (Figs. 8d,f). This may partially explain why radar data assimilation using the WRF-Var system with reduced length scales improved analyses and forecasts.
Fig. 8. The structures of the $u$ increments on the 25th model level for the single $u$ observation experiments (a) CV5-P1, (b) CV5-L05-P2, (c) CVA-L300-P1, (d) CVA-L300-P2, (e) CVA-L100-P1-VL1, and (f) CVA-L100-P2-VL1. The color scale $u$ runs from $-0.5$ to $+0.5$ m s$^{-1}$.

The vertical localization scheme [Eq. (12)] works well to keep observation information close to the observation location, as shown in Figs. 9e and 9f. Compared to Figs. 9c and 9d, the negative $u$ increment in the low-level atmosphere does not occur. At the 25th level where that observation is placed, the information can be spread about 5 km in the vertical direction (Fig. 7a).

2) MULTIVARIATE ANALYSES BY ASSIMILATING A SINGLE $u$ OBSERVATION

The features of the multivariate analysis increments are shown in Figs. 10 and 11. In Fig. 10, the $v$ increments from the experiments CV5-P2, CVA-L300-P2-VL1, and
CVA-L100-P2-VL1 are plotted. The pattern of the $\nu$ increment in CV5-P2 (Fig. 10a) results from the isotropic $u$ increment shown in Fig. 8a through the balance response of the CVT in CV5. In contrast, the patterns from the two CVA experiments clearly show irregularity corresponding to the anisotropic BE in CVA. Comparing experiments CVA-L300-P2-VL1 (Fig. 10b) and CVA-L100-P2-VL1 (Fig. 10c), it is obvious that the smaller
FIG. 10. The structures of the $v$ increments at the 25th model level in the single $u$ data assimilation experiments (a) CV5-L05-P2, (b) CVA-L300-P2, and (c) CVA-L100-P2. The color scale runs from $-0.1$ to $+0.1$ m s$^{-1}$.
FIG. 11. Similar to Fig. 10, but for the $T$ increments. The color scale runs from $-0.1$ to $+0.1$ K.
horizontal localization length scale limits the increment to a local area around the single observation. Similar to Fig. 10, Fig. 11 shows $T$ increments from the three experiments. Again, the patterns from the two CVA experiments suggest anisotropic characteristics. It is also noted that the amplitudes of the $T$ increments are small, indicating that the correlations between temperature and $u$-wind are weak. Large sensitivity to the horizontal localization scales (Figs. 11b,c) is found when CVA is used. It appears that some noises in the CVA experiments exist because of the limited sample size. However, it is expected that the use of data from a dense observing network, such as radar, satellite, and surface stations, will alleviate these sampling noises. In addition, the use of an additional flow-dependent ensemble of forecasts valid at the assimilation time could also help reduce the sample noises.

c. Sensitivity to vertical localization schemes

As shown in Fig. 7a, the default vertical localization scheme [Eq. (12)] has a small vertical localization radius in the low-level atmosphere. Experiments with the vertical localization scheme [Eq. (13)] are conducted to show the analysis sensitivity to vertical correlation matrix specifications. In the experiments using the vertical localization matrix [Eq. (13)], the localization radius is 3 km, which has been previously used for radar radial velocity assimilation (Li et al. 2012). The three experiments CVA-L100-P2-VL2, CVA-L100-P3-VL1, and CVA-L100-P3-VL2 are listed in Table 2. The location P3 stands for a single observation location at (364, 150, and 15) in the model grid.

The vertical west–east section of the $u$ increments for the single $u$ observation experiments CVA-L100-P2-VL2, CVA-L100-P3-VL1, and CVA-L100-P3-VL2 are shown in Fig. 12. Comparing the experiments CVA-L100-P2-VL1 and CVA-L100-P2-VL2 (Fig. 9f and Fig. 12a), the vertical correlation schemes produce almost the same analysis increment structures. But for an observation at the 15th model level, the observation spread in the vertical direction is very different (Figs. 12b,c). Compared to the default formulation Eq. (12), Eq. (13) might be more efficient to spread observation vertically in the low-level atmosphere.

5. Real data assimilation and forecasting experiments

In this section, the impact of CVA on data assimilation and forecasts is examined using real data from a convective case that occurred on 21 July 2012 in Beijing. First, the synoptic background of the case will be briefly described, and then the experimental setup and results will be presented.

a. The Beijing “7.21” extreme rainfall event

A warm-season intense convective system occurred in Beijing on 21 July 2012 that caused extreme precipitation, with hourly rainfall rates exceeding 85 mm.
The 700-hPa large-scale circulations and moisture fields from the National Centers for Environmental Prediction (NCEP) Global Forecast System (GFS) 0.5° × 0.5° analyses at 0000 UTC and 1200 UTC 21 July (shown in section 5c, in Fig. 15 below) indicate that the heavy rainfall took place in the favorable synoptic environment of a southwesterly monsoonal flow ahead of a low-pressure vortex with high low-level moisture over the southwest region of Beijing. It is seen that a northeast–southwest-oriented trough axis and a well-defined wind-shift line associated with a low-level vortex were moving northward into southern Beijing. The southerly flow supplied moisture for this extreme precipitation event.

b. Experimental setup

Cycling data assimilation and forecast experiments are conducted to investigate the impacts of proposed background error formulation on analyses and forecasts. All the data assimilation and forecast experiments are over the inner domain of the BJ-RUC operating at the Beijing Meteorology Bureau, since the background error statistics presented in the previous sections are carried out over this domain. The initial and boundary fields are interpolated from NCEP GFS 0.5° × 0.5° analyses and forecasts at 1200 UTC 20 July 2012. The 3-hourly data assimilation starts at 1800 UTC 20 July 2012 and ends at 0000 UTC 21 July 2012. The GFS 6-h forecast at 1800 UTC 20 July 2012 provides the background in the first cycle of the data assimilation experiment.

Five numerical experiments (Table 2) are conducted to examine the impacts of the background error covariance modeling on analyses and forecasts. The control experiment (CV5-L05) is the 12-h 3-km forecast initiated from the WRF-Var analysis using CV5 at 0000 UTC 21 July 2012. In CV5-L05, the operational tuning factor with a value of 0.5 is used for each control variable. The forecasts in the experiment using CV5 without tuning is not shown in this paper because it is less accurate than CV5-L05. The four data assimilation experiments using CVA are named CVA-L100, CVA-L60, CVA-L100-VL2, and CVA-L60-VL2, respectively. These experiments are used to show the sensitivity of data assimilation to the horizontal localization length scale with and without the vertical localization scheme [Eq. (13)]. The precipitation forecast skill in an experiment with the length scale of 300 km using CVA is not as good as those with the smaller length scales, and its result is not shown. This is possibly caused by noises introduced into analysis increments with large horizontal localization length scale (Figs. 9c,d, 10b, and 11b). In the convective scale data assimilation using high-density observations, the small length scale may be effective to extract small-scale information in the observations. It has been reported that a length scale with a value of 60 km gives good forecasts for radar radial data assimilation (Li et al. 2012).

A 12-h forecast is made for each analysis at 0000 UTC 21 July 2012. In addition, we do not run cycles at later times (e.g., at 0300 UTC) because the precipitation began at 0200 UTC 21 July 2012. The forecasts after that time have less value to forecasters. All the operational data, including radar radial winds, are assimilated. The examples of the distributions of observations assimilated are shown in Fig. 13. The operational observations include radiosonde, surface synoptic observations (SYNOP), automated weather system (AWS) surface
observations, global positioning system precipitable water (GPS PW), aircraft data, and radar radial velocity.

c. Results

1) SYNOPTIC WEATHER SYSTEM ANALYSIS AND FORECAST

We first examine the analysis increments to show the influence of CVA on data assimilation. The wind increments at 700 hPa and precipitable water increments from the experiments CV5-L05 and CVA-L100 in the first cycle at 1800 UTC 20 July 2012 are shown in Figs. 14a and 14b. At the time, the SYNOP, AWS, GPS PW, and radar radial wind data are assimilated. The surface data and GPS PW data have almost the same distribution as those at 0000 UTC 21 July shown in Fig. 13a.

From Figs. 14a and 14b, it is seen that, though the locations of maximum analysis increments for the wind and humidity in the two experiments are similar, the patterns of the increments are quite different. The location-dependent error modeling benefits the analysis in CVA-L100-VL2 by producing the increments with large amplitudes in the north region of the model domain. The reason is that, in general, the error variances in the north region are larger than those in the south region at the low model level (Figs. 1 and 2). It is also seen that using location-dependent error covariance capable of accounting the correlation of humidity to other variables leads to significant differences in the humidity analysis increments in terms of amplitudes, as well as horizontal structures.

The analyses and 12-h forecasts are compared to the GFS analyses. The GFS analyses are produced by the NCEP hybrid variational–ensemble data assimilation system at T574 resolution (~27 km). It is seen that the low-pressure vortex system and associated large wind speeds are well represented in the analyses using CVA (Figs. 15c,e,g). The main differences between the CVA and CV5 experiments are in humidity analyses (Figs. 15c,e,g). The experiments using CVA produce better humidity analyses in terms of precipitable water (PW) that are closer to the GFS analysis. As for the 12-h forecast, the experiment CV5-L05 failed to predict the low-pressure vortex that is well predicted by the experiments using CVA. All the experiments yield good precipitable water forecasts. The maximum wind speed forecasts in all the experiments have a southward location bias. However, the location bias is reduced in the experiments using CVA.

2) PRECIPITATION VERIFICATIONS

The precipitation forecasts are verified against radar quantitative precipitation estimates (QPE). Neighborhood-based fractions skill score (FSS; Roberts and Lean 2008) is used to examine the forecast skill of hourly precipitation. If a forecast is perfect, the value of FSS is 1.0. The hourly radar QPE that is produced operationally at the Beijing Modeling Bureau (BMB) is taken as the observation for verifying the model forecasts. Figure 16 shows FSS for the thresholds of 1 mm h$^{-1}$ (Fig. 16a) and 10 mm h$^{-1}$ (Fig. 16b). Overall, the experiments using CVA produce improved FSSs over the CV5-L05 experiment. The two experiments with vertical localization yield better forecasts than the ones that do not use it.
FIG. 15. The 21 Jul 2012 (left) 0000 UTC analyses and (right) 1200 UTC forecasts [except for (b)]. (a), (b) The GFS analyses, (c)–(d) CV5-L05, (e)–(f) CVA-L100-VL2, and (g)–(h) CVA-L100. Precipitable water (mm) is given by the color-shaded contours, wind speed > 15 m s⁻¹ is given by red contours, and geopotential height is shown by blue contours.
Figure 17 shows the 12-h accumulated precipitation and radar QPE at 1200 UTC 21 July 2012. It is seen that the experiment CV5-L05 misses the heavy precipitation (above 50 mm per 12 h as shown by the red shade) within the Beijing city limits by a southwest displacement error. In comparison, the CVA-L100-VLoc and CVA-L60-VLoc experiments reduce the location error in the heavy precipitation. For the hourly precipitation (not shown), for example, at 1200 UTC, the CV5-L05 has a southwest displacement bias, which is noticeably reduced in the experiments using CVA. In summary, the results show that using CVA improves the synoptic weather system and precipitation forecasts up to 12 h.

6. Summary and discussion

In this paper, the features of background error modeling via the NMC method are investigated for the WRF-Var system. The aim of this work is to further improve the performance of the WRF-Var system through the best use of the climatological background error covariance estimation. A new CVT is proposed and described to incorporate climatological via the NMC method in WRF-Var. To the authors’ knowledge, this is the first work using the proposed CVT to study climatological BE modeling in the context of the WRF-Var system. The new CVT scheme will also benefit other components, such as 4DVar and hybrid...
Var-ensemble data assimilation in the WRF community data assimilation system (Barker et al. 2012; Huang et al. 2013).

The features of CV5 and CVA are investigated in detail using the short-term regional 3-km-resolution forecasts in June, July, and August 2012 from BJ-RUC. The results show that the location-dependent background error variances vary from month to month and also have a feature of diurnal variations in the low-level atmosphere. WRF-Var CV5 BE modeling underestimates wind error variance but overestimates wind error length scale.

The main work related to CVA is summarized as follows:

![Figure 17](image-url)
• CVA gives good variance modeling in the NMC ensembles, as shown by the single observation assimilation experiments.
• CVA has the benefits of incorporating geographically dependent covariance information and producing multivariate analysis.
• CVA is capable of accounting for the error correlation between humidity and other analysis variables.

The results from a real data assimilation and forecast study for a real convective case show that the use of CVA improves the synoptic weather system and precipitation forecast for up to 12 h. The new proposed CVT will be investigated with more cases in future research. In WRF-Var, the climatological statistical correlations between relative humidity and other control variables can be taken into account with CV option 6 (CV6; Chen et al. 2013). The CVA provides another candidate to model climatological statistical correlations between relative humidity and other analysis variables. Moreover, the diurnal variations of variances can be considered in climatological BE modeling in WRF-Var using CVA. The BE covariance accounting for the diurnal variation and geographically dependent error variance may benefit the surface data assimilation. This will be investigated in future studies.

In reality, the BE covariance may be substantially flow dependent. The current BE statistics using the NMC method may not be optimal to provide the BE covariance of the day for mesoscale and convective-scale data assimilation. A super ensemble, including the NMC ensemble and short-term ensemble forecasts, could be used to blend climatological error covariance and flow-dependent error covariance of the day in a hybrid system.

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REFERENCES


