Geometrical optics phase matching of radio occultation signals

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[1] Remote measurements of the atmospheric state can be performed by radio occultation between satellites, a GPS satellite transmitting a radio signal to a receiving low Earth orbit (LEO) satellite, or between LEO satellites. The bending angle of the traversing optical ray can be measured by detecting the Doppler shift of radio signals. The bending angle is an integrated measure of the refractive index in the atmosphere traversed by the optical ray. With a time series (profile) of the bending angle, it is possible to perform an inversion to obtain the refractive index. Various techniques for the retrieval of the bending profile already exist. The most recent methods have solved the problem of multipath, i.e., when the atmosphere allows several rays to exist simultaneously. The paper presents a new method for the reconstruction of bending angle as a single-valued function of impact parameter from complex radio occultation signal under multipath propagation conditions. The method utilizes the assumption of spherical symmetry of refractivity, i.e., $n(\vec{r}) = n(r)$, and the principle of synthetic aperture, thus allowing sub-Fresnel resolution. A distinctive feature of the method, as compared to previously known methods, is direct applicability for arbitrary orbits of transmitting and receiving satellites, without intermediate propagation of complex electromagnetic field to circle or straight line. This comes at the expense of impossibility to reduce the method to a FFT. The method can be useful for inverting radio occultation signals and for validation of other radio holographic methods. *INDEX TERMS:* 6904 Radio Science: Atmospheric propagation; 6964 Radio Science: Radio wave propagation; 6969 Radio Science: Remote sensing; 6974 Radio Science: Signal processing; *KEYWORDS:* radio occultation

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1. Introduction

[2] In general terms, the radio occultation technique relies on the bending of radio waves caused by refractive index gradients in a planetary atmosphere [e.g., *Kursinski et al.*, 1997, 2000]. In Global Navigation Satellite System (GNSS) radio occultations the observations are based on the bending of radio waves traversing the Earth's atmosphere from a GNSS satellite to a Low Earth Orbit (LEO) satellite. The basic observation is the Doppler shift of the radio wave from which the impact parameter can be retrieved and subsequently the bending angle with the knowledge of the positions and the velocities of the satellites. The retrieved pairs of bending angle and impact parameter can be inverted through the Abel transform to yield the index of refractivity as function of height [*Fjeldbo et al.*, 1971].

[3] The propagation of radio waves can be described as rays according to geometrical optics when diffraction effects in the refractive medium can be neglected. The methods of processing radio occultation data based on Fourier Integral Operators, such as Canonical Transform [*Gorbunov*, 2002a, 2002b] and Full Spectrum Inversion [*Jensen et al.*, 2003], use an asymptotic solution, which generalizes the standard geometrical optics (M. E. Gorbunov et al., Comparative analysis of radio occultation processing approaches based on Fourier integral operators, submitted to *Radio Science,* 2004) (hereinafter referred to as Gorbunov et al., submitted manuscript, 2004). This solution has a weaker limitation of applicability as compared to the standard geometrical optics. For GPS frequencies, the asymptotic description is valid for inhomogeneities with scales exceeding 60 m (Gorbunov et al., submitted manuscript, 2004). Inhomogeneities with smaller scales

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cannot be resolved. Radio waves transversing the stratosphere and upper troposphere are generally subject to moderate ray bending, and the measured signals from these portions of the atmosphere are normally single tone signals as only one ray connects the transmitter and the receiver at the same instant.

[4] In single ray regions, the computation of bending angles is straightforward as they are unambiguously related to the instantaneous frequency of the received signal. Therefore radio occultation measurements have traditionally been based on the instantaneous frequency. However, radio signals propagating through the lower troposphere may have a very complex structure due to atmospheric multipath effects caused mainly by water vapor structures [*Gorbunov et al.*, 1996; *Gorbunov and Gurvich*, 1998; *Gorbunov et al.*, 2000; *Sokolovskiy*, 2001]. In regions that exhibit multipath, the bending angles cannot be derived directly from the instantaneous frequency of the measured signal because this frequency will be related, not to a single pair of bending angle and impact parameter, but to a number of pairs. Another drawback of retrieving the bending angle directly from the instantaneous frequency is that the vertical resolution is limited by the size of the Fresnel zone. Thus there has been much effort in developing techniques with high vertical resolution that are capable of correctly retrieving the bending angle profile in multipath regions [*Gorbunov*, 2001]. So far, six high resolution methods have been proposed for processing of radio occultation signals in multipath regions: (1) back-propagation [*Gorbunov et al.*, 1996; *Hinson et al.*, 1997, 1998; *Gorbunov and Gurvich*, 1998], (2) radio-optics, [*Lindal et al.*, 1987; *Pavelyev*, 1998; *Hocke et al.*, 1999; *Sokolovskiy*, 2001; *Gorbunov*, 2001], (3) Fresnel diffraction theory [*Marouf et al.*, 1986; *Mortensen and Høeg*, 1998; *Meincke*, 1999], (4) canonical transform [*Gorbunov*, 2001, 2002a, 2002b], (5) a Fourier transform method [*Jensen et al.*, 2002a], and (6) the Full Spectrum Inversion (FSI) method [*Jensen et al.*, 2003]. In terms of resolution and ability to handle multipath, the most efficient radio holographic methods are currently the Fourier operator based methods; canonical transform and FSI. The canonical transform requires that the electromagnetic field is back-propagated to a straight line before the canonical transformation can be applied. Due to the inherent back-propagation the canonical transform is demanding in terms of CPU power.

[5] The Fourier transform method has theoretical limitations in the respect that it only works perfectly with circular satellite orbits. However, corrections developed in the aforementioned reference, which lead to the FSI method, can extend the validity area of the Fourier method to cover near-circular/realistic satellite orbits. In simulation studies, using realistic orbits, we have not experienced situations where the validity of these corrections has been exceeded.

[6] This paper presents a new high-resolution radio-holographic method, which uses a phase matching or a transform of the occultation signal along the receiver orbit. The advantage of this technique is that it automatically accounts for the shape of the satellite orbits without any approximate corrections. The phase matching technique can be described in general terms, but the method can only be used with an explicit phase matching function. Here two explicit, or special forms of the phase function, are considered. In the first form, the method is identical to the Fourier method [*Jensen et al.*, 2002b], whereas the second and new form is a model derived from the geometrical optical phase in vacuum. The phase matching method can also be viewed as Fourier Integral Operator (FIO) method and is theoretically related to the canonical transform method developed by *Gorbunov* [2002a, 2002b]. In *Gorbunov and Lauritsen* [2002] a canonical transform without back-propagation has been considered. In this work, the differential equations for the specialized case mentioned above have been stated, but no solutions were obtained. However, the method presented here can be shown to represent a solution to these differential equations (M. E. Gorbunov, personal communication, 2003).

[7] The theory of phase matching is based on the synthetic aperture concept and the method of the stationary phase originally used in the FSI method. It will be shown, that the phase matching method has the same high-resolution properties as the Fourier method, is applicable for noncircular orbits as the FSI method, and in addition, makes a perfect cancellation of the defocusing factor.

[8] The new method is strongly related to the Fourier and the FSI method, as will be revealed in the following. There are two basic concepts in the Fourier and the FSI methods: the path described by the receiving antenna is considered as a synthetic aperture so the temporal signal is being processed as a whole in order to achieve the maximum resolution in space. The second concept is that each ray is uniquely identified by its Doppler frequency when an unambiguous relation between impact parameter and ray Doppler frequency exists. This is the case for circular and nearly circular orbits when the corrections embedded in the FSI method are applied. When the above mentioned concepts are utilized a Fourier transform of the entire occultation signal will yield the ray Doppler frequency as function of time [*Jensen et al.*, 2003]. From the measured Doppler frequency as function of time the bending angle as function of the impact parameter, and finally the refractivity profile are retrieved using the Abel inversion technique.

[9] The novelty of the phase matching method, compared to previous work, is firstly its direct application to a radio occultation signal measured along a noncircular orbit, and secondly, the explicit form of the derived phase matching function. Like the Fourier method and the FSI method the phase matching method will be derived within the framework of pure geometrical optics, which compared to earlier analysis both is a considerable simplification and also emphasizes the ray nature of radio occultation signals. In *Jensen et al.* [2002b], scales and resolution for the Fourier method have been derived from geometrical optics and from simple diffraction consideration. The use of simple geometrical optics does not, however, describe all properties related to radio occultation signals, but it is important to clarify what can be described with geometrical optics, and what are remaining of diffraction problems.

2. Phase Matching

[10] In the following, the general concept of phase matching in relation to radio occultation signals will be established. This technique has emerged from analysis of the physical nature of the radio occultation signal based geometrical optics, as will be shown in the last part of this section.

[11] The occultation signal represents the field of the received radio signal by the LEO satellite from the transmitting GPS satellite. The field at the receiving aperture is in general a superposition of the fields from all the optical rays that have traversed the atmosphere. So, the occultation signal can be considered as a sum of the detected fields from each ray path characterized by the Doppler frequency of the individual rays. The case of only one ray is referred to as single path, whereas the case of several rays is referred to as a multipath situation.

[12] Given a complex (radio occultation) time signal, expressed as a sum of all individual rays received at the LEO satellite, $f(t) = \sum A_j(t) \exp(i\psi_j(t))$ we can define a phase transform as follows:

$$M(c) = \int_0^T f(t)\exp(-i\psi_o(c,t))dt$$
$$= \sum \int_0^T A_j(t)\exp\Big(i\psi_j(t) - i\psi_o(c,t)\Big)dt, \quad (1)$$

where the phase $\psi_0(c, t)$ is a function of time t, the parameter c defines the transform space. $A_j(t)$ and $\psi_j(t)$ are the amplitude and the phase of the ray numbered by j, and T is the duration of the occultation. The parameter c can in principle take any value and the phase transform can be viewed as a transformation from the time domain to a c parameter domain. In addition, associated with the phase, an amplitude factor (slowly varying) could be used so the transform will be a phase as well as an amplitude match; this will be done later.

[13] The phase matching technique can be considered as an advanced use of least squares fitting; e.g., in single path regions a least squares fit between the detected phase and the phase function given by equation (4) will result in zero rms erros in the time intervals where the model phase variation is identical to the variation in the signal phase. However, in multipath regions this approach will not be useful as the signal phase in these regions has a very complex structure. As will be shown in the following, the phase matching has the ability to match the ray having the same Doppler frequency as the phase function also in multipath regions for some specific phase functions.

[14] If the phase in an integral of the type $\int A(t)e^{i\psi(t)}dt$ is rapidly varying compared to the amplitude, the result of the integration will be zero. Even if a phase matching function is subtracted from the phase of the integrant as shown in (1 the result could still be zero if the resulting integrand do not contain any stationary phase points, on the other hand, in case of a perfect phase match, the result will be the integrated amplitude. In other cases, the result of the integral can be interpreted as the weight of the mode represented by the applied phase function. This is the case for a Fourier transform, where the applied phase function is linear in time or space, which yields the spectral energy of a signal with respect to the specific mode given by the Fourier frequency, i.e., contributions to a spectral component come from all time-intervals where the time derivative of signal phase matches the Fourier frequency. At these points (or intervals) the phase difference can be said to be stationary.

[15] A convenient technique to evaluate the phase integral (equation (1)) is to make use of the stationary phase method [*Born and Wolf*, 1999]. This method utilizes the assumption that, due to the oscillating structure of the integrand, the main contribution to the integral occurs near a stationary point, where variations of the amplitude and the higher order terms of the expansion of phase can be neglected. Furthermore, for the application of the stationary phase method in this context we assume that one and only one stationary point must be present.

[16] In the following, the summation introduced in equation (1) will be omitted in order to simplify the calculations, and the signal used should be considered as one of the signals in the sum. Implicit is hereby assumed that the other rays in the sum only give vanishing results,

i.e., no stationary point exists for the specific phase matching function in agreement with the assumptions described above. The phase matching function $u(c)$, can be brought on an explicit form, if the integrand fulfils the assumptions stated above. By expanding the phase to second order $M(c)$ yields:

$$
\begin{aligned}
M(c) &= \int_0^T A(t)\exp(i(\psi(t) - \psi_o(c,t)))dt \\
&= \int_0^T A(t)\exp(i[\psi(t_1) - \psi_o(c,t_1) + (\dot{\psi}(t_1) \\
&\quad - \dot{\psi}_o(c,t_1))(t - t_1) + {}^1/_2(\ddot{\psi}(t_1) \\
&\quad - \ddot{\psi}_o(c,t_1))(t - t_1)^2] + \ldots)dt \\
&\cong \sqrt{\frac{2\pi i}{\ddot{\psi}(t_1) - \ddot{\psi}_o(c,t_1)}} A(t_1)\exp(i[(\psi(t_1) \\
&\quad - \psi_o(c,t_1))])|_{\dot{\psi}(t_1) = \dot{\psi}_o(c,t_1)}, \qquad (2)
\end{aligned}
$$

where all derivatives are with respect to time. In the expansion of the phase above, it is noticed that the integral is stationary if the first order term in time is vanishing [*Born and Wolf*, 1999], i.e., when $\dot{\psi}(t_1) = \dot{\psi}_o(c, t_1)$.

[17] The phase of $M(c)$, $\psi(t_1) - \psi_o(c, t_1)$, is a function of the transforming space parameter c, and by differentiating the phase with respect to c, an auxiliary function on parametrical form can be found:

$$
\left[c, \frac{\partial(\psi - \psi_o)}{\partial c}\right] \qquad (3)
$$

which expresses a relation between the transform space parameter and the partial derivative of the matching phase.

[18] Equation (3) can be used to design some wanted property of the matching function from, the structure of the signal phase. This will not be described systematically, but instead some examples will be discussed. A special case is the Fourier method, where the matching phase function is $\psi_o(c, t) = ct$, i.e., in this case, the phase transform becomes identical to the Fourier transform, and the auxiliary function $[c, -t_1]$ (equation (3)) yields the Doppler frequency as a function of time. This assumes that only one stationary point exists, which unfortunately is not always the case if the satellites have noncircular orbits. For that reason the application of Fourier transformation to noncircular orbits requires modifications, as described by *Jensen et al.* [2003]. These modifications are a part of the FSI method. In order to overcome these modifications we seek a phase matching function that is capable of handling noncircular orbits. The form of such function can be deduced by studying the geometrical optical phase and Doppler frequency of an arbitrary ray. The ray phase, ψ, can be expressed as:

$$
\begin{aligned}
\psi = k\int n ds &= k\sqrt{r_L^2 - a^2} + k\sqrt{r_G^2 - a^2} + ka\alpha \\
&\quad - 2k\int_a^\infty \sqrt{u^2 - a^2}\frac{d\ln(n)}{du}du, \qquad (4)
\end{aligned}
$$

in which $u = rn(r)$ is the refractional radius. Whereas the ray Doppler frequency, ω, is given by

$$
\omega = \frac{d\psi}{dt} = k\left(\dot{r}_L\sqrt{1 - \frac{a^2}{r_L^2}} + \dot{r}_G\sqrt{1 - \frac{a^2}{r_G^2}} + a\dot{\theta}\right), \qquad (5)
$$

where a is the impact parameter, r is the distance from the local center of curvature to a point on the ray trajectory, n is the index of refraction, r_G and r_L are the radii of the GPS and LEO satellites, respectively, and θ is the angle between r_G and r_L. The last term in (4) can be recast into

$$
2k\int_a^\infty \sqrt{u^2 - a^2}\frac{d\ln(n)}{du}du = \int_a^\infty \alpha da, \qquad (6)
$$

in which α is the bending angle. Now, from equations 2–6 it follows that if we choose a phase matching function of the following form:

$$
\psi_o(c,t) = k\left(\sqrt{r_L^2 - c^2} + \sqrt{r_G^2 - c^2} + c\beta\right), \qquad (7)
$$

in which

$$
\beta = \theta - a\tan\left(\frac{\sqrt{r_L^2 - c^2}}{c}\right) - a\tan\left(\frac{\sqrt{r_G^2 - c^2}}{c}\right), \qquad (8)
$$

then the time derivative of $\psi_o(c, t)$ is equal to the ray Doppler frequency for $c = a$ and at the stationary point, as determined by $\dot{\psi}(t_1) = \dot{\psi}_o(c, t_1)$, we have

$$
\begin{aligned}
\dot{\psi}(t_1) - \dot{\psi}_o(c,t_1) &= k\left(\dot{r}_L\sqrt{1 - \frac{a^2}{r_L^2}} + \dot{r}_G\sqrt{1 - \frac{a^2}{r_G^2}} + a\dot{\theta}\right. \\
&\quad \left. - \dot{r}_L\sqrt{1 - \frac{c^2}{r_L^2}} - \dot{r}_G\sqrt{1 - \frac{c^2}{r_G^2}} - c\dot{\theta}\right) = 0 \qquad (9)
\end{aligned}
$$

When the radial velocities are sufficiently smaller than tangential velocities, which is the case for any realistic

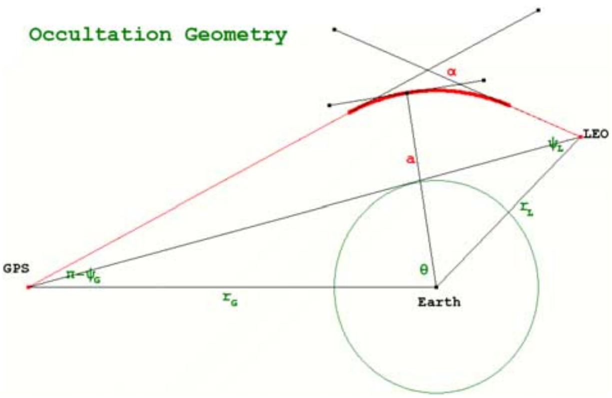

Figure 1. Occultation geometry. The figure has been sketched with the use of equation (4), which can be considered as an approximation to the geometrical optics ray path. α is the bending angle, ψ_G, ψ_L are the angles between the ray path and the radius vector at the GPS and LEO positions, θ is the angle between the LEO and GPS, r_G, r_L are the distances to the Earth's center from the GPS and LEO, respectively, and a is the impact parameter. As noticed, the angles are connected by the relation $\alpha = \theta + \psi_L - \psi_G$.

orbits, this equation is approximately linear and there will only be one real solution namely; $c = a$. That is, phase matching using equation (7) is a direct mapping from the time domain to the impact parameter domain, as $c = a$. Furthermore, the derivative of the phase of the transformed field yields the bending angle multiplied by the wave number since

$$\frac{\partial(\psi - \psi_o)}{\partial c} = k\alpha. \qquad (10)$$

To summarize, by applying the phase matching function given by equation (7) the transformed field can be computed for a desired range of impact parameters and the corresponding bending angles can be determined from the transformed field using equation (10). The technique will be valid for all orbits for which equation (9) has only one real solution.

[19] The phase ψ_o (c, t) can be viewed as an approximation to the true optical ray path derived from equation (4) by neglecting the last term in this expression. From a geometrical point of view, ψ_o (c, t) corresponds to the phase of a ray trajectory described by two straight lines starting at the LEO and the GPS positions, respectively, and connected by a circular arc of the size β and radius c; this is illustrated in Figure 1.

[20] The phase matching method is closely related to the FSI technique, which is best illustrated by approximating the phase function in equation (7) by a linear function containing a constant c_0. In this case the FSI phase matching function can be written as:

$$\psi_{FSI}(c, c_0, t) = k\Bigg(\sqrt{r_L^2 - c_0^2} + \sqrt{r_G^2 - c_0^2} - c_0\left(a\tan\left(\frac{\sqrt{r_L^2 - c_0^2}}{c_0}\right) + a\tan\left(\frac{\sqrt{r_G^2 - c_0^2}}{c_0}\right)\right) + c\theta\Bigg). \qquad (11)$$

[21] Because the phase ψ_{FSI} is linear in the variable c the FSI method is essentially a Fourier transform in c. For the implementation of the FSI technique suggested in equation (11) the constant c_0 can be chosen in the midtroposphere roughly representing the center of the impact parameter range in which multipath occurs.

[22] Now, to finish the development of this specific phase matching technique, the equivalent matching amplitude $C(c, t)$, has to be calculated. This is done in Appendix A, together with an analysis of the benefits of using amplitude in the phase matching. The result from Appendix A shows that the use of the phase in equation (4) cancels the defocusing factor perfect, which together with the amplitude function makes the output amplitude of the phase transform $u(c)$ a constant in the absence of absorption in the atmosphere. This last property and the uniqueness of the solution to the Doppler equation (equation (9)) are the main advantages of the phase method compared to the FSI method.

[23] The analysis of the specialized geometrical optics phase matching has been rather lengthy, so a short recap of the practical use of the method is appropriate. The complex measured occultation signal is denoted $f(t)$ and the matching function $C(c, t)\, e^{-i\psi_0(c,t)}$, where $C(c, t)$ is the matching amplitude. The matching phase (equation (7)) is given by ψ_0 (c, t). The amplitude $C(c, t)$ is chosen so the amplitude of the result of the transformed field is constant. An explicit expression of $C(c, t)$ is given in Appendix A. The resulting phase of the phase match is denoted $\Psi(c)$.

[24] Implementation of the phase matching method follows three steps:

[25] 1. For each impact parameter, c, phase matching is performed on the occultation signal:

$$u(c) = \int_0^T f(t)C(c,t)e^{-i\psi_o(c,t)}dt = |u(c)|e^{i\psi(c)}$$

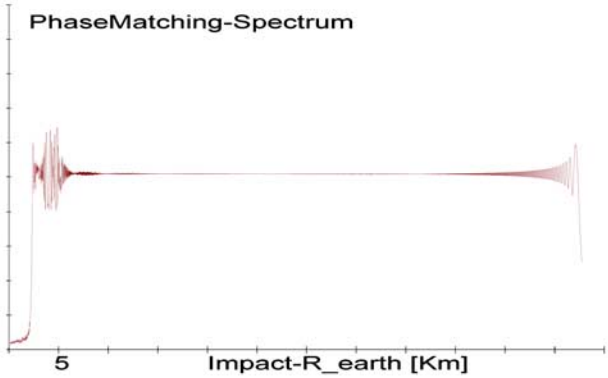

Figure 2. Amplitude spectrum of the phase matching result, i.e., $|u(c)|$ from the simulated radio occultation signal. The impact height is the impact parameter minus the Earth radius.

[26] 2. The phase of the transformed field is differentiated with respect to c and yields the bending angle:

$$\frac{d\Psi}{dc} = k\beta$$

[27] 3. The resulting bending angle profile is Abel transformed thereby giving the refractive index. So, the usage of the method is just a set of integrations followed by differentiation of the phase of the transformed field.

[28] The recipe for the FSI method is nearly the same, except that phase function, ψ_{FSI} (equation (11)), is used instead of ψ_0 (equation (7)) and that the differentiation in point 2 is with respect to frequency and yields the Doppler frequency instead of the bending angle. Most important in the case of the FSI the integrations can be performed using a FFT.

[29] The number of c (impact parameters) values used will depend on the magnitude of the bending angle in the sense that the phase variation should be sufficiently small to make the differentiation correct. As phase matching using equation (7) cannot be implemented as an FFT evaluation of the integral is demanding in terms of CPU time and for practical implementation, it might be advantageous to evaluate the integrals in the vicinity of the stationary phase point using the same techniques normally used in the evaluation of the back-propagation integral [see, e.g., *Gorbunov et al.*, 1996]. However, this attempt has not been applied in this study where the phase matching method has been implemented in a straightforward (nonoptimal) manner with integrations over the entire time series.

[30] To illustrate the use of the method, it has been applied to an example of an occultation signal. For a given refractive profile and satellite positions (i.e., at a given time), the angle between the satellites has been calculated as function of the impact parameter. Now since the angle is also given at this specific time, it is possible to find the actual impact parameter(s) from the aforementioned calculated function. With this information the phase and amplitude can be calculated. This signal propagating method is based on pure geometrical optics and is limited to handle a spherical symmetric atmosphere only. The signal has been constructed using an artificial refractive index profile. The refractive index profile is an exponential curve, which is a least squares fit of a real radiosonde profile, modulated by a 'bump' at a certain height. The strength of the modulation can be varied from no multipath to critical refraction. The amplitude spectrum of the phase matching result, i.e., $|u(c)|$ form the simulated radio occultation signal, is shown in Figure 2, and the equivalent bending angle is depicted in Figure 3. The bending angle, computed from geometrical optics, is compared to the 'measured' bending angle. It is seen that the two curves are completely identical.

3. Conclusion

[31] The technique of geometrical optical phase matching introduced here is a specialization of the previous developed Fourier transform method taking the form of the optical phase into account. Both methods utilize the synthetic aperture formed by the moving LEO satellite to

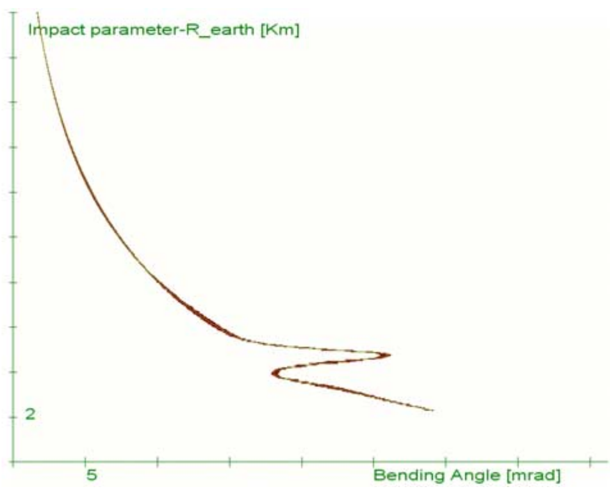

Figure 3. Bending angle. Two curves are mapped: the geometrical optical bending angle (in green) and the 'measured' bending angle (in red), i.e., measured from the result of the phase matching method on the simulated signal. It is seen that the two curves are practical identical. The small oscillations seen on the red curve can be reduced by using smaller steps in phase matching parameter c. The impact height is the impact parameter minus the Earth radius.

give a high resolution of the measured impact parameter and both rely on the method of stationary phase. The FSI method can be considered a hybrid of the matching method and the Fourier method, utilizing the speed of the Fourier transform and the ability to work with noncircular satellite orbits. Like the Fourier and the FSI methods the phase matching method utilizes the assumption of spherical symmetry of refractivity and the principle of synthetic aperture, thus allowing sub-Fresnel resolution. A distinctive feature of the phase matching method, as compared to previously known methods, is the direct applicability for realistic orbits of transmitting and receiving satellites, without some intermediate propagation of complex electromagnetic field to a circle or a straight line. This comes at the expense of impossibility to reduce the method to FFT. The phase matching method is new in application sense and also methodically relatively to the recent analysing methods within radio occultation, in the respect that only simple geometrical optics has been applied in the analysis.

[32] Initial simulations, using realistic orbits, of the phase matching technique show no advantage compared to the FSI method, which is to be expected, especially as long as the task is to determine the bending angle as function of the impact parameter. The phase matching gives a flat spectrum with no absorption in the atmosphere, which theoretical is not always the case for the FSI method. In this case phase matching could have an advantage compared to the FSI if absorption measurement were important. The computation time of the phase matching method is highly increased compared to the FSI method. This is due to the fact that the FSI method makes use of a FFT for the computation of the spectrum, which is not possible in the phase matching method. The value of the use of phase matching method in practice is therefore limited, despite its theoretical importance. However, as a reference method, in cases where there is doubt about results produced by the FSI, it is important.

Appendix A: Calculation of the Amplitude Factor in the Geometrical Optics Phase Match

[33] The amplitude factor in the phase matching method can partly be defined arbitrarily as long as it is slowly varying. However, it is convenient to use an amplitude factor, which makes the amplitude of the phase match constant, if absorption along the ray is absent. If the detected amplitude is not constant it will indicate that absorption is present. To do this, equation (2) and the geometric optical intensity [*Eshleman et al.*, 1980; *Jensen et al.*, 2003; S. S. Leroy, private communication, 2001] will be used for the synthesis.

[34] The intensity yields:

$$I_L = \frac{P_G}{2\pi} \frac{a}{r_G r_L \sin(\theta) \sqrt{r_G^2 - a^2} \sqrt{r_L^2 - a^2} \left(\frac{d\theta}{da}\right)_L}, \qquad \text{(A1)}$$

where P_G is the intensity at the transmitting GPS satellite and $(d\theta/da)_L$ the defocusing factor at the receiving LEO satellite. In all the references above, the defocusing factor has been derived as a total differential quotient in a stationary geometry with respect to the transmitter and the receiver positions. When the satellites are moving this must be interpreted as a partial derivative, i.e., $(d\theta/da)_L = \partial\theta/\partial a$. The expression for $\partial\theta/\partial a$ has been derived by M. E. Gorbunov and K. B. Lauritsen (Analysis of wave fields by Fourier integral operators and their application for radio occultation, submitted to *Radio Science,* 2003), and it yields:

$$\frac{\partial\theta}{\partial a} = \frac{1}{\dot{a}} \left(\dot{\theta} - \frac{\dot{r}_L}{r_L} \frac{a}{\sqrt{r_L^2 - a^2}} - \frac{\dot{r}_G}{r_G} \frac{a}{\sqrt{r_G^2 - a^2}} \right) \qquad \text{(A2)}$$

For practical purposes $\partial\theta/\partial a \approx \dot{\theta}/\dot{a}$, unless the radial velocities are very large.

[35] The second time derivative of the phase in equation (1) can be expressed as:

$$\ddot{\psi}(t_1) - \ddot{\psi}_o(c, t_1)|_{a=c} = k\dot{a} \left(\dot{\theta} - \frac{a\dot{r}_L}{r_L \sqrt{r_L^2 - a^2}} - \frac{a\dot{r}_G}{r_G \sqrt{r_G^2 - a^2}} \right). \qquad \text{(A3)}$$

In equation (2) the square of the intensity (equation (A1)) is divided by the square root of the derivative of the phase (equation (A3)). The result of this is that $\dot{a}$ disappears and the matching amplitude can be defined so the result of the phase matching (equation (2)) has constant amplitude. Besides the elegance of this, it has the property that it can reveal and detect absorption along the ray path [*Gorbunov*, 2002b; M. S. Lohmann et al., manuscript in preparation, 2003]. This property is the main advantage of the geometrical optics phase match method compared to the FSI method, where a perfect cancellation only is possible when the radial velocities are zero. By defining the matching amplitude as follows

$$C(c, t) = \sqrt{\frac{k}{c} r_L r_G \sin(\theta) \frac{1}{c} \sqrt{r_G^2 - c^2} \sqrt{r_L^2 - c^2}} \cdot \left(\dot{\theta} - \frac{\dot{r}_L}{r_L} \frac{c}{\sqrt{r_L^2 - c^2}} - \frac{\dot{r}_G}{r_G} \frac{c}{\sqrt{r_G^2 - c^2}} \right), \qquad \text{(A4)}$$

this purpose is realized.

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