Two case studies on NARCCAP precipitation extremes

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We introduce novel methodology to examine the ability of six regional climate models (RCMs) in the North American Regional Climate Change Assessment Program (NARCCAP) ensemble to simulate past extreme precipitation events seen in the observational record over two different regions and seasons. Our primary objective is to examine the strength of daily correspondence of extreme precipitation events between observations and the output of both the RCMs and the driving reanalysis product. To explore this correspondence, we employ methods from multivariate extreme value theory. These methods require that we account for marginal behavior, and we first model and compare climatological quantities which describe tail behavior of daily precipitation for both the observations and model output before turning attention to quantifying the correspondence of the extreme events. Daily precipitation in a West Coast region of North America is analyzed in two seasons, and it is found that the simulated extreme events from the reanalysis-driven NARCCAP models exhibit strong daily correspondence to extreme events in the observational record. Precipitation over a central region of the United States is examined, and we find some daily correspondence between winter extremes simulated by reanalysis-driven NARCCAP models and those seen in observations, but no such correspondence is found for summer extremes. Furthermore, we find greater discrepancies among the NARCCAP models in the tail characteristics of the distribution of daily summer precipitation over this region than seen in precipitation over the West Coast region. We find that the models which employ spectral nudging exhibit stronger tail dependence to observations in the central region.


1. Introduction

In recent years, the literature on climate change has devoted increasing attention to potential changes in the frequency and severity of extreme weather events. Several studies have projected significant changes in extremes due to climate change [Frei et al., 2006; Beniston et al., 2007; Allan and Soden, 2008; Karl and Melillo, 2009], and recent efforts have linked some individual extreme events to human-induced warming [Peterson et al., 2012]. Because the aforementioned and many other studies employ deterministic simulation models to study extreme events, it is important to understand if and how extreme weather events manifest themselves in such models. In some cases, it has been found that climate models are unable to reproduce extreme weather statistics from the past observed record [Wehner, 2013]. Additionally, because extreme weather events are by definition rare, there is great uncertainty involved in estimating their magnitudes and frequencies.

This study introduces novel methodology to evaluate the ability of regional climate models to reproduce past extreme precipitation events seen in observations. One approach used to examine climate model performance in the simulation of extremes is to estimate summary statistics from the model via a fitted probability distribution and compare them to those seen in observations. Wehner [2013] employs the generalized extreme value (GEV) distribution to compare estimates of return values from various sources. While this approach allows one to compare characteristics of the upper tails of the distributions (i.e., the climatology of model output and observations, it does not provide insight into whether a climate model is able to reproduce specific extreme events seen in observations. This study extends beyond the climatological behavior of precipitation extremes to an examination of correspondence (or, weather)
of daily extreme precipitation between model output and observations at a given time and location. Examination of correspondence of extremes allows us to indirectly study whether atmospheric conditions which led to an extreme precipitation event in the observed record on a given day are reproduced in reanalysis-driven regional climate models (RCMs) to the extent that the models also simulate an extreme precipitation event on that day.

[4] We study output of the six RCMs of the North American Regional Climate Change Assessment Program (NARCCAP) driven by reanalysis. Previous studies have examined NARCCAP precipitation extremes from different perspectives. Schliep et al. [2010] perform statistical analyses using spatial hierarchical modeling to fit a GEV distribution to annual maximum precipitation at each model grid box. Mailhot et al. [2011] examine future changes in annual maxima of precipitation measurements over Canada from the NARCCAP ensemble. Gutowski et al. [2010] study monthly extreme precipitation events in two regions of North America, and Wehner [2013] also studies NARCCAP precipitation extremes.

[5] In order to evaluate the distribution of their daily extreme precipitation events, as in the aforementioned studies, we first describe the marginal behavior of daily extreme precipitation produced by each of the six NARCCAP RCMs, as well as that seen in an observational product and the driving reanalysis itself. Studies of extremes of univariate climate variables have an extensive history and Von Storch and Zwiers [2002, section 2.9] give an overview of appropriate statistical methodologies, all of which analyze a subset of data deemed to be extreme. One approach with an extensive history is to model seasonal maximum precipitation amounts using the GEV distribution [Cunnane, 1989; Hosking and Wallis, 1997; Wehner, 2013]. Alternative methods exist for modeling exceedances over a threshold, and we employ a threshold exceedance approach using the Generalized Pareto distribution (GPD). An advantage of the GPD is that using daily exceedances will offer reduced uncertainty over the GEV, which retains only one data point per year or season. This is important considering the precipitation record studied here covers a relatively short time period of 23 years. More importantly, our approach preserves the daily matching of model output and observations, which is essential for an examination of correspondence of extremes from each source.

[6] Our extension to the study of correspondence of extremes requires techniques from multivariate extreme value theory and has not previously appeared in the climate literature. Specifically, we formulate the strength of this correspondence via the statistical concept of tail dependence. Typically, measures such as correlation or covariance are quite useful for summarizing dependence in climate and weather data. As they measure spread from center, these statistics are effective at capturing dependence in the center of a multivariate probability distribution, but they are not well-suited to measure dependence in the joint tail. Here we examine tail dependence through an established probability framework designed specifically for the joint tail of a multivariate distribution [Resnick, 2007]. To summarize tail dependence, we employ measures which gauge the extent of correspondence of the largest events in each margin. For this portion of the analysis, it is irrelevant that tail characteristics of the RCM output may not match those of the observations due to factors such as differing resolutions, because the examination of correspondence is done after accounting for marginal effects. We restrict attention to RCM output driven by reanalysis so that daily correspondence between the two sources is sensible.

[7] Our analysis is also fundamentally different from conditional approaches, which model the parameters of the GEV or GPD to be functions of covariates. A conditional approach is sensible when the covariate is measured on relatively long time scales; Sillman et al. [2011] use a monthly atmospheric blocking indicator as covariate for extreme daily minimum temperatures, while Katz [2010] and Zhang et al. [2010] use year as a covariate when modeling temporal trends in extreme behavior. Such methodology requires that one first condition on a value of one variable and then extract the corresponding extremes of the other variable. Because the quantities we wish to relate are both observed daily, it is not clear how to determine these conditionally extreme values. Instead, we treat daily precipitation amounts from model output and observations as emerging from a bivariate probability distribution.

[8] We employ the methodology in an evaluation of NARCCAP RCM-simulated extreme precipitation phenomena in two case studies, over different regions and different seasons. The first is a study of extreme precipitation events on the Pacific coast, which was partially explored in Weller et al. [2012]. A portion of that study examined 19 years of past daily winter season precipitation over a West Coast region of North America in regional climate simulations produced by the Weather Research and Forecasting (WRF) model, one of the NARCCAP RCMs, and an observational product [Maurer et al., 2002]. It was found that strong tail dependence in precipitation amounts existed between the WRF model output and observations. We extend that work here by first examining the tail dependence in daily precipitation amounts between the driving reanalysis product and an observational data set, with the aim of indirectly examining whether conditions which led to an extreme event in the observed record are present in the reanalysis. We then study the tail dependence between the observations and all six NARCCAP RCMs for precipitation in this region over both winter and summer, with the goal of evaluating the ability of the RCMs to simulate these past extremes, given the boundary conditions from the reanalysis.

[9] We perform a similar investigation of precipitation extremes over the prairie (Corn Belt) region of the U.S., examining both summer and winter precipitation. While winter precipitation is often driven by strong large-scale systems, central U.S. summer precipitation extremes are typically associated with convective systems influenced by processes that are more local in scale. Due to difficulties in regional model parameterization of convective precipitation [Molinari and Dudek, 1992], results may differ between summer and winter precipitation over this region. After performing marginal analyses, we investigate the daily correspondence in extremes between the observations and the reanalysis, as well as the observations and each RCM.

[10] The outline of the paper is as follows: In section 2 we describe the NARCCAP program in more detail and introduce the model output and observational data sources used in this work. Section 3 details an examination of the ability
of the NARCCAP models to simulate past observed precipitation extremes along a West Coast region of North America in summer and winter. This section also includes a review of our statistical techniques, which are based in extreme value theory. In section 4 we employ similar techniques to study the models’ representations of extreme precipitation events over a central U.S. region. We conclude with a summary and discussion in section 5.

2. NARCCAP Models, Reanalysis, and Observations

[11] NARCCAP is an international coordinated effort to investigate uncertainties in high-resolution dynamical simulations of regional climate over North America [Meerns et al., 2009]. NARCCAP consists of a suite of six RCMs run over a common spatial domain at similar resolutions (~50 km) and over common time periods. Phase I of the experiment involves running each RCM for the period 1979–2004 with boundary conditions provided by the National Center for Environment Prediction (NCEP)–Department of Energy (DOE) global reanalysis II product [Kanamitsu et al., 2002]. In Phase II, the regional models are run with boundary conditions provided by four different fully coupled general circulation models (GCMS) in a fractional factorial design, for the years 1981–2003 (control period) and 2041–2070 (future period), under the Intergovernmental Panel on Climate Change A2 scenario [Nakicenovic et al., 2000]. We wish to study past daily correspondence between NARCCAP model output and observations, we only use output from Phase I.

[12] The six RCMs and their major references are listed in Table 1. The Canadian Regional Climate Model (CRCM) and the Experimental Climate Prediction Centre’s version of the Regional Spectral Model (ECP2) are the only two RCMs that include some form of interior nudging (a push toward the large-scale driving conditions in the interior of the domain). Additional details on NARCCAP and the configuration of the models can be found in Meerns et al. [2012], on the program website at http://www.narccap.ucar.edu, and in the provided references.

[13] We also study output of the NCEP reanalysis product directly. The reanalysis is a data assimilation product that uses past observed weather as inputs. Precipitation amounts from this product are recorded four times daily over a global T62 Gaussian grid. This results in an approximately 2° latitude/longitude resolution of reanalysis output. Precipitation is not an assimilated observation but rather a modeled quantity in the reanalysis product. The reanalysis output was provided by the Physical Sciences Division of the Earth System Research Laboratory in Boulder, Colorado, USA, from their website at http://www.esrl.noaa.gov/psd/.

[14] We compare precipitation output of each NARCCAP RCM and the NCEP reanalysis to the gridded observational product produced by Maurer et al. [2002]. This product consists of spatially gridded precipitation amounts interpolated from weather station measurements over the continental U.S. and southern Canada for the years 1949–2010. In constructing this product, the PRISM technique [Daly et al., 1997] was employed to correct for elevation, which is particularly important in mountainous regions. It is gridded at 1/8° resolution and on a daily temporal scale, with each day defined to begin at midnight local time. This product has been previously employed as a verification tool for model output [Liang et al., 2004; Wood et al., 2004; Wehner, 2013]. We compare daily precipitation output from each of the six NARCCAP models driven by the NCEP reanalysis, as well as the reanalysis directly, to daily precipitation in the observational product for the years 1981–2003.

[15] A challenge in comparing output from the NARCCAP models, the NCEP reanalysis, and the Maurer et al. [2002] gridded observations arises from the differences in spatial resolutions of the three sources. Wehner [2013] noted significant differences in precipitation return values over some regions and seasons between the Maurer et al. [2002] gridded product and a coarser observational product produced by the NOAA Climate Prediction Center [Higgins et al., 2000]. Exploratory analyses (not shown) indicate that the observational product captures some small-scale precipitation phenomena in greater detail than the NARCCAP model output and reanalysis. When comparing output from the three sources, we will define quantities of interest which represent output over approximately common spatial areas. After comparing the tail behavior of the marginal distributions of each source, we will study the correspondence of the sources’ extreme events on a common scale, achieved via marginal transformation.

3. Pacific Region Precipitation Extremes

[16] We begin with an analysis of daily extreme Pacific region precipitation events seen in the observational record, each of the NARCCAP models forced by the NCEP reanalysis product, and the reanalysis product itself. We first examine the nature of the upper tails of the probability distribution of precipitation produced by each source. Second, we study the tail dependence in precipitation events between each model and the observational data; the objective here is to determine the extent to which the largest

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Name</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRCM</td>
<td>Canadian RCM</td>
<td>Caya and Laprise [1999]</td>
</tr>
<tr>
<td>ECP2</td>
<td>Experimental Climate Prediction Center’s version of the Regional Spectral Model</td>
<td>Juang et al. [1997]</td>
</tr>
<tr>
<td>HRM3</td>
<td>Third-generation Hadley Centre RCM</td>
<td>Jones et al. [2003]</td>
</tr>
<tr>
<td>MM5I</td>
<td>Fifth-Generation Pennsylvania State University–National Center for Atmospheric Research (NCAR) Mesoscale Model</td>
<td>Grell et al. [1994]</td>
</tr>
<tr>
<td>RegCM</td>
<td>RegCM version 3</td>
<td>Giorgi et al. [1993a, 1993b] and Pal et al. [2007]</td>
</tr>
<tr>
<td>WRFG</td>
<td>Weather Research and Forecasting model with Grell convective scheme</td>
<td>Skamarock et al. [2005]</td>
</tr>
</tbody>
</table>
precipitation events in the observed record correspond to the largest events produced by the RCMs and the reanalysis. We examine daily precipitation occurring in the months of November, December, January, and February for the years 1981–2003, which results in a winter sample of $T_w = 2765$ days, as well as daily summer (June-July-August (JJA)) precipitation over the same time period, resulting in a summer sample of $T_s = 2116$ days. We include November in the winter season as precipitation extremes occurring in this month over the region share features with winter extremes; e.g., many are atmospheric river events [Dettinger et al., 2011].

3.1. Exploratory Analysis

[17] Our study region is defined as the area between 32°N and 53°N latitude and from 118°W longitude to the Pacific coast; this is the same region studied in Weller et al. [2012]. This region, shown in Figure 1, captures extreme precipitation events affecting the coastal regions of California and the Pacific Northwest, as well as heavy precipitation in the mountainous regions of the Cascades and Sierra Nevada. Examination of temporally averaged daily precipitation amounts in each season over this region (not shown) indicates that on average, the NARCCAP models are able to reproduce the patterns of spatial variability in precipitation over the region. However, their coarse resolution relative to the observational product does not allow them to capture localized areas of heavy precipitation seen in observations, such as over the Olympic peninsula in Washington. Due to its coarse resolution, output of the reanalysis does not capture localized features of precipitation seen in the outputs of NARCCAP RCMs and observations. A full comparison of seasonal means to observations was presented in Mearns et al. [2012].

[18] Our primary interest here is in extreme daily precipitation output from the three sources. For winter events, as in Weller et al. [2012], we extract the spatial maximum total precipitation over an area of approximately $200 \times 200$ km$^2$ (4 x 4 RCM grid cells, 17 x 17 grid cells from the observational product). Because summer extreme precipitation is typically smaller scale in nature, for summer events, we extract precipitation amounts from a smaller area, approximately $100 \times 100$ km$^2$. As the reanalysis product is of coarser resolution, for both seasons, we extract the maximum precipitation amount from a single grid cell of the reanalysis output on each day. Thus, the quantity we examine for a given day is the aggregated precipitation amount over a footprint of fixed size and located such that this amount is maximized. A way to think of this footprint is that it selects the total precipitation amount from the largest storm over the region on a given day. These size footprints were chosen in order to adequately capture the spatial extent and intensity of extreme precipitation events in the region in winter and summer, respectively. Define $X_t^j$ to be this extracted quantity on day $t$ from model output product $j$, where $j = 1, \ldots, 6$ corresponds to the six NARCCAP models, while $j = 7$ corresponds to the NCEP reanalysis. Let $Y_t$ be this quantity extracted from the observational data product on day $t$. An example of the footprint selected from each product on one winter day is given in Figure 1. The selection of this spatial footprint is done independently for each RCM, the observational product, and the reanalysis; while temporal correspondence is maintained, we do not require spatial matching of footprints from model output and observations on a given day. Because the primary interest in this study is to examine daily correspondence of extreme precipitation events produced by multiple sources, we wish to account for the possibility that an extreme precipitation event which
occurred on a given day in the observational data was seen in a slightly different location in the region by the NCEP reanalysis and each RCM. We examine spatial mismatching further in section 3.4.

[19] We examine the Spearman rank correlation between $X_j$ and $Y$ for $j = 1, \ldots, 6$. Though we calculate the estimated correlation coefficients from the whole of the data, we use the Spearman correlation as opposed to the usual Pearson correlation due to its robustness to extreme values. For winter precipitation, correlations range from 0.70 (RegCM) to 0.76 (CRCM); for summer the range is from 0.32 (HRM3) to 0.58 (ECP2). While Spearman correlation measures dependence over all levels of the distribution of $(X_j, Y)$, our primary aim is to examine tail dependence; we thus proceed to examine the upper tails of the distributions of $Y$ and the $X_j$ for $j = 1, \ldots, 7$.

### 3.2. Comparison of Marginal Tail Behavior

[20] We first examine the upper tails of $X_j$, $j = 1, \ldots, 7$, and $Y$. The aim is to study the consistency of the different models' representations of extreme precipitation, as was done via the GEV in Wehner [2013]. We fit the Generalized Pareto distribution (GPD) to the precipitation quantities extracted from both the observational product and each NARCCAP model. The GPD is suggested by asymptotic theory for univariate threshold exceedances [Coles, 2001]. More specifically, given that a random variable $X$ satisfies a mild condition, the exceedances of a suitably high threshold $u$ approximately follow a GPD:

$$
\mathbb{P}(X > x | X > u) \approx \left( 1 + \alpha \frac{x - u}{\psi_u} \right)^{-1/\xi},
$$

where $x_j = x$ if $x > 0$ and 0 otherwise. The parameter $\psi_u$ depends on the threshold $u$, while $\xi$ does not. The parameter $\xi$ determines the tail behavior of the random variable $X$: $\xi < 0$ corresponds to a distribution with finite upper limit, $\xi = 0$ (taken as a limit) corresponds to a light (exponentially decreasing) tail, and the case $\xi > 0$ implies a heavy (decaying as a power function) tail. The heavy-tail case is seen quite often in daily precipitation measurements from both point source and gridded measurements [Cooley et al., 2007; Furrer and Katz, 2008; Schieple et al., 2010].

[21] After examining diagnostics to determine appropriate thresholds [Coles, 2001, chapter 4], we fit the GPD to exceedances of the 0.955 and 0.94 quantiles for summer and winter, respectively, of each $X_j$ and $Y$. A summary of chosen thresholds, maximum likelihood estimated parameter values, and their standard errors is given in Table 2. Due to differences in resolution between the observational product and each regional model, the thresholds $u_j$ are not directly comparable to the thresholds chosen for the observations.

### Table 2. Threshold Selected and Maximum Likelihood Parameter Estimates (Standard Errors) From GPDs Fit to Pacific Region Precipitation Quantities From Each Source*

<table>
<thead>
<tr>
<th>Model</th>
<th>$u_j$</th>
<th>$\psi_u$ (se)</th>
<th>$\xi$ (se)</th>
<th>$\delta_{20}$ (CI)</th>
<th>$\delta_{50}$ (CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter (November–February)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRCM</td>
<td>863</td>
<td>172.5 (21.6)</td>
<td>0.02 (0.09)</td>
<td>102.3 (93.0, 125.7)</td>
<td>111.3 (98.6, 148.0)</td>
</tr>
<tr>
<td>ECP2</td>
<td>1129</td>
<td>325.9 (43.8)</td>
<td>0.12 (0.11)</td>
<td>124.5 (115.6, 148.0)</td>
<td>132.5 (114.2, 161.6)</td>
</tr>
<tr>
<td>HRM3</td>
<td>1032</td>
<td>273.9 (32.3)</td>
<td>0.13 (0.08)</td>
<td>159.0 (135.0, 222.5)</td>
<td>184.0 (148.3, 293.9)</td>
</tr>
<tr>
<td>MM5I</td>
<td>1026</td>
<td>246.7 (33.3)</td>
<td>0.11 (0.10)</td>
<td>151.6 (136.4, 192.4)</td>
<td>165.4 (144.9, 228.7)</td>
</tr>
<tr>
<td>RegCM</td>
<td>1093</td>
<td>325.2 (42.4)</td>
<td>0.06 (0.09)</td>
<td>153.8 (138.4, 193.1)</td>
<td>167.7 (147.2, 228.0)</td>
</tr>
<tr>
<td>WRFG</td>
<td>1086</td>
<td>339.8 (43.2)</td>
<td>0.06 (0.09)</td>
<td>88.3 (81.3, 105.0)</td>
<td>95.1 (86.1, 120.1)</td>
</tr>
<tr>
<td>NCEP</td>
<td>46</td>
<td>10.4 (1.2)</td>
<td>0.07 (0.08)</td>
<td>84.9 (71.6, 120.1)</td>
<td>98.1 (79.0, 157.3)</td>
</tr>
<tr>
<td>(Obs)</td>
<td>14969</td>
<td>3938.5 (554.6)</td>
<td>0.00 (0.11)</td>
<td>161.1 (102.4, 154.8)</td>
<td>128.8 (109.5, 192.1)</td>
</tr>
<tr>
<td>Summer (June–August)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRCM</td>
<td>87</td>
<td>29.1 (3.6)</td>
<td>0.04 (0.09)</td>
<td>59.4 (51.4, 79.4)</td>
<td>67.6 (56.4, 100.3)</td>
</tr>
<tr>
<td>ECP2</td>
<td>158</td>
<td>41.0 (5.8)</td>
<td>0.12 (0.11)</td>
<td>79.8 (62.3, 121.0)</td>
<td>97.6 (71.0, 185.4)</td>
</tr>
<tr>
<td>HRM3</td>
<td>88</td>
<td>40.1 (5.8)</td>
<td>0.12 (0.11)</td>
<td>84.9 (71.6, 120.1)</td>
<td>97.6 (71.0, 185.4)</td>
</tr>
<tr>
<td>MM5I</td>
<td>110</td>
<td>42.3 (5.6)</td>
<td>0.06 (0.10)</td>
<td>93.9 (81.4, 127.7)</td>
<td>106.4 (88.5, 163.4)</td>
</tr>
<tr>
<td>RegCM</td>
<td>123</td>
<td>52.5 (7.2)</td>
<td>0.10 (0.11)</td>
<td>110.2 (89.8, 164.3)</td>
<td>130.8 (100.4, 224.9)</td>
</tr>
<tr>
<td>WRFG</td>
<td>96</td>
<td>56.0 (6.8)</td>
<td>0.09 (0.08)</td>
<td>77.7 (68.8, 100.1)</td>
<td>85.8 (74.4, 119.8)</td>
</tr>
<tr>
<td>NCEP</td>
<td>26</td>
<td>63.3 (8.8)</td>
<td>0.03 (0.08)</td>
<td>53.3 (48.3, 65.2)</td>
<td>58.1 (51.6, 76.7)</td>
</tr>
<tr>
<td>(Obs)</td>
<td>2003</td>
<td>620.3 (86.0)</td>
<td>0.12 (0.11)</td>
<td>73.4 (60.6, 108.7)</td>
<td>86.7 (67.4, 149.1)</td>
</tr>
</tbody>
</table>

*Also shown are estimated 20 and 50 year return levels with 95% profile likelihood confidence intervals. Return values and confidence intervals have been normalized to grid box-level values.
to be grid box-level values; that is, values were obtained by dividing the estimates obtained for footprint precipitation amounts by the total number of grid cells whose values were aggregated to obtain the studied quantity. We see that slight differences in estimates of the tail parameter $\xi$ produce larger differences in the return levels as the return period increases. Also shown are 95% profile likelihood confidence intervals for the return values, which reflect the relatively large uncertainty in these estimates. Large uncertainty is seen even for the 20 year return value, which corresponds to a time frame slightly shorter than that of the NARCCAP simulations studied here. Owing likely to its coarse resolution, the reanalysis produces lower estimates of return values than those seen in the observational data. For summer season, the values computed from the RCMs generally match the observations better than does the reanalysis itself, while in winter, the relative magnitudes of biases in these quantities are similar for RCMs and the reanalysis.

[24] Due to differences in the study region and the quantity of interest, the 20 year return values reported in Table 2 are not directly comparable to results reported in Wehner [2013]. Twenty year return values were reported by Wehner [2013] for precipitation over the western United States (defined to be west of 100°W longitude), and these values were compared to those obtained from the Maurer et al. [2002] observational data. The 20 year return values in Table 2 are consistent with Wehner [2013] in that the models generally exhibit wet biases in terms of this quantity. A notable exception is CRCM, which Wehner [2013] found to have the smallest disagreement with this observational product over the western U.S. in these two seasons.

3.3. Tail Dependence

[25] Having examined the marginal behavior of our precipitation quantities $X_i$ and $Y$ from the model products and observations, respectively, we turn attention to the tail dependence between each $X_i$ and $Y$. When an extreme precipitation event is seen in the observed record on a given day, we expect that to a certain extent, the atmospheric conditions which produced that event will be reflected in the NCEP reanalysis for that day. We first aim to learn whether the extreme precipitation event was seen in the reanalysis product, which may indicate that the boundary conditions for such an event were fed into the RCMs. We then examine whether the RCMs themselves are likely to produce an extreme precipitation event in response.

[26] A first step in examining tail dependence between two study variables is to determine whether they are asymptotically dependent or asymptotically independent. Two random variables $Z_1$ and $Z_2$ with a common marginal distribution are said to be asymptotically dependent if

$$\chi := \lim_{z \to z^*} P(Z_2 > z \mid Z_1 > z) > 0,$$

where $z^*$ is the (possibly infinite) right endpoint of the support of $Z_1$ and $Z_2$. Asymptotic dependence occurs when $\chi = 0$. Asymptotic dependence implies that the very largest events in one margin exhibit some correspondence to the largest events in the other margin. This is an important feature, and it is different from dependence in the usual sense of correlations and covariances. For example, a bivariate Gaussian distribution with any correlation less than 1 exhibits asymptotic independence [Sibuya, 1960]. In practice, a determination that asymptotic dependence is present must be made from the data.

[27] As an exploratory step, we examine the level of tail dependence between each model product and observations via a metric introduced by Coles et al. [1999]:

$$\chi(q) := 2 - \frac{\log P(Z_2 < z_q \mid Z_1 < z_q)}{\log P(Z_2 < z_q)},$$

where $z_q$ is the $q$ quantile level of the common marginal distribution. It can be shown that $\lim_{q \to 1} \chi(q) = \chi$ in (2), and Coles et al. [1999] suggest an empirical estimator of $\chi(q)$ for large values of $q$ as a diagnostic for assessing tail dependence.

[28] Figure 2 shows plots of estimated $\chi(q)$ plotted against $q$ for $q \in [0.75, 1)$ for each model to observations comparison, for both summer and winter precipitation. Examining winter precipitation, we see that all six regional models exhibit similar tail dependence with observations, with the $\chi(q)$ estimates fluctuating near 0.5 for large $q$. Additionally, the estimates of $\chi(q)$ for the RCMs for winter season extremes tend to be higher than the estimate computed from the reanalysis directly. Confidence intervals (shown only for CRCM) constructed via the delta
method and a normal approximation reveal no significant differences in \( \chi(q) \) between the different sources and show great uncertainty in these estimates for \( q \) close to 1 due to the limited data exceeding high quantiles. Tail dependence between each model and observations tends to be lower when considering summer extremes in this region, with estimates fluctuating near 0.3; nonetheless, the plots provide strong evidence that \( \chi > 0 \) for each model; that is, each bivariate pair \((X_j, Y)_j\) exhibits asymptotic dependence when considering both summer and winter extremes in this region. In other words, some correspondence is seen between the very largest daily precipitation amounts from the observational product and each of the six models. In winter, the tail dependence estimates between each RCM and observations are slightly greater than that between the reanalysis output and observations. When considering summer extremes in this region, this is only true for the CRCM and ECP2 models, which employ spectral nudging.

As seen in the definition of \( \chi \) in (2), an examination of tail dependence requires that each component of a given random vector has a common marginal distribution. In order to further examine the tail dependence in \((X_j, Y)_j\), we apply probability integral transformations \( Z_j := G_j(X_j) \) and \( Z_{\text{obs}} := G(Y) \) using the fitted GPD above the chosen thresholds and the empirical distribution function below the thresholds [see Coles and Tawn, 1991]. These transformations result in the \( Z_j \) and \( Z_{\text{obs}} \) having common unit Fréchet marginal distributions, with distribution function \( F(z) = \exp\{-z^{-1} \}, \ z > 0 \). One can think of this transformation as analogous to computing a \( z \) score to standardize a normal variate; however, the transformation here is to a very heavy-tailed distribution.

Figure 3 displays winter precipitation amounts from both the NCEP reanalysis and the WRFG regional model, each plotted against observations on both the original scale and after transformation to Fréchet scale. As seen in the figure, the transformation to a heavy-tailed distribution visually magnifies the very largest precipitation amounts, while nonextreme realizations cluster near the origin. Furthermore, many of the largest precipitation events from both the reanalysis and the climate model correspond to the largest events seen in the observations, indicating the presence of tail dependence.

As both \( Z_j \) and \( Z_{\text{obs}} \) are heavy tailed, we assume that \((Z_j, Z_{\text{obs}})\) is a multivariate regular-varying distribution. Multivariate regular variation is a probabilistic framework used to characterize tail dependence and is strongly linked to classical multivariate extreme value theory. Greater detail on multivariate regular variation is given in Appendix A. This framework suggests a polar coordinate transformation, and for each bivariate pair \((Z_j, Z_{\text{obs}})\), we transform to polar coordinates under the \( L_1 \) norm by defining \( R_j = Z_j + Z_{\text{obs}} \) and \( W_j = Z_j / R_j \). One can think of the radial component \( R_j \) as measuring the overall size of a realization from \((Z_j, Z_{\text{obs}})\), and the angular component \( W_j \) as measuring the relative contribution of \( Z_j \) (the model output) to the total size. Under the regular variation framework, \( W_j \) and \( R_j \) become independent as \( R_j \) gets large; furthermore, tail dependence is described by a probability distribution governing \( W_j \) for realizations with large radial components. Many realizations with angular component values near 1/2 would indicate strong tail dependence, as this means both the model and observations see extreme precipitation amounts; conversely, many values near 0 and 1 would indicate weak correspondence of the two
Table 3. Median Location Biases (Degrees) of Selected Footprint From Each NARCCAP Model and Reanalysis Relative to Observations, for the Largest 127 Winter Events in Terms of $R_w$, and Proportion of Events for Which the Distance Between Selected Footprints Exceeds 500 km (Pacific Region)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Delta^\circ E$</th>
<th>$\Delta^\circ N$</th>
<th>$p_{&gt;500}$</th>
<th>$\Delta^\circ E$</th>
<th>$\Delta^\circ N$</th>
<th>$p_{&gt;500}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRCM</td>
<td>−0.12</td>
<td>−0.23</td>
<td>0.05</td>
<td>−0.01</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>ECP2</td>
<td>−0.38</td>
<td>−0.69</td>
<td>0.05</td>
<td>−0.14</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>HRM3</td>
<td>−0.45</td>
<td>2.21</td>
<td>0.18</td>
<td>−0.30</td>
<td>2.17</td>
<td>0.49</td>
</tr>
<tr>
<td>MM5I</td>
<td>0.15</td>
<td>−0.39</td>
<td>0.07</td>
<td>−0.27</td>
<td>0.71</td>
<td>0.23</td>
</tr>
<tr>
<td>RegCM</td>
<td>−0.24</td>
<td>−0.68</td>
<td>0.11</td>
<td>−0.01</td>
<td>0.51</td>
<td>0.26</td>
</tr>
<tr>
<td>WRFG</td>
<td>0.06</td>
<td>−0.70</td>
<td>0.09</td>
<td>−0.31</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>NCEP</td>
<td>−1.50</td>
<td>−1.34</td>
<td>0.11</td>
<td>0.50</td>
<td>−0.79</td>
<td>0.16</td>
</tr>
</tbody>
</table>

3.4. Location Biases of Simulated Extremes

Due to the differing resolutions and characteristics of the precipitation data sources, we did not require that the selected footprint described in section 3.1 be matched spatially from each source. We examine biases (relative to the observations) in the locations of simulated extreme precipitation events from each RCM and the reanalysis. As in section 3.3, we consider the largest 127 events in terms of $R_w$, for each comparison $j = 1, \ldots, 7$. Table 3 displays median longitude and latitude biases by season for footprints selected from each RCM and the reanalysis compared to those selected from the observational product. To provide some sense of the variability of the differences in footprint locations, we also display the proportion of days for which the nautical distance between the footprints selected from the two sources for each comparison exceeds 500 km.

With the exception of the HRM3 model, all of the NARCCAP RCMs have median latitude and longitude biases of less than 1° in both seasons, and relatively few events exhibited a spatial mismatch of greater than 500 km. The HRM3 model exhibits a northward bias in its simulation of precipitation extremes in both seasons over this region; this bias is quite severe relative to those of the other five NARCCAP RCMs. All NARCCAP models and the reanalysis product generally exhibit greater precision in the locations of their simulated extremes in winter, as compared to summer.

4. Prairie Region Precipitation Extremes

In this section, we turn our attention to extreme precipitation over the prairie region defined in Bukovsky [2011]. This region was chosen due to its relatively homogeneous terrain, as well as its central location within the NARCCAP domain. While the Pacific region studied in section 3 was near the boundary of the NARCCAP simulations, this region is far from the boundary. Thus, it is possible that the boundary conditions will have less influence on regional model precipitation output in this region than in the Pacific region. The prairie region stretches from approximately 38° to 49°N latitude and 86° to 98°W longitude, excluding the Great Lakes. We study daily precipitation from both the summer months (JJA) and winter (DJF) for the years 1981–2003, resulting in sample sizes of $T_s = 2116$ and $T_w = 2075$ days, respectively.

There are reasons to expect seasonal differences in the way the NARCCAP RCMs simulate extreme precipitation events in the prairie region, as well as differences between this region and the Pacific region. Primarily, the nature of extreme precipitation events in the Pacific region tends to be much different than that of extremes over the region studied here. Most of the annual precipitation in the Pacific Northwest falls during the winter, and it has very distinct wet and dry seasons. Winter precipitation over the West Coast has been linked to global phenomena such as the El Niño/Southern Oscillation [Castello and Shelton, 2004; Ropelewski and Halpert, 1987] and is often associated with strong synoptic-scale forcing features such as atmospheric rivers [Leung and Qian, 2009; Dettinger et al., 2011; Weller et al., 2012]. As these features generally exhibit themselves in large-scale global models [Dettinger, 2011] and coarse-resolution reanalyses, the results of section 3.3 may not be surprising.

On the other hand, the prairie region obtains most of its annual total precipitation during the warm season, and a substantial fraction of this precipitation comes from mesoscale convective systems [Fritsch et al., 1986; Trenberth et al., 2003; Schumacher and Johnson, 2006] which are not well resolved in coarse-resolution global climate models and even many finer-resolution regional climate model simulations. Not only is convection poorly resolved in many cases, with problems mainly compounded by the use of convective parameterization, but the mesoscale
Figure 4. Precipitation fields (millimeters) for 16 June 1990 over the prairie region from the observational product, NCEP reanalysis, and each NARCCAP regional climate model driven by reanalysis.

processes that produce favorable conditions for convection are not always well resolved or well simulated either [Anderson et al., 2003; Davis et al., 2003; Cook et al., 2008; Bukovsky and Karoly., 2011]. Winter in the prairie is cold and relatively dry compared to the Pacific region. However, as in the Pacific region, winter precipitation is associated with stronger, more organized, larger-scale forcing than in summer, usually in conjunction with synoptic-scale low-pressure systems.

Some previous examinations of precipitation from NARCCAP models also suggest that we may see differences in extreme events over this region. While not directly studying extremes, Sain et al. [2011] used a functional analysis of variance approach to study variability in precipitation fields produced by regional and global model couplings in the NARCCAP experiment. Their work found that summer precipitation fields over this region exhibited complex interactions between global and regional models, suggesting that different regional models may produce significantly different precipitation fields, even given the same boundary conditions. Schliep et al. [2010] found some differences in marginal behavior of extreme precipitation among NARCCAP RCMs driven by the NCEP reanalysis product. However, no previous analysis has directly examined the tail dependence in precipitation between NARCCAP regional models and observations.

4.1. Exploratory Analysis

In contrast to the Pacific region, extreme precipitation events over the prairie region tend to be of greater intensity in summer than in winter. We illustrate the differences in the precipitation patterns on 16 June 1990, as represented by the six RCMs, the NCEP reanalysis, and the observational product. This day saw the second largest total summer season precipitation amount over the study region in the observed record. Figure 4 shows the precipitation pattern over the region on this day from the reanalysis and gridded observational product. The figure indicates several very intense precipitation pockets over eastern Iowa, which caused severe flash flooding over the region [Barnes and Eash, 1994]. The event does appear to be partially captured at coarse resolution by the reanalysis; however, the most intense precipitation amounts in the observations are highly concentrated over an area that would be covered by only a few RCM grid boxes.
In Figure 4 we also plot the spatial precipitation fields for 16 June 1990 from each of the six NARCCAP models driven by the NCEP reanalysis. Generally speaking, Figure 4 shows that the regional models do not capture this event very well. WRFG captures this event best, simulating large precipitation amounts over eastern Iowa. The MM5I and RegCM models do exhibit significant precipitation amounts; however, they fail to adequately capture the location of the observed rainfall event. The other three models do not exhibit large precipitation events; in fact, the HRM3 model indicates nearly zero rainfall over most of the region on this day.

### 4.2. Comparison of Marginal Tail Behavior

As in section 3.1, we select a precipitation footprint with the aim of targeting extreme precipitation events in each season. As summer extreme precipitation events in this region are often associated with relatively small-scale convective systems, following the methodology of section 3.1, we select a footprint of approximate size 100 × 100 km² for summer events and of size 200 × 200 km² in winter. This quantity on day \( t \) from each regional climate model is denoted by \( X_{jt} \) for \( j = 1, \ldots, 6 \) and by \( Y_t \) from the observational product. We also extract the one-cell daily maximum amount from the NCEP reanalysis over this region; denote this for day \( t \) as \( X_{jt} \). Spearman correlations between RCM output and observations range from 0.43 (MM5I) to 0.61 (CRCM) for winter, and from 0.16 (HRM3) to 0.41 (CRCM) in summer, an indication of seasonal differences in model skill in simulating precipitation.

As in section 3.2, we fit the GPD to the exceedances of a high quantile in each margin separately. After checking diagnostics, we choose the 0.94 empirical quantiles for each estimation procedure. A summary of maximum likelihood parameter estimates and scaled estimates of 20 and 50 year return values is shown in Table 4. Despite smaller estimates of \( \xi \), estimates of return values of this quantity from the NCEP reanalysis are quite comparable to those obtained from the observations for both summer and winter. Turning to the RCMs, we see greater variability between the six models in terms of marginal tail behavior of our computed precipitation quantity in summer; tail parameter estimates range from –0.12 (MM5I) to 0.15 (WRFG). In winter, estimates of \( \xi \) and return values are more consistent between the RCMs, with the exception of HRM3. The effects of the varying parameter estimates are most clearly seen in extrapolation: the point estimates of the 50 year summer precipitation return value from each RCM cover a wide range. Due to the relatively large uncertainty in its estimation, we cannot conclude that significant differences in the tail parameter \( \xi \) exist between the six RCMs; however, less consistency among the RCMs is seen in these estimates for summer precipitation than in winter.

### 4.3. Tail Dependence

As an exploration of the level of tail dependence in precipitation amounts between climate model output and observations, we again employ the Coles et al. [1999] estimator of \( \chi \) as defined in (2). Figure 5 shows empirical estimates of \( \chi(q) \) in (3) for \( q \in [0.75, 0.97] \) and for each season examined. For summer precipitation in this region, this estimator decreases toward 0 as \( q \) increases for most of the NARCCAP models. This provides strong evidence that asymptotic independence is present; that is, there is little to
no correspondence between the largest summer precipitation events over the region produced by any of the model products and those seen in observations. Interestingly, estimates of this parameter tend to be higher in both seasons for the NCEP reanalysis product than for the RCMs, though the differences are not significant. This is in opposition to estimates for winter Pacific region precipitation shown in Figure 2.

We also add 95% confidence bands for the MM5I (summer) and ECP2 (winter) models to the plots to illustrate the uncertainty in these estimates. In contrast, estimates of $\chi(q)$ tend to be much larger for all models over this region in the winter season. The NARCCAP models and reanalysis exhibit greater skill in capturing extreme winter precipitation events in this region than summer extremes. In both seasons, estimates of $\chi(q)$ tend to be higher for the two nudged models, CRCM and ECP2. This is particularly noticeable for winter extremes, for which a separation in $\chi(q)$ is seen between two groups: a group consisting of the reanalysis and the two nudged models, and a group composed of the remaining four NARCCAP models.

One can further see evidence for asymptotic independence in summer precipitation by applying transformations.
to marginal distributions so that each is unit Fréchet and examine the angular component values for those points with large radial components (see Section 3.3). Figure 6 shows a scatterplot of Fréchet-transformed summer precipitation amounts over the region from both the NCEP reanalysis and CRCM output, each plotted against observations. For both comparisons, nearly all points with large values in either margin are near the axes of the plot. This stands in contrast to the Pacific region winter precipitation comparisons in Figure 3, which contain many large points on the interior of the quadrant. Figure 6 (right column) shows the corresponding histograms of angular components for the largest 127 sample values in terms of the radial component, for both the NCEP reanalysis and CRCM output against the observations. The histograms are heavily U-shaped, indicating a lack of correspondence of large precipitation events in observations with large precipitation events from either the reanalysis or CRCM output. Similar results are seen for each of the six NARCCAP models, indicating asymptotic independence of summer precipitation amounts produced by the NARCCAP models and those seen in observations, as well as asymptotic independence of the reanalysis output and observations. In contrast, asymptotic dependence appears to be present for winter precipitation over this region. Histograms of angular components for all six RCM-observation comparisons in both seasons are available in the supporting information.

While it was found that summer precipitation amounts seen in observations over this region are asymptotically independent of those amounts produced by the NARCCAP models and the reanalysis, the empirical Spearman correlations between each model and observations suggest that there is some positive dependence in each bivariate pair \((X_j, Y_j)^T\). From an extremes perspective, a conclusion of asymptotic independence drawn from data does not tell the whole story about dependence at observable levels. For example, a bivariate Gaussian distribution with correlation \(\rho = 0.9\) possesses asymptotic independence but still exhibits tail dependence at finite levels. It is useful to address the strength of this second-order dependence in the distribution’s tail.

\[ \tilde{\chi}(q) := \frac{2 \log P(Z_1 > z_q)}{\log P(Z_1 > z_q, Z_2 > z_q)} - 1, \]  

where \(z_q\) is the \(q\) quantile level of \(Z_1\) and \(Z_2\). For \(q \in [0, 1]\), \(\tilde{\chi}(q) \in (-1, 1)\) serves as a measure of the level of tail dependence in the asymptotic independence setting. Specifically of interest is the limiting behavior, and Coles et al. [1999] define \(\tilde{\chi} = \lim_{q \to 1} \tilde{\chi}(q)\). As an example, if \((Z_1, Z_2)^T\) follows a bivariate Gaussian distribution with correlation \(\rho \in (-1, 1)\), it follows that \(\tilde{\chi} = \rho\).

We apply empirical estimates of \(\tilde{\chi}(q)\) to our samples from \((X_j, Y_j)^T, j = 1, \ldots, 7\), for summer precipitation. Figure 7 shows these estimates plotted against \(q \in [0.75, 0.97]\). There are several notable features of Figure 7. First, the NCEP reanalysis shows a larger estimate of \(\tilde{\chi}\) than any of the RCMs; that is, there is stronger correspondence between the extreme events in the reanalysis and observations than between any of the RCMs and the observations. Second, of the RCMs, the CRCM and ECP2 models exhibit stronger dependence to observations than the other four; these two models employ nudging techniques that force them to more closely follow the evolution of the large-scale forcing in the reanalysis product in the interior of their domains and not just at their boundaries. Third, the four non-nudged models have estimates of \(\tilde{\chi}\) near 0. For perspective, a situation in which \(X_j\) and \(Y\) were exactly independent would correspond to \(\tilde{\chi} = 0\). Figure 7 suggests then that for four of the six NARCAP models, the extreme summer precipitation events in the prairie region produced by the RCMs occur nearly independently of the observed extremes. However, the NARCCAP models do not produce summer precipitation over this region which is more strongly tail dependent to the observations than output of the reanalysis product itself.

### 4.4. Location Biases of Simulated Extremes

In Table 5 we display the median latitude and longitude biases of locations of selected footprints from each RCM and the reanalysis, relative to observations, for the largest 127 precipitation days from each season in terms of radial component values. We also display the proportion of days of these 127 for which the nautical distance between the observational footprint and the model/reanalysis footprint exceeds 500 km. None of the biases are severe; however,

<table>
<thead>
<tr>
<th>Model</th>
<th>Summer</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(^\circ E)</td>
<td>(^\circ N)</td>
</tr>
<tr>
<td>CRCM</td>
<td>-0.01</td>
<td>1.62</td>
</tr>
<tr>
<td>ECP2</td>
<td>0.38</td>
<td>1.70</td>
</tr>
<tr>
<td>HRM3</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>MM5I</td>
<td>-0.01</td>
<td>-0.24</td>
</tr>
<tr>
<td>RegCM</td>
<td>-0.64</td>
<td>0.80</td>
</tr>
<tr>
<td>WRFG</td>
<td>0.08</td>
<td>0.60</td>
</tr>
<tr>
<td>NCEP</td>
<td>-0.63</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

*Also displayed is the proportion of events for which the distance between selected footprints exceeds 500 km.

Coles et al. [1999] introduce a metric to quantify the strength of this dependence. For a bivariate random vector \((Z_1, Z_2)^T\) with common marginal distributions, they define

\[ \tilde{\chi}(q) := \frac{2 \log P(Z_1 > z_q)}{\log P(Z_1 > z_q, Z_2 > z_q)} - 1, \]
we see a larger proportion of summer days for which the footprint selected from the RCM or reanalysis was a great distance from that selected from the observational product, as compared to the examination of winter extremes in section 3.4. The precision of the spatial locations of extreme events produced by the models over winter in this region is comparable to that over the Pacific region in summer. While the two nudged models, CRCM and ECP2, produced relatively few extremes falling at great distances from the observations in both seasons, these two models also exhibited median northward biases of over 1.5° in the location of simulated extreme summer events.

5. Summary and Discussion

[50] This work applies novel statistical techniques in two case studies on the ability of NARCCAP RCMs, driven by the NCEP reanalysis, to reproduce past observed extreme precipitation events. Precipitation is analyzed from the perspective of a storm over a region, rather than at fixed locations. The methodology goes beyond the climatological comparisons of tail behavior between model output and observations which have been made in previous studies. By examining tail dependence, we are able to investigate whether reanalysis-driven model output is at its most extreme on the same days that the observed precipitation is most extreme. Because the tail dependence investigation is done after accounting for marginal effects, differences in scales due to varying product resolutions do not create difficulties. Our case studies illustrate that tail dependence can differ considerably depending on the phenomenon of interest.

[51] The NARCCAP models are able to simulate past observed precipitation extremes over the West Coast region of North America reasonably well, particularly during the wet winter season. These results lend confidence to the idea that the RCMs, when provided boundary conditions that are conducive to an extreme precipitation event over this region, can downscale these appropriately and exhibit this event in their precipitation output. When such conditions are present in a future scenario run of a global model, the RCM can then produce the extreme event in its simulation from the boundary conditions it is given. The nature of West Coast precipitation, which is often driven by strong synoptic-scale processes and orographic features, lends itself to adequate simulation by the RCMs.

[52] Mixed results were found when studying extreme precipitation events over the prairie region. Tail dependence was found between model output and observations for winter precipitation, which often falls as snow. This suggests that the RCMs are successful at translating and evolving the larger-scale forcing present in the boundary conditions for these events through the very large NARCCAP domain in space and time. In contrast, extreme summer precipitation events over the prairie region produced by the NARCCAP RCMs forced by the NCEP reanalysis show little to no correspondence with such events seen in the observed record; in fact, no improvement in tail dependence over precipitation from the reanalysis itself is provided by the RCMs. This may be due to the still-too-coarse resolution of the NARCCAP RCMs for these convective events and some of their driving processes or errors related to the parameterization schemes that influence the triggering of convection and its intensity. There may also be errors in the timing of events in the RCMs that would place them earlier or later and not within the daily totals as defined in the observations.

[53] It was found that the two nudged NARCCAP models exhibited stronger tail dependence to observations than the other four models studied, in both summer and winter seasons. Differences were most notable over the prairie region, which is much farther from the boundary of the NARCCAP simulations than the Pacific region studied in section 3. That the nudged models do a better job is not surprising, since they receive information on the forcing for these events in the interior of the domain. Thus, they are less likely to have errors in the timing and spatial evolution of these extreme events.

[54] Despite studying different quantities and employing different statistical techniques, the results of marginal analyses conducted here are consistent with the findings of [Wehner, 2013]. Twenty year return values computed from the NARCCAP models over each region and season studied here exhibited wet biases relative to observations, consistent with [Wehner, 2013]. Marginal analyses provide a comparison of return values from each source; however, the novelty of this work is our examination of tail dependence, which quantifies the extent of daily correspondence of extremes from two sources.

[55] The analyses performed in this study are limited spatially and temporally to areas which have relatively homogeneous characteristics of extreme precipitation. Extending the methodology temporally, such as examining an entire year of precipitation extremes, would require the application of techniques to account for the seasonality in features of extreme precipitation over a region. Statistical methodology required to expand this study spatially, such as over an entire continent, is still under development. Nonetheless, the methodology introduced here provides a useful way to compare the extremes of weather variables produced by two sources over a region and quantify the ability of climate models to simulate extremes, via the concept of tail dependence.

[56] While the work here provides an exploration of the ability of NARCCAP RCMs to simulate extreme precipitation events over two different regions in two seasons, the increased scientific interest in potential changes in extreme weather events under climate change motivates further examination of the ability of climate models to simulate such events. Future studies may apply the techniques used here to study extremes of precipitation or other extreme phenomena (e.g., temperatures, wind speeds) over different regions and further study uncertainties in climate model representations of extreme weather events.

Appendix A: Multivariate Regular Variation

[57] It is through the framework of regular variation that we describe dependence in sections 3.3 and 4.3. Regular variation [Resnick, 2007] provides a useful framework for describing the joint tail of a multivariate distribution and performing estimation from multivariate threshold exceedance data. In fact, the very definition of regular variation is given in terms of the tail of a distribution. The idea of regular variation is that the joint tail of a random vector decays.
like a power function. A decomposition into polar coordinates naturally arises, and dependence is described by a probability measure on the unit sphere. A random vector \( \mathbf{Z} = (Z_1, \ldots, Z_d) \) taking values in \([0, \infty)^d\) is said to be regularly varying on the cone \( C = [0, \infty) \setminus \{ 0 \} \) if there exists a nondecreasing sequence \( \{ a_n \} \) with \( a_n \to \infty \) and nonnegative finite Radon measure \( \nu \) such that

\[
\nu \left( \frac{\mathbf{Z}}{a_n} \right) \sim \nu(\cdot),
\]

on \( C \) as \( n \to \infty \), where \( \sim \) denotes vague convergence of measures. The limit measure \( \nu \) has the property that for any measurable set \( A \subseteq C \) and \( r > 0 \),

\[
\nu(rA) = r^{-\alpha} \nu(A),
\]

where \( \alpha > 0 \) is called the tail index. It is in (A2) that one sees the power-function joint tail behavior. In this work, we perform transformations so that \( \mathbf{Z} \) has unit Fréchet marginal distributions, which imposes a tail index of \( \alpha = 1 \).

The homogeneous tail decay on multiples of sets in (A2) suggests a transformation to polar coordinates. Defining radial and angular components \( R = \| \mathbf{Z} \| \) and \( \mathbf{W} = \| \mathbf{Z} \|^{-1} \mathbf{Z} \), where \( \| \cdot \| \) is any norm on \( C \), the regular variation condition (A1) can be rewritten

\[
\nu \left( \frac{R}{a_n} > r, \mathbf{W} \in \cdot \right) \sim r^{-\alpha} \nu(\cdot),
\]

where \( H \) is a nonnegative finite Radon measure on the unit sphere under the chosen norm. The measure \( H \) is called the angular measure and completely characterizes the tail dependence in \( \mathbf{Z} \). By appropriate choice of normalizing sequence \( a_n \), \( H \) can be made to be a probability measure.

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