Gamma Distribution Parameters for Cloud Drop Distributions from Multicylinder Measurements

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ABSTRACT

The liquid water content and drop diameters in supercooled clouds have been measured since the 1940s at the summit of Mount Washington in New Hampshire using a rotating multicylinder. Many of the cloud microphysics models in the Weather Research and Forecasting Model (WRF) assume a gamma distribution for cloud drops. In this paper, years of multicylinder data are reanalyzed to determine the best-fitting gamma or monodisperse distribution to compare with parameters in the WRF cloud models. The single-moment cloud schemes specify a predetermined and constant drop number density in clouds, which leads to a fixed relationship between the median volume drop diameter and the liquid water content. The Mount Washington drop number densities are generally larger and best-fit distributions are generally narrower than is typically assumed in WRF.

1. Introduction

The liquid water content and drop diameter in supercooled clouds have been measured at Mount Washington Observatory since the 1940s (Arenberg 1943; Langmuir 1944) using the rotating multicylinder. In this paper we reanalyze measurements made for these multicylinder observations to determine the cloud liquid water content and median volume drop diameter for a range of specified widths for the gamma distribution, which is assumed in the Milbrandt, Morrison, and Thompson cloud microphysics models in the Weather Research and Forecasting Model (WRF; Milbrandt and Yau 2005; Morrison et al. 2005; Thompson et al. 2004, 2008). In contrast to precipitation drops (e.g., Čurić and Janc 2011), these small cloud drops have negligible terminal velocity and move only when carried by the wind.

We use these results as well as those from assuming a monodisperse drop distribution to choose the distribution that best fits the multicylinder measurements. For best-fitting drop distribution we determine the number density of the cloud drops for that multicylinder observation. This allows us to compare measured drop number densities and distribution widths with those assumed in the WRF cloud microphysics models.

2. Background

The multicylinder method for determining the diameter of cloud drops and the liquid water content of
supercooled clouds exploits the variation in collision efficiency of cloud drops with cylinders over a range of diameters. The collision efficiency of a drop depends on the drop diameter, the cylinder diameter, and the wind speed. It is the fraction of drops of a given diameter in the swept volume of the cylinder that collides with the cylinder rather than being carried around it. Staff at the Mount Washington Observatory used a graphical method (Clark 1946; Howe 1991) to qualitatively describe the width of the drop distribution, ranging from narrow (A) to broad (J), and determine the liquid water content of the supercooled cloud and the size of the drops. Langmuir and Blodgett (1946) provided graphs of collision efficiency as a function of nondimensional numbers \( K \) (inertia parameter) and \( \phi \), where \( \phi = 0 \) for Stokes flow. Finstad et al. (1988) repeated the Langmuir and Blodgett (1946) analysis with more accurate formulations of the drag coefficient of the drops at low Reynolds number and using a digital computer rather than a differential analyzer. They provided collision efficiency formulas that are based on fitting spline curves to the data and are convenient for use in computer algorithms. In this paper, we use the Finstad collision efficiency formulas with some revisions to analyze the multicylinder data to determine the properties of the supercooled clouds.

We describe the method we use to determine the drop size and liquid water content for an assumed monodisperse cloud drop distribution. We then assume, instead, that the drop distribution is a gamma distribution and determine the distribution parameters in an extension of our method based on the monodisperse assumption. From the best-fitting gamma/monodisperse distribution we determine the cloud liquid water content, the median volume drop diameter, and the number density of cloud drops. We compare these results with the assumptions of the Milbrandt, Morrison, and Thompson cloud microphysics models in WRF.

3. Multicylinder method and analysis

When exposed in supercooled clouds, the multicylinder (Fig. 1) rotates at 1 or 2 rpm (depending on which motor is used) to generate layers of ice that are uniformly thick around the circumferences of the six stacked cylinders. The observer records the duration of the exposure, average wind speed and direction, air temperature, and atmospheric pressure. At the end of the exposure, the observer disassembles the multicylinder in a cold laboratory (air temperature less than 0°C and no wind) and measures the mass and thickness of the ice on each cylinder. Those data are used in a computer program that finds the parameters of the assumed drop distribution that minimize the sum of the absolute errors in the mass of ice on each cylinder.

a. Monodisperse and gamma drop distribution

For a monodisperse distribution, the multicylinder algorithm sweeps through drop diameters \( d_j \) from 3 to 90 \( \mu \text{m} \) in 0.1-\( \mu \text{m} \) increments and calculates for the \( i \)th cylinder the normalized ice mass

\[
\hat{m}_{ij} = E(U, d_j, D_i)UL_iD_i \Delta t, \tag{1}
\]
which includes all factors except the unknown cloud liquid water content. The quantities $D_i$ and $L_i$ are the measured mean iced cylinder diameter and length of the $i$th cylinder, $U$ is the average wind speed during the multicylinder exposure with duration $\Delta t$, and $E(U, d_i, D_i)$ is the collision efficiency calculated from the Finstad et al. (1988) formulas. The algorithm then computes a liquid water content $W_j$ for each of the cylinders from the measured ice mass $m_i$ and the calculated normalized ice mass,

$$W_j = m_i / \hat{m}_{ij},$$

and estimates the cloud liquid water content $W_j$ by averaging the $W_i$. The measure of the goodness of fit for this pair of $d_i$ and $W_j$ is the sum of the absolute value of the difference in the calculated and measured ice masses:

$$m_{\text{err},ij} = \sum_i |W_j \hat{m}_{ij} - m_i|.$$  (3)

The drop diameter $d_j$ for which $m_{\text{err},ij}$ is a minimum is then the diameter of all the cloud drops and the corresponding $W_j$ is the supercooled cloud liquid water content.

We modified this approach to calculate the parameters of a gamma distribution with density function

$$F(d) = \frac{N}{(\alpha - 1)\beta} \left( \frac{d}{\beta} \right)^{\alpha - 1} e^{-d/\beta}. \quad (4)$$

The width of the drop distribution is defined by $\alpha$, while $\beta$ determines the rate of the exponential decrease with diameter $d$. Note that the Khrgian–Mazin distribution (e.g., Pruppacher and Klett 1980) and the exponential distribution are gamma distributions with $\alpha = 3$ and $\alpha = 1$, respectively. The integral of $F(d)$ over all diameters is the drop number density $N$. The integral of $F(d)$ multiplied by the drop volume and the density of water $\rho_w$ is the cloud liquid water content

$$W = \rho_w \frac{\pi}{6} \int_0^\infty d^3 F(d) \, dd \approx \rho_w \frac{\pi}{6} N \beta^3 (\alpha + 2)(\alpha + 1) \alpha. \quad (5)$$

The median volume drop diameter $d_{\text{MVD}}$ for the distribution, which is defined as the diameter for which half the liquid water content is in smaller drops and half is in larger drops, depends on $\alpha$ and $\beta$:

$$d_{\text{MVD}} = \beta (\alpha + 2.67). \quad (6)$$

For a specified integer value of $\alpha$, the multicylinder algorithm steps through values of $\beta$. Collision efficiency is computed for 1-µm increments in drop diameter across $F(d)$. The calculated normalized ice mass

$$\hat{m}_{i,\beta} = U L_i D_i \Delta t \sum_{k=0}^{k_{\text{max}}} E(U, d_k, D_i) \Delta \hat{W}_{k,\beta}$$

is determined by summing over the increments of normalized liquid water content:

$$\Delta \hat{W}_{k,\beta} = \rho_w \pi \frac{\pi}{6} d^3 F(d) \frac{dd}{N}.$$  (8)

The estimated drop number density for each cylinder is the ratio of the measured ice mass and the calculated normalized ice mass

$$N_{i,\beta} = m_i / \hat{m}_{i,\beta}.$$  (9)

For that value of $\beta$, the estimate of the drop number density $N_{i,\beta}$ is the average of the $N_{i,\beta}$. The estimated cloud liquid water content and ice masses on the cylinders are then

$$W = N_{\beta} \sum_{k=0}^{k_{\text{max}}} \Delta \hat{W}_{k,\beta}$$

and

$$m_i = N_{i,\beta} \hat{m}_{i,\beta}. \quad (10)$$

For each value of $\beta$, the sum of absolute value of the difference between the calculated and measured ice masses on the cylinders is calculated as in Eq. (3). The value of $\beta$, and associated $N_{i,\beta}$, for which the mass error is a minimum are then the distribution parameters for the specified value of $\alpha$.

Errors in the multicylinder observations affect the liquid water content and median volume drop diameter resulting from this analysis. The critical measurements are the observation time, the wind speed, and the mass of ice on each cylinder. The other observation parameters, air temperature and pressure, are used in the calculation of collision efficiency. Errors of a few degrees Celsius or millibars have a negligible effect on the results. If the recorded observation time is longer (shorter) than the actual time, the liquid water content will be correspondingly smaller (larger) than the actual value. If the recorded wind speed is smaller (larger) than the actual wind speed, then both the liquid water content and median volume diameter will be larger (smaller) than the actual value. The mass of ice on each of the cylinders is measured to 0.01 g with an electronic balance and the diameters of the ice-covered cylinders are measured to 0.1 mm with a micrometer. The recorded
values are checked for consistency by plotting \( D_I \) versus \( m_I/(U_L D_I \Delta t) \) on a log–log scale. This is the plot used in the manual method for reducing the multicylinder data described in Howe (1991). Any errors in the measurements of the ice on the cylinders, or in entering the data, are shown by the departure of these points from a smooth curve. Cylinders with errors that cannot be corrected and cylinders that deviate from the curve because of spotty ice accretions are removed from the analysis.

b. Collision efficiency

The collision efficiency for these calculations of the monodisperse and gamma distribution is based on Finstad et al. (1988), with the exception that we allow the collision efficiency to be 0. In that paper it is argued that the collision efficiency is never 0; that the history term, if it were included, would increase the \( E = 0 \) they obtained for small values of the inertia parameter \( K \). The collision efficiency subroutine in the Finstad multicylinder analysis program sets a minimum value of 0.01 for \( E \). However, multicylinder observations frequently result in the largest cylinder, and sometimes the next largest, covered only with what appears to be frost. That layer is very thin and translucent, in contrast to the typical white rime ice on the other cylinders. Therefore, we allow \( E = 0 \). Because the fitted Finstad et al. (1988) spline curves do not fit the data well for small \( K \), we use the numerical results in their Table 3 for \( K < 0.256 \).

c. Sublimation and deposition

We tested the Makkonen (1992) approach for determining the deposition of frost and the sublimation of rime ice to correct the measured ice masses on the cylinders. For each multicylinder observation we did a heat balance calculation for each cylinder, balancing the convective and evaporative cooling, latent heat of fusion, heat used to warm the impinging drops to 0°C, and heat released when the accreted ice cools to the surface temperature of the cylinder, using the formulations in Jones (1996). The air in the cloud is assumed to be saturated with respect to water, while the air at the surface of the cylinder is saturated with respect to ice. This iterative calculation results in estimated cylinder surface temperatures and the mass of rime ice sublimated or frost deposited for each cylinder. For most multicylinder observations this analysis indicated sublimation from the two smallest cylinders and deposition on the two largest cylinders. On the other two cylinders deposition and sublimation were equally likely. We corrected the measured ice masses with these results and then used the corrected masses in the algorithms described above. However, the mass errors for both the monodisperse distribution and the gamma distributions using the ice masses corrected for sublimation and deposition were all larger than the errors using the measured ice masses, so this correction was abandoned.

4. Results

a. CRREL multicylinder observations

In the 1990s, 2011, and 2013 Jones and Claffey made 249 multicylinder observations in supercooled clouds...
at the Mount Washington Observatory. During these observations the visibility was very low. For the multicylinder observations with a weather observation in the same hour, the reported visibility was either 0 or 100 m. The wind speed and air temperature during the observations are shown in Fig. 2a. For typical exposure durations (10th, 50th, and 90th percentile values are 7, 12.8, and 22 min), the multicylinder averages over a significant distance. The range (10th–90th percentile) in the product of wind speed and duration is 7–24 km. Using the lengths and diameters of the six cylinders, this amounts to cloud sample volumes ranging from 80 to 290 m³. The drop diameters, assuming a monodisperse distribution, and the supercooled cloud liquid water contents for these observations with ice on at least three cylinders, using Eqs. (1)–(3), are shown in Fig. 2b.

We analyzed the multicylinder data assuming also that the cloud drop distribution was described by a gamma distribution for eight values of \(\alpha\) ranging from 2 to 99. The best fitting distributions for each \(\alpha\) for multicylinder observation F on 27 November 1992 (marked with a circle in Fig. 2b) are shown in Fig. 3a with the corresponding drop volume distribution in Fig. 3b. The observations are identified by the month, day, and year (MMDDYY) followed by a letter (A–Z) that indicates the order of the observation on that date. The drop number densities \(N\) for this case vary from 3700 cm⁻³ for \(\alpha = 2\) to 490 for \(\alpha = 99\), as compared with 450 cm⁻³ for a monodisperse distribution. The variation of the median volume drop diameter and cloud liquid water content for this range of \(\alpha\) as and for a single drop size is shown in Fig. 4 for this multicylinder observation as well as for two other observations with different characteristics. In all cases, for a narrower assumed distribution, the best-fit median volume drop diameter is larger and the liquid water content is smaller.

The measured ice masses on the six cylinders and the calculated best-fit ice masses for \(\alpha = 2, 10, \text{ and } 99\), representing broad, moderate, and narrow distributions, are shown in Fig. 5 for the three cases in Fig. 4. These cases are discussed in detail in the next three paragraphs.

For the observation on 27 November 1992, the best fit \((d_{\text{MVD}} = 13.8 \mu m; W = 0.64 \text{ g m}^{-3})\) to the ice masses on the cylinders is provided by the gamma distribution
with $\alpha = 99$, with a total ice mass error of 1.2% as compared with 3.1% for the worst fit with $\alpha = 2$. All the distributions produce nearly the correct ice masses on cylinders 2 and 5. The mass errors for cylinders 1, 3, and 4 decrease as the distribution narrows (Fig. 5). Cylinder 6 was not used in the analysis because of difficulties in measuring the ice accretion. In this case, the fits provided by the monodisperse and $\alpha = 50$ distributions are nearly as good as for $\alpha = 99$.

For the observation on 14 March 1994, the gamma distribution with $\alpha = 10$ and $N = 850 \, \text{cm}^{-3}$ provides the best fit ($d_{\text{MVD}} = 11.8 \, \mu\text{m}; W = 0.47 \, \text{g m}^{-2}$) with a mass error of 1.5%, while the monodisperse distribution (error = 10.6%; $N = 210 \, \text{cm}^{-3}$) is the worst fit. The ice masses for $\alpha = 2$ and $\alpha = 99$ shown in Fig. 5 are typical of the drop distributions that are too wide and too narrow, respectively, with too much (little) ice on cylinders 1 and 6, and too little (much) ice on cylinders 3–5. The relatively low wind speed (12.0 m s$^{-1}$) for this observation causes a wide range of collision efficiencies for the range of cloud drop diameters and emphasizes the difference between the narrow, moderate, and broad distributions.

The gamma distribution with $\alpha = 2$ and $N = 470 \, \text{cm}^{-3}$ is the best fit ($d_{\text{MVD}} = 21.4 \, \mu\text{m}; W = 0.57 \, \text{g m}^{-2}$) for the observation on 10 March 2011 with an error of 2.1%, compared an error of 3.8% for the worst-fitting monodisperse distribution ($N = 80 \, \text{cm}^{-3}$). The pattern of increasing ice mass with cylinder diameter (Fig. 5) is typical of conditions with relatively large drop diameters and high wind speeds. The calculated ice masses on cylinders 4 and 5 approach the measured ice masses as the...
assumed distribution becomes wider. An even better fit is provided by an exponential drop distribution \((\alpha = 1)\).

Overall, the narrow distributions (monodisperse and gamma with \(\alpha = 50\) or 99) fit the data better than the broader distributions. The average absolute mass error and bias for each of the six cylinders for all the Cold Regions Research and Engineering Laboratory (CRREL) observations are shown in Fig. 6 for gamma distributions with \(\alpha = 2, 10,\) and 99. The monodisperse distribution is slightly better than \(\alpha = 99\); the absolute error and bias are smaller or equal to the values for the gamma distribution except for cylinder 2 (larger absolute error), cylinder 5 (both values larger), and cylinder 6 (larger bias). Individually, the narrow distributions provide the best fit for 73\% of the multicylinder observations. The broadest distributions \((\alpha = 2, 4,\) or 7) are best for 16\% of the observations, and the moderate distributions \((\alpha = 10, 13,\) or 16) are best for the remaining 11\%.

b. Wind speed variations

The wind speed used for the multicylinder analyses outlined in section 3a is the average for the duration of the observation. To determine the effect of the varying wind speed during a multicylinder observation, we also analyzed the observations from 2013 using the 1-min-average wind speeds that the observatory now archives, assuming a monodisperse distribution. For this analysis, we stepped through each observation in time steps matching the start and end times of the 1-min increments. For each time step, each bare cylinder diameter was increased using the final ice thickness for that cylinder multiplied by the fraction of the total observation time that had elapsed. The collision efficiency of the drops was calculated from that iced diameter and the wind speed in the time step. As for the analysis using the average wind speed, the minimum of Eq. (3) defined the best drop diameter and liquid water content for the observation.

Time series of the 1-min wind speeds and the mean wind for the multicylinder observations are plotted in Fig. 7. There is a gap early on 3 April when the data from the pitot–static anemometer were affected by ice. The maximum difference between the 1-min winds and the mean wind for the 24 multicylinder observations ranges from 1 to 4.8 m s\(^{-1}\), or between 5\% and 28\% of the mean wind. On average the calculated liquid water content is 0.6\% smaller and the calculated drop diameter is 0.1\% larger relative to the values based on the average wind speed. The largest fractional change in liquid water content using the 1-min winds is an increase from 0.21 to 0.22 g m\(^{-3}\), with a decrease in drop diameter from 9.1 to 9 \(\mu\)m. The largest fractional change in drop diameter is a decrease from 20.7 to 20.4 \(\mu\)m with no change in liquid water content. These small differences indicate that using 1-min winds for the multicylinder analysis does not provide a significantly different result from that obtained by using the average wind speed.

c. Howe multicylinder observations

The Mount Washington Observatory has used multicylinder observations to characterize the properties of supercooled clouds since 1941. A few decades ago the observatory provided the handwritten data sheets for the observations from 1969 through the spring of 1988 to CRREL and we transferred the data to electronic format. During those 20 years, the observations were made
by observers on the observatory staff, with J. Howe analyzing the data or checking the analyses done by others. The graphical analysis method, which includes an evaluation of the width (A–J) of the cloud drop distribution, is described in Howe (1991). Distribution A is a single drop size. Distribution B is based on a measured distribution and C–J (no distribution I) are progressively broader. The distributions are approximately Gaussian with respect to the volume of the drops (Table 1).

The frequency of the 644 observations varied significantly. More than 75% were made in the winter season of 1972/73 and the four winters between October 1984 and May 1988. Fewer than 10 multicylinder observations were made in each of eight winters. Since 1979 the observations have been made from the tower of the Sherman Adams Summit Building, which replaced the old summit building. Most observations occurred in the cold weather months from November through April, but 8% were made in October and May. For 83% of the observations the wind direction is between southwest and northwest, indicating a continental air mass. The air temperature for the Howe observations ranges from $-21^\circ$ to $-29^\circ$C, and the maximum wind speed, drop diameter, and supercooled cloud liquid water content are 42 m s$^{-1}$, 45 $\mu$m, and 1.4 g m$^{-3}$, respectively. The monodisperse distribution A is chosen for 19% of the observations with the other eight distributions chosen for between 4% and 14% of the cases. The three narrowest distributions account for 47% of the observations.

We reanalyzed the Howe observations using the approach in section 3a to determine the cloud properties based on monodisperse and gamma distributions, minimizing the cylinder mass errors. As for the more recent CRREL multicylinder observations, the narrow drop distributions generally provide the best fits to the cylinder masses: 63% of the observations fit the monodisperse distribution or the gamma distributions with $\alpha = 50$ or 99. The broadest distributions ($\alpha = 2, 4,$ or 7) are best for 22% of the observations, and the moderate distributions ($\alpha = 10, 13,$ or 16) are best for the remaining 15%. A chi-square test of the 644 observations indicates that the Howe A–J measures of distribution width and our monodisperse and gamma distribution widths are related at the 0.001 level.

In these analyses, three of the cases result in large median volume diameters and two result in large liquid water contents for the best-fit distribution. The large median volume diameters are between 45 and 90 $\mu$m. The air temperature during these observations ranged from $-4^\circ$ to $-8^\circ$C and there was clear glaze ice on the cylinders. In all three cases, the Howe analyses found that the J distribution provided the best fit, and we found that the broad gamma distribution with $\alpha = 2$ was best. The observations with large supercooled liquid water contents of 1.25 and 1.48 g m$^{-3}$ had large $d_{MVDS}$ (24 and 18 $\mu$m) in narrow ($\alpha = 50$) and broad ($\alpha = 7$) drop distributions with air temperatures of $-6^\circ$ and $-9^\circ$C, respectively.

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<th>Distribution</th>
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<th>B</th>
<th>C</th>
<th>D</th>
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d. Weather and cloud parameters

The width of the cloud drop distributions is related to the conditions in the cloud (Table 2). The mean weather and cloud parameters for the observations with narrow drop distributions are statistically significantly different (at the 0.0001 level) from those for moderate and broad distributions combined. On average, narrow cloud drop distributions are associated with lower air temperatures, lower pressures, and higher wind speeds. In these weather conditions the cloud supercooled liquid water contents and drop diameters are smaller than they are in milder weather. Comparing broad and moderate drop distributions, the only significant difference (at the 0.001 level) is between the means of the median volume drop diameter, which is 14.7 μm for broad and 12.8 μm for moderate distributions.

The differences between the conditions associated with narrow drop distributions and those associated with moderate or broad distributions may be related to the air mass or front at the summit during the multiclyinder observation. Whipple (1948) compiled multiclyinder data for January–April, November, and December for 1944–46. During that time weather observations were made at the summit of Mount Washington every 3 h. At observation times with fog and air temperature below freezing, a multiclyinder observation was also done. For all observations times, the type of air mass or front that prevailed was determined from surface synoptic charts obtained from the Weather Bureau in Boston, Massachusetts. All observations were divided into three air-mass categories and three frontal categories. The dominant air mass was continental polar (cP) (74% of observations), with maritime polar (mP) and maritime tropical air masses each occurring 10% of the time, and fronts accounting for the remaining 6%. Observations with icing had different occurrence rates, with cP air still dominant at 65% of icing observations, mP air accounting for 16%, and fronts (mostly cold and occluded fronts) 11%. The relatively cold temperatures, low liquid water contents, and small drops sizes associated with cP air described in Whipple (1948) are consistent with the conditions associated with the narrow drop distributions (Table 2), which account for 66% of the CRREL and Howe multiclyinder observations.

There are no data on the cloud type or thickness from the weather observations at the summit of Mount Washington. However, Whipple (1948) discusses the cloud conditions at the summit that typically occur with the different air masses and fronts. The cP air masses in migrating polar anticyclones are associated with fair weather, with the summit in a cap cloud and the general cloud cover in the region ranging from clear to overcast. The mP air masses originate in storms over the

Atlantic and produce low stratus clouds, while nimbostratus clouds are typical of occluded fronts. As part of the multiclyinder observations made in the 1940s, the cloud-base height below the summit was estimated using the method in Langmuir (1944) that assumes the cloud is formed by air forced to rise over the summit. We calculated the cloud-base height below the summit for each of the CRREL and Howe multiclyinder observations using the measured pressure, air temperature, and cloud liquid water content, following Thorkildson et al. (2009). The mean and standard deviation are 430 and 180 m, respectively.

e. Comparison to WRF cloud drop distributions

We can compare these two sets of multiclyinder observations with the assumptions underlying the Morrison, Milbrandt, and Thompson cloud microphysics models in WRF. In the single-moment cloud water variable only Morrison and Thompson models, the number density N of cloud drops is fixed and, therefore, the relationship between median volume diameter and cloud liquid water content is also fixed:

\[ W = \frac{\rho_w \pi N d_{MVD}^3 (\alpha + 2)(\alpha + 1)\alpha}{6(\alpha + 2.67)^3} \]  

(11)

Note that in the limit \( \alpha \to \infty \), Eq. (11) is the relationship between W and \( d_{MVD} \) for a monodisperse distribution. Both Thompson and Morrison base their cloud drop distributions on observations in Martin et al. (1994). In the Thompson cloud microphysics module, users can specify N, which in turn determines the value of \( \alpha \). The default is a maritime air mass with \( N = 100 \text{ cm}^{-3} \), which results in \( \alpha = 13 \). The suggested (Thompson et al. 2008) continental airmass value of \( N = 250 \text{ cm}^{-3} \) results in \( \alpha = 7 \). The maximum allowed value \( \alpha = 16 \) is reached for \( N \leq 77 \text{ cm}^{-3} \). As N gets large, \( \alpha \) approaches its minimum value of 3. In the Morrison module, \( \alpha \) depends on N (with a default value of 250 cm\(^{-3}\)), air

| Table 2. Means and standard deviations (in parentheses) of weather and cloud parameters for broad/moderate and narrow drop distributions. |
|---------------------------------|-----------------|------------------|
| Number of observations         | 306             | 587              |
| Air temperature (°C)           | −8.8 (4.3)      | −12.2 (6.1)      |
| Wind speed (m s\(^{-1}\))      | 19.6 (7.2)      | 23.2 (8.0)       |
| Pressure (hPa)                 | 794.2 (7.7)     | 792.0 (7.9)      |
| Supercooled liquid water content (g m\(^{-3}\)) | 0.51 (0.24)  | 0.42 (0.21)      |
| Median volume drop diameter (μm) | 14.0 (5.6)  | 11.3 (5.1)       |
temperature, and pressure, with lower and upper bounds of 3 and 11. In Fig. 8 the Howe and CRREL median volume diameter and liquid water content pairs for the best-fit distributions are plotted along with the Thompson maritime, continental, and $N = 1000 \text{ cm}^{-3}$ ($\alpha = 4$) curves and Morrison curves for $N = 50, 250,$ and $1000 \text{ cm}^{-3}$ for temperatures and pressures typical for the multicylinder observations. The vast majority of the measurements at Mount Washington are to the left of the curves for $N = 250 \text{ cm}^{-3}$, indicating that the Morrison and Thompson default values of $N$ do not represent the summit cloud properties in winter conditions.

The double-moment Milbrandt cloud physics module assumes a broad distribution of cloud drops ($\alpha = 2$) and allows $N$ to vary from nominal values of $100 \text{ cm}^{-3}$ for maritime air masses and 200 and $800 \text{ cm}^{-3}$ for continental and polluted continental air masses, respectively. Comparisons of the Milbrandt results with the observed $d_{\text{MVD}}-W$ pairs for the best-fit distributions for three of the CRREL intensive multicylinder observation periods are shown in Fig. 9. The values of modeled $d_{\text{MVD}}$ and $W$ are from the data underlying Fig. 13 in Yang et al. (2012) from simulations done by Yang using the Canadian mesoscale model Global Environment Multiscale Limited Area Model (GEM-LAM) with Milbrandt cloud physics. For the March 1994 case, a polluted continental air mass is assumed. The diagnosed $d_{\text{MVD}}-W$ pairs fall on both sides of the nominal curve, while most of the observed values are to the left of the curve, indicating somewhat larger cloud drop number densities than obtained by the model. For March 1996 a continental air mass is assumed. Most of the diagnosed $d_{\text{MVD}}-W$ pairs cluster along the curve for $N = 900 \text{ cm}^{-3}$. A continental air mass is also assumed for the March 2011 observation period. Back trajectories calculated for this period are consistent with a storm passing northwest of New England on 11–12 March. On 10–11 March, winds were from the southeast as the warm front moved through, bringing in

![Figure 8](image-url)
a marine air mass. The multicylinder observations during this time (circles) show relatively low liquid water contents, while the simulated cloud drops cluster along the $N = 450$ cm$^{-3}$ curve. A cold front crossing New Hampshire around 0000 UTC 12 March turned the winds westerly for 12–13 March, which gradually replaced the marine air mass with a continental air mass. The corresponding multicylinder observations (squares) have $d_{\text{MVD}}$s clustered around 15 $\mu$m and relatively high liquid water contents. The simulated cloud drops fall along the $N = 900$ cm$^{-3}$ curve with larger $d_{\text{MVD}}$ than were observed.

For both the continental and polluted continental assumptions, the observed drop number densities tend to be higher than the simulated values, as was seen in the comparison with the single-moment schemes in Fig. 8.

f. Cloud drop number densities

Using the best-fit distributions from monodisperse and gamma distributions with $\alpha$ ranging from 2 to 99, we determined the drop number density for the 893 Howe and CRREL multicylinder observations. We ranked them from smallest to largest $N$ and assigned a nonexceedance frequency equal to the rank divided by 894 to obtain the cumulative distribution plotted in Fig. 10. The median drop number density is 700 cm$^{-3}$ and the 10th and 90th percentile values are 220 and 2490 cm$^{-3}$. The large values of $N$ tend to be associated with broad distributions, which include many very small drops. For $\alpha = 2$, for example, 95% of the drops are smaller than the median volume diameter, and half are smaller than 0.36$d_{\text{MVD}}$. These small drops make up very little of the mass of ice on the larger cylinders of the multicylinder because of their small volumes and low collision efficiencies. The monodisperse distribution provides the lower bound on $N$ and results in 10th, 50th, and 90th percentile values of 120, 480, and 1520 cm$^{-3}$ for these observations.

We discussed the effects of errors in the multicylinder observations in section 3a. An error in recording the observation duration would have the most significant effect on the calculated drop number density because it would change the liquid water content with no change.
in either the median volume diameter or the fit to the gamma distributions. The error in measuring exposure duration is less than ±5 s, resulting in 10th and 90th percentile errors in duration of ±1.2 and 0.3% and errors of the same magnitude but opposite sign in both W and N.

5. Conclusions

Supercooled cloud drops are observed at the summit of Mount Washington at temperatures ranging from −30°C to just below 0°C. Multicylinder observations of these supercooled clouds indicate that 1) there is no relationship between the median volume drop diameter and the supercooled cloud liquid water content, 2) the drop distributions tend to be narrow, and 3) the drop number densities are relatively high in comparison with the defaults in the WRF microphysics modules. The underlying reasons for the narrow drop distribution and high number densities are not known. They may be related to cold temperatures throughout the boundary layer, enhanced aerosol counts associated with winter weather, or the proximity of the supercooled cloud to the ground in the complex terrain around Mount Washington. Some of the high drop number densities may be an artifact of the assumed gamma distribution shape, with a different distribution possibly resulting in lower values of N. However, the minimum N is provided by the monodisperse distribution.

Users of WRF can change the source code in the microphysics schemes to use the value of N appropriate for their simulation. In the Milbrandt scheme, alpha_c specifies the gamma distribution parameter and the air mass, with associated cloud number density N_c_SM given by cloud condensation nuclei type. In the Morrison scheme, changing the number density NDCNST changes PGAM. Similarly, in the Thompson scheme changing Nt_c changes the gamma distribution parameter mu_c.

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