Comparison of AMIE-modeled and Sondrestrom-measured Joule heating: A study in model resolution and electric field–conductivity correlation

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1 Joule heating by high-latitude ionospheric electric fields is thought to be underestimated by models, and it has been conjectured that the source of the underestimation is “electric field variability,” which is often defined as electric field structure below the resolution of the model. We investigate this and related issues by (1) comparing the Joule heating measured by the Sondrestrom incoherent scatter radar during a 40 h period containing a storm with that modeled by the Assimilative Mapping of Ionospheric Electrodynamics (AMIE) procedure and (2) employing an magnetosphere-ionosphere (M-I) coupling model to analyze the theoretical dependence of Joule heating estimates on the spatial resolution of the inputs. We find that as compared with Sondrestrom measurements, a much larger contribution from correlation between conductance and squared electric field (positive for AMIE and negative for Sondrestrom) partially compensates for a much smaller mean-squared electric field, such that the overall average Joule heating rate modeled by AMIE is 29% less than measured by Sondrestrom. The underestimation of the mean-squared electric field was not associated with small-temporal-scale variability. Surprisingly, the M-I coupling model finds that coarse spatial resolution causes overestimation of the Joule heating rate, owing to the finding that the subresolution-scale spatial fluctuations in conductance and squared electric field are anticorrelated. When comparing estimates of the total Joule heating over a period of time, the increased Joule heating arises as a larger contribution from temporal correlation between conductance and squared electric field, which overcompensates for the reduced mean-squared electric field. Therefore, the difference in the Sondrestrom and AMIE correlation contributions might be explained by a difference in spatial resolution.


1. Introduction
1.1. Background

[2] Codrescu et al. [1995] first raised the question of the contribution of electric field variability to Joule heating. They noted that the Joule heating rate is proportional to the mean of the squared effective electric field \( \langle |\vec{E}|^2 \rangle \), and that the mean of the square may be much larger than the square of the mean, owing to large fluctuations of the electric field about its mean \( \vec{E}^2 = \vec{E} + \vec{v}_n \times \vec{B} \), where \( \vec{E} \) is the electric field, \( \vec{v}_n \) is the neutral gas velocity, and \( \vec{B} \) is the magnetic field. Therefore, modeling of the mean electric field \( \langle \vec{E} \rangle \) must be supplemented by modeling of the variability of the electric field \( \langle |\vec{E} - \langle \vec{E} \rangle|^2 \rangle \).

[3] Codrescu et al. [1995] raised this issue in the context of modeling thermospheric temperature and flow using general circulation models (GCMs). GCMs are physics-based global models of the thermospheric circulation and temperature that originally were driven only by solar radiation [e.g., Kohl and King, 1967], but which subsequently added energy input from interaction with magnetospherically driven ion flows in the high-latitude ionosphere [Richmond and Matsushita, 1975; Fuller-Rowell and Rees, 1980; Dickinson et al., 1981; Roble et al., 1982], to reconcile discrepancies with wind measurements. The effect of ion flows was included through a model of the high-latitude electric field [e.g., Foster et al., 1986; Weimer, 2001], that
is, through a mean $\langle \vec{E} \rangle$. However, discrepancies continued to be pointed out [e.g., Hedin et al., 1994], prompting Codrescu et al. [1995] to compare the physics-based Coupled Thermosphere-Ionosphere GCM model (CTIM) [Fuller-Rowell et al., 1994] with the empirical Mass Spectrometer Incoherent Scatter Radar (MSIS) model [Hedin et al., 1977; Hedin, 1987], and conjecture that the high-latitude source term was underestimated in CTIM. They suggested electric field variability as the missing source.

[4] To address electric field variability Codrescu et al. [2000] used the Millstone Hill radar database to estimate $\langle |\vec{E} - \langle \vec{E} \rangle|^2 \rangle$ for the high-latitude ionosphere. They found that the variability was on the order of the mean. The Codrescu et al. [1995] and Codrescu et al. [2000] studies inspired subsequent studies by Crowley and Hackert [2001], by Matsuo et al. [2003, 2005] and Matsuo and Richmond, 2008], by Shepherd et al. [2003], and by Cosgrove and Thayer [2006], utilizing the Assimilative Mapping of Ionospheric Electrodynamics (AMIE) procedure [Richardson and Kamide, 1988], DE2 satellite data [Hansen et al., 1981; Heeis et al., 1981], SuperDARN [e.g., Greweald, 1995], and the Sondrestrom incoherent scatter radar, respectively. For more complete summaries of previous research on variability, see Matsuo et al. [2003] and Johnson and Heeis [2005].

1.2. AMIE and Sondrestrom

[5] In the first half of this article we investigate variability and related issues that arise in the modeling of Joule heating by comparing the Joule heating measured by the Sondrestrom incoherent scatter radar during a 40 h period containing a storm with the Joule heating modeled by the Assimilative Mapping of Ionospheric Electrodynamics (AMIE) procedure, after removing the Sondrestrom data from the AMIE assimilation. The comparison employs a resolution dependent decomposition of the Joule heating rate into terms with a physically transparent interpretation, so that the ability of the model to capture various aspects of the physics can be analyzed.

[6] The AMIE procedure [Richardson and Kamide, 1988; Richardson et al., 1988] is a procedure for combining a statistical model of electric field and field line integrated conductivity (conductance) with present observations, to obtain an improved estimate of the global electric field and conductance. It can be thought of as an application of Bayes’ theorem. An initial or prior Gaussian probability distribution function (PDF) for a set of model parameters is determined from a statistical electric field model. A likelihood is determined as a Gaussian PDF for observations conditioned on the model parameters. From the likelihood and the prior, Bayes theorem determines an improved or posterior PDF for the model parameters conditioned on the measurements.

[7] The Sondrestrom radar is an incoherent scatter radar located in Sondrestrom Fiord, Greenland. In what is known as three-position mode, it measures the line-of-sight plasma drift velocity sequentially in three different directions. From these are obtained a vector velocity $\vec{v}$, which is used to compute the electric field $\vec{E}$ under the assumption that $\vec{v} - v_\parallel \vec{B} = \vec{E} \times \vec{B}/B^2$, where $\vec{B} = BB$ is the magnetic field, and $v_\parallel$ is the field-aligned component of velocity.

[8] The radar measurements and AMIE modeling will be compared for a 40 h period during 9–10 January 1997, which contained a moderate storm. This storm period has been studied previously by many authors. The modeling study by Lu et al. [1998] describes the estimation of electric field and conductance using the AMIE procedure, which is rerun without the Sondrestrom data for the present study.

[9] This study will be able to compare the resolved-scale mean-squared electric field, small-temporal-scale variability, and temporal correlation between conductance and squared electric field in the model and measurements. Because the measurements have a smaller spatial resolution than the model, there will remain some ambiguity with regard to the contribution of small-temporal-scale variability.

1.3. Simplest-Case Magnetosphere-Ionosphere Coupling Model

[10] The measurement-model comparison reveals a difference in the way that Joule heating is realized: the measurements have a larger contribution from the mean-squared electric field; and the model has a larger contribution from temporal correlation between the conductance and squared electric field. Although this difference may be the result of model error and/or a small (40-h) statistical sample, we take them as motivation to pursue a possible physical explanation, specifically, an explanation in terms of differing resolution.


[12] Following earlier work on M-I coupling, we consider a simplest-case model in which variable ionospheric electric fields are realized by driving a variable ionospheric conductivity with a constant magnetospheric generator, which has a finite (and constant) internal conductance. By varying the internal conductance the magnetosphere varies between behaving as a voltage source (which gives no electric field variability) and as a current source (which gives maximal electric field variability). The ionospheric conductance is taken to have a one-dimensional sinusoidal variation.

[13] We find that the difference in the contributions from temporal correlation can be explained by a difference in spatial resolution. However, an alternative explanation is found in effects related to AMIE’s reliance on magnetometers.

[14] In addition, the model finds that coarse spatial resolution leads to overestimation of the Joule heating rate, in the presence of small-temporal-scale variability! While the model is probably too simple to warrant a prediction based on this unexpected result, it does show that simultaneous spatially resolved measurements of conductance and electric field are needed to determine even the sign of the contribution to Joule heating from small-temporal-scale variability.

[15] In section 2 we give a decomposition of the Joule heating rate into resolved-scale and small-scale contributions. In section 3 we present the measurement-model comparison, in terms of small-scale, resolved-scale, and correlation-related contributions. In section 4 we analyze theoretically the effects of small-temporal-scale variability on
the modeling of Joule heating, and interpret the measurement-model comparison. In section 5 we summarize our findings, and suggest future work.

2. Resolution-Dependent Decomposition of the Joule Heating Rate

[16] We begin by writing the Joule heating rate of neutral species due to collisions with the ionized form \( f' \) as

\[
f' = m_i n_i |\vec{v}_i - \vec{v}_n|^2 = \frac{en_i}{B} \frac{\rho_i}{B} |\vec{E}|^2 + \frac{en_i}{B^2} |\vec{E}_i|^2
\]

\[
\simeq \sigma_p |\vec{E}_i|^2 = \sigma_p (\vec{E}_i \cdot \vec{E}_i + |\vec{v}_n \times \vec{B}|)^2,
\]

where \( \sigma_p \) is the Pedersen conductivity, \( \vec{v}_n \) is the neutral velocity, \( \vec{B} = BB \) is the magnetic field, \( \rho_i \) is the ion-neutral collision frequency to the ion gyro frequency, \( n_i \) is the ion number density, \( \nu_m \) is the ion-neutral collision frequency, \( m_i \) is the ion mass, \( e \) is the absolute value of charge on an electron, \( \vec{v}_i \) is the ion velocity, and \( \vec{E}_i \) and \( E_i \) are the perpendicular and parallel to \( \vec{B} \) electric field components. We drop the wind contribution, and assume the electric field maps along \( \vec{B} \). Then integrating along \( \vec{B} \) replaces \( \sigma_p \) with the field line integrated conductivity \( \Sigma_p \) (the conductance), and gives the field line integrated Joule heating rate as

\[
J' = \Sigma_p |\vec{E}_i|^2.
\]

[17] Small-scale variability refers to variability on scales below the resolution of some relevant model. (The term “small-scale” will be used when it is not necessary to distinguish between spatial and temporal, either because it applies to both (as at present), or because it is clear from the context (as will occur later).) Consider the model as a mapping from a finite set of “resolution cells” to values for physical quantities. Each resolution cell is a small space-time region sized to the resolution (spatial and temporal) of the model, and together the cells are a tiling of the experimental domain. To investigate the contribution of small-scale variability we obtain a separation of scales by writing quantities as sums of the simple average \( \langle \cdot \rangle_i \) over the \( i \)th resolution cell, and a variation about it:

\[
\Sigma_p = \Sigma_{p0} + \delta \Sigma_{p}, \quad \Sigma_{p0} = \langle \Sigma_p \rangle_i,
\]

\[
\vec{E} = \vec{E}_i + \delta \vec{E}_i, \quad \vec{E}_i = \langle \vec{E}_i \rangle.
\]

Substituting the decompositions (3) into the Joule heating rate equation (2), and taking the mean \( \langle \cdot \rangle_i \) over the \( i \)th resolution cell, gives the mean Joule heating rate in the cell:

\[
J'_i = \langle J' \rangle_i = \Sigma_{p0} E_i^2 + \Sigma_{p0} (\delta E_i^2)
\]

\[
+ 2 \langle \delta \Sigma_p \delta E_i \rangle \cdot \vec{E}_i + \langle \delta \Sigma_p \delta E_i \rangle,
\]

\[
= \Sigma_{p0} E_i^2 + \Sigma_{p0} (\delta E_i^2)
\]

\[
+ 2 \sigma_{\Sigma p} \sigma_{\delta E_i} C_{\Sigma p \delta E_i} \delta E_i + \sigma_{\Sigma p} \sigma_{\delta E_i} C_{\Sigma p \delta E_i},
\]

where \( \delta E_i \) is the standard deviation of \( \delta \Sigma_p, \delta E_i \) is the standard deviation of \( \delta E_i, \delta \Sigma_p \) is the trans-

terpose of a vector containing the standard deviations of the two components of \( \delta E_i, C_{\Sigma_p \delta E_i} \) is the correlation coefficient between \( \delta \Sigma_p \) and \( \delta E_i \), and \( C_{\Sigma_p \delta E_i} \) is a diagonal matrix containing the small-scale correlation coefficients between \( \delta \Sigma_p \) and the two components of \( \delta E_i \). The last equality in equation (4) makes use of the definition for the correlation coefficient:

\[
\langle ab \rangle = \langle a \rangle \langle b \rangle + \sigma_a \sigma_b C_{ab}, \quad \sigma_a = \langle (a - \langle a \rangle)^2 \rangle, \quad \sigma_b = \langle (b - \langle b \rangle)^2 \rangle.
\]

and \( C \) is a quantity between \(-1\) and \(+1\) known as the correlation coefficient.

[19] In equation (4), the first term is the contribution from the resolved-scale electric field and conductance. The second term is the contribution from the small-scale variability of the electric field, weighted by the resolved-scale conductance, which was identified by Codrescu et al. [1995] as a potentially important missing term in the modeling of Joule heating. The last two terms are the contribution from small-scale correlations between the electric field and conductance, which are dependent on standard deviations and correlation coefficients computed over the \( i \)th interval, and which are either positive or negative depending on the sign of the respective correlation coefficient. Note that because the last two terms may be negative, it is not necessarily the case that small-scale variability (the sum of the last three terms) makes a positive contribution to Joule heating.

3. Comparison of Sondrestrom Measurements and AMIE Modeling for a Moderate Storm

[19] On the basis of the assumption that the electric field measured by the Sondrestrom radar represents the actual electric field (resolved up to some spatiotemporal resolution), we will evaluate the ability of AMIE to model the Joule heating during a 40 h period containing a moderate storm, for which AMIE had good data inputs. Previous AMIE studies of this period included the Sondrestrom measurements among those assimilated. However, in order to test the characteristics of the model at a position spatially removed from assimilated electric field measurements, the AMIE procedure has been reapplied with the Sondrestrom measurements omitted. During this period the Sondrestrom radar operated in what is called three-position mode, which means that the electric field is only measured at one position in space. Therefore, the comparison will be based on a single 40-h time series, with the AMIE output taken at the position of Sondrestrom, so that only temporal features can be directly studied. Specifically, we will study the small-temporal-scale variability, and the contribution from resolved-scale temporal correlation between electric field and conductance.

3.1. Analysis Method

[20] To investigate the importance of small-temporal-scale variability, we would like to set the time period over which the mean \( \langle \cdot \rangle_i \) in (4) is taken to the model’s temporal resolution, and use Sondrestrom data to assess the importance of the small-temporal-scale terms, which cannot be computed from model outputs. However, since the AMIE model can be run at any time step (for which input data is available), it is
unclear what we should consider to be the temporal resolution of the model. Should we consider it to be limited only by the temporal resolution of the measurement of the model input parameters, in which case the temporal resolution of AMIE and Sondrestrom are approximately equal?

[21] Since there is no natural choice for what scale constitutes small-temporal-scale, we will consider the temporal resolution as a variable. This will allow us to evaluate at what resolution small-temporal-scale effects become important in the Sondrestrom measurements. It will allow us to compare the temporal resolution of AMIE with the temporal resolution of Sondrestrom, by comparing their small-temporal-scale terms. It will allow us to compare measurement and model at the same temporal resolution, over a range of resolutions. This will provide some comparative information in the Fourier domain, and allow us to isolate small-temporal-scale effects, in spite of the fact that the measurements are not evenly sampled.

[22] Results will be presented as a function of the temporal interval length (the temporal resolution) over which the mean $\langle \cdots \rangle$ in (4), which we will call the small-scale mean, is computed. Because the data are obtained at only one spatial location, the spatial interval lengths for $\langle \cdots \rangle$ will not be adjustable, and will be fixed at the spatial resolutions of the Sondrestrom radar, and the AMIE model, respectively. Standard deviations and correlation coefficients will be computable only in the temporal dimension. These represent temporal statistics of spatial averages, over one spatial resolution cell, with different spatial resolutions for the measurement and model. The effect of this difference will be analyzed in section 4.

[23] We will work with 40 h of Sondrestrom data subdivided into between 1 and 160 intervals, corresponding to interval lengths (resolutions) of between 40 and 1/4 h. For each subdivision the Joule heating rate terms of equation (4) are computed in each interval (by averaging the data in the interval to estimate the small-scale means $\langle \cdots \rangle$), and then these averaged over all the intervals in the subdivision, using an average weighted by the number of data points in the subdivision. We refer to the latter as the $N$-interval mean, and denote it by

$$\langle \phi_i \rangle_N = \frac{1}{\sum_{i=1}^{N} K_i \sum_{i=1}^{N} K_i \phi_i},$$

where $\phi_i$ is one of the terms of equation (4), $N$ is the number of intervals (determines the resolution), $K_i$ is the number of data points in the $i$th interval, and the subscript $i$ is used on the left hand side to remind the reader that $\langle \cdots \rangle_N$ operates on functions defined on the intervals.

[24] The weighting by $K_i$ ensures that the $N$-interval mean Joule heating rate $\langle J' \rangle_N$ does not depend on the resolution $N$, that is,

$$\langle J' \rangle = \langle J' \rangle_N$$

is the simple average over all the data points, for all $N$. However, the relative contributions of the terms in equation (4) depend on the resolution. When the resolution is so fine ($N$ so large) that $K_i$ is either one or zero, then the small-scale variability terms vanish, and the resolved-scale quantities $\Sigma_{\phi_i}$ and $E_i$ completely determine the mean Joule heating rate $\langle J' \rangle$. For smaller $N$ there is a nonzero contribution from the small-scale variability terms. By plotting all the terms versus the resolution $N$, the error due to ignoring the small-scale variability terms for a given resolution can be determined.

[25] In order to make a quantitative investigation of temporal correlation effects, we will employ equation (5) to separate the mean Joule heating rate $\langle J' \rangle_N$ into contributions from uncorrelated $N$-interval means, and quantities proportional to correlation coefficients. Substituting equations (4), (6), and (5) into equation (7) gives

$$\langle J' \rangle_N = \langle \Sigma_{\phi_i} E_i^2 \rangle_N + \langle \Sigma_{\phi_i} (\delta E_i^2) \rangle_N$$

$$+ \langle 2\Sigma_{\phi_i} \sigma_{\phi_i} \sigma_{E_i} C_{\phi_i E_i} \rangle_N$$

$$= \langle \Sigma_{\phi_i} E_i^2 \rangle_N + \sigma_{\phi_i} \sigma_{E_i} C_{\phi_i E_i}$$

where $\langle \Sigma_{\phi_i} \rangle_N$ is independent of the resolution $N$. It is, in fact, the simple mean conductance, over all the data points. Note that the standard deviations and correlation coefficients in Terms 1B and 2B are defined over the resolved scale, that is, using the $N$-interval mean $\langle \cdots \rangle_N$ in equation (5), whereas the standard deviations and correlation coefficients in Term 3 are defined over the small scale, that is, within each interval using the mean $\langle \cdots \rangle_i$. The superscripts $i$ have been added to indicate that standard deviations and correlation coefficients are computed only with respect to time, of quantities averaged over one spatial resolution length.

[26] Equation (8) is a temporal-resolution-dependent decomposition of the mean Joule heating rate $\langle J' \rangle$ (mean over the entire 40 h experiment) into terms with a clear physical interpretation. Term 1A is the contribution from the resolved-scale squared electric field, which is generally regarded as the most important source term. Term 1B is the contribution from correlations between the resolved-scale conductance and resolved-scale squared electric field. It may be positive, negative, or zero, according to the correlation coefficient $C_{\phi_i E_i}$. The sum of Terms 1A and 1B is the total contribution from resolved-scale quantities, which is the part that can be modeled, by definition. Term 2A is the contribution from the mean over small scales of the square of the electric field variation, itself averaged over 40 h. Term 2B is the contribution from correlations between the resolved-scale conductance and the mean over small scales of the squared electric field variation. It may be positive, negative, or zero, according to the correlation coefficient $C_{\phi_i E_i}$. The sum of Terms 2A and 2B is the direct contribution of the small-temporal-scale electric field variability (that is, the part that does not rely on small-scale correlations). Term 3 is the contribution from correlations over small temporal scales between the conductance and electric field.

### 3.2. Data and Basic Results

[27] We will work with 40 h of Sondrestrom data taken during a World Day, which contains a CME-driven moderate...
storm. The data ingested by AMIE include cross-track plasma drift measurements from the DMSP F12 and F13 satellites, plasma drift measurements from six SuperDARN radars at about 2-min resolution, and 5-min averaged magnetic perturbations observed at 119 magnetometer stations (see Lu et al. [1998] for more details). The standard Sondrestrom data products for this period allow computation of all the terms in equation (8).

\[28\] During the storm, the Sondrestrom radar operated in a three position mode with 5 min dwells at each position. Vector velocities were computed every 5 min by triangulation, as part of the standard Sondrestrom data processing. The vector velocities are converted to electric field vectors using the assumption that the plasma motion in the plane perpendicular to \(\vec{B}\) is given by \(\vec{E} = \vec{B}/B^2\). This provides a minimum 5-min sampling of the electric field, although the actual resolution is somewhat greater than this, because two thirds of the data used to compute the vector velocity is shared with the previous vector velocity, and one third with the velocity twice removed. In addition, the average sample spacing has been increased by error screening. The Sondrestrom velocity fitter outputs an error estimate. We have only used velocity data that has either an absolute error less than 80 m/s, or a relative error less than 20%.

\[29\] The spatial resolution of the Sondrestrom measurements is determined by the beam geometry. One of the three beams is directed up the magnetic field, and therefore sees a line-of-sight velocity determined mainly by the neutral wind. The two oblique beams measure the velocity components determined by the electric field. Except when the electric field is small, the two oblique beams provide the zeroth order plasma velocity, and the neutral wind contribution represents a first order correction. Therefore, the spatial resolution is determined by a two beam geometry; along the separation vector it is roughly the beam spacing (~250 km, at the centroid scattering altitude), while in the direction perpendicular to the separation vector it is roughly the beam width (~5 km, at the centroid scattering altitude). Since auroral arcs are generally elongated in the direction perpendicular to the magnetic meridian, the beam separation vector is placed in this direction. This realizes a spatial resolution of a few kilometers in the magnetic meridian, which is the direction that contains the most structure. In contrast, the AMIE grid size in the meridional direction is a little under 200 kilometers (referenced to the centroid scattering altitude). Some possible effects of this large difference in spatial resolution are discussed in section 4.

\[30\] The eastward and northward electric field components measured by Sondrestrom, and modeled by AMIE, during the 40 h experiment period, are plotted in Figure 1. The AMIE grid spacing is 5 min (1.25°) in local time, and 1.67° in latitude. Since Sondrestrom drifts through local time, bilinear interpolation is applied to the two nearest local times on the AMIE grid, to obtain a value at the local time of Sondrestrom. The nearest latitude to Sondrestrom on the AMIE grid is 0.5° to the north, and we simply use this point. The period of the experiment is 0 h, 4 min, 9 January 1997, to 15 h, 51 min, 10 January 1997, which is about 40 h. There is considerable correspondence between Sondrestrom and AMIE, but there are also significant differences. One point of general agreement is that the component means over the 40 h period are both less than 10 mV/m. On the other hand, the Sondrestrom-measured field exhibits more dramatic shifts, and assumes larger extrema.

\[31\] Ultraviolet images of the aurora from the Polar spacecraft (Polar-UVI) were available during the last 6 h of the 40 h period. As seen in Figure 2, assimilation of Polar-UVI improves the modeled conductance. The effect of the assimilation of Polar-UVI will be examined in section 3.4. Otherwise, in order to simplify interpretation of the results, we will omit the Polar-UVI data.

![Figure 1. Eastward and northward electric field components measured by Sondrestrom, and modeled by AMIE for the Sondrestrom location, during the 40 h experiment. Time = 0 is 0 h, 4 min, 9 January 1997.](image-url)
Figure 3 shows the terms of equation (8) computed from standard Sondrestrom data products, and from the AMIE model. The horizontal axis is the number \( N \) of (equal length) subdivisions of the 40-h experiment period used to compute the \( \bar{\cdot} \) in equation (8). As explained above, we regard the horizontal axis as the resolution, with larger \( N \) corresponding to finer resolution. The sum of all the terms is also shown, which is independent of \( N \), according to equation (7). Figure 3 shows the expected result that the mean contribution (Term 1A) increases, and the variability contribution (Term 2A) decreases, with improved resolution. The remaining terms are proportional to correlation coefficients, and will be discussed in the following sections.

The overall mean Joule heating rate modeled by AMIE was 29% lower than that measured by Sondrestrom (green lines in Figure 3). We find below that this occurred because the model found a much smaller value for the mean squared electric field, and in spite of the fact that the model found a much larger contribution from correlation between conductance and squared electric field; although the latter effect partially compensated for the former. Small-temporal-scale variability was not a factor.

### 3.3. Conductance Correlation

Term 1B, which is the contribution from correlations between the resolved-scale conductance and the resolved-scale squared electric field, is shown for Sondrestrom in Figure 3a, and for AMIE in Figure 3b. For Sondrestrom, Term 1B is opposite in sign and less than 20% the magnitude of Term 1A (the mean squared electric field). In contrast, for AMIE, Term 1B is positive and 50% of Term 1A. This result constitutes a clear departure of the model from the measure-
ments, which greatly affects the AMIE prediction for Joule heating.

[36] The correlation coefficients $C_{\Sigma, E}$ associated with Term 1B, for Sondrestrom and AMIE, are shown in Figure 4. The correlation coefficient for Sondrestrom is around $-0.18$, and independent of resolution for resolutions less than about $2\ h (N > 20)$. (Note that when the number of subdivisions $N$ becomes very small the computation of the $N$-interval correlation coefficient suffers from poor statistics, and necessarily gives $+1$ when $N = 2$.) This result is consistent with the general physical argument that ionospheric conductance loads the magnetospheric generator.

[37] At a resolution of $15\ min$ the correlation coefficient for AMIE is $+0.59$, and it increases with degrading resolution (i.e., decreasing $N$).

[38] As a check on this result, the conductance and electric field magnitude are plotted in Figure 5, for Sondrestrom and AMIE. Visual inspection of Figure 5 affirms the quantitative results in Figure 4. A possible explanation for this result is given in section 4.3.

[39] The contribution of small-scale correlation between conductance and electric field (Term 3) is also shown in Figure 3, for both Sondrestrom and AMIE. It is essentially zero for AMIE. For Sondrestrom, it acts to slightly reduce the total contribution of the small-scale terms, indicating a negative correlation coefficient on small scales.

[40] The contribution of correlation between resolved-scale conductivity and small-scale electric field variability (Term 2B) is also shown in Figure 3. It is very small for both measurement and model.

3.4. Effect of Assimilation of Polar-UVI

[41] The fact that AMIE uses assimilation of magnetometer data to infer both the conductance (through an empirical relation derived by Ahn et al. [1983, 1998]) and electric field (through Ohms law and assumptions about the current geometry) might lead to an artificially large value for the correlation coefficient between conductance and squared electric field. The relation developed by Ahn et al. [1983, 1998] basically predicts that conductance increases with magnetometer deflection. Therefore, it is clear that if ionospheric current increases because of an increased magnetospheric electric field, AMIE will find an increased ionospheric conductance, even though such may not be the case. Therefore, the correlation coefficient has been computed with and without assimilation of Polar-UVI, for the last $6\ h$ of the $40\ h$ observation period (which is the only time Polar-UVI was available), and the results shown in Figure 6.

[42] Figure 6a shows that assimilation of the Polar-UVI reduced the correlation coefficient by about $0.35$, and from a positive value to a negative value. However, the Sondrestrom-measured correlation coefficient, which was around $-0.6$ over this period, remained smaller than the modeled value by about $0.4$.

[43] It seems likely that the correlation coefficients in Figure 6 were smaller than the corresponding ones in Figure 4 because the shorter temporal period emphasized the shorter-temporal-scale fluctuations. The correlation coefficient...
with assimilation of Polar-UVI became positive for resolutions coarser than about 24 min. Hence, it is very possible that a positive value would still have been obtained over the 40 h observation period, even if Polar-UVI had been available over the entire period.

Figure 6b shows the contributions of the correlations to the Joule heating (Term 1B). Both modeled values are very small, indicating again that analysis of this 6 h period may not be very relevant to analysis of the full 40 h period.

Figure 6b also shows the contributions from the resolved-scale squared electric fields, over the 6 h period (Term 1A). The lowering of the modeled conductance caused by assimilation of Polar-UVI (Figure 2) leads to an increased value for the modeled Joule heating. This occurs because AMIE uses Ohms law to compute the modeled electric field, using currents implied from magnetometer deflections. Decreasing the modeled conductance, while holding the magnetometer deflections constant, increases the modeled electric field. Because the electric field is squared in the Joule heating expression (2), the increased electric field can dominate the decreased conductance, leading to an overall increase in Joule heating.

The mean conductance over the entire 40 h period modeled by AMIE was 54% higher than that measured by Sondrestrom. This overestimation may be responsible for a good portion of the 29% underestimation of Joule heating noted in the introduction to section 3. More research is needed on the effects of assimilating Polar-UVI, with regard to estimating Joule heating, and the contribution to it from correlation between conductance and squared electric field.

3.5. Small-Temporal-Scale Variability

Figure 7 shows the total contribution from resolved-scale terms (Term 1A + Term 1B), and the total contribution from small-temporal-scale terms (Term 2A + Term 2B + Term 3), measured by Sondrestrom (solid lines) on the same axis. What is noteworthy is that the small-scale terms do not appear to be of great significance. The small-scale contribution does not reach 50% of the resolved-scale contribution until the resolution is coarser than 2 h ($N = 20$), and does not equal the resolved-scale contribution until the resolution is coarser than 8 h ($N = 5$). However, energy from spectral components below the 5–10 min resolution of the Sondrestrom measurements may effect the conclusion.

The comparison in Figure 7 between AMIE (dashed lines) and Sondrestrom (solid lines) is more meaningful. The discrepancy between AMIE and Sondrestrom occurs only for the resolved-scale terms, over a wide range of resolutions; the small-temporal-scale contribution to Joule heating is well reproduced by AMIE. Therefore, the fact that the mean Joule heating rate modeled by AMIE is 29% lower than that measured by Sondrestrom is not related
to temporal resolution, that is, it is not related to small-
temporal-scale variability.

This finding supports the idea (noted above) that the
temporal resolution of AMIE is the resolution of the mea-
urements of the input data, which is similar to the resolu-
tion of the Sondrestrom measurements.

4. Effects of Small-Spatial-Scale Variability
Derived From M-I Coupling Model

According to the above data analysis, the modeled
Joule heating contribution from the N-interval mean of the
resolved-scale squared electric field is around 38% of the
measured contribution, over a range of temporal resolutions
(with the measurement and model temporal resolutions
made equal). On the other hand, the contribution to Joule
heating from temporal correlation between conductance and
squared electric field as modeled by AMIE is large and
positive, whereas that measured by Sondrestrom is negative
and of comparably small magnitude. The much larger con-
tribution from correlation partially compensates for the much
smaller mean-squared electric field, such that the modeled
average Joule heating rate is 71% of the measured value. In
sections 4.1 to 4.4 we examine whether these results may
derive from a difference in spatial resolution.

4.1. Effect of Small-Spatial-Scale Variability
on Joule Heating Rate

The spatial resolution of the Sondrestrom measure-
ments in the meridional direction is much finer than that of
AMIE (section 3.2). Therefore, the last 3 terms of equation (4)
are more significant for AMIE than for Sondrestrom (assum-
ing the same temporal resolution, which is achieved by the
analysis of section 3). To assess the likely contribution of
these small-scale terms we will evaluate the Joule heating rate
with and without them, in terms of a simple electrostatic M-I
coupling model.

A model of the spatial electric field structure in the
high-latitude ionosphere requires essentially a model of
auroral arc electrodynamics, a topic that has been much
studied [e.g., Evans et al., 1977; de la Beaujardière and
Vondrak, 1982; Marklund, 1984; Mallinckrodt and Carlson,
1985; Robinson, 1984; Lysak, 1985; Vickrey et al., 1986;
Kan and Cao, 1988]. The high-latitude ionospheric electric
field is generally considered to be driven by a magneto-
spheric generator that is independent of the ionosphere.
However, most authors have noted that the conductance and
electric field magnitude are generally anticorrelated in an auro-
ral arc system (although there are exceptions [Marklund,
1984]). This is explained by postulating that there is a sig-
nificant electrical load isolating the ionosphere from the mag-
netosphere [e.g., Evans et al., 1977], which may be considered
as an internal conductance associated with the magnetospheric
generator.

This internal conductance includes the conductance
along magnetic field lines connecting the magnetosphere
and ionosphere, and hence decreases with decreasing scale
size (owing to increased current crowding [Lysak, 1985]).
On small scales it is much less than the ionospheric (trans-
verse) conductance, so that the magnetosphere behaves essen-
tially as a constant-current source [Vickrey et al., 1986] (which
implies strong anticorrelation between conductance and elec-
tric field magnitude). On larger scales the field-aligned cur-
cents on the edges of arcs are a significant percentage of the
transverse ionospheric current (so that the transverse iono-
spheric current is not constant), and the magnetospheric
conductance is on the order of the transverse ionospheric
conductance.

For present purposes we wish to assume the sim-
pest possible model that captures this basic system. Hence,
we assume a uniform magnetospheric electric field \( E_g \), and
a uniform magnetospheric conductance \( \Sigma_g \), which drives a
structured ionospheric conductance \( \Sigma_p \). This implies that the
ionospheric electric field is given by the voltage divider

\[
E = E_g \frac{\Sigma_g}{\Sigma_g + \Sigma_p}.
\]

Structure in the ionospheric conductance arises from auroral
arcs, through the process of auroral acceleration. A struc-
tured electric field results from polarization of the arcs, through the voltage divider (9).

This model has a few obvious shortcomings: it does not
account for variations in \( \Sigma_g \) associated with "anomal-
ous" resistivity in regions of parallel potential drop; it is
one dimensional, and hence is only applicable to auroral
arcs that are highly elongated; and it is valid for one scale
size only (i.e., \( \Sigma_g \) is a function of scale size). However, the
model (9) is descriptive in a general sense, and therefore
it is instructive to apply it to estimating the statistical
quantities in equation (4).

Since the model (with fixed \( \Sigma_g \)) is valid for one scale
size only, we assume that the ionospheric conductance has the sinusoidal form

\[
\Sigma_p = \Sigma_0 (1 + \varepsilon \cos(kx));
\]

different scale sizes can be considered by varying \( \Sigma_g \). We
will compare the Joule heating rate estimated by first averag-
ing \( \Sigma_p E \) over one wavelength of space \( (\langle \Sigma_p E \rangle, E) \), with
the actual Joule heating rate averaged over one wavelength
\( (\langle \Sigma_p E \rangle, E) \). The former quantity is the first term in equa-
tion (4) \( \langle \Sigma_p \rangle = \langle \Sigma_p \rangle_0 = \langle \Sigma_p \rangle_0 \), \( E_1 = (E_1) \), and represents what
would be computed by a model with spatial resolution equal
to one wavelength of the conductivity variation (equation (10)).
The latter quantity is the sum of all the terms in equation (4),
and represents the actual Joule heating rate.

Through order \( \varepsilon^3 \) we find

\[
\langle \Sigma_p \rangle_0 (E_0)^2 = E_0^2 \Sigma_0 \left( \frac{\Sigma_g}{\Sigma_g + \Sigma_0} \right) \left( 1 + \varepsilon^2 \left( \frac{\Sigma_0}{\Sigma_g + \Sigma_0} \right)^2 \right) + o(\varepsilon^4),
\]

\[
\langle \Sigma_p E \rangle_0 (E_1)^2 = E_0^2 \Sigma_0 \left( \frac{\Sigma_g}{\Sigma_g + \Sigma_0} \right) \left( 1 + \varepsilon^2 \frac{\Sigma_0}{\Sigma_g + \Sigma_0} \left( \frac{\Sigma_0}{\Sigma_g + \Sigma_0} \frac{\Sigma_0}{\Sigma_g + \Sigma_0} \right) \right) + o(\varepsilon^4),
\]

where the average is taken over one period of the sine
wave. For positive \( \Sigma_g \) and \( \Sigma_0 \), \( \Sigma_0 > \Sigma_0/2 - \Sigma_g \), so that
\( \langle \Sigma_p \rangle_0 (E_0)^2 > \langle \Sigma_p E \rangle_0 \), that is, coarse spatial resolution leads
to overestimation of the Joule heating! The conclusion
holds for any generator impedance \( \Sigma_g \), and hence for any
spatial scale.
Therefore, the anticorrelation of polarization electric fields and conductance over space leads to overestimation of the Joule heating rate by models with coarse spatial resolution, in spite of the fact that the mean squared electric field is underestimated.

As explained above, the model is only descriptive of the actual physical system in a gross, qualitative sense, and may not be reliable in a quantitative sense. Our conclusion, then, is that we cannot know whether small-spatial-scale structure leads to underestimation or overestimation of the Joule heating rate without measuring all of the terms in equation (4). Although the second term in equation (4) is always positive, the third and fourth terms may be negative, depending upon the signs of the correlation coefficients \( C_{\xi P/E_x} \) and \( C_{\xi P/E_x} \), where we have added the superscript \( x \) to indicate that we are referring to the spatial part of the average in \( \langle \cdots \rangle \), (the temporal resolution being assumed equal). Hence, the sum of the second through fourth terms may be negative, as has happened in this simplest-case M-I coupling model.

### 4.2. Exchanging Spatial Resolution for Temporal Correlation Heating

The Sondrestrom-AMIE comparison took place in the temporal domain and found (1) that the Joule heating contribution from temporal correlation between the conductance and squared electric field was much larger for the AMIE model than for Sondrestrom and (2) that the temporal correlation coefficient between conductance and squared electric field is large and positive for AMIE but negative and of small magnitude for Sondrestrom. Let us now investigate what the model results of section 4.1 imply regarding temporal correlation.

To begin, take the temporal means \( \langle \cdots \rangle \), of \( \langle \sigma_p \rangle \langle E \rangle \) and \( \langle \Sigma E^2 \rangle \):

\[
\langle \Sigma_\delta P \rangle \langle E \rangle \rangle_x = \langle \langle \Sigma_\delta P \rangle \rangle_x \langle \langle E \rangle \rangle_x + \langle \langle \sigma_\delta \rangle \rangle_x \langle \langle E \rangle \rangle_x C_{\Sigma_\delta P \langle E \rangle \rangle_x,}
\]

\[
\langle \Sigma_\delta P \rangle \langle E \rangle \rangle_x = (\langle \langle \Sigma_\delta P \rangle \rangle_x \langle \langle E \rangle \rangle_x + \langle \langle \sigma_\delta \rangle \rangle_x \langle \langle E \rangle \rangle_x C_{\Sigma_\delta P \langle E \rangle \rangle_x,}
\]

\[
= (\langle \langle \Sigma_\delta P \rangle \rangle_x \langle \langle E \rangle \rangle_x + \langle \langle \sigma_\delta \rangle \rangle_x \langle \langle E \rangle \rangle_x C_{\Sigma_\delta P \langle E \rangle \rangle_x,}
\]

\[
+ \sigma_{\xi E} C_{\Sigma_\delta P \langle E \rangle \rangle_x,}
\]

where \( \sigma \) is used to denote a standard deviation, \( C \) is used to denote a correlation coefficient, the subscripts denote the respective variables, and the superscripts denote whether over space (\( x \)) or time (\( t \)). Here we have in mind that the spatial averaging length (the length over which the spatial averages are taken) is equal to the spatial resolution, whereas the temporal averaging length is much longer than the temporal resolution (as would be applicable to the Sondrestrom-AMIE comparison). Combining the two relations in equation (13) gives

\[
\sigma_{\xi_\delta} \sigma_{\xi E} C_{\Sigma_\delta P \langle E \rangle \rangle_x,}
\]

\[
The left-hand side represents the contribution to Joule heating from temporal correlation between conductance and squared electric field (Term 1B of equation (8)) computed with coarse spatial resolution (i.e., computed from quantities averaged over one spatial resolution cell). The first term on the right-hand side represents the contribution to Joule heating from temporal correlation between conductance and squared electric field computed with fine spatial resolution (and afterward averaged over a spatial resolution cell). The second term on the right-hand side is positive by equation (12). The third term on the right-hand side is positive by equation (11). Assuming that the temporal correlation length divided by the temporal averaging length is much smaller than the spatial correlation length divided by the spatial averaging length (which is the spatial resolution), then \( \sigma_{\xi_\delta} \ll \sigma_{\xi P} \) and \( \sigma_{\xi E} \ll \sigma_{\xi P} \), so that the last term on the right-hand side has a much smaller magnitude than the second term on the right-hand side, and can be ignored. (This follows because an average over many uncorrelated quantities is thought of as an average of many uncorrelated quantities, the number increasing with the temporal averaging length. Thus the spatial variance of the resulting quantity can be reduced arbitrarily by increasing the temporal averaging length.)

We conclude that \( \sigma_{\xi_\delta} \sigma_{\xi E} C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) that is, the contribution to Joule heating from temporal correlation between conductance and squared electric field computed with coarse spatial resolution is larger than the same computed with fine spatial resolution. Equation (14) shows that this difference is in compensation for both \( (\langle E \rangle_x - \langle E \rangle) > 0 \) and \( (\langle \Sigma_\delta P \rangle \langle E \rangle_x - \langle \Sigma_\delta P \rangle \langle E \rangle_x) > 0 \). Therefore, the increased Joule heating contribution from temporal correlation between conductance and squared electric field found for AMIE, as compared with Sondrestrom, may be caused by the difference in spatial resolution.

### 4.3. Effect of Spatial Resolution on Temporal Correlation Coefficient

The result given in equation (14) does not specifically constrain the relationship between the temporal correlation coefficients \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) and \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \). The magnetospheric generator \( E_x \) may be temporally correlated with auroral activity in a large spatial-scale sense, and this could cause a positive temporal correlation coefficient when measured with coarse spatial resolution \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) even while the temporal correlation coefficient measured with fine spatial resolution \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) is negative (owing to, for example, drifter, polarized, small-spatial-scale structure). However, as shown in section 4.2, the conclusion that \( \sigma_{\xi_\delta} \sigma_{\xi E} C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) \( \sigma_{\xi_\delta} \sigma_{\xi E} C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) applies whether or not \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) > \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \). To investigate the relationship between the temporal correlation coefficients \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \) and \( C_{\Sigma_\delta P \langle E \rangle \rangle_x,} \), and to exemplify the conclusions of section 4.2, we will add a temporal dimension
to the M-I coupling model, and consider temporal modulations of the magnetospheric electric field and auroral activity with a varying phase relationship. The model will be evaluated numerically.

To form the model, assume a magnetospheric electric field $E_g$ with a temporal dependence $\cos(\omega t/2 + \theta)$ that oscillates over a long time period, and modulate the ionospheric conductance of section 4.1 by the temporal dependence $(1 + \varepsilon \cos \omega t)$ (note that $\cos^2(\omega t/2) = (1 + \cos \omega t)/2$). This suggests the model

$$E_g = E_0 \cos(\omega t/2 + \theta)$$
$$\Sigma_P = \Sigma_0(1 + \varepsilon \cos \omega t)(1 + \varepsilon \cos(kx + \phi(t))), \quad \varepsilon < 1,$$  

where we have added the time dependent phase $\phi(t)$ to the spatial oscillation, which models a finite temporal correlation length by way of a drifting conductivity structure. (A similar phase term is not included for the temporal oscillation of the conductance, because we assume the spatial correlation length is greater than the spatial interval length to be analyzed, which is assumed equal to the spatial resolution.) When $\theta = 0$, $\varepsilon = 1$ the temporal correlation coefficient between $E_g^2$ and the spatial integral of $\Sigma_P$ over one wavelength is $+1$. When $\theta = \pi/4$, $\varepsilon = 1$ the same correlation coefficient is $-1$. The magnitude $\Sigma_0$ and modulation factor $\varepsilon$ will be set to reproduce the desired mean and standard deviation for $\Sigma_P$. (To keep the number of tuneable parameters to a minimum, and avoid an overly specialized model, we force the temporal and spatial modulation factors ($\varepsilon$) to be the same.) The ionospheric electric field is derived as in section 4.1, using equation (9).

The model has been evaluated using a spatiotemporal grid 6 wavelengths on a side. To begin we set $\theta = 0$, $\Sigma_0/\Sigma_0 = 0.3$, and $\varepsilon$ and $\Sigma_0$ so that the mean and standard deviation of the conductance has the values measured during the 40 h experiment. Figure 8 shows the resulting conductance and electric field surfaces. On the basis of the assumption that AMIE has a spatial resolution coarser than the spatial scale for arc structure, we calculate the temporal correlation coefficient of the spatially averaged conductance and the square of the spatially averaged electric field $C_{\Sigma_P},(E)^2$, as representative of the temporal correlation coefficient modeled by AMIE. On the basis of the assumption that Sondrestrom has a spatial resolution finer than the spatial scale for arc structure, we calculate the temporal correlation coefficient of the conductance and the squared electric field at each position, and then average the result over space $\langle C_{\Sigma_P}(E)^2 \rangle_x$, to represent the correlation coefficient measured by Sondrestrom. The result is

$$C_{\Sigma_P}(E)^2 = 0.8, \quad \langle C_{\Sigma_P}(E)^2 \rangle_x = -0.3,$$

independent of the generator electric field $E_0$. The results (16) hold for almost any nonconstant phase function $\phi(t)$, including for the simple ramp $\phi(t) = \alpha \omega t$, for any $\alpha$ greater than 0.2. The standard deviation of $C_{\Sigma_P}(E)$ (over $x$) is also small. However, for $\alpha$ approaching zero, such that there is little decorrelation of the spatial structure over the modeled time period, the averaged correlation coefficient $\langle C_{\Sigma_P}(E)^2 \rangle_x$ deviates from $-0.3$ and becomes positive.

Adjusting $\Sigma_0/\Sigma_0$ adjusts the correlation coefficients. With $\Sigma_0/\Sigma_0 = 0.9$ (reduction in the amount of polarization) we get $C_{\Sigma_P}(E)^2 = 0.9$ and $\langle C_{\Sigma_P}(E)^2 \rangle_x = -0.1$. As $\Sigma_0/\Sigma_0 \rightarrow 0$, which means that the magnetosphere approaches a current source, the correlation coefficients approach $C_{\Sigma_P}(E)^2 = 0.4$ and $\langle C_{\Sigma_P}(E)^2 \rangle_x = -0.5$. Therefore, it is plausible that the positive correlation coefficient found by AMIE, and the contrasting negative correlation coefficient found by Sondrestrom, may be the results of a difference in spatial resolution.

To examine the temporal correlation coefficient and the terms of equation (14) as a function of spatial resolution, we employ an analysis method similar to that of section 3.1, and subdivide the 6-wavelength spatiotemporal grid in the spatial dimension, into between 1 and 40 subregions. Each subregion is 6 wavelengths long in the temporal direction, and 6/$N_x$ wavelengths long in the spatial direction, where $N_x$ is the number of subregions in the subdivision. For each subdivision the correlation coefficient $C_{\Sigma_P}(E)^2$ and terms of equation (14) are computed for each subregion, and the results averaged over all the subregions of the subdivision.

The results with $\Sigma_0 = 1.0, \Sigma_0 = 3.3, E_g = 0.03$ V/m, $\phi(t) = \omega t/3$, and $\varepsilon$ set so that the mean and standard deviation...
of the conductance has the values measured during the 40 h experiment, are shown in Figure 9.

Figures 9a and 9b show the case with \( q = 0 \). With coarse spatial resolution (small \( N_x \)), the conductance and squared electric field are highly correlated, owing to the high correlation between \( S_P \) and \( E_g^2 \). However, with fine spatial resolution, the drifting polarized conductance structure becomes dominant, so that the conductance and squared electric field become negatively correlated.

The arguments surrounding equation (14) are illustrated in Figure 9b. The term \( \langle \sigma'_{S_P} \sigma'_{E_g} C_{S_P, E_g} \rangle_x \) is negligible, and \( \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \) is larger than \( \langle \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \rangle_x \) by an amount that increases with decreasing spatial resolution.

Figures 9c and 9d show the case with \( q = \pi/4 \). When \( S_P \) and \( E_g \) are anticorrelated the correlation coefficient is everywhere negative. However, this does not affect the fact that \( \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \) is larger than \( \langle \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \rangle_x \), by an amount that increases with decreasing spatial resolution. Nor is the difference between \( \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \) and \( \langle \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \rangle_x \) decreased by the anticorrelation between \( S_P \) and \( E_g \). The term \( \sigma_{S_P} \sigma_{E_g} C_{S_P, E_g} \) is negligible because the spatial structure decorrelates over a time period short compared with the experimental time period, while the temporal structure remains correlated over one spatial-resolution length, and therefore the discussion of equation (14) is valid.

In conclusion, we have just found that regardless of whether there is an increase in the temporal correlation coefficient, the contribution to Joule heating from temporal correlation between the conductance and squared electric field increases with increasing spatial resolution, in such a way as to compensate for the decrease in the mean-squared electric field. In addition, if the magnetospheric sources of electric fields and auroral precipitation are correlated (\( q = 0 \),...
then the temporal correlation coefficient also increases with increasing spatial resolution.

4.4. Discussion of Model Results

[77] Given the results of the comparison between correlation coefficient determined with and without assimilation of Polar-UVI, described in section 3.4, it is likely that AMIE overestimates the temporal correlation coefficient $C_{\langle \sigma_E \sigma_C \rangle}$. Therefore, we cannot conclude that the large positive value for the correlation coefficient is a result of coarse spatial resolution.

[74] Would this same effect artificially enhance the overall contribution to Joule heating from temporal correlation between the conductance and squared electric field? We have seen in the model that the overall contribution $(\sigma_{\langle \sigma_E \sigma_C \rangle})^2$ increases with resolution, independent of the behavior of the correlation coefficient. Therefore, we do not have a clear answer.

[75] Model uncertainty (on the resolved scale) is another likely source for the underestimation of Joule heating [Matsuo and Richmond, 2008; Cosgrove and Codrescu, 2009]. Electric field variability and model uncertainty: A classification of source terms in estimating the squared electric field from an electric field model, submitted to Journal of Geophysical Research, 2009]. This occurs simply because uncertainty in the electric field model means that we should consider the electric field as a random variable, with an associated probability distribution function (PDF), where the modeled value specifies the mean of the PDF, and the model uncertainty specifies the variance of the PDF. The expectation value for the squared electric field is the second moment of the PDF, which is larger than the square of the mean value by the variance of the PDF. Hence, using the modeled value for the electric field leads to underestimation of the squared value.

[76] Given that there are other sources for the underestimation of Joule heating by AMIE (for example, overestimation of the conductance as discussed in section 3.4, and model uncertainty) we do not need to assign the 29% underestimation of the Joule heating found above to small-spatial-scale variability. Hence, there is room for the implications of this modeling exercise: that is, small-spatial-scale electric fields are likely polarizing electric fields, and therefore negatively correlated with conductance (over space), they may not lead to underestimation of Joule heating. Simultaneous spatially resolved measurements of conductance and electric field are needed to resolve this matter.

5. Conclusions and Discussion

[77] The results and conclusions are summarized as follows:

[78] 1. The contribution to Joule heating from the mean-squared electric field as modeled by AMIE over the storm period was about 38% of the Sondrestrom-measured contribution, over a wide range of temporal resolutions, when the effects of temporal correlation between the electric field and conductance are not included (i.e., Term 1A for AMIE was 38% of Term 1A for Sondrestrom). The underestimation is probably a result of either (1) overestimation of the conductance (section 3.4), (2) resolved-scale model uncertainty (section 4.4) [Matsuo and Richmond, 2008; Cosgrove and Codrescu, submitted manuscript, 2009], and/or (3) small-spatial-scale variability. Resolved-scale model uncertainty exists because the model parameters do not uniquely determine the resolved-scale convection electric field. Small-spatial-scale variability can contribute to the comparison because the resolution of the Sondrestrom measurements is presumably much finer than the spatial resolution of the AMIE model.

[79] 2. The Sondrestrom-measured contribution to Joule heating from temporal correlation between field line integrated conductivity (conductance) and the resolved-scale electric field was around $-20\%$ of the total measured Joule heating, over a wide range of temporal resolutions. The same contribution derived from the AMIE procedure was around $+28\%$ of the total modeled Joule heating, over the same range of temporal resolutions. Inclusion of the correlation terms in the AMIE-Sondrestrom comparison transformed a 62% underestimation of the average Joule heating rate into a 29% underestimation.

[80] 3. Comparison with Sondrestrom measurements offers no evidence that AMIE fails to adequately represent Joule heating from small-temporal-scale electric field fluctuations. The fact that the mean Joule heating rate modeled by AMIE is 29% lower than that measured by Sondrestrom is not related to temporal resolution, that is, it is not related to small-temporal-scale variability. This finding supports the idea that the temporal resolution of AMIE is the resolution of the measurements of the input data, which is similar to the resolution of the Sondrestrom measurements.

[81] 4. The Sondrestrom-measured temporal correlation coefficient between the conductance and the squared electric field was around $-0.18$, over a wide range of temporal resolutions. The same correlation coefficient derived from the AMIE procedure was around $+0.6$, over the same range of temporal resolutions.

[82] 5. In order to explain the above results, we derive in section 4.1 the ionospheric electric field $E$ assuming a uniform magnetospheric electric field $E_g$ and a uniform magnetospheric conductance $\Sigma_g$, applied to a one-dimensional structured ionospheric conductance $\Sigma_P$. This simplest-case M-I coupling model contains small-spatial-scale variability of the ionospheric conductance and electric field, where the electric field variability arises from polarization of ionospheric conductance structures. For this model, we show that coarse spatial resolution leads to underestimation of the Joule heating rate, in spite of underestimation of the mean-squared electric field. Although this model may be too simple, the result emphasizes that it cannot be known whether small-spatial-scale variability leads to underestimation or overestimation of the Joule heating rate, until a careful measurement of the spatial correlation between conductance and electric field has been made. Equation (4) gives a decomposition of the Joule heating rate into physically distinct small-scale terms; the third and fourth terms may be either positive or negative, which shows that either underestimation or overestimation is possible.

[83] 6. The time dependent M-I coupling model of section 4.3 involved the propagation of the conductance structures of the model of section 4.1. The propagation ensures that the time average of a quantity resolved on a small-spatial-scale is independent of position in space over a larger spatial
scale. With the time-average of the conductance the same as the full average over space and time, the manifestation of the assumption that the calculated Joule heating does not depend on spatial resolution is that as the spatial resolution is increased, the associated reduction in the mean-squared electric field must be compensated for by an increase in the contribution to Joule heating from temporal correlation between conductance and squared electric field. This, combined with item 5, suggests that the different correlation contributions for AMIE and Sondrestrom, noted in item 2, might be due to the fact that the spatial resolution of AMIE is more coarse than that of Sondrestrom. In section 4.2 we gave a generalization of this argument, which was demonstrated by the examples of section 4.3.

[84] 7. In section 4.3 we found that if the magnetospheric source for electric fields is correlated with the magnetospheric source for auroral precipitation, then the temporal correlation coefficient between conductance and squared electric field may be strongly dependent on spatial resolution. The correlation coefficient may be positive when resolved with coarse spatial resolution, but become negative as the spatial resolution decreases. (Note that the conclusions of the previous two items do not depend on correlation between the magnetospheric source for electric fields and the magnetospheric source for auroral precipitation.)

[85] 8. The modeling exercise was motivated by the AMIE-Sondrestrom comparison, and provides a potential explanation for the results of the comparison, with respect to the correlation contribution. However, any assertion of this explanation is complicated by the fact that there is an alternative explanation; the heavy reliance by AMIE on magnetometers, to infer both the conductance and the electric field, likely leads to increased correlation between conductance and squared electric field. Nevertheless, the results of the modeling exercise stand on their own.

[86] The results suggest a way that non-self-consistent statistical models of electric field and conductance, with coarse spatial resolution, can be used to model Joule heating. By adding a model of the temporal correlation coefficient, equation (8) yields a self-consistent (with respect to electric field and conductance) computation of the Joule heating rate. Item 5 gives hope that small-spatial-scale variability will not be a significant contaminant, as long as the temporal correlation coefficient is matched to the spatial resolution of the models. Item 3 suggests that the temporal resolution of the electric field model may not be an issue. The only remaining bias toward underestimation is resolved-scale model uncertainty of the electric field model, which would have to be estimated separately.

[87] By projecting a fan of many beams, the AMISR radar can sample plasma density, temperature, and line-of-sight drift velocity over both time and space. By working with such a data set and making reasonable assumptions, it should be possible to produce a spatial-resolution-dependent model of the temporal correlation coefficient between conductivity and squared electric field. Also, the Sondrestrom radar database includes a large collection with the radar in scanning mode [Cosgrove and Thayer, 2006] which may be applicable. Satellite data could be used to measure the spatial correlation coefficient, and to compare the terms in equation (4) as a function of spatial resolution, which would determine if there is a significant contribution from small-spatial-scale variability.

[88] We note in closing that there are other possible sources for the apparent underestimation of atmospheric heating noted by Codrescu et al. [1995] besides Joule heating, and these should also be investigated. The tendency for atmospheric energy deposition from auroral electron precipitation to replace Joule heating inside auroral arcs was noted by Evans et al. [1977]. Baker et al. [2004] have recently found that the total contribution of the former may be comparable to that of the latter. It is also possible that atmospheric heating results from the component of the electric field parallel to the magnetic field, that is, from wave heating. Elevated electron temperatures are consistently found in the auroral $E$ region, and the source is thought to be heating from parallel electric fields associated with unstable Farley-Buneman waves [e.g., Schlegel and St.-Maurice, 1981; St.-Maurice and Schlegel, 1982; Dimant and Milikh, 2003, Milikh and Dimant, 2003; Bahcivan et al., 2006; Bahcivan, 2007]. Can the heating of electrons lead to the heating of neutrals? This possibility has not been fully investigated.

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