Offshore Propagation of Coastal Precipitation

YANPING LI
University of Saskatchewan, Saskatoon, Saskatchewan, Canada

R. E. CARBONE
National Center for Atmospheric Research,* Boulder, Colorado

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ABSTRACT

This work focuses on the seaward propagation of coastal precipitation with and without mountainous terrain nearby. Offshore of India, diurnal propagation of precipitation is observed over the Bay of Bengal. On the eastern side of the bay, a diurnal but nonpropagating signal is observed near the west coast of Burma. This asymmetry is consistent with the inertio-gravity wave mechanism. Perturbations generated by diurnal heating over the coastal mountains of India propagate offshore, amplify in the upwind direction, and dissipate in the downwind direction relative to the steering wind, owing to critical-level considerations. A linear model is applied to evaluate sensitivity to gravity waves, as these affect deep moist convection and propagation. Analyses are performed for various heating depths, mountain widths, stability, Coriolis effect, background mean wind, and friction. Calculations reveal how these factors affect the amplitude, dissipation, initiation phase, and propagation speed of the diurnal disturbance. The propagation of precipitation triggered by land–sea breezes is distinguishable from that triggered by a mountain–plains circulation. Convection resulting purely from mountain heating begins earlier, propagates slower, and damps faster than that of the land–sea breeze. For mountains near a coast, slower propagation and stronger earlier convection result from a resonance-like combination of two dynamical mechanisms. The propagation of precipitation is initially triggered by the mountain breeze near the coastal mountain. Over the open ocean, the dominant signal propagates as that of the land breeze but with stronger convection.

1. Introduction

Future progress in weather and climate prediction requires improvements in the understanding and representation of the space–time distribution of precipitation. One important goal is to understand the mechanisms associated with offshore propagation of diurnally excited precipitation in tropical regions. Diurnal propagating precipitation systems are commonly observed over continents and coastal areas (e.g., Carbone et al. 2000, 2002, Laing et al. 2008, Wang et al. 2004; Levizzani et al. 2006; Pereira Filho et al. 2010; Keenan and Carbone 2008; Houze 2012). At midlatitudes, for example, the warm-season diurnal convective precipitation originates over the Rockies and propagates eastward over the Great Plains and Midwest (Carbone et al. 2002), which is explained by the potential vorticity (PV) mechanism (Li and Smith 2010b). Diurnal elevated heating over the Rockies generates diurnal PV pulses that are conserved and drift eastward with the midtroposphere mean wind. In the presence of vertical mean wind shear, this then generates a vertical disturbance in the lower troposphere, triggers moist convection, and systematically produces eastward-propagating precipitation systems in the warm season. In the tropics, gravity waves are implicated in the propagation of convection (Crook 1988). However, other factors may also be involved, including the internal dynamics of organized convection (Rotunno et al. 1988).

Over tropical coastal areas, such as the rainy western coast of Colombia (Mapes et al. 2003a), strong diurnal convection propagates westward out to sea from the Andes ridge. Propagation speed is \( \sim 15 \text{ m s}^{-1} \) for up to
700 km, occurring most regularly in August and September. The speed and propagation distance are difficult to explain by a simple land breeze given the rate of dissipation for atmospheric density currents over the water. The observed phenomenon may be a remote response to the strong, locally forced diurnal convection over the tropical Andes through gravity waves. A gravity wave mechanism is proposed by Mapes et al. (2003a) for the Panama Bight, which involves the generation of thermally forced gravity waves in the deep heated boundary layer over the nearby Andes. Daytime heating over the Andes triggers gravity wave motion, resulting in ascent that propagates westward over the Pacific Ocean. Recently the international field campaign the Variability of the American Monsoon Systems (VAMOS) Ocean–Cloud–Atmosphere–Land Study (VOCALS) Regional Experiment was held in a nearby coastal region (Muñoz 2008). The offshore diurnal waves and their impact on the stratocumulus deck over the southeastern Pacific have been studied by Jiang (2012).

A similar phenomenon occurs during the onset of the Asian monsoon (Yang and Slingo 2001; Houze 2012; Kikuchi and Wang 2008; Xie et al. 2006; Johnson 2011). Convection over the Bay of Bengal and the South China Sea typically forms along the northern coastlines of those areas around midnight or early morning. The convection propagates southward over great distances (~500 km or more) before dissipating over the central or southern portions of those areas. Yang and Slingo (2001) showed that cold cloud tops appear around midnight. Webster et al. (2002) provided evidence of propagation over the Bay of Bengal in a time–latitude diagram of brightness temperatures. On some occasions, precipitation systems (inferred from the cold cloud tops) were found to propagate from the Indian coast from near 20°N to the equator over a 2-day period. Radar data from the NOAA Research Vessel Ron Brown in the Bay of Bengal indicates that the convection associated with the diurnal signal has characteristics of squall lines with trailing stratiform precipitation (Webster et al. 2002). The mechanisms responsible for these diurnally propagating signals over the Bay of Bengal and the South China Sea are not well understood.

In the Bay of Bengal, the phase of the daily maximum brightness temperature propagates southeastward for several hundred kilometers parallel to the coast in summer with a speed of ~15–20 m s⁻¹. This progression suggests a diurnally excited gravity wave mechanism with origin over land (Yang and Slingo 2001). The Eastern Ghats provide an elevated diurnal heat source, thereby exciting gravity waves, which presumably propagate seaward. Adveective effects can be ruled out because these systems propagate approximately at right angles to the southwesterly monsoon flow. Density current dynamics appear to be an unlikely explanation because the speed of propagation (~15 m s⁻¹) is too fast to be accounted for by the relatively weak cold pools observed in this region. At the eastern side of the bay, the prevailing monsoon flow is onshore toward coastal mountains. Responses such as the upstream blocking by the pressure deceleration mechanism of Grossman and Durran (1984) and/or gravity waves emanating off of the land may be active and could explain the observed behavior.

Numerous simulations of sea breezes and mountain–plain circulations have been performed over the last 20 years (Mahrer and Pielke 1977; Banta 1986; Mapes et al. 2003b; Crosman and Horel 2010; Qian et al. 2009; Jiang 2012). A combined sea breeze with slope flow may lead to an earlier inland sea-breeze frontal passage (Ookouchi et al. 1978). A slope of sufficient steepness can act to block inland penetration of the sea-breeze front and decrease the horizontal and vertical velocities and scales $u, w, l, \text{and } h$ (Asai and Mitsumoto 1978; Segal et al. 1983; Neumann and Savijarvi 1986; Porson et al. 2007). The critical slope angle at which the coupling of thermally driven slope flow outweighs the terrain-blocking effect is variable and depends on the vertical stability profile, magnitude of slope heating (e.g., vegetation type, aspect), the total length of the slope, the distance from the coastline to the mountain, and the absolute height of the mountain (Abbs 1986; Segal et al. 1997; Abbs and Physick 1992).

This paper focuses on the seaward propagation of diurnal organized convective systems over two different geographic settings: 1) tropical coasts in a monsoon region (e.g., the Bay of Bengal and the west coast of Burma); and 2) tropical coastal mountainous regions, such as the western coasts of Mexico and Colombia. Two questions are addressed:

1) For tropical coasts in monsoon regions: Is the offshore-propagating precipitation intrinsically determined or influenced by gravity waves?
2) For tropical coastal mountainous regions: Which offshore propagation mechanism is dominant, the mountain–plain circulation or the land–sea breeze? How do the environmental factors affect propagation speed and strength of the convection?

To address the physical factors that control the propagation of the offshore convection, a dry 2D linearized Boussinesq model is used to study the early-stage forcing of local storms and to evaluate some related factors. These include the size and height of the mountain, the environmental mean wind and stability, the thickness of the thermal boundary layer. The
inertio-gravity wave (IGW), as modified by the Doppler shift from the environmental wind and the Coriolis effect, is discussed and used to explain the observations. The differences between the propagation of the organized convection, as caused by mountain–plain circulations and/or land breeze–sea breeze mechanisms, are further studied. Combinations of the mountain breeze and land breeze are used to reconcile the observations with theory.

2. Observational data and analysis method

The Climate Prediction Center (CPC) morphing technique (CMORPH) precipitation estimates are produced by a Kalman filter approach based on the NWS CPC morphing technique (Joyce et al. 2004). This technique produces global precipitation analyses at relatively high spatial and temporal resolution. It uses quantitative precipitation estimates derived from low-orbit satellite microwave observations. Location information is obtained by temporal interpolation of geostationary satellite IR data. CMORPH data are available every 3 h and gridded at 0.25° resolution. Eight-hourly NCEP–NCAR reanalysis data (www.esrl.noaa.gov/psd/data/reanalysis/reanalysis.shtml) are used to characterize the undisturbed environment at a spatial resolution of 0.5°. In this study CMORPH data are primarily used to estimate the diurnal propagation of precipitation.

In Li (2009), Li et al. (2009), and Li and Smith (2010a), harmonic analysis is used to extract the diurnal component (i.e., 24-h harmonic) from hourly observations. Herein we calculate the monthly average precipitation rate $\text{RAIN}_n(t)$ for eight 3-h intervals $t$, where $n$ refers to a specific grid point. The diurnal component of precipitation $\text{RAIN}_n(t)$ at a specific grid point $n$ is then calculated according to $\text{RAIN}_n(t) = C_n \cos \left(2\pi \left(\frac{t}{24} - \left(\frac{\psi_n}{24}\right)\right)\right) + e$, where $t$ represents local time (LT) in hours. The term $\psi_n$ is the calculated phase (LT) of the diurnal component of $\text{RAIN}_n(t)$. This is equivalent to the time $t_{\text{max}}$ at which the diurnal signal reaches its maximum. Note that $t_{\text{max}}$ is related to the diurnal harmonic, not the full signal. The term $e$ represents that portion of the daily variance not explained by the diurnal component.

The phase and amplitude are calculated by

\[
\psi_n = \tan^{-1} \left(\frac{B_n}{A_n}\right) \times \frac{24}{2\pi}, \quad C_n = \sqrt{A_n^2 + B_n^2},
\]

with $A_n = \sum_{t=3,6}^{24} \text{RAIN}_n(t) \cos \left[2\pi \left(\frac{t}{24}\right)\right]$ and $B_n = \sum_{t=3,6}^{24} \text{RAIN}_n(t) \sin \left[2\pi \left(\frac{t}{24}\right)\right]$. The summation is conducted over a period of 24 h ($t = 3, 6, 9, 12, 15, 18, 21$, and 24).

![Fig. 1. (a) Diurnal amplitude and (b) phase (LT) of precipitation over Bay of Bengal in July 2009.](image)

**a. Diurnal propagating precipitation systems over a tropical coastal area in a monsoon region with mountains near the coastline (Bay of Bengal and the west coast of Burma): IGW mechanism**

A 4-yr time series of CMORPH data (2006–10) was examined and found to have similar diurnal precipitation features at each of the selected inspection locations. Here, we show the results from year 2009, a year with more rain in general and more prominent diurnal precipitation features when compared with other years. For July 2009, the analysis of CMORPH precipitation shows that strong diurnal convection is observed over the Indian monsoon region, defined by the Bay of Bengal (AoA' in Figs. 1, 2), and the west coast of Burma (BoB' in Figs. 1, 2) during the wet season [June–August (JJA)]. Offshore propagation of precipitation is observed from the India coast (Fig. 2a). Figure 2a shows two identical diurnal cycles, which result from the average of precipitation at each time ($t = 3, 6, 9, \ldots, 24$ LT) for all days in July 2009. The phase distribution of diurnal precipitation (Fig. 2b) illustrates propagation, exclusively offshore, toward the southeast. On the landward side, the phase reaches a diurnal peak at ~1800–2200 LT. Over the ocean, phases originate from 0000 to 0600 LT. The propagation phase speed here is ~10 m s⁻¹, consistent
with Fig. 4 in Yang and Slingo (2001). Yang and Slingo used 3-hourly, 0.5° latitude–longitude gridded brightness temperature $T_b$ from multiple satellites developed by the European Union Cloud Archive User Service (CLAUS). Both Nesbitt and Zipser (2003) and Biasutti et al. (2012) studied the global diurnal cycle of precipitation using the TRMM satellite dataset. They mainly focused on the diurnal brightness temperature–frequency phase distribution and did not evaluate offshore propagation.

Near the west coast of Burma, the precipitation is purely diurnal and nonpropagating (Figs. 2c,d; 88°–92°E). The peak of diurnal convection is $\sim$1200–1400 LT. Landward (94°–98°E), the phases start from 1800 LT, exhibiting northeast movement of $\sim$2 m s$^{-1}$.

Environmental wind from the NCEP reanalysis shows strong easterly winds above 500 hPa are dominant in summer over the Indian Ocean (Fig. 3a). In the middle and lower troposphere $\sim$750 hPa, a monsoon trough with northwesterly flow ($\sim$5 m s$^{-1}$) persists near India (Fig. 3a). Southwesterly flow ($\sim$5 m s$^{-1}$) prevails near Burma (Fig. 3b).

### 1) ON THE INDIA COAST

The data presented are fully consistent with the presence of IGWs. IGWs are gravity waves inclusive of a significant Coriolis effect (Fig. 4). For the Indian monsoon region (circa 22°N), a Coriolis effect is likely. Perturbations generated by diurnal convective heating over the coastal Eastern Ghats propagate in a manner consistent with the intrinsic frequency band of IGWs.

Given a Doppler shift by the background environmental wind, eastward-propagating (westward propagating) IGWs may survive if the background wind is westerly (easterly) or weak easterly (westerly), that is, $<3$ m s$^{-1}$ in this case. The IGW propagates upwind with respect to the mean wind (the IGW propagates in the same direction of the mean wind but faster than the mean wind), and the intrinsic frequency after Doppler shift will be $|\sigma + Uk| > f$ (becomes higher). When the IGW propagates downwind with respect to the mean wind (the IGW propagates in the opposite direction of the mean wind, or the IGW propagates in the same
direction of the mean wind but slower than the mean wind) the intrinsic frequency after the Doppler shift will be \(|\sigma - Uk| < f\) (becomes lower). It follows that a negative sign is used for the downwind case. Therefore, the upwind side (relative to the background wind) is \(|\sigma + Uk| > f\), while the downwind side (relative to the background wind) is \(|\sigma - Uk| < f\). Close to the eastern coast of India, Fig. 3a shows a weak to moderate midtropospheric steering wind and no critical level with respect to the eastward propagation of IGWs. Since IGW propagation speeds are \(\sim 10\, \text{m s}^{-1}\) (Fig. 2b) and the environmental northwesterlies are \(\sim 5\, \text{m s}^{-1}\) (Fig. 3a), the IGWs are propagating upwind \((|\sigma + Uk| > f)\) with respect to the midtropospheric westerlies. Perturbations caused by IGWs exist on the offshore side, propagating in the direction of the background wind. These waves actually propagate upwind relative to the background wind (northwesterly). Therefore, heating energy is transported away as gravity wave disturbances.

2) ON THE BURMA COAST

At the Burma coast a critical level exists in the lower troposphere (Fig. 3b) for landward-propagating IGWs. The critical level is where \(|\sigma - U \cdot (\sigma/C_p)| = f\). The IGW wave equation \([\text{Eq. (A12)}]\) goes to singularity and wave absorption occurs. Applied to this case, and assuming the inland propagating speed is \(2\, \text{m s}^{-1}\) (Fig. 2b), the critical level occurs where \(U(z) = 4\, \text{m s}^{-1}\). Since westerly \(U(z)\) decreases with height, the propagation of inland IGWs is confined within the wave ducting layer from ground to 800 hPa (Fig. 3b). Seaward-propagating IGWs dissipate owing to \(|\sigma - Uk| < f\) (Fig. 4). Therefore, diurnal precipitation is local and nonpropagating near the Burma coast.

The phase of diurnal precipitation is mainly determined by diurnal destabilization. This is consistent with the observed precipitation phase (Fig. 2d) over the ocean near Burma’s west coast, reaching its maximum in the afternoon when the marine atmospheric boundary layer (MABL) becomes unstable. Typically, the diurnal cycle of rainfall reaches its maximum in the early morning over the oceans and has a maximum in the afternoon over land (Fig. 11 in Nesbitt and Zipser 2003). However, this may not be the case in the Indian Ocean north of the equator, where a northern ocean current transports cool surface water to the west coast of Burma. With relatively cool SSTs near the coast, a stable MABL forms, only to be destabilized by radiative heating in the afternoon.

While the northeastward seasonal propagation of the Indian summer monsoon is likely controlled by the large-scale meridional thermal contrast, the movement of diurnal precipitation is associated with local wind–terrain interactions. Orographic precipitation is analogous to
waves (propagating southeastward) in a wave packet (propagating northeastward); the latter is consistent with progression of the monsoon.

b. Diurnal propagating precipitation systems in tropical coastal areas with significant mountains near the coastline (mountain–plain, land–sea contrast)

Inhomogeneous sensible heating during daytime between land and water surfaces results in a horizontal pressure gradient that drives sea breezes (Steyn 2003; Carbone and Li 2015). With significant mountains near the coast, the interactions between boundary layer stability and topography can act to block the late-afternoon inland acceleration of the sea-breeze front and remove the low-level baroclinicity (Ookouchi et al. 1978). Here, a detailed examination is made for coastal regions where mountain–plain and land–sea contrasts coexist.

1) West Coast of Mexico (~22°N)

Near the west coast of Mexico, westward propagation of diurnal convection is observed over the North American Monsoon Experiment region (Figs. 5, 6, 7). Convection is triggered during the afternoon over the peaks and foothills of the Sierra Madre Occidental (SMO), propagating slowly westward (~7 m s⁻¹, 1–4 m s⁻¹ in excess of steering winds) from the SMO after sunset. This weakly organized convection is commonly associated with enhanced easterly wind shear at low- and/or midtropospheric levels, as diagnosed from soundings (Lang et al. 2007).

The analysis of CMORPH data also shows that along the west coast of Mexico, near 22°N, the offshore propagation of precipitation is most active in June, July, August, and September (Fig. 6). The longitudinal phase distribution illustrates a starting phase of precipitation at ~1800 LT with a westward propagation speed ~9 m s⁻¹ (Fig. 7a). The longitudinal amplitude distribution reveals decreasing precipitation amplitude for approximately 500 km (Fig. 7b, 100°–110°W). The environmental wind (from NCEP monthly mean) (Fig. 8) shows weak westerlies (~2 m s⁻¹) below 850 hPa and moderate easterlies aloft (approximately ~7 m s⁻¹). Weak westerlies in the PBL help to form the confluence line and force a regeneration of convection on the west side of the previous mesoscale convective systems (MCSs). Since propagation of the disturbance is faster than that of the environmental wind, there is no critical level for westward-propagating IGWs. Easterly wind shear facilitates the organization of westward-propagating precipitation systems owing to favorable horizontal vorticity considerations (Rotunno et al. 1988). From November through May the background wind is often strong westerly with deep westerly shear in the troposphere (from NCEP reanalysis, not shown here). These winds together with reduced precipitable water suppress offshore propagation in winter (Fig. 6).

2) West Coast of Colombia (5°N)

Near the west coast of Colombia, strong westward propagation of diurnal convection is observed from May to December (Fig. 9). The longitudinal phase distribution shows the starting phase of the diurnal precipitation is near midnight (~0000 LT), with westward propagation speed of ~10 m s⁻¹ (Fig. 10a). The longitudinal amplitude distribution shows that amplitudes decrease significantly only after 10° longitude (1000 km) (Fig. 10b; 75°–85°W). The NCEP monthly mean winds (Fig. 11) reveal the presence of weak westerlies below 875 hPa (~4 m s⁻¹) and moderate easterlies above (approximately ~7 m s⁻¹). The
propagation speed of the disturbance is faster than that of the environmental wind. Considering the concave shape of the Colombia coast, the blocking effect of the coastal mountains, and sufficient moist static energy, local sea-breeze circulations are enhanced. These factors, among others, facilitate offshore-propagating precipitation most of the year (Fig. 9). In February and March there is no obvious offshore propagation. In these months the mean environmental wind is westerly with strong westerly shear, which confines convection to the coastal zone near foothills.

3. Linear model analysis

A linear model with a heating-generated disturbance (see the appendix) is introduced for two purposes: 1) to demonstrate the IGW condition when the Coriolis effect is no longer negligible at relatively high latitude and 2) to compare gravity waves generated by local diurnal elevated heating versus land/sea breezes.

a. Gravity wave analytical solution and IGW condition

At relatively high latitudes—for example, 22°N—the Coriolis effect \( f = 5.46 \times 10^{-5} \text{s}^{-1} \) is no longer negligible compared to the diurnal frequency \( \sigma = 7.27 \times 10^{-5} \text{s}^{-1} \). A gravity wave is then considered an IGW. For an IGW to exist, the intrinsic frequency of the wave must be larger than the Coriolis parameter \( |\sigma| > f \). (See the appendix for a derivation of the IGW condition.) Given a background mean wind, the Doppler shift will decrease the intrinsic frequency of the IGW \( \sigma - UK \) if the IGW propagates downwind (with respect to the mean wind) and increase the intrinsic frequency of the IGW \( \sigma + UK \) if the IGW propagates upwind (with respect to the mean wind). When \( f \) and \( U \) are relatively large, such as over the Bay of Bengal (Fig. 4), situations may occur for \( |\sigma - UK| < f < |\sigma + UK| \) (assuming \( k = \sigma / C_p \)). In this case, IGWs propagate in one direction (offshore propagation from the east coast of India; Fig. 1) and dissipate in the other direction (no southwestward inland propagation from the west coast of Burma; Fig. 1).
b. IGWs from diurnal local elevated heating:

Analytical expression of amplitude $A$, phase speed $C_p$, and the decaying scale $1/l$ for the isolated diurnal elevated heating case

A series linear model simulation is performed for isolated diurnal elevated heating. This simulates the case with diurnal heating over a mountain. Diurnal elevated heating is located at the center of the domain. IGW beams appear on both upwind and downwind sides, at a relatively flat angle for the downwind side.

Assuming the vertical perturbation at the LCL ($z_0$) can be expressed as $w(x, z_{LCL}, t) = A \cdot \exp(-\lambda x) \cdot \exp[i(\sigma t - \pi)] \cdot \exp(-i\phi_0) \cdot \exp[i(\sigma x/Cp)]$, where $A$ is amplitude, $C_p = \sigma/k_\alpha$ is the phase speed with $k_\alpha$ as the dominant horizontal wavenumber, $\phi_0$ is the starting phase, and $1/A$ is the decaying scale. Diurnal heating (the net radiation) reaches its maximum at noon (when phase $= \pi$).

This exponential decay is imbedded within the linear model solution, since the introduction of the linearized friction term alpha [Eqs. (A1)–(A5)] will ultimately lead to a solution that decays exponentially with the distance away from the heating source. The horizontal exponential decay (or the introduction of friction) in the linear model solution is mainly used to represent the friction or dissipation effect within the PBL. The exponential function also matches the observation well (Figs. 7b, 10b). The exponential decay of the diurnal IGWs (precipitation) amplitude in the real world may also be caused by loss of perturbation energy due to upward wave propagation or a critical level, which is not included in the linear model.

We have found that exponential function works well in describing the decay of the perturbation away from the source, as shown in Fig. 7b (22°N case) or Fig. 10b (5°N case), for which the curve fit of the exponential function (green) represents the decay of the diurnal precipitation amplitude with the offshore distance. The introduction of an exponential function $w(x) \sim e^{-\lambda x}$ anticipates, and effectively assumes, the horizontal distribution of the diurnal vertical disturbances generated by the diurnal heating. Since $\lambda x$ is nondimensional, $1/A$ should have the unit of meters and represent the horizontal decaying scale.

A series of sensitivity tests were designed to obtain the approximate analytical solution of IGWs generated by diurnal local elevated heating [see the appendix, section b(1), for more details]. The sensitivity test results are shown in Figs. 12 and 13. In the first sensitivity test (Fig. 12), all the parameters are fixed ($N = 0.02$ s$^{-1}$, diurnal frequency $\sigma = 7.27 \times 10^{-5}$ s$^{-1}$, and vertical heating depth $H = 1$ km) except the mountain width/heating width $a_r$, which varies from 50 to 450 km. In the second sensitivity test (Fig. 13), all the parameters are fixed ($N = 0.02$ s$^{-1}$, $\sigma = 7.27 \times 10^{-5}$ s$^{-1}$, and $a_r = 100$ km) except the heating depth $H = 1/\beta$ of the elevated heating, which varies from 0.05 to 2 km. The linear model value of $N$ in this application is consistent with its dry atmosphere, which is considerably more stable than the natural atmosphere containing moisture. The linear model sensitivity tests for different
combinations of the parameters (Figs. 12, 13) help to estimate the approximate analytical expression of

$$w(x, z, t) = A \cdot f(z) \cdot \exp(-\lambda x) \cdot \exp[i(\sigma t - \pi)] \cdot \exp(-i\varphi_0) \cdot \exp[i(\sigma x/C_p)],$$

with

$$A \propto \frac{B_0}{2\sqrt{2\pi^2} \beta} \cdot \sigma^4 \cdot \sqrt{k_1} \cdot \cos(y_1 z_0),$$

$$\lambda = \sqrt{k_1^2 + k_0^2},$$

$$C_p = \frac{\sigma}{k_0} = \frac{\sigma}{\sqrt{2k_1 k_0}}.$$

Here, we assume that $w(x, z, t)$ can be separated into the product of several independent functions of $z$, $x$, and $t$ explicitly, where $f(z)$ is the vertical profile of $w$. Those parameters in these preassumed functions are then solved and expressed analytically with existing variables, which are known. In Eqs. (1)–(3), $H = 1/\beta$ is the vertical heating scale, and $k_1 = 1/a$, is the dominant horizontal wavenumber determined by the heating scale $a$, which is the horizontal scale of the Gaussian shape heating distribution. The expression $y_1 = N k_1/(\sigma^2 - f^2)^{1/2}$ represents the dominant IGW vertical wavenumber determined by the source of the heating. The expression $k_0 = [\beta^2(\sigma^2 - f^2)/N^2]^{1/2}$ is the intrinsic IGW horizontal wavenumber, determined by the environment.

The appendix provides detailed derivations. To obtain the analytical expression of the starting phase $\varphi_0$, we return to Eq. (A15) and set $z = z_0$, and compare its real and imaginary parts with $e^{-i\varphi_0}$:

$$\frac{i e^{\gamma H}}{\gamma(\sigma - f^2)} \cdot \frac{B_0 B_0(k) k^2}{(i \gamma - \beta)^2} \cdot \cos(y_1 z_0) \propto (\cos\varphi_0 - i \sin\varphi_0),$$

so that

$$\varphi_0 = \tan^{-1}\left(\frac{-\Im\varphi}{\Re\varphi}\right) = \frac{3\pi}{2} - \tan^{-1}\left(\frac{y_1}{\beta}\right).$$

Here, we assume that most of the energy concentrates on the value of $k_1$ and $y_1 = N k_1/(\sigma^2 - f^2)^{1/2}$, which are the dominant horizontal and vertical heating scales, respectively, for the diurnal elevated heating case. The physical explanation for the phase (Fig. 12b) is as follows: If the horizontal scale of the isolated heating is small, then $k_1 = 1/a$, is large with $N k_1/(\sigma^2 - f^2)^{1/2} > 1/H$, $\tan^{-1}(y_1/\beta) \to \pi/2$, and the starting phase $\varphi_0 \to \pi$. This indicates that the vertical disturbance reaches its maximum when the diurnal heating rate reaches its peak (1200 LT), thereby being in phase with each other. If the horizontal scale of the isolated heating is large, then $k_1 = 1/a$, is small with $N k_1/(\sigma^2 - f^2)^{1/2} < 1/H$, $\tan^{-1}(y_1/\beta) \to 0$, and the starting phase $\varphi_0 \to 3\pi/2$, which indicates that vertical disturbance reaches its maximum when diurnal heating ends (1800 LT). Elevation ensures the thermally generated gravity waves will fully develop and propagate outward and downward without restraint.

Figure 12 illustrates a sensitivity test for the width of the isolated diurnal elevated heating, while Fig. 13 shows a sensitivity test for the heating depth of the isolated diurnal elevated heating. How heating depth, environmental stability, and mountain width modify the disturbance in the far field is also given in Table 1. In general, the results are the same as expressed analytically in Eqs. (1)–(4). The amplitude of the vertical disturbance increases with heating depth (Fig. 13a) and latitude [Eq. (1)], and decreases with mountain width (Fig. 12a) and stability [Eq. (1)]. Starting phases are from 1200 to 2000 LT (Figs. 12b, 13b). Assuming that precipitation is about one-quarter period lag of $w$, then the precipitation maximum is after sunset (1800–0200 LT). The phase speed increases with mountain width significantly, as well as heating depth, although it increases slowly [Figs. 12c, 13c; Eq. (2)].

For the two observations discussed in section 2b—a tropical coastal area with significant mountains near the coastline, the west coast of Mexico (22°N; Figs. 6, 7) and the west coast of Colombia (5°N; Figs. 9, 10)—the amplitude, starting phase, propagating phase speed, and the horizontal decaying scale are estimated by curve fitting, with linear regression for phase distribution (Figs. 7a, 10a), and exponential curve fit for amplitude distribution (Figs. 7b, 10b). The results are also shown in Figs. 12 and 13. In general, the observations from these locations are close to the simulated case with horizontal width of the heating $a_0 = 100$ km, heating depth $H = 1/\beta = 1$ km, and lifting condensation level $z_{LCL} = 2$ km. The mean wind is set to zero for simplicity. Background $N$ is set to 0.02 s$^{-1}$, latitude is set to 5°N, and the friction term is $\alpha = 10^{-8}$ s$^{-1}$. The west coast of Mexico (22°N; Figs. 6, 7) case is more similar to the theory-predicted isolated diurnal elevated heating
c. IGW from land–sea breeze: Analytical expression of $A$, $C_p$, and $1/\lambda$ for the land-breeze/sea-breeze case

A series of sensitivity tests are designed to obtain the approximate analytical solution of IGWs generated by land–sea breeze [also see the appendix, section b(2), for the detailed derivation of the IGW analytical solution]. The sensitivity test results are also shown in Figs. 12 and 13. In the first sensitivity test (Fig. 12), all the parameters are fixed except the coastal transition zone/heating width $a_r$, with parameter settings the same as in section 3b. In the second sensitivity test (Fig. 13), all the parameters are fixed except the heating depth $H = 1/\beta$ of ground heating, with the parameter settings the same as in section 3b. The linear model sensitivity tests for different combinations of parameters (Figs. 12, 13) help to obtain the approximate analytical expression of amplitude, decaying scale, and phase speed for land–sea breeze with

$$A \approx \frac{B_0}{2\sqrt{2N^2\beta}} \cdot \sigma^2 \cdot \sqrt{k_0} \cdot \cos(\gamma z_0),$$

(5)

$$\lambda = k_0,$$

(6)

$$C_p = \frac{\sigma}{k_0},$$

(7)

$$\varphi_0 = \frac{3\pi}{2} + \tan^{-1}\left(\frac{\beta}{\gamma_1}\right).$$

(8)

Here, $k_0 = \frac{[\beta^2(\sigma^2 - f^2)/N^2]^{1/2}}{k_1 = 1/a_r}$, and $\gamma_1 = Nk_1/(\sigma^2 - f^2)^{1/2}$, which are the dominant horizontal and vertical scales, respectively, for gravity waves generated by diurnal land heating for the land breeze–sea breeze case.

The physical explanation for phase [Eq. (8); Figs. 12b, 13b] is that, if the vertical scale of the land PBL heating is small, $\beta = 1/H$ is large, $\tan^{-1}(\beta/\gamma_1) \to \pi/2$, then the starting phase $\varphi_0 \to 2\pi$, which indicates that vertical disturbance reaches its maximum at midnight (0000 LT). Assuming that precipitation is about one-quarter period lag of $w$, then the precipitation maximum is in the early
morning (0000–0600 LT). If the vertical scale of the heating is large, $\beta = 1/H \to 0$, then the starting phase $\phi_0 \to (3/2)\pi$. This suggests that the vertical disturbance reaches its maximum at sunset (1800 LT) and the precipitation maximum at night (1800–0000 LT). In this case the phases are similar to the phase of the continental tide (Fig. 14 in Li and Smith 2010a).

For diurnal land–sea heating, Figs. 12 and 13 illustrate how the width of the coastal transition zone and heating depth modify the disturbance in the far field. The results are also given in Table 1. In general, the results are the same as expressed analytically in Eqs. (5)–(8): the width of the coastal transition zone does not affect the results except the starting phase [Fig. 12; Eq. (8)]. Amplitude increases with heating depth [Fig. 13a; Eq. (5)]. Starting phases are between sunset time and midnight at coastline, which is a one-quarter period later than that of the elevated local heating case [Figs. 12b, 13b; Eq. (8)]. Assuming that precipitation is about one-quarter period lag of $w$, then the precipitation maximum will be from midnight to early morning (0000–0600 LT). The phase speed increases with heating depth [Fig. 13c; Eq. (7)]. The phase speeds are significantly faster than that of the elevated heating case (Figs. 12c, 13c). This is because the intrinsic wavenumber $k_0 = [\beta^2(\sigma^2 - f^2)/N^2]^{1/2} = (10^{-3} \cdot 7.27 \times 10^{-5})/4 \times 10^{-2} = 2 \times 10^{-6}$, as determined by the environment, is usually much smaller than the dominant wavenumber determined by the isolated heating source [k$_d$ = 1/k$_x$ = 1/(100 × 1000) = 10$^{-5}$], while N[$\beta^2(\sigma^2 - f^2)$]$^{1/2}$ is also the horizontal scale of the sea breeze (Rotunno 1983).

Figures 12 and 13 exhibit the case for the west coast of Colombia (Fig. 5, 5°N; Figs. 9, 10), which is generally (but not always) closer to the theory-predicted diurnal land–sea breeze than that of the west coast of Mexico (22°N). Discussions of the linear model analytical solution and the sensitivity test for the parameters are mainly for understanding the differences in amplitude, phase speed, decaying scale, and starting phases between isolated elevated heating and the land breeze–sea breeze case. Also of importance is how the phase speed, among others, changes with different horizontal scales and the environment. Here, the decaying scales derived from the observations need to be treated with caution, since the nonlinearity of
moist convection and precipitation amount is not only a function of the vertical disturbance caused by IGWs but also local CAPE, convective inhibition (CIN), instability, etc.

4. Discussion and conclusions

This work focuses on the offshore propagation of precipitation in the tropics with or without coastal mountains.

Table 1. Diurnal elevated heating vs land–sea breeze \( \{W(x, z, t) = A \cdot \exp(-\lambda x) \cdot \exp[i(\sigma t - \varphi)] \cdot \exp(-i\phi_0) \cdot \exp[i(\sigma z/C_p)]\} \).

<table>
<thead>
<tr>
<th>Case I heating: Local elevated diurnal heating</th>
<th>Case II heating: Land–sea breeze</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heating profile</strong></td>
<td></td>
</tr>
<tr>
<td>( B_h(x) = \exp(-X^2/a_r^2) )</td>
<td>( B_h(x) = \begin{cases} 0 &amp; x &lt; -a_r \ 0.5 + 0.5 \times \sin \left( \frac{\pi}{2} \frac{x}{a_r} \right) &amp; -a_r \leq x \leq a_r \ 1 &amp; x &gt; a_r \end{cases} )</td>
</tr>
<tr>
<td><strong>Heating region</strong></td>
<td></td>
</tr>
<tr>
<td>( B_h(z) = \begin{cases} 0 &amp; z &lt; H \ {z - H}e^{-\beta(z-H)} &amp; 0 \leq z &lt; H \ 1 &amp; z \geq H \end{cases} )</td>
<td>( B_h(z) = e^{-\beta z} )</td>
</tr>
<tr>
<td><strong>Horizontal heating scale</strong></td>
<td></td>
</tr>
<tr>
<td>( k_1 = \frac{1}{a_r} )</td>
<td>( k_0 = \sqrt{\frac{\beta^2(\sigma^2 - f^2)}{N^2}} )</td>
</tr>
<tr>
<td><strong>Vertical heating scale</strong></td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 = \frac{Nk_1}{\sqrt{\sigma^2 - f^2}} )</td>
<td>( h = \frac{1}{\beta} )</td>
</tr>
<tr>
<td><strong>Amplitude</strong></td>
<td></td>
</tr>
<tr>
<td>( A \propto \frac{B_h}{2\sqrt{2\sigma^2N^2}} \cdot \sigma^4 \cdot \sqrt{k_1} \cdot \cos(\gamma_1 z_0) )</td>
<td>( A \propto \frac{B_0}{2\sqrt{2\sigma^2N^2}} \cdot \sigma^4 \cdot \sqrt{k_0} \cdot \cos(\gamma_2 z_0) )</td>
</tr>
<tr>
<td><strong>Decaying scale</strong></td>
<td></td>
</tr>
<tr>
<td>( \lambda = \sqrt{k_1^2 + k_0^2} )</td>
<td>( \lambda = k_0 )</td>
</tr>
<tr>
<td><strong>Phase speed</strong></td>
<td></td>
</tr>
<tr>
<td>( C_p = \frac{\sigma}{k_0} = \frac{\sigma}{\sqrt{2k_1k_0}} )</td>
<td>( C_p = \frac{\sigma}{k_0} )</td>
</tr>
<tr>
<td><strong>Initiation time</strong></td>
<td></td>
</tr>
<tr>
<td>( \phi_0 = \frac{3\pi}{2} - \tan^{-1} \left( \frac{\gamma_1}{\beta} \right) )</td>
<td>( \phi_0 = \frac{3\pi}{2} + \tan^{-1} \left( \frac{\beta}{\gamma_1} \right) )</td>
</tr>
</tbody>
</table>
In the South Asian monsoon region, the Bay of Bengal, both the diurnal offshore propagation of precipitation near India and the diurnal nonpropagating signal near Burma are indicative of an inertia-gravity wave (IGW) mechanism. The perturbations generated by the diurnal elevated heating over the Eastern Ghats of India propagate offshore in the upwind direction with respect to the mid-tropospheric westerlies (using environmental wind as the reference). In the case of Burma, such waves dissipate because these violate the IGW condition. In tropical coastal regions with steep mountains nearby (e.g., northwest Mexico and Colombia), strong offshore-propagating diurnal precipitation signals are observed, which is broadly consistent with the gravity wave mechanism.

A linear model is used to confirm the plausibility of gravity waves that result from inhomogeneous heating, such as mountain–plain contrast and land–sea contrast. Approximate analytical expressions are derived for starting amplitude and phase, propagating phase speed, and the decaying scale of the diurnal disturbance, by means of sensitivity tests with the parameters. The test results are shown to be consistent with observations as a function of the mountain width, diurnal heating depth, coastal transition zone width, friction, Coriolis acceleration, background mean wind, and stability. The results suggest that two horizontal scales are involved, the width of the mountain and the intrinsic horizontal sea-breeze scale.

For the land–sea case, only the intrinsic horizontal sea-breeze scale is important. Isolated diurnal elevated heating and sea breezes are distinguishable. The phase speed of the land–sea breeze is much faster than that generated by the isolated diurnal elevated heating. The convection caused by the isolated elevated heating tends to start earlier, propagate slower, and damp faster than that of the land–sea breeze. The combination of these two forcings is such that, for those near the coastal mountains, perturbations are mainly generated by the mountain-elevated heating. Over the ocean, far away from the coast, the dominant signal propagates as that of the land breeze together with stronger convection. Slower propagation and the larger amplitude of diurnal convection near the coast are greater than the linear combination of both forcings, including a resonance effect and the nonlinear effects of the terrain blocking.

Future work will examine whether the gravity wave mechanism is adequately represented in current regional climate models. A high-resolution WRF Model will be used to study the interactions of these diurnal gravity waves with the background mean wind shear. Dry dynamics will be used to determine whether an idealized simulation can reproduce the gravity wave mechanism. The results will be compared to the magnitude and phase of the generated vertical motion, with the nonprecipitation day average from reanalysis data. Once the gravity wave mechanism is validated, moisture and latent heat can be introduced to evaluate the timing and the amount of the daily precipitation. Other factors, such as the mountain width (mountain vs plateau), mountain slope, and the distance from the mountain to the coastline, will also be examined in detail.

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APPENDIX

The Analytical Derivation for GW and IGW Generated by Diurnal Heating

a. The analytical solution for gravity wave generated by diurnal heating

A linear model with Boussinesq approximation including three momentum, one buoyancy, and one incompressibility condition(s) is solved by the fast Fourier transform (FFT) method (Smith and Lin 1982; Robinson et al. 2008; Li 2009). It is employed here to demonstrate how gravity waves are modified by environmental parameters, such as stability, horizontal and vertical heating scale, the Coriolis effect, and background mean wind:

\[
\begin{align*}
    u_t + U u_x + V u_y - f v &= -p'_s - \alpha u, \\
    v_t + U v_x + V v_y + f u &= -p'_y - \alpha v, \\
    w_t + U w_x + V w_y &= -p'_z + b - \alpha w, \\
    b_t + U b_x + V b_y + N^2 w &= B - \alpha b, \quad \text{and} \\
    u_x + v_y + w_z &= 0,
\end{align*}
\]

with the isothermal background atmosphere \( p' = [p - f(z)]/\rho, \) buoyancy \( b = g(T - T_0 + g z / C_p) / T_0, \) and scaled heating rate \( B(x, y, z, t) = B_0(x, y, z) e^{\omega t}, \)

\[
B_0(x, y, z) = B_0 B(y, x) B(z)
\]

\[
= B_0 \exp\left(-\left(a_x x^2 + a_y y^2\right)\right) B_0(z),
\]

and \( B_0 = g Q / \rho_0 C_p T_0, \) where \( Q \) is the heating rate in watts; \( B_0(x, y) \) and \( B_0(z) \) are the horizontal and vertical structure of the heating, respectively. A small damping \( \alpha \) is added to remove singularities.
Using Fourier transform $\hat{f}(k) = 1/2\pi \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$ [inversion Fourier transform $f(x) = \int_{-\infty}^{\infty} \hat{f}(k) e^{ikx} dk$], Eqs. (A1)–(A5) are converted to

\begin{align*}
  i\sigma \ddot{u} &= -ik\dot{p} + f_v, \\
  i\sigma \ddot{v} &= -il\dot{p} - f_u, \\
  i\sigma \ddot{w} &= -\dot{\bar{p}}_z + \bar{b}, \\
  ik\dot{\bar{p}} + i\dot{\bar{b}} + \ddot{w}_z &= 0, \\
  \end{align*}

respectively, with intrinsic frequency

$$\sigma = \sigma + U k + V l - i\alpha,$$  \hspace{1cm}  \mbox{(A11)}

which reflects the Doppler shift by background mean wind and damping.

From Eqs. (A6)–(A10), we get a single ODE for vertical velocity,

$$\ddot{w}_z + \gamma^2 \dot{w} = \frac{\hat{B}_h(k^2 + \beta^2)}{\sigma^2 - \beta^2}$$  \hspace{1cm}  \mbox{(A12)}

with the vertical wavenumber given by

$$\gamma = \sqrt{\frac{(N^2 - \sigma^2)(k^2 + \beta^2)}{\sigma^2 - \beta^2}}.$$  \hspace{1cm}  \mbox{(A13)}

Once $\ddot{w}(k, l, z)$ is solved, the other variables can be recovered.

\textit{b. IGW from diurnal local elevated heating: Effect of $H$, $N$, $f$, $U(z)$, and $\alpha$}

1) \textbf{ANALYTICAL SOLUTION FOR IGW GENERATED BY THE DIURNAL LOCAL ELEVATED HEATING}

We assume that diurnal local elevated heating has the vertical structure

$$B_v(z) = \begin{cases} 
0 & 0 \leq z < H \\
(z - H_m) e^{-\beta(z-H_m)} & z \geq H_m 
\end{cases}$$  \hspace{1cm}  \mbox{(A14)}

with horizontal structure $B_h(x) = \exp(-X^2/\alpha_x^2)$, where $\alpha_x$ is the horizontal scale of the Gaussian shape heating distribution. The term $H_m$ is equivalent to the mountain height.

The background atmosphere has stability $N$ and mean wind $U$. The boundary conditions are lower boundary: $z = 0$, $\ddot{w}_1(k, 0) = 0$; upper boundary: decay, $\ddot{w}_2(k, z)$ is finite when $z \to \infty$.

The analytical solution for $w(x, z)$ in Fourier space is

\begin{itemize}
  \item[(i)] $0 \leq z < H$
  \item[(ii)] $z \geq H$
\end{itemize}

\begin{align*}
  \ddot{w}_1(z) &= \frac{i e^{i\gamma H}}{2\gamma(\sigma^2 - \beta^2)(i\gamma - \beta)} \left( e^{i\gamma H} - e^{-i\gamma H} \right) e^{i\gamma z} \\
  \ddot{w}_2(z) &= \frac{iB_0\hat{B}_h(k)k^2}{2\gamma(\sigma^2 - \beta^2)(i\gamma - \beta)} \left( e^{i\gamma H} - e^{-i\gamma H} \right) e^{i\gamma z} \\
  &\quad + \frac{B_0\hat{B}_h(k)k^2}{(\sigma^2 - \beta^2)(\beta^2 + \gamma^2)} \left\{ (z-H)(\beta^2 + \gamma^2) + 2\beta \right\} e^{-\beta(z-H)} - 2\beta e^{i\gamma(z-H)} \hspace{1cm} \mbox{(A16)}
\end{align*}

Assuming LCL = $z_0$ and no friction $\alpha = 0$, the vertical perturbation at the LCL can be expressed as

$$w(x, z_{\text{LCL}}, t) = A \exp(-\lambda x) \cdot \exp[i(\sigma t - \pi)] \cdot \exp(-i\phi_0) \cdot \exp[i(\sigma t/C_p)],$$

where $A$ is amplitude, $C_p = \sigma/k_\sigma$ is the phase speed, $\phi_0$ is the starting phase, and $1/\lambda$ is the decaying scale. Diurnal heating (the net radiation) reaches its maximum at noon (when phase $\approx \pi$).

According to Eq. (A15), the vertical perturbation at $z_0 = \text{LCL}$ can be also expressed as

\begin{align*}
  w_1(x, z_0) &= \int_{-\infty}^{\infty} \frac{ie^{i\gamma H}}{2\gamma(\sigma^2 - \beta^2)} \cdot \frac{B_0\hat{B}_h(k)k^2}{(i\gamma - \beta)} \cdot 2\cos(\gamma z_0) e^{ikx} dk \\
  &= A e^{-\lambda x} e^{i\phi_0} e^{ikx} \left( \text{with } k_x = \frac{\sigma}{C_p} \right) \\
  &= f(x) = \int_{-\infty}^{\infty} F(k) e^{ikx} dk. \hspace{1cm} \mbox{(A17)}
\end{align*}
Based on convolution theorem $f(t) * g(t) = \int f(\tau)g(t - \tau) d\tau$ and $f(x)g(x) = F(F^{-1}G(k)) = \int F(m)G(k - m) dm$, and applied to this case, with $e^{-\lambda x} \rightarrow (2/\pi)^{1/2} \cdot [\lambda/(\lambda^2 + k^2)]$ and $e^{ikx} \rightarrow \delta(k - k_0)$, so that $e^{-\lambda x} \cdot e^{ikx} \rightarrow [2/\pi]^{1/2} \cdot [\lambda/(\lambda^2 + k^2)] \cdot \sqrt{2\pi\delta(k - k_0 - m)} dm = (2\Lambda/l)[\lambda^2 + (k - k_0)^2].$

Then $F(k) = 2\Lambda/l[\lambda^2 + (k - k_0)^2]$ and

$$B_h \cdot \hat{B}_h(k)^2 \cos(\gamma z_0) \gamma N^2(k^2 + k_0^2) \Rightarrow 2\Lambda/l \lambda^2 + (k - k_0)^2.$$ (A18)

2) ANALYTICAL SOLUTION FOR IGW GENERATED BY THE DIURNAL LAND/SEA BREEZE

The derivation is similar to section 3b, but we change the heating function to

$$B_h(x) = \begin{cases} 0 & x < -a_r, \\ 0.5 + 0.5 \sin(\pi x/a_r) & -a_r \leq x \leq a_r, \\ 1 & x > a_r. \end{cases}$$ (A19)

and $B_h(z) = e^{-Bz}$.

The analytical solution for $w(x, z)$ in 2D Fourier space is then

$$\hat{w}(z) = \frac{B_h \cdot \hat{B}_h(k)^2}{(\sigma^2 - f^2)(\hat{\beta}^2 + \gamma^2)} \cdot e^{\gamma z} + e^{-Bz}. \quad \text{(A20)}$$

Here, $k_0 = [\beta^2(\sigma^2 - f^2)/N^2]^{1/2} \quad \text{(when} \quad \beta = \gamma \text{)}$ is the intrinsic IGW horizontal wavenumber determined by the source environment that supplies the majority of energy for $B_h(k)$.

REFERENCES


