ABSTRACT

Mountain waves and rotors in the lee of the Medicine Bow Mountains in southeastern Wyoming are investigated in a two-part paper. Part I by French et al. delivers a detailed observational account of two rotor events: one displays characteristics of a hydraulic jump and the other displays characteristics of a classic lee-wave rotor. In Part II, presented here, results of high-resolution numerical simulations are conveyed and physical processes involved in the formation and dynamical evolution of these two rotor events are examined. The simulation results reveal that the origin of the observed rotors lies in boundary layer separation, induced by wave perturbations whose amplitudes reach maxima at or near the mountain top. An undular hydraulic jump that gave rise to a rotor in one of these events was found to be triggered by midtropospheric wave breaking and an ensuing strong downslope windstorm. Lee waves spawning rotors developed under conditions favoring wave energy trapping at low levels in different phases of these two events. The upstream shift of the boundary layer separation zone, documented to occur over a relatively short period of time in both events, is shown to be the manifestation of a transition in flow regimes, from downslope windstorms to trapped lee waves, in response to a rapid change in the upstream environment, related to the passage of a short-wave synoptic disturbance aloft.

The model results also suggest that the secondary obstacles surrounding the Medicine Bow Mountains play a role in the dynamics of wave and rotor events by promoting lee-wave resonance in the complex terrain of southeastern Wyoming.

1. Introduction

A boundary layer is a region of fluid flow that is strongly affected by the no-slip condition along the interface with a solid boundary. The aspect of boundary layer dynamics that represents the greatest difficulty theoretically is the flow separation (Batchelor 1967). Laboratory experiments show that boundary layers exhibit no tendency to separate where the flow external to the boundary layer accelerates. Conversely, where the external flow is strongly retarded, the boundary layer will separate from the solid surface (Lighthill 1986). In stratified fluids, such as the atmosphere, separation-causing deceleration can be triggered, among other means, by the adverse pressure gradient force due to pressure perturbations induced by internal gravity waves launched in flow over complex terrain. If that is the case, then boundary layer separation (BLS) is said to be wave induced (Scorer 1958; Baines 1997).

In orographically forced atmospheric flows, in general, the occurrence of wave-induced boundary layer separation is a manifestation of nonlinear processes. In
flows over terrain with vertically uniform stability and wind profiles, nonlinearity displays itself as large-amplitude waves at or near the mountain tops. Flows in which either the stability, the wind, or both change with height may too be conducive to boundary layer separation through formation of trapped lee waves or of a hydraulic jump at low levels (Jiang et al. 2007).

Attendant to wave-induced boundary layer separation is the occurrence of terrain-induced atmospheric rotors (Doyle and Durran 2002). The latter has been traditionally described and schematically represented as horizontal vortices with an axis parallel to a mountain ridge (Grubišić et al. 2008) and are often characterized by severe or extreme turbulence (Doyle and Durran 2002, 2007; Strauss et al. 2015). The interplay of baroclinic vorticity generation along stable layers embedded in mountain waves and shear vorticity generation within the terrain-adjacent boundary layer has been shown to exert a significant impact on the rotor structure and strength. According to Hertenstein and Kuettner (2005), rotors tend to take the form of hydraulic jumps when baroclinically generated vorticity prevails, whereas lee-wave-type rotors are expected to occur when shear-generated vorticity dominates.

BLS in stratified flows over two-dimensional (2D) hills has been examined in laboratory experiments (Baines and Hoinka 1985) and described by theoretical studies (Ambaum and Marshall 2005). These investigations reveal a range of separation regimes (Baines 1997), dependent on the parameters $NH/U$ (nonlinearity parameter or nondimensional obstacle height) and $H/L$ (aspect ratio or slope steepness), where $N$, $U$, $H$, and $L$ are, respectively, the buoyancy frequency, the upstream wind speed (both assumed to be constant with height), the maximum height of the mountain, and its half-width. In the limit of neutral stratification ($NH/U \sim 0$) no separation occurs on shallow hills ($H/L \sim 0$). As the aspect ratio is increased ($H/L \geq 0.1$), salient-edge BLS is expected to occur at the hilltop. For $0 < NH/U \leq 1$ and low aspect ratios, small-amplitude internal gravity waves cannot induce BLS because the flow acceleration on the lee slope and deceleration farther downstream are rather weak. With strong stratification and nonlinearity ($NH/U \geq 1$), the amplitude of the pressure perturbations related to gravity waves in the inviscid flow above the boundary layer becomes large enough to force the flow to separate in a wide range of $H/L$ values.

A large number of mountain wave studies emphasize the profound impact that boundary layer dynamics may have on downslope windstorms, rotors, and lee waves. For instance, a boundary layer can effectively limit trapped-lee-wave propagation by absorbing the reflected, downward-propagating wave beams (Jiang et al. 2006; Smith et al. 2006; Smith 2007). Furthermore, the boundary layer diurnal cycle of diabatic heating and cooling modulates downslope windstorms by inducing changes both in the turbulence intensity and in the horizontal buoyancy gradients within waves (Jiang and Doyle 2008). A number of studies, with a special focus on lee-wave-type rotors, examined the sensitivity of BLS and rotor formation to several other factors such as surface roughness, wavelength, and wave amplitude (Doyle and Durran 2002; Vosper et al. 2006; Jiang et al. 2007). The role played by elevated inversions in the formation of rotors was investigated by Vosper (2004) and, with laboratory experiments, by Knigge et al. (2010). The effect of a downstream mountain range on BLS and rotor formation was investigated by Grubišić and Stiperski (2009) and Stiperski and Grubišić (2011).

Beyond numerical, laboratory, and theoretical investigations, several field measurement campaigns were recently conducted to study the structure and dynamics of atmospheric rotors and their surface signatures in the lee of different mountain ranges, including the Falkland Islands (Mobbs et al. 2005), the Pennines (Sheridan et al. 2007), and the Dinaric Alps (Gohm et al. 2008). The most extensive of these are the pair of campaigns in the Sierra Nevada—the Sierra Rotors Project (Grubišić and Billings 2007) and the Terrain-Induced Rotor Experiment (T-REX; Grubišić et al. 2008). Despite these significant efforts, the observational evidence of boundary layer separation in terrain-induced flows has been lacking until now. French et al. (2015, hereafter Part I) describe observations of two events of wave-induced boundary layer separation and rotors in the lee of the Medicine Bow Mountains (MBM) in southeastern Wyoming (Fig. 1). These events occurred on 26 January and 5 February 2006 and were documented during the NASA06 field campaign (Part I). In this paper, we use a mesoscale numerical model to simulate these two events in order to place the Part I observations in a broader mesoscale context and to examine physical processes involved in their formation and temporal evolution.

The paper is organized as follows. The details of the modeling approach are presented in section 2. Sections 3 and 4 contain our analysis of numerical simulation results for the 26 January and 5 February cases. Summary and conclusions are presented in section 5.

2. Numerical simulations

a. Model setup

The simulations presented here were performed using version 3.3 of the ARW Model (Skamarock and Klemp 2008) in a nested configuration with two domains
having a horizontal grid spacing (Δx = Δy) of 2 km and 400 m in the outer (d01) and inner (d02) domains, respectively. Initial and boundary conditions for the simulations were interpolated from the operational ECMWF analyses, available at 6-hourly intervals with native TL511L62 resolution. The model top was located at the 100-hPa pressure level, roughly corresponding to an altitude of 15 700 m. Rayleigh damping was applied to the vertical velocity in the upper 5000 m of the domain, in order to absorb vertically propagating gravity waves and prevent their downward reflection from the model top, which was treated as a rigid lid.

The terrain-following grid uses the σ coordinate introduced by Laprise (1992) and features 61 vertical levels in both domains. The grid spacing is 0.003σ near the lower boundary and increases toward higher altitudes with a constant stretching factor of about 5% (i.e., each model level is about 5% thicker than its lower neighbor). The vertical grid spacing near the lower boundary is chosen so as to be on the same order of (and not much smaller than) the maximum terrain height increment across one element of the horizontal grid (Mahrer 1984; De Wekker 2002). The former (0.003σ) corresponds to about 21 m in our case, while the latter (Δz tan α, where α is the local slope angle) is well below 60 m throughout most of the domain. Under these conditions, the grid setup should not cause large errors in the computation of near-surface pressure gradients. Additional simulations with different near-surface vertical resolutions (0.001σ and 0.1σ) did not produce appreciably different results from those presented herein.

A third-order time-split Runge–Kutta scheme was used for the discretization of time derivatives. Spatial derivatives were treated with upwind-biased fifth- and third-order schemes along the horizontal and vertical directions, respectively, providing implicit artificial diffusion. Given the fully compressible nature of the WRF solver, isotropic divergence damping and forward biasing of the vertically implicit acoustic-time-step terms were adopted in order to damp acoustic modes in the solution.

Fluxes in the surface layer and vertical subgrid-scale fluxes in the PBL are parameterized with theEta Mellor–Yamada–Janjić scheme (MYJ; Janjić 2002), while horizontal diffusion is handled with a Smagorinsky first-order closure. The Morrison two-moment scheme (Morrison et al. 2009) is used for microphysical processes and atmospheric radiation is treated with the RRTM scheme (Mlawer et al. 1997) for the longwave and the Dudhia (1989) scheme for the shortwave component.

The fairly high grid resolution of the inner domain lies in the so-called terra incognita range (Wyngaard 2004), where the application of traditional BL parameterizations, such as the MYJ scheme, might be questionable. Consequently, we explored the sensitivity of model results to boundary layer parameterizations by performing a few additional runs with alternative schemes (not shown). Overall, the general characteristics of our results were unaltered, increasing our confidence in the appropriateness of the selected model setup.

The 26 January and 5 February simulations were both initialized at 0000 UTC on respective dates and carried forward for 36 h. The first few hours of each run are regarded as spinup time and were not taken into account in the analysis of results.

b. Model verification

For model verification, simulation results were compared with available measurements from both surface-based and airborne sensors. Airborne measurements with the University of Wyoming King Air (UWKA) were made between 2020 and 2230 UTC 26 January and between 1350 and 1545 UTC 5 February and include both level flight tracks as well as ramp soundings (Part I). Surface-based measurements include observations from the Medicine Bow wind profiler, located approximately 55 km north of MBM, and a weather station...
at Laramie Regional Airport, some 50 km downstream of the MBM peak (Fig. 1). For comparison with the surface-based measurements, model output was extracted from the nearest-neighbor grid points to the surface sites. For the wind profiler, the model data were extracted from the outer domain, whereas for the Laramie surface station, the model data come from the inner domain. The periods of comparison for the surface measurements for the two events extend from 1200 UTC 26 January to 1200 UTC 27 January and from 0600 UTC 5 February to 0600 UTC 6 February. For comparison with airborne measurements, the model data from the inner domain were linearly interpolated along the aircraft trajectory.

The observed and simulated vertical profiles of wind speed and direction at the Medicine Bow wind profiler are presented in Fig. 2, which shows a good agreement between simulation results and observations, in particular for 26 January. On both of these days, the passage of an upper-tropospheric anomaly is apparent at altitudes between 6000 and 8000 m MSL. On 26 January, the passage of this anomaly occurs between 1400 and 2200 UTC and is evident in the turning of the wind from southwest to northwest and back to west, accompanied by a temporary drop of the wind speed at these levels, all compatible with the passage of a synoptic short-wave trough as described in Part I. The overall evolution of this event, including the timing of the short-wave-trough passage, is captured well by the simulation. On 5 February, rapid wind turning from west to north-northeast and then back to northwest is evident in observations between approximately 1000 and 2000 UTC. In this case, it appears there is a time lag of about 2 h between the onset of the wind turning aloft in the observations and the simulation results. At low levels, the simulation results show sustained northwest winds whereas the observations display a sharp turn to the northerly wind soon after 1200 UTC and a subsequent gradual shift to the northwesterlies. By 1800 UTC, the model is starting to catch up with the observations and, by 0000 UTC the next day, the two appear to be in a fairly good agreement. Although we cannot ascertain the cause of this delay without additional numerical experiments, it is likely that the mesoscale model acquired this timing error through the initial and boundary conditions from the global model analyses.
The comparison between surface observations at the Laramie airport and model predictions for this site is shown in Fig. 3. The overall tendency of surface weather parameters is well reproduced for both events. In particular, the onset of high and sustained wind speeds between 1800 UTC 26 January and 0000 UTC 27 January is captured almost perfectly, as is the second peak in wind speed between 0300 and 0600 UTC 27 January. On 5 February, the simulation misses a sharp peak in wind speed at around 1400 UTC, the time of the aircraft research mission. An initial underprediction of near-surface air temperatures is also apparent.

A comparison between the observed and simulated pseudovertical atmospheric profiles from the UWKA ramp soundings for the two events is shown in Fig. 4. Profiles are taken along descending flight segments, starting slightly downwind or over the top of the MBM and extending over its upwind slope. Generally, the rather coarse model resolution does not allow the small-scale spatial variability, apparent in the observations, to be resolved. However, the most important features of the 26 January event are reproduced reasonably well, including the midtropospheric west-northwest wind direction, the wind veering with height in the lowest kilometer above the ground, the presence of a
low-level jet with wind speeds exceeding 15 m s\(^{-1}\) (slightly overestimated by the model), and a relatively constant vertical gradient of virtual potential temperature of about 4.4 \times 10^{-3} \text{ K m}^{-1} throughout the troposphere. The greatest differences between the model output and the ramp sounding on 26 January occur at heights above 5.5 km MSL. This range of altitudes coincides with a flight segment in which the UWKA was flying slightly downstream of the MBM within a wave breaking region (cf. section 3a below).

On 5 February, there is less of an agreement between the model results and observations along the ramp sounding. Although the wind direction is reproduced reasonably well, the wind speed and temperature profiles compare less favorably. In particular, the layered structure of the observed wind speed and temperature profiles below about 3.2 km MSL are not captured, including a low-level jet of about 18 m s\(^{-1}\) in a shallow neutral layer topped by a strong inversion (\(\sim 16 \times 10^{-3} \text{ K m}^{-1}\)) at 2.75 km and a neutral layer above it with a wind speed of about 15 m s\(^{-1}\) topped by another, weaker inversion at 3.2 km MSL. Overall, at the time of the observations, the model is showing an underestimation of wind speed below 2.75 km MSL and an overestimation above this altitude. Two hours later, while the agreement of the wind speed profiles aloft improves, the discrepancy at low levels remains and even increases. Given the impact of the wind speed profile curvature (through the Scorer parameter) and of elevated inversions (Vosper 2004) on mountain wave response, it is reasonable to anticipate the simulation results for 5 February to show less favorable agreement with observations.
A comparison of observations and model output along the cross-mountain flight legs is shown in Fig. 5. To illustrate the degree of unsteadiness in the simulations, model output is represented by a set of profiles spanning a period of $\pm 1.5$ h around the central time. The set of simulated profiles on 26 January is centered at 2030 UTC, the time the flight leg was flown on that day, and at approximately 2 h after the observations, that is, at 1615 UTC 5 February. The 2-h difference in the latter case accounts for the delayed evolution of the synoptic-scale flow diagnosed in that simulation.

On 26 January, the amplitude and spatial variability of vertical velocity and potential temperature perturbations along the UWKA-level legs are well captured by the simulation. The descent of potentially warmer air in the mountain wave on the lee side of the MBM and the updraft–downdraft couplet with the attendant potential temperature anomaly farther downstream are in overall good agreement between the observations and the model output. Clearly, the observed profiles have more pronounced finescale structure, especially in the lee updraft–downdraft couplet on 26 January. We note also that the model underestimates the acceleration of the wind speed in the lee of the MBM.

On 5 February, both the aircraft data and the model results show evidence of short-wave perturbations along the entire flight track, both downstream of the MBM as well as upwind of it, where a short-wavelength wave train generated by Elk Mountain is clearly visible. Although the amplitude of these short waves is well
reproduced by the model, the horizontal wavelength appears to be less so. A large spread of the model solutions is an indication of a high degree of unsteadiness in the flow at this time. The examination of the individual model profiles shows a closer agreement between the observed and the simulated waves in the lee of Elk Mountain at the beginning of this time period, around 1630 UTC. Toward the end of this time window, instead, a closer agreement of the observed and simulated short waves is found in the lee of the MBM. As will be discussed in section 4, the high degree of unsteadiness in this case has implications for the location of the BL separation and the structure of the model-predicted rotors. Finally, we note that the model is predicting colder conditions upwind of the MBM and underestimating the horizontal wind speed on the lee side of the MBM in this case.

Despite the described shortcomings of the 5 February simulation, we consider the overall agreement between the observations and simulation results satisfactory. In the next two sections we use the simulation results, in particular those for the 26 January event for which the agreement is closer, to examine the dynamics of these two observed rotor events.

3. Analysis of model results: 26 January

a. Flow structure and boundary layer separation

Vertical cross sections of horizontal wind speed, vertical velocity, potential temperature, and subgrid-scale turbulent kinetic energy (SGS TKE) shown in Fig. 6 reveal several key features of this case. These cross sections were extracted from the inner domain along line AB, which passes through the top of the MBM with heading of 98° from north (Fig. 1). The selected time, 2145 UTC 26 January, best matches the time range of the available observations.

The four panels of Fig. 6 show a deep layer of the troposphere, between approximately 3 and 6 km MSL, flowing over the MBM and getting accelerated on the lee slope, forming a low-level jet that is confined to a much shallower layer underneath a deep stagnant region aloft. A warm potential temperature anomaly, near-neutral atmospheric stability, and nonzero values of...
SGS TKE are all associated with that deep stagnant region aloft. The almost vertical orientation of isentropes and the presence of patches of reversed flow at the height of about 6 km MSL (Fig. 6a) suggest that the acceleration of the downslope flow is occurring beneath a breaking mountain wave. Since there is no environmental critical level, wave breaking is likely the result of a large-amplitude mountain wave reaching the critical steepness (cf. section 3c). The location of the model-predicted wave breaking is consistent with what was observed as described in Part I. Another wave breaking region with strong downslope flow underneath it is evident farther downwind, over the lee slope of the Laramie Mountains.

The downstream edge of the MBM lee-slope windstorm reaches some 20 km downwind from the MBM peak, where a sudden adjustment occurs through a flow region that resembles an undular hydraulic jump, with a series of short waves on top of the jump between 3 and 4 km MSL. Underneath the first of those short waves one finds a region of reversed flow near the ground. The strong flow through these short waves appears to connect to the downslope flow on the lee slopes. Together, these features are consistent with BLS occurring at that point in time at the downstream edge of the downslope winds over the MBM slope. Downward vertical motions on the steepest stretches of the lee slope, as well as strong updraft– downdraft couplings associated with short waves farther downstream (x = 20 km), are clearly visible in Fig. 6b. The structure of this vertical velocity couplet, with near-ground reversed flow underneath it, is indicative of the presence of an atmospheric rotor. While the layer of downslope flow is stably stratified, the flow within the rotor region is almost neutral (Fig. 6c). Figure 6d shows the SGS TKE and reveals higher values of SGS TKE in the boundary layer, with a clearly pronounced maximum at the leading edge of the rotor region.

The vertical velocity component and SGS TKE at 2145 UTC 26 January are shown in Fig. 7 at a horizontal plane at 3000 m MSL, which partially intersects the topography. The horizontal wind vectors clearly show a strong acceleration of the wind in the immediate lee of MBM. They also reveal a degree of flow splitting around the MBM and a distinct wake downwind of it. Strong vertical velocity perturbations are visible downwind of the MBM terrain (Fig. 7a), including a relatively wide zone of gentle downdraft that corresponds to the downslope flow and the sharp vertical velocity couplet farther downwind associated with the rotor. The wake with reduced horizontal momentum is evident downstream of the rotor and to the north of line AB (Fig. 7a). The increased SGS TKE in the immediate lee of the mountain is most likely resulting from shear production associated with the low-level wind maximum in the downslope flow (Fig. 7b). An even more prominent SGS TKE maximum is visible farther downstream and corresponds to the leading edge of the rotor. As shown by Doyle and Durran (2002, 2007), turbulence in this zone is generated by intense shear between the detached low-level jet and the underlying, approximately stagnant, rotor region.

Evident in these cross sections is the elongated structure of the vertical velocity and SGS TKE fields in the lee of the obstacle. The longitudinal axis of these bands is nearly perpendicular to the upstream wind direction and approximately parallels the mountain ridge, only the highest portion of which is visible in this figure.
This suggests that three-dimensional aspects of the flow past MBM are only of secondary importance for the dynamic evolution of this event—although, not completely negligible, as evidenced in the lee-side vertical velocity couplet and SGS TKE being more pronounced in the lee of the elevated Northern terminal of MBM, north of line AB, downwind of which the wake forms.

In summary, our simulation results suggest that the 26 January event is marked by the presence of a large-amplitude mountain wave over the lee slope of MBM whose breaking leads to the formation of an undular hydraulic jump at low levels, at or near the mountain top, which in turn induces the BLS and rotor formation near the surface. Given the accurate prediction of the large-amplitude mountain wave, we find that the mesoscale model simulation of this event reproduces the observed dynamical evolution of the atmospheric rotor with excellent timing. This point is further elucidated in the following section.

b. Evolution of surface fields and rotor morphology

In this section we focus our attention on the time evolution of the mountain wave and rotor signatures. Figure 8 presents a set of Hovmöller diagrams, in which the horizontal axis shows the distance along line AB (Fig. 1) and the vertical axis displays time from 1200 UTC 26 January to 1200 UTC 27 January—that is, from hour 12 to hour 36 of the simulation. The MBM top lies at \( x = 0 \). Horizontal lines delimit the period during which airborne measurements were taken.

Fig. 8. Hovmöller diagrams (x–t) illustrating the evolution of atmospheric parameters on 26 Jan 2006 along line AB. (a) Section-parallel 10-m wind speed (m s\(^{-1}\)), (b) surface pressure perturbation (hPa), (c) vertical velocity (m s\(^{-1}\)), and (d) potential temperature (K). Fields in (c) and (d) are shown at \( z = 3000 \) m MSL. Hatching in these two panels indicates the intersection with the terrain. The MBM top lies at \( x = 0 \). Horizontal lines delimit the period during which airborne measurements were taken.
the topography. The pressure perturbation in Fig. 8b is computed by reducing surface pressure to the lowest terrain height along line AB and subtracting the minimum value for all points along the line. Consequently, at all times, $p^* = 0$ at the location of the pressure minima and $p^* > 0$ elsewhere.

Given the along-flow orientation of cross-section AB, the normal wind component is generally negligible (not shown). Flow acceleration is apparent in the lee of both the MBM and the Laramie Mountains (Fig. 8a). Upstream blocking is visible on the windward side of both ranges before 1800 UTC 26 January and upstream of the MBM after 0300 UTC 27 January. Stationary bands of alternating positive and negative horizontal wind speed, evident after 0300 UTC over the Laramie Valley, are signatures of trapped lee waves that form there. The signature of these resonant waves is also evident in the pressure perturbation field (Fig. 8b) and vertical velocity and potential temperature fields (Figs. 8c and 8d).

Zooming in on the lee side of the MBM, one can see that the BLS point—the downstream edge of the lee-side windstorm being its proxy—is anything but stationary before 0300 UTC 27 January. On 26 January, this edge extends as far as $x = 30$ km before 1800 UTC and retreats to $x = 15$ km between 1800 and 2100 UTC and even farther back to $x = 10$ km shortly after 2100 UTC. This movement of the BLS point agrees remarkably well with the observations presented in Part I. The most intense near-surface reverse flow occurs immediately downstream of the BLS point at 2100 UTC, in combination with the strongest updrafts and downdrafts (Fig. 8c). The absence of the downdraft in the immediate lee of the MBM at 3000 m MSL around 0000 UTC 27 January is consistent with the BLS point lying farther upstream at this time (Fig. 8a). The downdraft, and the attendant downslope wind, at 3000 m MSL set in again at 0300 UTC 27 January, in connection with the onset of lee waves. It is worthwhile to point out that the upstream shift of the BLS point in the lee coincides with a pulse of stronger wind speed on the upstream side, between 1800 UTC 26 January and 0000 UTC 27 January (Fig. 8a).

The time evolution of the near-surface winds is strongly correlated with the surface pressure perturbations and potential temperature at or near the MBM top. Strong downslope winds in the lee of the MBM persist as long as there is a significant pressure gradient across the MBM, a positive pressure anomaly upwind of the MBM and a lee-side minimum over the Laramie Valley (Fig. 8b) contributing to it. The attendant wave-induced pressure gradient force favors the flow acceleration to the west of the pressure minimum and the sharp deceleration and, eventually, the detachment of the BL to the east of it. The flow reattaches again farther downwind in the Laramie Valley. Another area of strong flow acceleration is evident in the lee of the Laramie Mountains ($x > 70$ km). The upstream shift of the BLS point is correlated with the upwind retreat of the lee-side pressure minimum in the lee of the MBM. Concurrent with that is a contraction and disappearance of a pronounced lee-side warm potential temperature anomaly, which is clearly evident at 3000 m MSL before 1800 UTC 26 January. After that time, a much colder air mass appears to move into the area, in conjunction with a strong pulse of momentum between 1800 UTC 26 January and 0000 UTC 27 January.

The results presented so far suggest that the unsteadiness in the low-level flow around 2145 UTC 26 January documented in Part I coincides with a change in the mountain-wave regime—that is, with a transition from a large-amplitude mountain wave (up to about 2100 UTC 26 January) to resonant trapped lee waves (after about 0300 UTC 27 January). The observed upstream shift of the BLS point happens as the primary wave vanishes and before steady resonant lee waves develop.

In Fig. 9 we illustrate and contrast these two wave regimes. Whereas at 2000 UTC 26 January (Fig. 9a) the lee side flow is characterized by the breaking gravity wave above the MBM lee slopes and the hydraulic jump at low levels, at 0400 UTC 27 January (Fig. 9b) the isentropes over the Laramie Valley show a regular structure of trapped lee waves. From these cross sections, it is evident that the MBM and the Laramie Mountains form a double ridge, which shapes the flow response over the Laramie Valley and on the lee side of both mountain ranges. As shown by Grubišić and Stiperski (2009), the horizontal wavelength of lee waves that form in between two ridges that are close or equal in height corresponds to one of the higher harmonics of the primary orographic wavelength, which is equal to the ridge separation distance. This process produces a lee-wave spectrum that, while dictated by the atmospheric vertical structure, is fine-tuned by the orographic spectrum and leads to an integer number of horizontal wavelengths over the valley. Although the ratio of heights of the two ridges here is closer to $\frac{1}{3}$, the same dynamics appears to be at play, causing five wave crests and troughs to form in between the two ridges at this time (Fig. 9b). Furthermore, a strongly asymmetric flow in the lee of the downstream ridge, associated with high drag, is indicative of constructive interference of lee-wave trains generated by the two obstacles (Stiperski and Grubišić 2011).

To illustrate vorticity dynamics in these two wave regimes, in Fig. 9 we show also the spanwise vorticity component, $\eta = \partial u/\partial z - \partial w/\partial x$. In both of these regimes, strong shear-generated positive (clockwise) BL
vorticity is evident. The positive vorticity sheet conforms to the terrain up to the lee-side separation point. At that location, the BL vortex sheet detaches from the ground and becomes a free vortex sheet that forms the leading edge of the rotor. In Fig. 9b, the identity of that detached BL vortex sheet can be traced through several lee-wave crests and rotors downwind. Above the boundary layer in both of these regimes, one finds negative values of vorticity embedded within the waves. Negative (counterclockwise) vorticity is generated there by strong positive buoyancy gradients across the stable layer, between the strongly accelerated downslope flow and the weaker flow above it. The magnitude of the baroclinically generated vorticity is much higher in the large-amplitude mountain wave than in the resonant lee waves. As a consequence, the former overturns and breaks, as evidenced by a patchy structure of vorticity and isentropes that curl backward over themselves (Fig. 9a), as opposed to isentropes that smoothly follow the wave pattern in the lee waves (Fig. 9b).

c. Upstream environment and downstream response

The dynamical evolution of the lee-side flow, described in the previous section, can be understood by examining the time evolution of the upstream environment. To aid in this examination, Figs. 10 and 11 display the temporal change of the wind speed and stability profiles at points p1 and p3 (Fig. 1), upstream and downstream of the MBM, respectively. Point p3 corresponds to the location where a rapid upstream motion of the rotor was detected between 2130 and 2200 UTC in the UWKA measurements (Part I).

At point p1 (Fig. 10), the rotation of the horizontal wind vector, due to the passage of a short-wave trough described in section 2b, causes the cross-section-parallel wind component to increase in strength in a deep layer between 3 and 5 km MSL between 1800 UTC 26 January and 0300 UTC 27 January. An attendant low-level jet is also evident as a pulse of stronger wind around 2100 UTC 26 January with a maximum of approximately 18 m s$^{-1}$ at about 3000 m MSL. The near-surface flow appears to react quickly to this forcing. While both before 1800 UTC 26 January and after 0300 UTC 27 January, blocking is present at p1 below 3000 m MSL owing to stable stratification in the BL, in between these times, enhanced BL shear due to the low-level jet and the downward mixing of momentum seem sufficient to erode the stable layer near the surface. Consequently, the whole upstream air column flows over the MBM.

The results reported in section 3b suggest that a large-amplitude mountain wave forms above the lee slopes of MBM and breaks there at midtropospheric altitudes between 1800 and 2100 UTC 26 January. The upstream wind was approximately parallel to line AB during this time and the cross-section-parallel wind component shown in Fig. 10a confirms that there is no mean-state critical level for gravity waves above p1. Before 1800 UTC, on the other hand, it appears as if there is a mean-state critical level at 8 km MSL with small negative values of cross-section-parallel wind between 8 and 10 km MSL. However, given that the upper- and lower-level winds at that time were, respectively, from the south and west-southwest, the zero-wind line in this cross section does not represent a critical level for gravity waves generated by the flow over the MBM at that time. Consequently, as previously hypothesized, wave breaking between 1800 and 2100 UTC 26 January must have resulted from waves reaching their critical steepness and overturning above the lee slopes.

The local nondimensional mountain height $NH/u$ shown in Fig. 10c, which is computed by keeping $H$...
constant (and equal to 1350 m, the approximate height difference between the MBM top and the surrounding plain) and by using the local values of section-parallel wind speed $u$ and stratification $N$, confirms this conjecture. Since flow past the MBM shows some three-dimensional characteristics, we assume that $NH/u$ exceeding a threshold value of unity is a good indicator that wave breaking aloft can be expected (Baines 1997). From Fig. 10c, it is evident that $NH/u$ is larger than unity throughout the air column before 1900 UTC, in fair agreement with the estimate of $NH/u \approx 1.3$ obtained from the UWKA ramp soundings (Part I). Subsequently, and starting at low levels, $NH/u$ decreases below the critical threshold because of a combined effect of increasing wind speed and decreasing stratification and leads to the cessation of midtropospheric wave breaking.

The onset of lee waves hours later is likely related to the presence of an almost neutral layer above 6000 m MSL between 2100 UTC 26 January and 0600 UTC 27 January, which appears right after the passage of the upper-tropospheric wave. In conjunction with the positive vertical shear of the wind profile (Fig. 10b), the neutral layer aloft acts to trap wave energy at lower levels, leading to the formation of lee waves.

The response of the flow downstream of the MBM peak, at point p3, is illustrated in Fig. 11. Shallow, stably stratified, strong downslope flow is apparent there until 2100 UTC 26 January. The strength of the downslope flow increases with the onset of wave breaking at this location between 1800 and 2100 UTC 26 January. Wave breaking is evidenced through stagnation, static instability, and a low gradient Richardson number,

**FIG. 10.** Hovmöller diagrams ($t$–$z$) of the temporal evolution of the upstream atmospheric structure on 26 Jan 2006 at point p1 (Fig. 1). (a) Wind component parallel to section AB (m s$^{-1}$), (b) Brunt–Väisälä frequency (s$^{-1}$), and (c) local nonlinearity parameter. Vertical lines delimit the period during which airborne measurements were taken.
$\mathrm{Ri}_x = (g/\theta)(\partial \theta / \partial z)[(\partial u / \partial z)^2 + (\partial v / \partial z)^2]$, between 4 and 6 km MSL. As wave breaking ceases, the downstream extent of the downslope flow decreases and remains confined to higher levels on the mountain flank. As a consequence, point p3, which was embedded in downslope flow until about 2100 UTC, finds itself in the rotor region later on. Low wind speeds, high turbulence, and neutral stability in the rotor region are apparent between 2100 UTC 26 January and 0300 UTC 27 January. During this period, the separated BL flow is clearly evident above the rotor.

The preceding discussion emphasizes the role that the upstream environment plays in the wave and rotor dynamics. However, recent studies point out that a significant modulation of these phenomena can be caused by changes in the atmospheric environment on the downstream side of the primary orographic obstacle. In particular, the diurnal cycle of the downstream BL, with the attendant periodic variations of the lee-side atmospheric stability, has been shown to have a dramatic impact on the penetration of downslope winds along the lee slope and on the amplitude of mountain waves (Jiang and Doyle 2008; Mayr and Armi 2010), in particular in the afternoon hours.

Given that the local sunrise and sunset in this area occur at 1415 and 0015 UTC 26 January, respectively, the most interesting part of the 26 January event fell during the afternoon hours. To examine whether the BL diurnal cycle plays a role in the 26 January event, a sensitivity test with zero surface heat fluxes was performed. The results (not shown) were not appreciably different from the control run. Both the intensity of the
downslope flow and the overall evolution of the event were substantially unchanged. Given low sensible heat fluxes (generally less than 50 W m\(^{-2}\)) on a cold winter day, this result is not surprising.

4. Analysis of model results: 5 February

As discussed in section 2b, the simulation of the 5 February event agrees less favorably with observations (Figs. 2–5). Bearing in mind discrepancies between the observations and the simulation results described in section 2b, we focus the discussion here on the overall dynamic evolution of this event.

Vertical cross sections along line CD, which is parallel to the mean flow and the cross-mountain flight legs on 5 February (Fig. 1), are shown in Fig. 12 at four different times during the course of this event. Two additional orographic obstacles are visible in this transect: Elk Mountain, located about 35 km upstream of the MBM, and Sheep Mountain, about 25 km downstream of the MBM (centered at \(x = 0\)). At the time of airborne observations at 1430 UTC 5 February (Fig. 12a), there are only minor perturbations in the flow over the three peaks. Partially blocked flow upstream of Sheep Mountain is apparent, but there is no evidence of flow reversal there or anywhere else between the MBM and Sheep Mountain. Two hours later (1630 UTC; Fig. 12b), the flow is much more perturbed, with short waves in the lee of Elk Mountain, an incipient wave overturning and the flow stagnation between 5000 and 6000 m MSL over the MBM lee slope, and another strong set of perturbations in the lee of Sheep Mountain. At this time, the model is predicting near-surface flow reversal in the lee of both the MBM and Sheep Mountain. Taking the 2-h delay in the onset of the simulated short-wave-trough passage into account, this flow should roughly correspond to what was observed. Yet, at this time, there is no evidence in the simulation results of smooth lee waves between the MBM and Sheep Mountain as reported in Part I. Furthermore, the location of the flow reversal and the simulated rotor do not appear to match the observations either; that is, the model places the rotor at about 20 km downstream of the MBM summit, instead of about 10 km as indicated by the measurements (cf. Figs. 5 and 9 in Part I).

The examination of the temporal evolution of the key atmospheric parameters along line CD, shown in Fig. 13,
provides further insight into the temporal evolution of the simulated flow. From Fig. 13, it is evident that the reversed flow at this particular location ($x \approx 20$ km) in the lee of the MBM is strongest at 1630 UTC and that the downstream edge of the downslope flow, which forms the leading edge of the rotor at 1630 UTC, shifts upwind in time and weakens between 1600 and 1800 UTC (Fig. 13a). As is the case with the January 26 event, this retreat is concurrent with the erosion of a pressure minimum (Fig. 13b) and the dissolution of an elevated positive potential temperature anomaly in the lee of the MBM after 1630 UTC (Fig. 13d). Both of these are associated with a large-amplitude mountain wave that becomes critically steep and overturns in the altitude range 4000–6000 m MSL above the MBM lee slopes between 1600 and 1800 UTC. In addition, overall cooling starts at that time as a much colder air mass moves into the area (Fig. 13d). Concurrent with the wave breaking occurring aloft above the MBM lee slopes, the flow over the valleys, both upwind and downwind of the MBM, rearranges into a series of shorter trapped lee waves (1800 UTC; Fig. 12c). These simulated short lee waves are not strong enough to induce BLS and surface flow reversal in either the lee of Elk Mountain or the MBM (Fig. 13b). Although lacking the smoothness and strength of the observed waves, these short-wave perturbations in the lee of the MBM bear more resemblance to the flow documented in Part I than the flow at the time of the aircraft observations. As noted in section 2b, the horizontal wavelength of short waves in the lee of Elk Mountain is closer to that observed at 1630 UTC, and in the lee of the MBM at 1800 UTC, although the flow in the lee of the MBM continues to be unsettled at that time. The BLS and the surface flow reversal in the lee of the MBM reemerge again around 1930 UTC on the upper MBM lee slopes, after a steady short-lee-wave pattern has set up there as well, over the valley in between the MBM and Sheep Mountain (1930 UTC; Fig. 12d). These trapped lee waves are shorter than those observed and persist throughout the remainder of
the simulation period, producing intermittent surface reversals there (Fig. 13a).

As in the 26 January case, the upwind retreat of the BLS point is associated with midtropospheric wave breaking. In this case, the timing of the wave breaking is determined by the changing characteristics of the upstream environment aloft, after approximately 1500 UTC. Those characteristics include the wind speed that decreases with height (Fig. 14a), a stable layer centered at approximately 5 km MSL, and a layer of low stability above it (Fig. 14b). These elements combined lead to the increase of local nonlinearity in the layer between 4000 and 6000 m MSL, which promotes wave breaking (Fig. 14c).

To further elucidate the differences between the observed and the simulated vertical atmospheric structure on 5 February as it pertains to conditions conducive to trapped lee-wave formation, in Fig. 15 we compare the observed and simulated vertical profiles of the wind speed and the Scorer parameter at point p4, above the Saratoga Valley, at 1430 and 1630 UTC 5 February with the profile derived from the observations. The Scorer parameter, which depends on the first derivative of potential temperature and on the second derivative of wind speed, was estimated after treating the original profiles with a low-pass filter. The observations on 5 February suggest that trapped lee waves were present downstream of the MBM between 1400 and 1500 UTC and that the ducting mechanism was related to the curvature (a minimum) of the wind profile, causing a marked decrease with height of the squared Scorer parameter \( \left( c^2 = N^2/u^2 - u^{-1}(\partial u/\partial z)^2 \right) \) above 3800 m MSL (cf. Fig. 4 in Part I). Curvature effects are indeed nonnegligible in this case and, in both the
profiles derived from the observations and the simulation, local minima of the wind speed profile correlate very well with local minima of $\ell^2$. Furthermore, several elevated layers with decreasing $\ell^2$ (and occasionally even with $\ell^2 < 0$) are apparent in the observed profile, which is known to lead to wave energy trapping (e.g., Crook 1986). Comparing the profiles derived from the model simulation with that derived from the observations in Figs. 15a and 15b, it is evident that although both the 1430 and 1630 UTC model Scorer parameter profiles show a layered structure, neither matches the one derived from the observations particularly well. Perhaps the closest match to the observed profile, at least qualitatively, is shown by the model around 1200 UTC, 2 h ahead of the time of the observations (cf. Fig. 15c).

Finally, we comment on the structure of the lee-wave field at 1930 UTC illustrated in Fig. 12d. As previously discussed, by that time the short-lee-wave train had formed on the stable layer centered at 5 km MSL over the valleys, both upwind and downwind of the MBM peak. There appears to be a positive interference or resonance of lee waves between Elk Mountain and the MBM, which, according to Stiperski and Grubišić (2011), should produce a strong asymmetric flow field in the lee of the second mountain (here MBM) and a large-amplitude wave response there. In contrast, flow downwind of Sheep Mountain is rather disorganized and isentropes flatten at times, suggesting that a negative interference occurs there.

5. Summary and conclusions

The present study reports on the numerical simulations of complex mountain-wave and rotor events observed in the lee of the Medicine Bow Mountains (MBM) in southeastern Wyoming on 26 January and 5 February 2006 and complements the description of observational results provided in Part I.

Using high-resolution numerical simulations performed with the WRF Model, we demonstrate that the documented rotor events occurred during a complex transitional phase between different flow regimes in the lee of the MBM. The transition between strong downslope winds with hydraulic jumps and trapped lee waves in the lee of the MBM occurred in both of the examined events in response to a rapid change in the upstream environment, associated with the passage of a shortwave synoptic disturbance aloft. The rotor and the attendant surface-flow reversal in the 26 January case formed underneath an undular hydraulic jump, at the downstream end of a downslope windstorm. In the 5 February case, the same type of rotor was simulated in the early stages of the event. Yet, it appears that the observations on this day were collected during the subsequent transition into the trapped lee-wave regime. In both of these events, the lee-side rotors and surface-flow reversal were associated with wave-induced boundary layer separation. The observed and simulated upwind shift of the boundary layer separation point was related to the erosion of the pressure minimum on the lee side of the MBM, in response to the evolution of the wave field aloft.

The model simulation successfully reproduced the dynamical evolution of the 26 January event and generated results that are in good agreement with the observations, both in space and in time. The simulation of the 5 February event, which evolved in a strikingly
similar manner to the 26 January event, is in poorer agreement with the observations. In part, this disagreement stems from the delay in the model simulation of the passage of the synoptic short-wave trough—a timing error that the mesoscale model likely acquired through the initial and boundary conditions from the global model analyses. Furthermore, the evolution of such a short-wave trough cannot be expected to be properly handled by linear interpolation of the 6-hourly boundary fields, the only ones available from the ECMWF analyses. The difference in the fidelity of these two simulations, in which the predicted transition between different wave regimes is related to the wave breaking aloft, is an excellent illustration of the strong sensitivity that numerical simulations of terrain-induced flows display to initial conditions (Doyle and Reynolds 2008; Reinecke and Durran 2009).

In both of these events, the simulation results indicate that the secondary orographic obstacles surrounding the Medicine Bow Mountains, including Elk Mountain, Sheep Mountain, and the Laramie Mountains, play a role in promoting the resonance of lee waves that are well known to form frequently in the flow over the complex terrain of southeastern Wyoming (Marwitz and Dawson 1984).

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