Magnetosphere Sawtooth Oscillations Induced by Ionospheric Outflow

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Abstract

The sawtooth mode of convection of the Earth’s magnetosphere is a 2-4 hour planetary-scale oscillation powered by the solar wind-magnetosphere-ionosphere (SW-M-I) interaction. Based on global simulations of geospace, we show that ionospheric O⁺ outflows can generate sawtooth oscillations. As the outflowing ions fill the inner magnetosphere, their pressure distends the nightside magnetic field. When the outflow fluence exceeds a threshold, magnetic field tension cannot confine the accumulating fluid; an O⁺-rich plasmoid is ejected, and the field dipolarizes. Below threshold, the magnetosphere undergoes quasi-steady convection. Repetition and the sawtooth period are controlled by the strength of the SW-M-I interaction, which regulates the outflow fluence.

Report

Sawtooth oscillations were first observed in the fluxes of energetic charged particles at geosynchronous orbit (1,2) and are so called because the time series of the particle fluxes resemble the teeth of a saw blade, with a 2-4 hour periodic sequence of slow decrease followed by rapid increase. Many other geophysical processes have since been shown to vary in sync with periodic particle injections: magnetic fields at geostationary orbit, in the tail and at ground stations, auroral electrojet index, auroral precipitation and polar cap index (3-7). Whilst
observational studies of sawtooth oscillations have revealed a rich phenomenology of the magnetospheric response to quasi-steady driving by the solar wind, basic understanding of why they occur is lacking. This deficiency impacts our ability to model magnetospheric dynamics accurately and to forecast space weather.

Sawtooth oscillations occur during relatively stable solar wind conditions when external triggering by variability in the interplanetary medium is absent. This behavior suggests that the response of the magnetosphere-ionosphere (MI) system is conditioned by an internal mechanism. However, the mean sawtooth period of 3 hours (8,9) is much longer than any known electrodynamic cavity oscillation of the MI system, even when extended to include the bow shock as a boundary (10). The ionospheric and magnetospheric signatures of an individual sawtooth are similar to those of isolated substorms (5), i.e., episodic conversions of magnetic to plasma energy in the Earth’s magnetotail, with two noteworthy exceptions: the signatures of sawtooth substorms are more broadly distributed in local time, with particle injections, dipolarizations and ground signatures spanning from midnight to the dawn-dusk meridian and even into the dayside; and sawtooth signatures are more intense in the geostationary region than those of isolated substorms. The sawtooth mode is also distinguished from the steady magnetospheric convection (SMC) mode, which is driven by similar though somewhat weaker upstream conditions (11-13). Energy transfer from dayside and nightside magnetic reconnection is in near balance during SMC events (14), which do not exhibit the oscillations in convection characteristic of sawtooth events. It is not known why the solar wind-magnetosphere-ionosphere interaction settles into an SMC state versus a sawtooth oscillation for similar driving conditions, or why differences in driving conditions should lead to one state or the other.
Global simulations of magnetospheric dynamics based on equations of magnetohydrodynamics (MHD) provide physical insights into system behavior that are difficult to infer from in situ satellite measurements alone. Previous global simulations have been able to capture observed features of geomagnetic storms (15,16), isolated substorms (17,18), and other events. They produce the SMC mode (19,20), the stretching of magnetic fields associated with sawtooth oscillations (21) and fast periodic reconnections of the magnetotail when a non-MHD electric field is included (22), but they fail to give ~3-hour global-scale, quasi-periodic substorms that define the sawtooth mode. This shortcoming suggests that sawtooth dynamics are governed by a physical process not included in existing global models. Here, we use global simulations to show that the outflow of heavy ions from the Earth’s ionosphere can dramatically change the convection cycle of the magnetosphere and promote sawtooth oscillations.

The magnetosphere is a variable admixture of plasmas of ionospheric and solar wind origin (23), with the ionospheric source becoming more prevalent during the strong driving conditions typical of geomagnetic storms and sawtooth events (24). Heavy ion outflows from the ionosphere are most intense in the dayside cusp region (25) and near the polar cap boundary of the nightside auroral zone (26) where shear Alfvén waves (transverse electromagnetic waves that propagate energy along magnetic field lines in an MHD fluid) deposit electromagnetic power (27,28). Alfvén wave energy can be converted to ion energy either directly when the transverse scale of the wave becomes comparable to the ion gyroradius (29) or indirectly via secondary instabilities that excite ion gyroscale waves driven by the wave field-aligned current and velocity.
shear (30). Satellite studies estimate that 95% of heavy ion upflows are produced by low-frequency broadband turbulence typical of Alfvén waves and associated instabilities (31).

We used the multi-fluid Lyon-Fedder-Mobarry (LFM) global model (32,33) to simulate the solar wind-magnetosphere-ionosphere (SW-M-I) system including the effects of O\(^+\) ionospheric outflows (Supporting Online Material). The outflow boundary condition is specified in terms of an observed statistical correlation between (AC) Alfvénic Poynting flux \(S_{||}\) and accelerated O\(^+\) outflow flux \(F_{O^+}\):

\[
F_{O^+} = 2.97 \times 10^{10} S_{||}^{1.2}
\]

(1)

with \(F_{O^+}\) given in ions/cm\(^2\)-s and \(S_{||}\) in mW/m\(^2\). Although this statistical correlation exhibits considerable variance in observations and alone does not establish a direct causal link between \(F_{O^+}\) and \(S_{||}\), it does provide an empirical means of specifying an average outflow flux at the low-altitude simulation boundary for a given Poynting flux flowing downward through the boundary.

We evaluated the impact of the outflow on global convection from an ensemble of controlled simulations. The first set of investigations explored the dependence of convection on the outflow fluence \(F_{TOT}\) (in ions/s), calculated as the integrated flux in the northern hemisphere at a radial geocentric distance of 2.5 \(R_E\). To understand the effects of variance in the empirical relationship (1), we rescaled the equation such that \(S_{||} \rightarrow \alpha S_{||}\) and considered a range of constant \(\alpha \geq 0\) (34). The second set of experiments explored the dependence of convection state on varying solar wind conditions for fixed \(\alpha = 3.8\) (Table 1).
Studies using satellite measurements find that the outflow fluence for strong driving conditions is of order $10^{26}$ ions/s (35). The simulated fluences listed in Table 1 are thus representative of observations when $\alpha$ is in the range 1-6. The strength of interplanetary driving is measured by the solar wind-magnetosphere coupling parameter $\varepsilon \equiv V_{SW}B_{IMF}\sin^2(\theta/2)P_{\text{dyn}}^{1/6}$ (36) where $V_{SW}$ is the solar wind velocity, $B_{IMF}$ is the magnitude of the interplanetary magnetic field (IMF), $\theta$ is IMF clock angle in the y-z plane (= 180° for all simulations in Table 1), and $P_{\text{dyn}} = \rho_{SW}V_{SW}^2$ is the dynamic pressure of the solar wind of mass density $\rho_{SW}$. For fixed $\alpha$ (= 3.8), Table 1 shows that the outflow fluence increases with increasing $\varepsilon$ in this model. The trend is linear: $\langle F_{\text{TOT}} \rangle = 0.6*\varepsilon + 0.2$ with $\langle F_{\text{TOT}} \rangle$ in $10^{26}$ ions/s and $\varepsilon$ in (mV/m) (nPa)$^{1/6}$.

The outflow is located throughout the simulated auroral oval with a peak in the pre-midnight sector (Fig.1). The dawn-dusk asymmetry is a consequence of the asymmetry in the precipitation-induced ionospheric conductance derived from LFM’s electron precipitation model. The nightside morphology resembles statistical maps of both Alfvénic Poynting flux (27) and O\textsuperscript+ outflow flux (28) derived from satellite data; however, these simulations do not produce the persistent O\textsuperscript+ outflow fluxes typically observed in the dayside cusp. This difference is possibly due to the steady solar wind conditions in the simulations, which do not generate significant time-variable perturbations of the dayside magnetic field and accompanying Alfvén wave power flowing into the cusp ionosphere. Other simulations using a heuristic model for cusp outflows show that their interaction with the MI system is weak because the bulk of the cusp outflow intersects the plasma sheet tailward of the nightside reconnection line (33). Thus, cusp O\textsuperscript+ outflows are not expected to change the qualitative nature of the results reported here.
The magnetosphere in the baseline simulation (no outflow, $\alpha = 0$) settles into an SMC state, with magnetic reconnection occurring in the nightside plasma sheet at a tailward distance of approximately 25 RE. Inclusion of O$^+$ outflow inflates the magnetotail and causes reconnection to migrate further tailwards. For $\alpha = 1$ (run A), the tail is more dynamic with nightside reconnection variably located between 30 and 45 RE. Despite this increased variability the system remains in an SMC state. As the outflow fluence increases in runs B-F the nightside field lines become more stretched and nightside reconnection migrates even further tailward to 50-65 RE. This stretching originates from the additional pressure of the magnetospheric O$^+$ population, which enhances both the “ballooning” pressure force and the diamagnetic ring current that shears the ambient magnetic field. Both effects distend and stress the magnetic field lines. It is not clear at this point which is dominant. For sufficiently high outflow fluence, the nightside field lines are stressed to the point of inducing plasmoid ejection. The now unbalanced magnetic field tension forces the fluid and embedded field earthward to dipolarize in the inner region. This release of stored energy is manifested as a substorm. A large fraction of the O$^+$ fluid is lost downstream in the plasmoid and to the magnetopause during the convective surge associated with the substorm. The nightside ionospheric outflow then continues to fill the inner magnetosphere, stretching the field lines once again, resulting in another substorm (Fig. S2).

We analyzed the simulation magnetic inclination angle near geostationary orbit to estimate the substorm periodicity and to compare the simulation results with sawtooth substorms observed in satellite data (Fig. 2). The baseline run 0 and run J exhibit SMC behavior while the other three cases (C,H,I) in the figure exhibit sawtooth oscillations with decreasing period and increasing amplitude of the magnetic inclination angle evident as the strength of interplanetary driving ($\varepsilon$) is
increased. Observed SMC and sawtooth states both require quasi-steady solar wind drivers, but
the magnitude of the driver is observed to be stronger for sawtooth oscillations (37), as in the
simulations. From Table 1, an outflow fluence exceeding about $1.1 \times 10^{26}$ ions/s is a necessary
condition for the onset of sawtooth oscillations in the simulations. Although the SW-M-I system
is more complicated than the simulations, the relationship between $\varepsilon$, outflow fluence, sawtooth
versus SMC state, and sawtooth period can be tested observationally.

We used the power spectral density of the time variation in magnetic inclination angle to
determine the sawtooth period, with variance in period defined as the full width at half maximum
of the spectral peak defining the period. As the outflow fluence was increased in simulations B
through F, the period decreased and tended to a value of approximately 2 hours (Fig. 3).
Simulations C-E and H produce outflow fluences in agreement with the observed estimates of
order $10^{26}$ ions/s (35). They also yield sawtooth oscillations with periods ranging from 2 to 5
hours, consistent with the 2-4 hour range derived from satellite measurements (9).

The dipolarization time in the simulations ranged from 0.5-1 hour, compared to an average of 22
minutes in observed sawteeth, with standard deviation of 15 minutes (9). Thus, for simulated
events with periods approaching two hours or less the periodic waveforms resemble nonlinear
sinusoids rather than sawteeth. The apparent minimum period in Fig. 3 may be a consequence of
the artificially long dipolarization time in the simulations. This longer dipolarization may be a
feature of numerically induced magnetic reconnection in the simulations, which does not allow
for explosive local reconnection enabled by kinetic effects or a current-driven instability (38).
Alternatively, if the $O^+$ abundance of the reconnection inflow is anomalously high, giving a
lower Alfvén speed relative to that of the actual system, then the mass-loaded inflow may also
produce slower dipolarization in the simulations.

A distinguishing feature of the sawtooth mode compared to isolated substorms is the wide extent
in magnetic local time (MLT) (39) of signatures such as field line stretching and dipolarization.
We examined this feature by comparing a superposed epoch analysis of magnetic inclination
angle for all simulated sawtooth events in Fig. 3 with periods of 1.75 to 5 hours to a superposed
epoch derived from observed sawtooth events (5) (Fig.4). The simulated and observed results
exhibit a number of striking similarities: The minimum inclination angle of the simulated
sawteeth is 27° compared to 26° of observed sawteeth with both being lower than the average
minimum inclination angle of 43° observed at geostationary orbit for isolated substorms (5). The
observed and simulated sawteeth produce more intense signatures at geostationary orbit than
isolated substorms, and both exhibit dawn-dusk asymmetry with more pronounced features in the
premidnight sector. The asymmetry in the simulated onset is caused by the asymmetry in outflow
as discussed above. The MLT extent of the simulated sawteeth is not as broad as in observations,
e.g., the 44° contour at 0.0 UT in the simulated sawteeth spans 10 hours MLT, compared to 12
hours MLT for the observed sawteeth. However, the MLT extent in the simulations is still far
wider than the average MLT extent of 2 hours for an isolated substorm (5). This profound
difference arises from internal processes, which supplant and overcome the evolution of open
field lines in the Dungey convection cycle (40). These processes intensify the ring current
through addition of adiabatically energized O⁺ of ionospheric origin, which also augments the
radial pressure force to the point that magnetic tension in the geostationary magnetosphere can
no longer contain the fluid, not even in the dawn-dusk meridian. In this sense, the convection
cycle of a sawtooth substorm resembles the Vasyliunas cycle (41) at Jupiter, wherein corotating, mass-loaded flux-tubes pinch-off to form plasmoids in the premidnight sector, then cyclically refill after rotating around the magnetosphere again. The Vasyliunas cycle at Jupiter is periodic in space; its analogue at Earth is periodic in time. Jupiter’s periodic cycle is powered by planetary rotation; in contrast, Earth’s cycle is powered by the solar wind-magnetosphere dynamo and its propensity to produce ionospheric outflows.

Whilst we have shown that ionospheric outflow in MHD simulations can produce the sawtooth convection mode, we do not know if it is the only (or the most important) mechanism.

References and Notes


34. Variations in outflow flux per unit EM power input (i.e., \( \alpha^{1.2} \)) may be due to seasonal or solar cycle influences on the source population or to variations in preconditioning of the MI system prior to initiation of a sawtooth event. In addition, the simulation model may not accurately replicate the power typically observed in the Alfvénic band (see Supporting Online Material).


38. The reconnection in the LFM code is predominantly averaging error; opposing magnetic flux enters a single cell and is averaged (annihilated) out of existence (32). The rate of reconnection is sensitive to the conditions external to the actual reconnection region. In cases where the external flow toward the reconnection site is zero, the reconnection is effectively also zero. When reconnection is driven, i.e. forced by convergent flow, the rate can be quite high. The maximum rate is constrained by a Petschek-like inflow condition to be a fraction (~ 0.1) of the Alfvén speed in the inflow.

39. For zero dipole tilt angle in these simulations, MLT is similar to geographic local time, where 1200 MLT always directly faces the sun.


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Table 1: Simulation parameters: Shaded rows represent simulations with quasi-steady convection; unshaded rows represent simulations with quasi-periodic behavior. The solar wind density and temperature are constant at 5/cm³ and 10 eV; and the Earth’s dipole tilt angle is zero. The mean fluence $\langle F_{\text{TOT}} \rangle$ was derived from a 20-hour average in each simulation.

<table>
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<th>Run</th>
<th>$V_{SW}$ km/s</th>
<th>$B_z$ nT</th>
<th>$\alpha$</th>
<th>$\varepsilon$</th>
<th>$\langle F_{\text{TOT}} \rangle$ $10^{26}$/s</th>
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<td>3.7</td>
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<tr>
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</table>

Figures

Fig. 1. Morphology of ion outflow from simulation C, averaged over one hour, starting three hours after the simulation startup period. Auroral oval, defined by Feldstein (white) (42), is superimposed. For reference, the flux has been mapped along field lines to the ionosphere using $F_{O+}/B_d = \text{constant}$.

Fig. 2. Simulated magnetic inclination angle, $\theta_i = \sin^{-1}(B_z/B)$, as a function of simulation time at 2330 MLT, $(x^2 + y^2)^{1/2} = 6.6 R_E$, and $z = 0.5 R_E$. $B_z$ and $B$ are the $z$-component and total field magnitude at the simulation measurement point. Comparisons are shown for runs 0, C, H, I and J.
Fig. 3. Relationship between outflow fluence and sawtooth period for each simulation exhibiting quasi-periodic behavior. The colors represent solar wind conditions: $V_{SW} = 400$ km/s, $B_z = -10$ nT (blue), $V_{SW} = 600$ km/s, $B_z = -10$ nT (red), $V_{SW} = 400$ km/s, $B_z = -5$ nT (black). Squares indicate simulations with $\alpha=3.8$; circles indicate simulations with different values of $\alpha$.

Fig. 4. Superposed epoch analysis of magnetic inclination angle near geostationary orbit for simulated (left) and observed (right) sawteeth (5). The magnetic inclination angle is evaluated on the circle $(x^2 + y^2)^{1/2} = 6.6$ RE in the plane $z = 0.5$ RE for simulated sawteeth and at geostationary satellites for observed sawteeth. For each sawtooth, dipolarization onset (fiducial time 0 UT) is visually inferred as the last minimum before the rapid increase in magnetic inclination angle in the 2330 - 0030 MLT sector (5).
Magnetic Inclination, degrees

Time, hours

0 Baseline G –10 nT, 200 km/s C –10 nT, 400 km/s H –10 nT, 600 km/s I –5 nT, 400 km/s
Supporting Online Material for
Magnetosphere Sawtooth Oscillations Induced by Ionospheric Outflow

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This PDF file includes:

SOM Text
Figs. S1 to S2
**Lyon-Fedder-Mobarry Model**

In this study we use a multi-fluid extension of the Lyon-Fedder-Mobarry (LFM) global simulation model (32) to examine the effects of transversely accelerated O\(^+\) outflow on the solar wind-magnetosphere-ionosphere (SW-M-I) system. The LFM global simulation uses a finite volume method on a nonorthogonal grid to solve the MHD equations describing evolution of mass, momentum and energy for each ion species and magnetic flux. The multifluid version of the model (33) is based on two simplifying assumptions that are common to treatments of one-fluid plasmas: 1) a Maxwellian velocity distribution for each ion species in the frame moving with the species-dependent macroscopic velocity, \(V_\text{s} = V_{||\text{s}} + V_{\perp\text{s}},\) (|| and \(\perp\) indicate parallel and perpendicular to the local magnetic field and s denotes species); and 2) a \(V_{\perp\text{s}}\) that deviates only slightly from the electrical drift velocity, \(E \times B/|B|^2,\) which to lowest order is the common perpendicular velocity of all species. The accelerations of individual ion species parallel to \(B\) are coupled via the ambipolar electric field determined by the parallel gradient in electron pressure.

In the present study we treat the electrons as a cold neutralizing fluid at zero temperature, which removes the ambipolar electric field. As discussed in (33), inclusion of finite electron temperature is expected to have little impact on the simulation results since the outflow mainly affects the plasma sheet and inner magnetosphere which are largely governed by perpendicular forces. The computational grid of the simulation domain spans from 25 \(R_E\) upstream of earth to 300 \(R_E\) downstream along the \(x\)-axis (sun-earth line) with a 50 \(R_E\) radial extent in the \(y\) (dusk-dawn) and \(z\) (north-south) plane. The SW-M-I interaction is simulated by 1) imposing solar wind and interplanetary magnetic field (IMF) boundary conditions at the sunward boundary of the computational domain, 2) introducing a point dipole to represent the earth’s magnetic field, and 3) incorporating ionospheric electrodynamics, including a proxy for electron precipitation, at the inner (low-altitude) boundary (43). The inner boundary is a sphere at a geocentric radial distance of 2 \(R_E\) where the electric drift velocity derived from electrostatic coupling to the ionosphere is imposed as a boundary condition on the fluid velocity (43). The magnetosphere for each simulation was conditioned with a startup period with two hours of northward IMF (\(B_z = 5\) nT), followed by 2 hours of southward IMF at the strength of \(B_z\) used for each simulation. The
fiducial start time of 0 hours (e.g., Fig. 2) begins at the end of the 2 hour southward-IMF conditioning period.

**Ion Outflow Model**

The mechanisms that accelerate heavy ions to energies sufficient to escape gravity and outflow into the magnetosphere are poorly understood. The low-altitude motion of light ions resembles that of beads on a wire with the magnetic lines of force serving as the wires. Acceleration of the light ions is mainly due to magnetic field-aligned pressure gradients and leads to the well-known polar wind \( (44) \). The acceleration of heavy ions, such as O\(^+\), is more complex. The large gyroradius of an O\(^+\) ion makes it susceptible to transverse acceleration by turbulent electric fields, which facilitates upward lifting by the magnetic mirror force. The mirror force acts along the background magnetic field and is proportional to the ion perpendicular energy \( (45) \). As the perpendicular energy of the heavy ions increases via turbulent acceleration, the outward mirror force acting on them also increases. When it exceeds the gravitational force, the ions freely outflow into the magnetosphere. The kinetics of heavy-ion energization are multivariate \( (46, 47) \), and a general transport law relating outflow flux to its energy drivers has yet to be determined.

Lacking an established transport law, we have used in situ measurements from the FAST satellite to derive an empirical scaling relation between the average outflowing O\(^+\) number flux and the average earthward-flowing electromagnetic energy flux at ultralow frequencies (Alfvénic Poynting flux). Broadband low-frequency turbulence evidently produces 95% of observed O\(^+\) outflows \( (31) \). The empirical relationship is based on measurements recorded along the same satellite orbits near 4000-km altitude that were analyzed previously in \( (48) \) to derive relationships between electron precipitation, quasistatic Poynting flux and ion outflow flux. Quasistatic Poynting flux is distinguished from Alfvénic Poynting flux by the field impedance, which is proportional to the ratio of electric to magnetic field perturbations, both perpendicular to the ambient magnetic field and to each other. This ratio is equal \( (\mu_0 \Sigma_P)^{-1} \) for quasistatic fields and to the Alfvén phase speed \( (v_A) \) for one-fluid Alfvén waves \( (49) \); \( \mu_0 \) is the permeability of free space, and \( \Sigma_P \) is the height-integrated ionospheric Pedersen conductivity (in Siemen). To calculate the Alfvénic Poynting flux from measured fields, the cross product of the electric (in W/m\(^2\)) and magnetic (in nT) fields is formed, after both have been pass-band filtered in the range 0.125 to 0.5 Hz. The net magnetic field-aligned Poynting flux (downward minus upward) and
outflow flux are then averaged over outflow regions in 31 satellite orbits to give the scatter diagram shown in Figure S1. The straight line shows the least squares regression line. The lower cutoff (0.125 Hz) of the pass-band separates Alfvénic from quasi-static Poynting flux. The upper cutoff (0.5 Hz) has little effect on the trend in the data because most of the power resides at low frequencies, e.g., increasing the upper cutoff to 4 Hz yields essentially the same regression within the error bars.

Using a 95% confidence limit, the corresponding scaling relation between the number flux $F_{O^+}$ of outflowing ions and the downward-flowing, Alfvénic Poynting flux $S_\parallel$ is

$$F_{O^+} = 2.97 \times 10^{10 \pm 0.83} S_\parallel^{1.21 \pm 0.35}$$

(1)

with $F_{O^+}$ in ions/cm$^2$-s and $S_\parallel$ in mW/m$^2$, respectively. The field-aligned Poynting flux may be represented as

$$S_\parallel = \pm \delta E \times \delta B \cdot B_m/|B_m|\mu_0$$

(2)

in terms of the mean field ($B_m$), and pass-band filtered electric ($\delta E$) and magnetic ($\delta B$) fields. Positive (negative) $S_\parallel$ corresponds to downward Poynting flux in the northern (southern) hemisphere.

The data in Fig. S1 show that the bandpass-filtered Poynting flux is well correlated with the outflow flux, especially at higher values; however, large scatter about the regression line is also evident. The causes of this scatter are not known. Variables other than Poynting flux, e.g., the ion density distribution along the magnetic field, may influence the outflow rate for a given electromagnetic power input, and such variables may vary between measurements made at different points in space and time. Variable conversion of Poynting flux to electron energy flux above the measurement altitude may also introduce scatter in the relationship. We recognize that correlation does not necessarily imply causality. However, to the extent that the observed trend between bandpass-filtered Poynting flux and outflow flux is captured by the regression curve, scaling relation (1) may be used to specify the outflow flux for a given Poynting flux. Given the statistical uncertainties in relation (1) and in the correspondence between satellite measurements and MHD simulations of Alfvénic Poynting flux as discussed below, a variable scaling factor $\alpha \geq 0$ is introduced so that the effects of variations in scaling, $S_\parallel \rightarrow \alpha S_\parallel$, can be explored in the simulations.
The procedure for using (1) and (2) to introduce ionospheric O\(^+\) outflows in the multifluid simulations is as follows. The electric and magnetic fields are extracted from the simulated MHD fields on the grid surface closest to 3 \(R_E\) geocentric distance in the simulation domain, sufficiently far away from the low-altitude simulation boundary so as not to be corrupted by artificial boundary effects. The electric field in the simulation is calculated as \(E = -V \times B\), where \(V\) is the MHD fluid velocity. A 180-sec running average of \(E\) and \(B\) is calculated at each grid cell on the (3 \(R_E\)) fiducial surface. The fluctuating fields (\(\delta E, \delta B\)) in (2) are obtained by subtracting the mean fields from the instantaneous values of \(E\) and \(B\), stored at 5-s intervals on the fiducial surface. Mean \(B_m\) in (2) is taken to be the 180-s running average of \(B\). The 180-s averaging time separates Alfvénic from quasistatic Poynting flux in the simulation results, which are evaluated at higher altitudes than the FAST results in Fig. S1 (see discussion below). A 180-s averaging time was also found to differentiate Alfvénic from quasistatic Poynting flux in Polar-satellite measurements (27) recorded at geocentric altitudes of 4.9 – 6.9 \(R_E\). As in (27), the effects of standing waves in the simulated Poynting flux are eliminated by averaging \(S_\parallel\) over 1-minute intervals. If we assume Poynting flux is minimally dissipated or diverted as it flows from the fiducial surface to the low-altitude energization region, then \(S_\parallel\) may be projected along magnetic field lines (using \(S_\parallel/|B_m| = \text{constant}\)) to 4000-km altitude where equation (1) was derived. Since the mean magnetic field at low altitude is essentially dipolar, projection of \(S_\parallel\) is implemented for a dipole magnetic field with \(|B_m| = B_d\), the field of an earth-centered point dipole with moment 31,000 nT. With \(F_{O^+}\) now known at 4000-km altitude, the \(O^+\) number flux is then projected upwards along dipole field lines to the 2-\(R_E\) (geocentric) boundary of the simulation domain using \(F_{O^+/B_d} = \text{constant}\). The boundary condition of the simulation requires specification of the number density \((n)\), magnetic field-aligned velocity \((V_\parallel)\) and temperature of the \(O^+\) outflow (33). The outflow velocity and temperature are taken to be constants, 40 km/s and 100 eV, respectively, which typically vary less in observed outflows than the number density (50). The density is calculated from (1) as \(n = F_{O^+}/V_\parallel\).

The relationship between the 180-s averaging time and the 0.125 Hz pass-band cutoff separating the Alfvénic band in the Polar (27) and FAST data, respectively, may be understood by considering how transverse structure in Alfvénic fields influences satellite measurements at two different altitudes. If the time variations in low-frequency Alfvénic fields recorded at Polar and
FAST are due entirely to satellite Doppler shift of transverse spatial variations propagated along magnetic flux tubes by Alfvén waves, then the wave periods measured at Polar would be $\approx 20\times$ longer than at FAST, a consequence of Polar’s slower speed at high altitude and the divergence of the magnetic flux tube area sampled at Polar altitudes. The pass-band range used to derive the Poynting fluxes in Fig. S1 would then be lower in frequency at Polar by $\approx 1/20$. Thus when the Alfvén wave frequency is dominated by Doppler shift, the 0.125 Hz cutoff at FAST altitudes corresponds to a period of 160 s at Polar altitudes, which is essentially to the 180-s averaging time used in (27) to discriminate Alfvén wave fields from quasistatic fields.

The small-scale transverse structure responsible for satellite Doppler shifts in observed Alfvénic fluctuations is subgrid in the MHD simulations. The change in the Alfvén wave frequency caused by wave dispersion associated with the small-scale structure (49, pp. 80-83) is negligible except at very small scales, corresponding to higher frequency Doppler shifts where little wave power resides. In the dispersionless limit the intrinsic frequency (not Doppler-shifted) above which the inductive (Alfvénic) field dominates the quasistatic field is determined by the condition $f > (h \mu_0 \Sigma_p)^{-1}$ (51) where $h$ is the height above the ionospheric conducting layer. At the fiducial grid surface in the MHD simulations ($h = 2 \text{ R}_E$ corresponding to the 3 R_E geocentric radial distance used to calculate the Alfvénic power) and for a characteristic Pedersen conductance of $\Sigma_p \approx 10 \text{ S}$, the low-frequency cutoff of the Alfvén band is thus $f \approx 1/160 \text{ s}^{-1}$, which is essentially the inverse averaging time (180 s) used in the simulations. Since the bandpass-filtered Poynting flux extracted from the MHD simulations does not contain power at transverse scales smaller than a cell size (approximately 0.2 R_E at 3 R_E geocentric), the simulated Poynting flux in eqn. (1) will likely be less than the observed value. The variable scale factor $\alpha$ introduced above can be interpreted as a compensating factor.
Fig. S1. Correlation between ion outflow flux and Alfvénic Poynting flux. Each point corresponds to a single orbit average.
Fig. S2. Sequence showing the sawtooth mechanism for one oscillation from run H. Background color is log$_{10}$ of total number density with magnetic field lines superimposed in red, taken in noon-midnight plane. Field lines are traced from fixed positions along the x-axis starting at x=-5 R$_E$ with spacing of 2 R$_E$ and along the line (x=-5 R$_E$, y=0) starting at z=±5 R$_E$ in 2 R$_E$ increments. Plots show initial dipolar state (top left), stretched field lines caused by influence of ion outflow (top right), release of plasmoid (bottom left) and return to dipolar configuration (bottom right).