The Polychromatic Polarization Modulator

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ABSTRACT

An increasing number of astronomical applications depend on the measurement of polarized light. For example, our knowledge of solar magnetism relies heavily on our ability to measure and interpret polarization signatures introduced by magnetic field. Many new instruments have consequently focused considerable attention on polarimetry. For solar applications, spectro-polarimeters in particular are often designed to observe the solar atmosphere in multiple spectral lines simultaneously, thus requiring that the polarization modulator employed is efficient at all wavelengths of interest. We present designs of polarization modulators that exhibit near-optimal modulation characteristics over broad spectral ranges. Our design process employs a computer code to optimize the efficiency of the modulator at specified wavelengths. We will present several examples of modulator designs based on rotating stacks of Quartz waveplates and Ferroelectric Liquid Crystals (FLCs). An FLC-based modulator of this design was recently deployed for the ProMag instrument at the Evans Solar Facility of NSO/SP. We show that this modulator behaves according to its design.

Keywords: Polarimetry

1. INTRODUCTION

Our knowledge of solar magnetism relies heavily on our ability to detect and interpret the polarization signatures of magnetic fields in solar spectral lines. The Zeeman effect was first used to identify sunspots as regions of strong magnetism.1 Since then, the number of physical mechanisms applied to the interpretation of solar spectral line profiles has increased significantly. The Hanle effect observed in linear polarization has been exploited to diagnose weak turbulent magnetic fields.2–4 A rich spectrum of linear polarization observed near the solar limb named the “second solar spectrum”5 has been used to constrain magnetic fields and scattering physics. Observations of lines like Mn which display hyperfine splitting have been used to break the degeneracy between magnetic flux and field for weak magnetic fields.6–8 Information about plasma kinetics during solar flares is encoded in the linear polarization of H lines through impact polarization.9

Polarimeters are finding their way into other fields of astrophysics as well. Zeeman-Doppler imaging allows for disentangling magnetic field distribution on the surface a rotating star.10 A polarimeter was recently installed on the HARPS instrument for planet finding.11

There is now a strong drive toward instruments that observe over broad spectral ranges. In the context of understanding magnetism in the solar atmosphere, observations of spectral lines formed at different heights are needed to constrain the field geometry in three dimensions, and simultaneous observation of multiple spectral lines greatly enhances the diagnostic potential of Zeeman-effect observations through application of a line-ratio technique.12 Improving technologies (e.g., IR detector arrays) have made it possible to take advantage of the increased sensitivity of the Zeeman effect with wavelength through the observation of infrared spectral lines.13,14

It follows that the next generation of polarimeters must have the capability to observe in a variety of spectral lines over a wide wavelength range, coupled with the ability to observe several lines simultaneously. This is reflected in the design of the recently completed Spectro-Polarimeter for InfraRed and Optical Regions (SPINOR,15), installed at the Dunn Solar Telescope (DST) of the National Solar Observatory on Sacramento Peak (NSO/SP, Sunspot, NM), that can observe between 430 and 1600 nm. The instruments planned for the Advanced Technology Solar Telescope and the European Solar Telescope will span from the UV to the near-IR.16,17

One immediate instrument requirement stemming from this need for wavelength diversity is that the polarization modulation scheme must be efficient at all wavelengths of interest. Typically, one attempts to achieve this goal by achromatizing the polarimetric response of a modulator. Here, instead, we present a new paradigm for the design of efficient polarization
modulators that are not achromatic in the above sense, since they have polarimetric properties that vary with wavelength. However, they do so in such a way that they can be operated over a wide wavelength range with near optimal polarimetric efficiency. We refer to these modulators as polychromatic.

2. THEORY

The complete polarization state of light is described by the Stokes vector,

$$\vec{S} \equiv (I, Q, U, V)^T,$$  \hspace{1cm} (1)

where \(I\) is the intensity, \(Q\) and \(U\) are the net linear polarizations measured in two coordinate frames that are rotated by 45° with respect to each other, and \(V\) is the net circular polarization.

Since the Stokes vector contains four unknown parameters, any polarimeter must make a minimum of four measurements to determine it. Most polarimeters operate by employing retarders and polarizers in a configuration that encodes polarization information into a modulated intensity signal. This intensity modulation is then measured with a detector and analyzed to infer the input Stokes vector. An extensive treatment of the theoretical operation of Stokes polarimeters has been given by Collados\textsuperscript{18} and del Toro Iniesta and Collados\textsuperscript{19}. This work draws extensively from that formalism. Here, we summarize the most important points.

The polarization properties of a polarimeter in any state can be described by a 4×4 Mueller matrix. The first row of the Mueller matrix describes how the component transforms an input Stokes vector into an output intensity. So, in the typical case of an intensity-sensitive detector, only this first row of the polarimeter Mueller matrix (the modulation vector) is important. The modulation of intensity by a \(n\)-state polarimeter can then be captured in the \(n \times 4\) modulation matrix, \(\mathbf{M}\), which consists of the modulation vectors of the polarimeter in each of its \(n\) states. The modulation matrix is generally normalized by the \((1, 1)\) element.

The operation of a polarimeter can then be represented as

$$\vec{I} = \mathbf{M}\vec{S},$$  \hspace{1cm} (2)

and the input Stokes vector \(\vec{S}\) is obtained from the measured intensities \(\vec{I}\) by

$$\vec{S} = \mathbf{D}\vec{I},$$  \hspace{1cm} (3)

where \(\mathbf{D}\) is the demodulation matrix. For a given modulation matrix \(\mathbf{M}\), the optimal demodulation matrix is given by\textsuperscript{19} \(\mathbf{D} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\). The polarimetric efficiency of a modulation scheme can be derived from the demodulation matrix,\textsuperscript{18}

$$\epsilon_i = \left( \frac{n}{\sum_{j=1}^{n} D_{ij}^2} \right)^{-1/2},$$  \hspace{1cm} (4)

where the subscript \(i\) varies over the four elements of the Stokes vector. The polarimetric efficiency quantifies the noise propagation through the demodulation process. The noise in the \(i\)-th element of the inferred Stokes vector, \(\sigma_i\), depends directly on the efficiency,\textsuperscript{19}

$$\sigma_i = \frac{1}{\sqrt{n}} \frac{\sigma_f}{\epsilon_i},$$  \hspace{1cm} (5)

where \(\sigma_f\) is the uncertainty on the measured intensity. Thus, the efficiency is the most important measure of the performance of a modulator. Equation 5 implies that in order to measure a Stokes vector with high precision, a modulator with high efficiencies is desirable.

The efficiency on any Stokes parameter can be between 0 and 1, subject to the two independent constraints \(\epsilon_Q^2 + \epsilon_U^2 + \epsilon_U^2 \leq 1\), and \(\epsilon_V^2 \leq 1\). Many so-called magnetographs are designed to modulate only Stokes \(V\) with high efficiency. In the case of a modulation scheme that is balanced in the sense that the efficiencies of Stokes \(Q\), \(U\), and \(V\) are equal, the maximum (i.e., optimal) modulation efficiency for Stokes \(I\) is 1, and for Stokes \(Q\), \(U\), and \(V\), is \(1/\sqrt{3} \approx 57.7\%\).
3. THE POLYCHROMATIC MODULATOR WITH NEAR-OPTIMAL EFFICIENCY

To create polarimeters that operate over a large wavelength range, instrument developers have typically started their design from a canonical monochromatic solution, and then tried to select optical materials in order to make each element of the polarimeter achromatic. This, for instance, is the rationale behind the design of super-achromatic waveplates.\textsuperscript{20,21} As a result, the polarimeter modulation matrix becomes largely independent of wavelength. The success of such an approach depends on the choice of materials and on the desired wavelength coverage. In principle, this allows the user to apply a single demodulation scheme to infer the Stokes vector at any wavelength. In practice, some residual wavelength dependence remains and the application of a single demodulation scheme can only give a zeroth-order approximation to the true demodulation. Therefore, one still needs to perform a careful wavelength calibration of the polarimeter in order to infer the true Stokes vector within the requirements of polarimetric sensitivity dictated by the science. Previous work\textsuperscript{22,23} has also attempted to achromatize modulators by adding combinations of fixed and variable retarders, also with the goal of minimizing the wavelength dependence of the resulting Mueller matrix.

The preservation of a given form of the polarimeter’s Mueller matrix with wavelength is a very limiting, and arguably unnecessary constraint. The fundamental driver in the design of a polarization modulator for multi-line applications is the achievement of near-optimal modulation efficiencies in all Stokes parameters and at all wavelengths of interest. The modulation matrix of such a modulator can be completely arbitrary, and even strongly dependent on wavelength. Since calibration with wavelength is required, such wavelength dependence will not impact the precision of the polarimetric measurements. In addition, since the demodulation matrix can be calculated theoretically from the modulator design, it is straightforward to provide zero-order approximations of the measured Stokes vector at all wavelengths of operation.

We propose a new paradigm where polarization modulators are designed to satisfy the constraint of having optimal polarimetric efficiency at all wavelengths of interest. This generally results in polarimeters that have modulation and demodulation matrices that are strong functions of wavelength. We refer to these modulators as polychromatic.

We have developed a technique for optimizing polarization modulators using combinations of fixed and variable retarders. We employ a computer code that searches the parameter space (i.e., retardance and orientation of the optical components that comprise the modulator) for a solution that maximizes the Stokes modulation efficiencies over the desired spectral range. Using this code, we are able to find realizable configurations with a high degree of wavelength diversity. Below, we illustrate this concept with two types of polychromatic modulators for polarimetric applications based on Ferroelectric Liquid Crystals (FLCs) and a rotating stack of retarders. We have also designed polychromatic modulators using Liquid Crystal Variable Retarders (LCVRs), but for the sake of brevity we omit examples here.

As a first example, we start with the FLC modulator used by the Diffraction Limited SpectroPolarimeter (DLSP) of NSO/SP. The DLSP was designed to operate at 630 nm only. We define the region of acceptable performance of a polarimeter as the region over which the efficiency is greater than the optimal efficiency divided by \(\sqrt{2}\) in all Stokes parameters. Figure 1 (continuous curve) shows that the DLSP, as configured, has optimal and balanced efficiency at 630 nm with a spectral range of acceptable performance of 128 nm. We then optimized the efficiency of the DLSP modulator between 500 and 900 nm, by allowing the orientation angles of the FLCs to vary. The resulting solution (dotted curve) is a simple modification of the DLSP configuration, where the first FLC is rotated such that the new switching angles are 67.5° and 112.5°. With this new configuration, the modulator still provides optimal and balanced efficiency at 630 nm, however the usable range is now twice as large as before (259 nm). Next, we added a fixed retarder between the two FLCs, and allowed the orientations of the FLCs and the retardance and orientation of the fixed retarder to vary. The resulting solution (dashed curve) yields a polarimeter with optimal and balanced efficiency at 630 nm, but the usable range now spans 553 nm. It must be noted that one can obtain even larger usable ranges with this type of modulator, when the optimization is extended to all the retarding devices. A clear example of this (even if still subject to some external constraints) is the ProMag modulator illustrated in Sect. 4.

The design of the polarimeter for the Yunnan Solar Telescope\textsuperscript{25} involved a similar optimization of the efficiency over a broad wavelength range by adjusting the orientation of the fast axes of the retarding elements. Their modulator, in our opinion, is polychromatic in nature. However, they did not optimize the retardances of the elements in their polarimeter, nor did they adopt an optimal demodulation scheme. They also argue for the importance of minimizing crosstalk, which is not necessary to achieve an efficient polarimeter.

The second type of modulator is based on a stack of retarders glued in a fixed set of relative orientations, that is then rotated as a whole to the required set of positions to perform the measurement of the Stokes vector. We consider a...
Figure 1. Modulation efficiency curves for the four Stokes parameters between 400 and 1100 nm for modulators with two FLCs. Continuous curve: optimal and balanced solution at 630 nm (indicated by the central arrow) corresponding to the DLSP modulator. Dotted curve: polychromatic solution corresponding to a simple modification of the DLSP modulator, where the first FLC is rotated by an additional angle of 67.5°. Dashed curve: polychromatic solution obtained from the former solution through the addition of a fixed quartz retarder between the two FLCs. This solution was optimized between 500 and 900 nm (indicated by the two outmost arrows). The horizontal solid lines in the four panels indicate the maximum theoretical modulation efficiencies that can be achieved simultaneously for the four Stokes parameters (1 for I, and 1/√3 for Q, U, and V).

Figure 2. As Fig. 1, but now for a modulator consisting of a stack of three quartz retarders rotated over 8 discrete steps of 22.5 degrees. The modulator was optimized between 380 and 1600 nm (indicated by the two arrows).
stepping modulator, rather than a continuously rotating one. The adoption of a continuous integration scheme results in a small reduction of the modulation efficiency. For this type of rotating modulator, we consider a common scheme with 8 measurements at positions $k\pi/8$, with $k = 0, \ldots, 7$. The example in Fig. 2 shows the modulation efficiency of a stack of three Quartz retarders.

4. THE PROMAG POLychromatic MODULATOR

The polarimeter of the HAO Prominence Magnetometer (ProMag,\textsuperscript{27}) was re-designed using the methods described above. This instrument was conceived to observe solar prominences and filaments in the spectral lines of He I at 587.6 and 1083.0 nm, and also in Hα (656.3 nm). The re-design was prompted by a series of failed attempts at fabricating the ProMag modulator following the original design, which was based on a stack of six FLCs.\textsuperscript{27} The new design utilizes two FLCs followed by a fixed retarder, constrained to use one of the FLCs from the original design. The second FLC and the fixed quartz retarder were obtained according to the optimized solution. The retardances of all devices were measured in the laboratory. Since the measured retardances differed somewhat from the specified ones, a second optimization was performed using their measured retardances and varying only the orientations of the devices. Figure 3 shows the predicted efficiencies for the Stokes parameters resulting from the optimization.

The modulator was then constructed according to the design resulting from the second optimization. Actual efficiencies at 587.6 and 1083.0 nm were measured after deployment of the instrument at the Evans Solar Facility of NSO/SP, and they are in good agreement with the theory (see crosses in Fig. 3). The discrepancy observed at 1083.0 nm is likely caused by a less precise determination of the retardances of the modulator optics at that wavelength, possibly due to an undetected leak in the interference filter used during the measurement in the laboratory, combined with the different spectral responses of the ProMag NIR camera and the photo-diode used in the lab measurement.

The theoretical modulation matrix, $M$, is shown as a function of wavelength in Fig. 4. The first column is unity and therefore is omitted. Measured modulation amplitudes at 587.6 and 1083.0 nm are also shown to be in close agreement with the theory.

5. A RECONFIGURABLE MODULATOR

Specific science applications of polarimeters may require giving a higher priority to selected spectral lines and/or Stokes polarization parameters. For this reason, in our optimization code both the optimizing wavelengths and the four Stokes parameters can be attributed different weights in the search of a high-efficiency modulator configuration. This feature can be used to direct the optimization towards a specific solution. We can also use this feature to search for modulators that can be reconfigured to favor some Stokes parameters over others. For example, a modulator may be designed that can operate as

![Figure 3. Theoretical efficiency curves for the ProMag modulator, consisting of two FLCs followed by a quartz retarder. The modulator was optimized at 587.6, 656.3, 769.9, and 1083.0 nm (indicated by arrows). The crosses indicate the measured efficiencies at 587.6 and 1083.0 nm after deployment of the instrument.](image-url)
Figure 4. Theoretical modulation matrix of ProMag. The crosses indicate the measured modulation amplitudes at 587.6 and 1083.0 nm. The first column of the matrix is identically equal to 1 and is not shown.

Figure 5. As Fig. 1, but now for a modulator consisting of two stacks of two quartz retarders, rotate over 8 discrete steps of 45 and 67.5 degrees, respectively. The modulator was optimized between 500 and 700 nm (indicated by the two arrows). By changing the phase of the two rotation stages, the modulator may be configured for balanced operation (continuous curve), as a longitudinal magnetograph (dotted curve), or with emphasis on linear polarization (dashed curve).

an efficient longitudinal magnetograph (i.e., optimized for Stokes V) as well as a linear or full-Stokes polarimeter. Figure 5 shows such a modulator, consisting of two stacks of two quartz retarders each. The stacks are independently rotated in 8 discrete steps over 360 and 540 degrees, respectively. By changing the phase of the rotation of the second stack, the modulator may be tuned for efficient modulation in Stokes V only, in Stokes Q and U, or in all Stokes parameters. Though the optimization was carried out between 500 and 700 nm, the balanced scheme remains highly efficient beyond 900 nm.

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REFERENCES


