Response Time Characteristics of the Fast-2D Optical Array Probe Detector Board

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(Manuscript received 17 March 2016, in final form 14 September 2016)

ABSTRACT

Two-dimensional optical array probes are commonly used for imaging raindrops and ice particles on research aircraft. The ability of these probes to accurately measure particle concentration and size partly depends on the response characteristics of the detection system. If the response characteristics are too slow, then small particles are less likely to be detected and the associated effective sample volume decreases. In an effort to better understand the sample volumes of optical array probes at the National Center for Atmospheric Research, the temporal response of the Fast-2D optical array probe detector board from optical input on the detector to digitization was characterized. The analysis suggests that the board electronics have a response time constant consistently near 50 ns. However, there is also a slow decay term that conforms to a \(1/\sqrt{t}\) decay rate. The amplitude of this slow function can impact the probe response, varying the minimum detectable pulse width between 60 and 150 ns. Also, the amplitude of the slow function is largely dictated by the illumination angle of incidence. The effects of the response time characteristics are analyzed using a simulator for a 2D cloud (2D-C) probe with 25-\(\mu\)m photodiode spacing. The results show the greatest sensitivity to response time characteristics when particles are smaller than 150 \(\mu\)m, where 10% uncertainty in the slow fraction is likely to produce sample volume uncertainties near 10%. Ignoring response time effects may bias sample volume estimates in the small size regime by as much as 25%.

1. Introduction

Optical array probes (OAP) mounted on research aircraft are used to characterize the shape, size, and concentration of large cloud particles during flight. Such probes use a 1D linear array to capture the occultation of a laser beam by cloud particles (Knollenberg 1981). Typical probe models process each detector in the array to report a shaded (digital zero) or not-shaded (digital 1) state based on a predefined detector threshold. The digital state of each detector channel is latched in at regular intervals and a 2D 1-bit image of the particle’s shadow is recorded. An example of such 2D cloud (2D-C) images captured by a probe with 25-\(\mu\)m resolution during the High-Performance Instrumented Airborne Platform for Environmental Research (HIAPER) Pole-to-Pole Observations 2 (HIPPO-2) is shown in Fig. 1 (Earth Observing Laboratory 2011).

This observational technique has the benefit of recording data independent of particle shape, but the effective sample volume of the probe depends on the particle size. Small particles have a smaller effective sample volume than that dictated by the probe arm separation. This can be beneficial because it allows the probe to operate over a large concentration regime. The probe is most sensitive to larger particles, which are typically found in low concentration, and much less sensitive to small particles, which are typically found in high concentration. Conversely, this fact requires that one accurately know the particle size and corresponding sample volume. Thus, the size-dependent sample volume of OAPs can represent a source of error in size-resolved concentration estimates.

The size-dependent sample volume of an OAP depends at least partly on the physical characteristics of the probes. This includes the detector response time characteristics, which were the subject of analysis by Baumgardner and...
Korolev (1997) and Jensen and Granek (2002). Those works focused on the effect of a 0.4-μs time constant on the accuracy of the Particle Measuring Systems (PMS) 260X probe, but they do not detail whether or how the response characteristic is determined. In Lawson et al. (2006), a response time constant is reported for the SPEC 2D stereo (2D-S) probe, which is measured using a high-bandwidth laser, but again it is not clear whether the assumed exponential waveform is verified. Finally, in Strapp et al. (2001) the PMS OAP-2DC response characteristics were measured, including a direct capture of the response waveform published in the manuscript. The assumed exponential function was shown to reflect the actual data after correcting for the modulation signal’s limited bandwidth. These results were obtained for much slower detectors than what we currently employ in our instruments, and factors that we find to be significant may have been negligible in systems with slower response.

At NCAR, the use of these probes on the NSF Gulfstream V (GV), which operates at high airspeeds (greater than 200 m s$^{-1}$), makes the temporal response characteristics of our OAPs increasingly important for understanding their sample volumes. There is a need to characterize the detector response to better determine the sample volume and to understand realistic instrument capability. In particular, particle size and concentration measurements with a radius in the 20–100-μm (drizzle drop) size range have been scientifically desirable but difficult to measure with OAPs. Drops in this size range are believed to be important due to their potential effect on the lifetime of warm shallow clouds (Albrecht 1989). In this small size regime, the probes are increasingly sensitive to the physical characteristics of the probe. In an effort to better understand our OAP performance, we have developed a method for characterizing the response time of the detector system, starting with an optical input on the detector and measuring the corresponding digital output. In this way, the complete detection system is included in the analysis, and we are able to produce estimates of the detector system impulse response for inclusion in our OAP instrument models.

In this work we detail the method for acquiring OAP detector response data and its subsequent analysis. Section 2 contains a method for directly measuring the detector response characteristics from optical input to digital output of a Fast-2D detector board. Using a high-bandwidth (100 MHz) digital diode laser, we measure the characteristic step response of the detector circuit, including the digitizer (comparator). This is done by measuring the comparator transition times as we scan the comparator reference voltage by adjusting the illuminating laser duty cycle. We then compare these measurements to the minimum detectable pulse duration. Once the electronic response characteristics are determined, we apply the results to a model of a 2D probe in section 3 to estimate the size-dependent sample volume’s sensitivity to variations in the probe’s detector response time. Finally, a summary is given in section 4.

2. Measurements

The Fast-2D board [model ABD-0001, also called the cloud imaging probe (CIP) mono diode array] is manufactured by Droplet Measurement Technologies (DMT, Boulder, Colorado) and it consists of a linear photodiode array, where each photodiode current is converted to a voltage through a transimpedance amplifier. That voltage is then further amplified and drives a comparator circuit that generates a 1-bit not-shaded (digital 1) or shaded (digital 0) output state for the photodiode. A simplified schematic of the Fast-2D circuit is shown in Fig. 2.

Ideally the comparator output state is digital one when the amplifier voltage is greater than 50% of its average voltage, and digital zero when the amplifier falls below 50% of its average voltage. Practically, the comparator changes state when the voltage on the non-inverting terminal crosses the voltage on the inverting terminal. The presence of positive feedback from the comparator output generates hysteresis in the
comparator circuit, so the output state does not oscillate when the input signal is near the threshold voltage. This also results in a bias term in the threshold voltage (from the perspective of the amplifier driving the comparator input) where the comparator changes state. They asymptotically approach the design value as the average amplifier voltage increases. The Fast-2D amplifier saturates just below 4 V.

Eq. (1) is used to calculate the step response voltage waveform of the total detector system. Figure 3 shows the amount of shading required to cause a comparator transition as a function of the mean test point voltage (as measured on the analog test point right before the input resistor in Fig. 2). The circuit is designed to transition when the test point voltage crosses the 50% shading threshold. It asymptotically approaches this value as the optical power on the diode increases, but it always requires greater than 50% shading for any physically realizable mean test point voltage.

Each detector circuit has a corresponding DOF circuit that is identical to the standard comparator circuit, except that the comparator threshold is theoretically set to 67% shading (the amplifier output is split into the two comparator circuits). A particle is accepted as "in the sample volume" if at least one DOF comparator is triggered by the particle. In addition to the standard comparator output, Fig. 3 also shows the required shading for the DOF flag as a function of average amplifier voltage.

A likely point for characterizing the response time of the detector circuit is the analog test point immediately preceding the comparator circuit in Fig. 2; however, this omits the comparator response characteristics. The resistor $R_{in}$ isolates the analog test point from input capacitance in the comparator circuit, which could be significant. Attempts to probe the noninverting terminal of the comparator were unsuccessful because parasitic capacitance on the probe altered the dynamics of the waveform (the time at which the comparator output transitioned was noticeably different when the terminal was probed). This parasitic capacitance occurs because the probe’s own capacitive properties are added to that of the circuit when the two are in contact. In circuits with larger capacitance and input current, this is negligible and does not affect the measured waveform. In this case, because of the speed of the circuit and low current at the probe point, the probe capacitance dominates the measurement and the process of probing the waveform simultaneously alters it. Thus, the ideal point for measuring the full circuit dynamics is at the output of the comparator.

We have two measurements for assessing the dynamics of the detector system on the Fast-2D board. The test setup used for both measurements is shown in Fig. 4. A
high bandwidth (100-MHz or 2-ns rise/fall time) digital diode laser (Newport LQD660-110C) is modulated between (mostly) on and (very briefly) off states. The laser is collimated by a 30-mm lens and passed through a half-wave plate (HWP) and polarizing beam splitter (PBS), where one port is directed onto a high-bandwidth photodiode (Thorlabs DET10A with 1-ns rise time at 50 V load) as a time reference and monitor for the laser signal. The laser pulse demonstrates a fall time of approximately 3 ns when monitored with the DET10A. The HWP is used to control the split in laser power between the two PBS ports. The light exiting the other PBS port is directed onto the Fast-2D detector array through a periscope (not pictured). We then monitor the analog test point on the Fast-2D board to determine the steady-state voltage when the laser is on and off, and we make sure the input signal is not saturating any of the detection electronics. We monitor the output state of the comparator to determine whether the board is responding to the input pulse (see the next subsection) and to determine the time at which the voltage waveform crosses the threshold voltage (see section 2b). Table 1 summarizes all the measured and derived variables for characterization of the Fast-2D board.

### a. Minimum pulse width

In the first measurement, a diode laser illuminates the photodiode array. The laser is normally on except for very short periods where the laser is turned off to simulate shading from a particle passing through the beam. We progressively reduce the duration where the laser is off until the detection circuit does not respond fast enough to cause a change in state in the digital output. Contributions of zero-mean noise will blur this exact boundary, so we estimate the fraction of input pulses that result in a change in comparator output state and report here the 50% point (ideally where the limit would be if noise were absent). The histogram in Fig. 5 shows the distribution of minimum response times for all 64 channels on a Fast-2D board. The sizable spread would seem to suggest that there is significant variation in the response characteristics of the different detection channels. Furthermore, it seems to suggest the board does not meet its 50-ns time constant specification. We will show in the next section that an unexpected slow decay term is having a detrimental effect on the detector performance.

### b. Total voltage waveform

In our second test, we measure an effective voltage-versus-time waveform for a case when the laser is initially on and turned off very rapidly (with a bandwidth of 100 MHz, the laser has a fall time on the order of 2 ns). This step-down input is functionally described as \(1 - u(t)\), where \(u(t)\) is a unit step function. In this measurement, we change the voltage on the inverting terminal of the comparator by changing the duty cycle of the laser (the fraction of time during the waveform period where the laser is on divided by the waveform period) and recording the time at which the comparator changes state. Thus, where voltage waveforms are typically sampled at known time intervals, in this case, we are recording the time at known voltage intervals. Based on the duty cycle of the laser, we determine the effective threshold voltage at which the comparator changes state using Eq. (1) and

\[
V_- = \frac{1}{2} [DV_H + (1 - D)V_L],
\]

where \(D\) is the laser duty cycle, \(V_H\) is the analog test point voltage when the laser is on, and \(V_L\) is the amplifier voltage when the laser is off. This calculation assumes that the pulse repetition rate of the laser (10 kHz) is significantly above the cutoff frequency of the low-pass

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start voltage</td>
<td>Analog test point</td>
<td>(V_H)</td>
</tr>
<tr>
<td>Voltage bias</td>
<td>Analog test point</td>
<td>(V_L)</td>
</tr>
<tr>
<td>Digital output</td>
<td>Comparator output</td>
<td>(V_o)</td>
</tr>
<tr>
<td>(high and low)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition time</td>
<td>Comparator output</td>
<td>(t)</td>
</tr>
<tr>
<td>Inverting terminal voltage</td>
<td>Eq. (2)</td>
<td>(V_-)</td>
</tr>
<tr>
<td>Threshold voltage</td>
<td>Eq. (1)</td>
<td>(V_{th})</td>
</tr>
<tr>
<td>Duty cycle</td>
<td>Input waveform</td>
<td>(D)</td>
</tr>
<tr>
<td>Focus beam displacement</td>
<td>Ruled slide</td>
<td>(\Delta d)</td>
</tr>
<tr>
<td>Angle of incidence</td>
<td>Eq. (8)</td>
<td>(\alpha_i)</td>
</tr>
</tbody>
</table>

**Fig. 5.** Histogram of the minimum detectable input pulse width for all 64 channels on a Fast-2D board.
The physical effect responsible for the slow decay remains unclear. Capacitive coupling was considered, but we can change the amount of slow decay in the waveform by adjusting the laser angle of incidence (near-normal incidence significantly reduces the amplitude of the slow decay).

The slow term could be the result of scattering between a reflective mask over the photodiode array and the matte white surface on which the diode array is mounted. However, the relative separation between the two surfaces is on the order of millimeters, so a decay function on the order of 1 \( \mu \)s would have to experience approximately 300,000 reflections and require a surface reflectivity larger than 0.999997. Also, the decay function of most scattering is expected to conform to an exponential form.

Finally, it may be possible the laser excites electron-hole pairs in an inactive region of the detector and the carriers have very slow diffusion rates. If these regions are near the edges of the photodiode, it might explain the dependence on the angle of incidence. Whatever the physical cause of the slow decay function, it remains a significant factor in the characterization and analysis of the detector system of the Fast-2D.

We did not observe this slow decay term on the DET10A that was monitoring the laser optical output, so we are confident that it is an attribute of the 2D board and not an artifact of the test equipment.

Giving consideration to the different components of the threshold-time versus threshold-voltage waveforms, we developed the following fit function to describe the detector response to a step-down input:

\[
v(t) = A \exp\left(-\frac{t}{\tau}\right) + \frac{C \sqrt{1 + t_D}}{t_0 + t} u(t) + B. \tag{3}
\]

This equation can be broken into three parts. The first term is exponential and is the electronic response function with exponential amplitude \( A \) and electronic response time \( \tau \). The second term is the slow decay term with slow decay amplitude \( C \) and slow decay rise time \( t_0 \) and a slow decay time offset \( t_D \). Finally, the third term, \( B \), is a dc bias. A unit step function \( u(t) \) is used to indicate the dynamic terms of this function are only valid after \( t = 0 \). The electronic response function is in the form expected for first-order linear circuits and was explicitly used in Baumgardner and Korolev (1997), Jensen and Granek (2002), and Strapp et al. (2001), and implicitly in Lawson et al. (2006). The form of the second term is based on observations of the voltage waveforms, where it was noted that the data exhibited a long decay tail with a \( 1/\sqrt{t} \) behavior. The second term in Eq. (3) is not based on first principles or any physical effect known to the authors; however, it produces the best fits, decoupling electronic time constant retrievals from variability in the relative amplitudes of the two waveforms.

The energy of the total signal is split between the fast response electronic exponential and the slow decay function. The term slow fraction will be used to describe the fraction of signal power coming from the slow term in Eq. (3) and is represented in this work by the variable \( S_f \). The slow fraction can be determined by evaluating the ratio of the slow term to the total signal (less the bias term \( B \)) at the time of transition in the step response function in Eq. (3). Evaluating this ratio at \( t = 0 \) produces the relationship between \( S_f \) and \( C \) as follows:

\[
C = \frac{S_f t_D}{(1 - S_f) \sqrt{t_D}} \tag{4}
\]

and

\[
S_f = \frac{C \sqrt{t_D}}{t_0 A + C \sqrt{t_D}}. \tag{5}
\]
The relationship in Eqs. (4) and (5) is useful for simulation, where often $S_f$ is the parameter of interest, but $C$ is needed for model implementation.

Note also that

$$V_H = A + \frac{C\sqrt{t_D}}{t_0},$$

which provides the relationship between fit parameters needed to satisfy the initial condition of the input waveform.

Finally, the impulse response of the detection system can be obtained from the step-down response in Eq. (3) as follows:

$$h_D(t) = -\frac{1}{V_H} \frac{\partial}{\partial t} v(t)$$

$$= \frac{1}{V_H} \left[ \frac{C(t - t_0 + 2t_D)}{2(t + t_0)\sqrt{t + t_D}} + \frac{A}{\tau} \exp \left( -\frac{t}{\tau} \right) \right] u(t).$$

An example of the fit waveform is shown in Fig. 6. We fit Eq. (3) to all 64 detector channels of a Fast-2D board. Table 2 contains a summary of the fit parameters from Eq. (3). All amplitude terms in Table 2 are normalized (except $V_H$) to the initial voltage prior to the step function ($V_H$), so they are reported in unitless $V/V$ in the table. Note the electronic time constant $\tau$ is a very narrow distribution, but the mean and standard deviation are largely affected by a small number of outliers. Those outlier measurements of the electronic time constant correspond to a few cases of large fit error or instances of a large slow fraction, causing the fit to be insensitive to the electronic response.

A scatterplot of the electronic time constant versus slow fraction is shown as blue dots in Fig. 7. It is important to note that the electronic time constant is independent of the slow fraction. A poor fit function would likely couple these two terms. For comparison, Fig. 7 also shows the corresponding minimum detectable pulse widths described in the previous section. These pulse widths tend to be significantly larger than the electronic time constants and show a clear correlation with the slow fraction. This is expected, because as the slow fraction increases it will dominate the decay waveform and the effective response time of the system increases.

Our observations of the Fast-2D detector system (optical input to digital output) did not significantly differ from the response characteristics we would have obtained from analyzing the amplifier output at the analog test point. However, as the analog front end of the detection system is pushed to faster response times, the response characteristics of the digitizer will likely become more significant. The analysis shown here is important, because the response characteristics of a digitizer can be difficult to measure directly. A complete analysis should include all of the analog components, including the digitizer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Mean</th>
<th>Median</th>
<th>Std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start voltage</td>
<td>$V_H$</td>
<td>2.85 V</td>
<td>2.80 V</td>
<td>0.37 V</td>
</tr>
<tr>
<td>Exponential amplitude</td>
<td>$A$</td>
<td>0.53 V/V</td>
<td>0.56 V/V</td>
<td>0.17 V/V</td>
</tr>
<tr>
<td>Electronic time constant</td>
<td>$\tau$</td>
<td>51.7 ns</td>
<td>46.8 ns</td>
<td>24.8 ns</td>
</tr>
<tr>
<td>Slow term amplitude</td>
<td>$C$</td>
<td>5.4 V/V$\sqrt{t_D}$</td>
<td>5.03 V/V$\sqrt{t_D}$</td>
<td>1.77 V/V$\sqrt{t_D}$</td>
</tr>
<tr>
<td>Slow term rise time</td>
<td>$t_0$</td>
<td>121 ns</td>
<td>101 ns</td>
<td>73 ns</td>
</tr>
<tr>
<td>Slow term time offset</td>
<td>$t_D$</td>
<td>79.5 ns</td>
<td>78.4 ns</td>
<td>9.81 ns</td>
</tr>
<tr>
<td>DC bias</td>
<td>$B$</td>
<td>0.0126 V/V</td>
<td>0.0113 V/V</td>
<td>0.0282 V/V</td>
</tr>
<tr>
<td>Fit error</td>
<td></td>
<td>0.048 V/V</td>
<td>0.018 V/V</td>
<td>0.084 V/V</td>
</tr>
</tbody>
</table>

FIG. 7. Plot of the estimated electronic time constants (blue dots) and the measured minimum detectable pulse widths (red x’s) vs slow fraction for each photodiode channel. The electronic time constants are independent of the slow fraction and well centered around the design response time. The minimum detectable pulse widths are impacted by the amplitude of the slow decay term in the detected signal, which significantly slows the overall response of the detection system.
We have found that the electronic time constant of our Fast-2D boards is near its design specification of 50 ns. However, the effective response time of the detector is slower due to an additional response characteristic. Were the slow decay term not a factor, we would expect the minimum detectable pulse width to be less than the electronic time constant. Instead, we typically observed minimum pulse widths in the range of 70–150 ns.

There is substantial scatter in the slow fraction seen in Fig. 7. During the acquisition of the time constant data, we did not maintain a constant angle of incidence. Cursory observations seemed to suggest that the angle of incidence was a significant factor in the scattering fraction. To further investigate this, we took data on the slow fraction as a function of the angle of incidence for a few channels using the setup shown in Fig. 8. The high-bandwidth laser is collimated by lens L0 (40-mm focal length) and passes through an afocal beam expander composed of lenses L1 (30-mm focal length) and L2 (200-mm focal length). An iris is placed at the focal plane of the lens L1, and a beam splitter is placed between the iris and lens L2. The light exiting L2 is approximately 1-cm diameter. The beam position and angle relative to the Fast-2D board is adjusted using the periscope mirrors M3 and M4, and the test point is monitored with an oscilloscope. The laser is controlled using a function generator set in transistor–transistor logic (TTL) pulse mode. The initial reflection off the beam splitter is detected using the DET10A and is used as the reference channel in the oscilloscope. Upon impinging on the Fast-2D, most of the light is reflected by the reflective mask over the detector. The reflected light passes back through the optical system. The beam splitter picks off a portion of the reflected light after it passes through lens L2 and directs it onto a ruled slide that is located precisely at the focus of L2 (also the conjugate plane of the iris). A complementary metal–oxide–semiconductor (CMOS) camera (not shown) is used to image the focused spot on the slide and read off the displacement of the beam. The incident angle of the beam is given by

$$\alpha_i = \frac{1}{2} \frac{\Delta d}{f_{L2}},$$

where \(\alpha_i\) is the angle of incidence (rad), \(\Delta d\) is the displacement of the beam on the ruled slide, and \(f_{L2}\) is the focal length of lens L2. From the oscilloscope, we estimate the slow fraction by recording \(V_H\), \(V_L\), and the approximate point where the slow decay curve dominates the waveform. A plot of these observations is shown in Fig. 9.

In previous investigations we had found that adjacent even and odd array photodiodes seemed to have different angular sensitivity, suggesting that the mask over the detector array was slightly offset. In addition to the difference in angular sensitivity, it also seems that the scattering fraction has offsets that may be a similar result of a mask offset. Channels 17, 18, and 24 all seem to show a minimum in scattering fraction but at different angles of incidence. Meanwhile, channel 45 appears to be significantly offset from the minimum. All of the data were taken at the same absolute angles of incidence (shown in the left plot of Fig. 9). We fit the data to a quadratic function and allow the optimizer to adjust the angular offsets of each channel. This provides a more coherent picture of the angle of incidence versus the scattering fraction shown in the right plot of Fig. 9. The angle offsets of the tested channels are given in Table 3.

The Fast-2D scattering fraction seems to be quite sensitive to the angle of incidence. To obtain a scattering fraction of less than 0.1, it would seem that there is about a 1° tolerance in the angle of incidence. This assumes that the detected sensitivity is symmetric, which we could not confirm. The test setup cannot measure a wide range of incidence angles (limited to approximately \(\pm 0.3°\)), so we are subject to the alignment of the detector mask on this particular board. Thus, these results are not fully conclusive with regard to detector tolerancing, but it does suggest that the boards are quite sensitive in at least one incidence direction.

We should not expect all detectors, even on the same Fast-2D card, to have the same slow fraction. It would appear that slight deviations in mask alignment to the detector determine where the incident beam lies on the angle of incidence versus the slow fraction curve.
The angle of incidence that obtains a small slow fraction in one channel is very likely suboptimal for another channel.

3. Response time effect on probe sample volume

The step response characteristics measured in the sections above suggest the electronic response of the detection system is quite fast, but the slow decay term may significantly slow the overall detection system and adversely impact the instrument’s ability to measure small particles. To better understand the effects of response time, we use the results reported above in a 2D probe simulation.

a. Simulation description

The 2D probe simulation used here is largely composed of a diffraction model and includes a number of prior determined characteristics. It models the incident wave front as an elliptical Gaussian beam with 0.2-mrad vertical divergence and 0.6-mrad horizontal divergence based on laboratory measurements of the beam. Prior published OAP 2D simulations have been limited to plane-parallel waves. It also includes the optical receiver point spread function (PSF), which has been characterized as a 10-μm-diameter (null to null) diffraction-limited spot (or Airy disk). The PSF represents the smallest resolvable feature and is the spatial analog to the impulse response of a dynamic system. The PSF results in blurring of an image such that features smaller than the PSF typically cannot be

<table>
<thead>
<tr>
<th>Channel</th>
<th>Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.001°</td>
</tr>
<tr>
<td>18</td>
<td>−0.099°</td>
</tr>
<tr>
<td>24</td>
<td>0.074°</td>
</tr>
<tr>
<td>45</td>
<td>0.484°</td>
</tr>
</tbody>
</table>
resolved. Prior published OAP simulations do not include the PSF, which may be important in assessing the sample volume of small particles consisting of only a few pixels.

Particles are simulated passing through the probe sample volume as opaque disks as was done in Korolev et al. (1991). In this study, the electric field at the object plane is determined using a Fourier optics method detailed in Goodman (2005), where propagation of a field is calculated by multiplying the field’s Fourier transform by the transfer function of freespace such that the exiting wave’s Fourier transform is given by

\[ E(f_x, f_y; z) = H(f_x, f_y; z)E(f_x, f_y; 0), \]

where \( E(f_x, f_y; 0) \) is the Fourier transform of the input electric field; \( E(f_x, f_y; z) \) is the Fourier transform of the electric field after propagating a distance \( z; f_x \) and \( f_y \) are the spatial frequency axes corresponding to spatial dimensions \( x \) and \( y \), respectively; and \( H(f_x, f_y; z) \) is the transfer function of freespace, which is

\[
H(f_x, f_y; z) = \begin{cases} 
\exp\left[ \frac{j2\pi z}{\lambda} \sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2} \right] & \text{if } \lambda \sqrt{f_x^2 + f_y^2} < 1, \\
0 & \text{otherwise}
\end{cases}
\]

where \( \lambda \) is the wavelength of the illuminating laser, which is 660 nm in this work.

This method of propagating waves is practically similar to the Fresnel convolutions performed in Korolev et al. (1991). The Fourier transform method tends to be faster when Fresnel convolution kernels are near the same size as the total grid area, but it has the drawback of aliasing (where diffracted patterns that exit the edge of the modeled area wrap around).

The initial simulation runs at a much smaller spatial grid spacing than what is actually measured by the probe. Optical diffraction calculations are performed on a grid with a spacing of 1.5625 \( \mu m \). The grid spacing details are included in the simulation table.

We describe the electric field after the particle (modeled as an opaque circular disk here) is illuminated by the laser using

\[
E_p(x, y, t; z_p) = \{T_p(x, y)\delta[x - x_p(t), y - y_p]\}E_L(x, y; z_p),
\]

where \( E_p(x, y, t, z_p) \) is the total field after the laser is occulted by the particle. The particle has a position \( z_p \) along the optic axis; \( y_p \) along the detector array direction; and \( x_p(t) \) orthogonal to the array, which is a function of \( t \), reflecting that the particle is moving along this axis. The function \( T_p(x, y) \) is the transmission mask describing the particle centered about \( x = 0 \) and \( y = 0 \); \( \delta(x - x_p(t), y - y_p) \) is a Dirac delta function centered at \( x = x_p(t) \) and \( y = y_p \); and \( E_L(x, y, z_p) \) is the electric field of the incident laser beam at \( z = z_p \), which is described as an elliptical Gaussian beam in this simulation. For the purposes of this simulation, we assume the laser is centered on the detector array.

The elliptical Gaussian beam is described as the product of two 1D Gaussian beams (see Yariv and Yeh 2007, section 2.11),

\[
E_L(x, y; z) = \exp(jkz)E_y(x; z)e^2(x; z),
\]

where \( k \) is the wavenumber and is given by

\[
k = \frac{2\pi}{\lambda}.
\]

For brevity, we present only the definition of \( E_L(x; z) \), where \( E_y(x; z) \) follows the same form. The electric field definition is

\[
E_L(x; z) = E_0\sqrt{\frac{w_0}{w_x(z)}}\exp\left[ -\frac{x^2}{w_0^2(z)} \right] \exp\left[ jk\frac{x^2}{2R_x(z)} \right]
\]

\[ -\text{1}\ \text{arctan}\left( \frac{z - z_0}{z_0x} \right) \text{,}
\]

with the following definitions:

beam radius:

\[
w_x(z) = w_0\sqrt{1 + \frac{(z - z_0)^2}{z_0^2x^2}};
\]

beam waist:

\[
w_0x = \frac{\lambda}{\pi\theta_x};
\]

where \( \theta_x \) is the beam divergence along the \( x \)-coordinate axis.
Rayleigh range:
\[ z_0 = \frac{\pi w_0^2}{\lambda}, \quad (17) \]
beam radius of curvature:
\[ R_s(z) = \left( z - z_s \right) \left[ 1 + \frac{z_0^2}{(z - z_s)^2} \right], \quad (18) \]

In the case of the Gaussian beam description above, the term \( z_s \) refers to the location of the \( x \)-dimension beam waist. Notably this might be different from \( z_s \) when the beam is astigmatic. Note that in general, the \( y \) axis will have its own separate definition for the beam radius, waist, divergence, Rayleigh range, and radius of curvature.

The definition for an elliptical Gaussian beam provided above assumes the major and minor axes are aligned to \( x \) and \( y \) coordinates, respectively. Rotating the axes can be accomplished by substituting a rotated coordinate system into the above-mentioned definitions.

Once the electric field after the particle is obtained from Eq. (11), it is propagated, using Eq. (9), to the object plane of the probe (assumed to be in the center of the two probe arms) at \( z = 0 \) to obtain \( E_p(x, y, t; 0) \). The intensity on the detector is given by
\[ I_D(x, y, t) \propto |E_p(x, y, t; 0)h_{\text{optic}}(x, y)|^2, \quad (19) \]
where \( h_{\text{optic}}(x, y) \) is the optical system PSF that accounts for the optical system resolution. The PSF was measured directly by placing a charge-coupled device (CCD) in the object plane of the instrument, illuminating the detector with a 660-nm laser and measuring the Airy pattern from a small feature on the detector array. The first null in the Airy pattern is at a radial position of 5 \( \mu \text{m} \). The PSF for this simulation is given by
\[ h_{\text{optic}}(x, y) = \frac{J_1 \left( \frac{2\pi}{R_{\text{PSF}}} \right) r}{\left( \frac{0.61\lambda}{R_{\text{PSF}}} \right)^2}, \quad (20) \]
where \( J_1(x) \) is a Bessel function of the first kind, order 1, \( r = \sqrt{x^2 + y^2} \), and \( R_{\text{PSF}} \) is the radius of the first null in the PSF. For a diffraction limited system, the radius of the Airy disk is generally approximated using
\[ R_{\text{PSF}} = \frac{0.61\lambda}{\text{NA}}, \quad (21) \]
where \( \lambda \) is the wavelength of the light and \( \text{NA} \) is the numerical aperture of the collection optics.

Once the intensity on the detector is obtained from Eq. (19), we account for the circular apertures in the mask in front of each detector and perform spatial sampling (decimation) to obtain the intensity signal on the detectors as a function of time using
\[ I_s(t, n) = [I_D(x, y, t) * d(x, y)] \text{comb} \left( \frac{y}{\Delta y_D} \right) \delta(x), \quad (22) \]
where \( I_s(t, n) \) is the time domain signal on each detector at positions separated by \( \Delta y_D \), and \( d(x, y) \) is the description of the mask aperture function (a circ function 20 \( \mu \text{m} \) in diameter). The spatial variable \( y \) is replaced with \( n \) for the channel number. The Dirac delta function \( \delta(x) \) represents the fact that, as a linear array, the detectors only sample the intensity at \( x = 0 \). The function \( \text{comb}(y/\Delta y_D) \) is a periodic train of impulses positioned wherever the argument is an integer such that
\[ \text{comb}(x/\Delta x) = \sum_{k=-\infty}^{\infty} \delta \left( x - \frac{k}{\Delta x} \right). \quad (23) \]
This function is used to represent the discrete spatial sampling of the 64 detectors [for more information, see chapter 2 in Goodman (2005)].

The photodiode spacing on the instrument \( \Delta y_D \) is measured directly by placing a CCD in the object plane of the instrument and illuminating the detector array with a 660-nm laser. We have found the most effective way to measure the spacing is by Fourier transforming the CCD line along the reimaged detector array and measuring the location of the peak in spatial frequency.

Finally, the electrical signal includes the response time of the probe detector and electronics, and the 25-\( \mu \text{m} \) horizontal resolution through the following equation:
\[ V_D(t, n) = [I_s(t, n) h_D(t)] \text{comb} \left( \frac{t - \delta t}{\Delta t} \right), \quad (24) \]
where \( h_D(t) \) is the impulse response of the detection chain as determined in this work, and the comb function represents the decimating of the signal at \( \Delta t = \Delta y_D/v_{\text{air}} \), where \( v_{\text{air}} \) is the airspeed of the aircraft. The term \( \delta t \) is scanned in our analysis to account for possible variations in the particles’ arrival time relative to the clock on the probe, where for a particular particle \( 0 \geq \delta t \geq \Delta t \) and is uniformly distributed. For our analysis, we break this up into eight evenly spaced samples to numerically approximate averaging across all possible values of \( \delta t \).

Examples of a particle through the stages of analysis are shown in Fig. 10, where \( S_f = 0 \), and Fig. 11, where \( S_f = 0.3 \). The top-left panel shows \( I_D(x, y, t) \) for a
A 137.5-μm particle with \(z_p = 3\) cm, \(x_p(t) = 0\), and \(y_p = 0\) when it is illuminated by the elliptical Gaussian beam. The top-right panel shows the resulting intensity pattern on each detector \(I_s(t, n)\) recorded as the particle moves across the array in time. The bottom-left panel shows the resulting voltage waveform \(V_D(t, n)\) after sampling at the diode array resolution and imparting the detector response characterized in this work. Finally, the bottom-right panel shows the 1-bit digitized signal typical of OAP data. In this case, the DOF flag is triggered by 12 pixels, so the particle is regarded as in the sample volume.

In this analysis we use the DOF flag (requires 67% shading) to determine whether a particle is in the probe sample volume. If at least one pixel triggers the DOF flag, we count the particle as being in the sample volume. We then count the number of particles in the sample volume and divide by the total number of particles simulated to provide the probability of detection. Note that we do not require the particle be sized correctly, as this will depend on the specific OAP processing code. To determine the sample area of a particular particle size, we multiply the probability of detection by the total simulated area of the particles. The parameters used for the 25-μm Fast 2D-C simulation are described in Table 4.

### Simulation results

The sample area as a function of particle size is shown in Fig. 13 for different slow fractions. These slow fractions correspond to the range of values observed during the initial detector characterization. The black line in the figure is the case where there are no response time
effects in the probe. For comparison, the waveforms of those impulse responses are shown in Fig. 14.

As the amount of energy in the slow decay term increases, it can have a significant effect on particle sample volume. Figure 15 shows the fractional difference in sample volume compared to the case where response time is not included in the simulation. In this case the slow fractions used are more realistic for operational 2D probes with values between 0 and 0.1. Figure 15 shows the significant challenge in estimating the sample area (and therefore particle concentrations), particularly at small sizes.

We expand the simulation of our 25-μm 2D-C probe to include airspeeds from 80 to 250 m s⁻¹ with slow fractions between 0 and 0.1. We calculate the mean sample area as a function of particle diameter and airspeed, and the corresponding fractional spread in sample area (ΔSA/SA) due to variation in slow fraction. The results are plotted in Fig. 16, where the surface height corresponds to mean sample area and the color scheme corresponds to fractional error. Not surprisingly, the uncertainty due to slow fraction is larger for smaller particles; however, the airspeed dependence is relatively small. It is notable that the uncertainty due to slow fraction tends to exceed changes in sample volume due to airspeed. Overall, the uncertainty in sample area caused by uncertainty in slow fraction is on the same order or less than the slow fraction uncertainty or about 10%.

The fractional uncertainty due to slow fraction will add in quadrature with other fractional uncertainty terms (e.g., sizing error, Poisson statistics, and any sample volume uncertainty arising from other unknowns in the probe operation). This means the error shown in Fig. 16 represents a base fractional uncertainty in particle concentration. While the results reported here raise some basic concerns about sizing particles in the smallest size bins, it also offers some assurance that the error in particle concentration caused by response time uncertainty is less than 10% for particles larger than the third bin (diameter of 87.5 μm and larger). Furthermore, the uncertainty due to slow fraction is not fundamental. If the
slow fraction could be accurately determined for each individual channel on a specific probe, and we were confident the slow fractions would not change during flight, then the effects could be accounted for in the sample volume estimate. At this time, this does not seem practical because the analysis should include the probe’s illuminating laser, which does not currently have sufficient bandwidth for this analysis.

4. Conclusions

The response time characteristics of OAP detectors are known to have an impact on the accuracy of derived cloud and precipitation particle concentrations. While the effect of time constants has been discussed in other works, we have described a detailed method for characterizing the response behavior of the Fast-2D detector board. This analysis method includes the total detector system, from optical input to digitization, and includes a careful study of the observed waveforms.

We found that the electronic time constant of the Fast-2D is near its specified value of 50 ns. Where previously we have assumed that the electronic response time is the most important characteristic, we have found that an additional slow decay term appears to play a significant role in limiting the overall detection system bandwidth. We are able to adjust the amplitude of the slow decay term by adjusting the laser alignment to the detector board, but even at very near-normal incidence, typical slow fractions are between 0 and 0.1. This results in a spread of sample volume estimates ranging from less than 1% for particles larger than 150 μm to approximately 10% in the smallest size bins. To better constrain the spread in slow fraction across all channels on a board, the mask alignment to the photodiode array is critical.

Given the specific outcome of this work, the optical array probe community should be cautious about

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam divergence (vertical)</td>
<td>( \theta_b )</td>
<td>0.6 mrad</td>
</tr>
<tr>
<td>Beam divergence (horizontal)</td>
<td>( \theta_h )</td>
<td>0.2 mrad</td>
</tr>
<tr>
<td>Receiver numerical aperture</td>
<td>( R_{PSF} )</td>
<td>10 μm</td>
</tr>
<tr>
<td>Receiver minimum radius spot</td>
<td>( \Delta y_D )</td>
<td>25 μm</td>
</tr>
<tr>
<td>Detector pixel separation</td>
<td>( \Delta x_D )</td>
<td>6.2 cm</td>
</tr>
<tr>
<td>Probe arm separation</td>
<td>( t_0 )</td>
<td>50 ns</td>
</tr>
<tr>
<td>Airspeed</td>
<td>( t_D )</td>
<td>118 ns</td>
</tr>
<tr>
<td>Electronic response</td>
<td>( B )</td>
<td>73 ns</td>
</tr>
<tr>
<td>Slow term rise time</td>
<td>( t_{DOF} )</td>
<td>0 V</td>
</tr>
<tr>
<td>Slow term time offset</td>
<td>( \Delta f_x )</td>
<td>1.5625 μm</td>
</tr>
</tbody>
</table>
| Spatial resolution | \( (\Delta x, \Delta y) \) | 321.5 μm
| Spatial frequency resolution | \( (\Delta f_x, \Delta f_y) \) | 312.5 m⁻¹ |
| Grid size | \( N_x \times N_y \) | 2048 × 2048 points |

FIG. 13. Sample area of the 25-μm Fast 2D-C as a function of particle diameter for different slow fractions. The black line indicates the sample area if there is no response time effect in the probe. The near-horizontal lines in the upper-right corner correspond to the mechanical limitations due to the fixed distance between probe arms. The curves left of that region are where optical and electrical characteristics of the probe determine the particle sample area.
assuming their probe’s detection bandwidth is dictated solely by electronic circuit designs. Furthermore, instrument designers should be cognizant of other physical effects in the entire probe detection system (such as capacitive coupling, scattering, semiconductor detector response characteristics, and mask-to-photodiode array alignment). The effects of additional slow decay terms are of increasing concern for the design of state-of-the-art OAP components, since high-speed electronics cannot overcome bandwidth limitations imposed by other physical effects.

While there is considerable scientific motivation to push OAPs to characterize particles at smaller size ranges (10–100 \( \mu \text{m} \)), this work further emphasizes how small effects can have a significant impact on the performance of an OAP in this particle size range. Details that are insignificant at larger sizes can become dominating error sources in the smaller regimes. In this specific case we find there are previously unconsidered physical effects influencing the performance of the probe. We want to emphasize the importance of including physical effects not conventionally considered in OAP literature for OAP simulation, data analysis, and design in the small particle size range.

Finally, this work details only one piece of 2D probe characterization. Several other probe effects remain to be considered or ruled out in optical probe characterization. Comparator hysteresis, described briefly here, will likely cause the sample volume to have some dependence on the mean detector signal. The incident laser wave front, optical system PSF, and optical aberrations may impact the size-dependent sample volume and help explain some of the discrepancies in the depth-of-field (DOF) constant usually used to estimate the probe’s sample area. For a total probe characterization, a model using these (and possibly other) individual characterizations should be able to reproduce end-to-end test results [such as high-speed spinning disks or optical fiber described in Lawson et al. (2006)]. This validation process is essential, as the failure to obtain agreement indicates additional missing characterizations or possible mischaracterization of a component.

Acknowledgments. The authors thank Aaron Bansemer for providing exceptional feedback on the presentation and content of this work.

**Fig. 14.** Impulse responses (with maximum value normalized to one) for the slow fractions shown in Fig. 13.

**Fig. 15.** Fractional difference in the sample area for slow fractions between 0 and 0.1 relative to a probe with no response time effects.

**Fig. 16.** Sample area (surface height) and fractional uncertainty in the sample area due to uncertainty in slow fraction (color) as a function of particle diameter and airspeed. Note the color axis is logarithmic.
REFERENCES


