Fitting a Cartesian Prediction Model to Radial Velocity Data from Single-Doppler Radar

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ABSTRACT

Previous experiments with the adjoint technique for determining the three-dimensional wind and thermodynamic fields from single-Doppler radar data have assumed that the radar observes one component of the velocity in Cartesian coordinates. This technique is generalized to radial velocity observations by fitting a Cartesian prediction model to interpolating radial velocity data in Cartesian coordinates. The three-dimensional wind and temperature are determined by minimizing the difference between simulated single-Doppler observations of radial velocity and reflectivity in Cartesian coordinates and their predictions from a numerical model. An application of this technique to a simulation of dry convection gives successful results.

1. Introduction

Single-Doppler parameter retrieval has been recently studied by Sun et al. (1991, hereafter referred to as SUN91) and Kapitza (1991) using the technique of adjoint data assimilation. This technique attempts to determine the three-dimensional wind and thermodynamic properties by iteratively minimizing a cost function that is formulated as the difference between single-Doppler observations and the predictions from a numerical model. In SUN91 it was demonstrated, using simulated observations, that the three-dimensional wind and thermodynamic variables—that is, temperature and pressure—could be successfully retrieved. However, spherical geometry of the radar was not considered in these studies. The assimilation experiments were conducted in Cartesian coordinates assuming that the x-component velocity u is the observed component from a single-Doppler radar.

A single-Doppler radar provides observations of radial velocity and reflectivity sampled in spherical coordinates. To use these observations, the retrieval scheme shown in SUN91 must be modified. One could transform the coordinates of the prediction model, and hence the corresponding adjoint model, from Cartesian into spherical coordinates without changing the nature of the problem. The advantage of this methodology is that it can make the best use of the observations. The disadvantage, however, is the difficulty of coding the prediction model and its adjoint. If the radial velocity observations can be interpolated into Cartesian coordinates with reasonable accuracy, it is preferable to assimilate the radial velocity data on Cartesian grids without any change in the model’s coordinates. The use of radial velocity data in Cartesian coordinates is described below and results are presented.

2. Numerical model and control simulation

The numerical model consists of a three-dimensional domain bounded vertically by two solid plates, with the distance separating them much smaller than the density height scale of a dry-adiabatic atmosphere. Under this assumption, compressibility effects can be neglected. The governing equations, therefore, are reduced to those of the Boussinesq system, which simulates shallow dry convection:

\[
\frac{dv}{dt} + w \frac{\partial v}{\partial z} - \delta_{ij} \frac{\partial \theta}{\partial \theta} \frac{\partial v}{\partial z} + \nabla \pi + \nabla \nabla v = 0,
\]

\[
\frac{d\theta}{dt} + w \frac{\partial \theta}{\partial z} - \nabla \nabla \theta = 0,
\]

\[
\nabla \cdot v = 0,
\]

\[
\frac{dc}{dt} - \mu \nabla^2 c = 0,
\]

in which

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + v_h \cdot \nabla_h + v \cdot \nabla.
\]

Here, v is the perturbation velocity vector; v_h is the horizontal mean velocity vector; \pi is the perturbation pressure divided by a reference density; \theta is the perturbation potential temperature; \theta_0 is the horizontal mean potential temperature and its reference value on the surface, \theta_0, is set to 310°C; and c is the reflectivity...
in unit of dBZ. Also, \( \nu \) is the eddy viscosity, \( \kappa \) is the thermal diffusivity, and \( \mu \) is the diffusivity of reflectivity, all assumed constant.

The Boussinesq system can support a wide variety of motions under different assumptions. The physics chosen here is that of thermal convection initiated by low-level potential temperature perturbations. The mean state is specified by a unidirectional shear of \( 4 \times 10^{-3} \) s\(^{-1} \), and the stratified potential temperature profile as shown by Fig. 1. The mean velocity component in the \( x \) direction is assumed to be zero.

A control simulation is conducted on a domain of \( 1.6 \) km \( \times \) \( 1.6 \) km \( \times \) \( 1.0 \) km with periodic boundary conditions. This simulation is initiated with three thermal bubbles with amplitude of \( 1^\circ \)C and spatial dimension of \( 3 \times 3 \times 1 \) grid points, centered on grid points (15, 15, 2), (7, 7, 5), and (23, 23, 5), respectively. There is a total of \( 32 \times 32 \times 20 \) grid points, each separated by 50 m. The time step is 5 s. The simulation is run for 180 time steps before the integration of the reflectivity conservation equation is commenced. The initial reflectivity field is obtained by first taking a randomly distributed (white noise) field and then smoothing the field by a five-point spatial smoother. The portion of the simulation between time steps 200 and 215 is used as data for the retrieval experiments conducted on a smaller domain with a total of \( 20 \times 20 \times 20 \) grid points. The specification of the boundary conditions follows the direction of Oliger and Sundstrom (1976). Their analysis indicates that, for inviscid flow, all prognostic variables but one should be specified for inflow regions. For outflow regions, only one boundary condition should be specified. We specify the boundary conditions of \( u, v, w, \) and \( \theta \) for those boundary points at which the normal velocity \( U + u \) or \( V + v \) is directed into the domain, and then calculate the values of \( \theta \) from the motion equations. For the outflow points, we specify the boundary conditions of the normal velocities; all the other variables are obtained by linear extrapolation—that is, \( \phi_{N+1} = 2\phi_N - \phi_{N-1} \), where \( \phi \) represents any variable and \( N \) is the number of grid points in the \( x \) or \( y \) direction.

The specified boundary conditions are provided from the control simulation.

Figure 2 shows the control data fields of the three velocity components and the potential temperature \( \theta \) at the 200th time step (16.7 min) in a vertical cross section in the middle of the domain across one of the three bubbles. The bubble rises from the surface before it reaches the bottom of the stable layer, and in the meantime it is advected by the lower-level southerly wind. When the bubble impinges upon the stable layer, gravity waves are excited through the "obstacle effect" (Clark et al. 1986).

3. Method

The object of adjoint variational data assimilation is to adjust the input parameters so that the model solutions match the observations as closely as possible. Thus, a cost function, that provides an objective measure of the distance between the observations and the model solution, must be defined. In SUN91, because it was assumed that the \( x \)-component velocity \( u \) was completely known, the cost function was then defined by

\[
J = \int_T \int_{\Sigma} \left[ k_u (u - u^{ob})^2 + k_c (c - c^{ob})^2 \right] d\sigma dt, \tag{3.1}
\]

where \( u^{ob} \) and \( c^{ob} \) are the observed or simulated data of the \( x \)-component velocity and reflectivity, respectively. Here, \( u \) and \( c \) are their model counterpart, and \( \Sigma \) and \( T \) represent the spatial and time domains, respectively, over which the assimilation is performed.

The above definition of the cost function can be used only if the radar is scanning a volume that is distant enough so that the radar beams are essentially parallel, which is not a general assumption. In fact, a single-Doppler radar provides observations of radial velocity and reflectivity. There are no direct observations for any of the three velocity components \( u, v, \) and \( w \). Thus, the cost function should be defined in terms of the radial velocity. Let \( v \) denote the radial velocity and \( (x, y, z) \) be the Cartesian coordinate of a grid point. Assuming the radar is located at the origin, the distance between the grid point and the radar location is then given by

\[
r = (x^2 + y^2 + z^2)^{1/2} \tag{3.2}
\]

and the relationship between the velocity components \( u, v, \) and \( w \) and the radial velocity \( v \) are given by

![Fig. 1. Mean profiles of \( y \)-component velocity (solid line) and potential temperature (dotted line) in the control simulation.](image-url)
Given the observations of the radial velocity and reflectivity, the cost function is defined by

$$J = \int_{\tau} \int_{\mathcal{Z}} \left[ k_u (v_r - v_{r}^{ob})^2 + k_c (c - c^{ob})^2 \right] \, d\tau dt,$$

(3.4)

where $v_r^{ob}$ is the observed or simulated data of the radial velocity and $v_r$ is the radial velocity computed by (3.3), using the model output of $u$, $v$, and $w$. Here, $k_u$ and $k_c$ are weighting coefficients. Since the typical values for velocity and reflectivity in the simulation are 1 m s$^{-1}$ and 10 dBZ, respectively, $k_u$ is set to 1 and $k_c$ is set to 0.1 in order to give similar weights to these two fields.

The independent variables in the cost function $J$ are the initial conditions of $u$, $v$, $w$, and $\theta$ (c is not considered as an independent variable since it is observable). Their values at any later time can be obtained by integrating the model (2.1). Therefore, our object is to minimize $J$ by adjusting the input parameters, that is, initial conditions of $u$, $v$, $w$, and $\theta$. The minimization is performed iteratively using Nocedal’s limited memory quasi-Newton conjugate gradient algorithm VAI5AD (Liu and Nocedal 1988). The algorithm requires the knowledge of the gradients of the cost function with respect to the input parameters. These gradients are computed by integrating a set of time-dependent equations—the adjoint equations. Since the procedure of deriving the adjoint equations is the same as that described in SUN91 and Sun (1992), the reader is referred to them for details. Here, we will explain only the differences between the methodologies for the two kinds of data. With radial velocity data, the adjoint equations are similar to those with $u$ data except for the forcing terms associated with the data mismatch. In the original system of SUN91, the difference $u - u^{ob}$ is a forcing factor on the adjoint variable of $u$, but that variable does not directly affect optimization of the initial state. It affects the other adjoint variables indirectly through their interaction terms, and they control the initialization of $v$, $w$, and $\theta$. When using radial velocity data, the cost function directly forces
calculate the gradient of the cost function with respect to the control variables by integrating the adjoint equations backward, and (iv) find a new estimate of the input parameters toward the minimum of the cost function using the calculated cost function and its gradient. The above procedure is repeated until \( \Delta J = J_i - J_{i-1} \to 0 \), where \( i \) represents the \( i \)th iteration. At that point the assimilation is considered complete.

4. Experiments and results

Two experiments, one using radial velocity and reflectivity and one using \( x \)-component velocity and reflectivity as data, are conducted and compared. The relative rms errors of the retrieved fields after 95 iterations are listed in Table 1, for both experiments. In experiment 1, the cost function (3.4) is minimized to fit the numerical model described in section 2 to simulated radial velocity data \( v_{r}^{ob} \). Here, \( v_{r}^{ob} \) is calculated according to (3.3), where the velocity components \( u, v, w \) are obtained from the control simulation. The origin is placed in the southwest corner of the lowest level of the domain. The experiment is started with a zero first guess as the initial condition for all of the

<table>
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<th>Experiment</th>
<th>Data</th>
<th>( u )</th>
<th>( v )</th>
<th>( w )</th>
<th>( \theta )</th>
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<td>0.08</td>
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</table>

**TABLE 1. Summary of experiments.**

**FIG. 3.** Similar to Fig. 2 but for the retrieved fields from experiment 1 after 95 iterations.
four unknown fields $u$, $v$, $w$, and $\theta$. The initial condition of the reflectivity field is provided by the control simulation. Figure 3 shows the retrieved fields of $u$, $v$, $w$, and $\theta$ after 94 iterations. Comparing Fig. 3 with Fig. 2, we find that the patterns are nearly identical and the values of the maxima and minima between the corresponding fields are close. The final relative rms errors are 0.07 for $u$, 0.08 for $v$, 0.05 for $w$, and 0.28 for $\theta$. The results from this experiment are compared to those obtained from experiment 2, which uses $u$ instead of $v$, as the observed field, or in other words, which minimizes cost function (3.1). Poorer retrieval might be expected when radial velocity data are used because the need to retrieve all three Cartesian components results in more unknowns for the optimization system. It is found that the retrievals using radial velocity data are slightly better than those using a Cartesian component when the rms error in each field is compared individually. The final rms errors from experiment 2 are 0.11 for $v$, 0.07 for $w$, and 0.35 for $\theta$. However, since the $x$-component velocity is completely known in experiment 2 (it has no error), the quality of the retrieval, in terms of the total vector difference between the retrieved velocity and the actual velocity, is almost identical. In both experiments, the retrieved velocity fields have better accuracy than the temperature field. This is because neither a velocity component nor the radial velocity contain direct information about the temperature field.

5. Concluding remarks

The method for using the radial velocity data with the Cartesian prediction model has been presented and tested. The cost function was formulated by the difference between the observed radial velocity and the radial velocity calculated using the solutions from the Cartesian prediction model. Successful results obtained in this study indicate that the retrieval algorithm presented in SUN91 can be applied to real radial velocity data without changing the coordinate system of the prediction model.

Assimilating radial velocity data into a Cartesian prediction model has the advantage of being simple, but the data have to be interpolated from radar geometry to Cartesian coordinates. Efficient and accurate interpolation schemes have been developed by researchers (i.e., SPRINT, Mohr and Vaughan 1979). Such schemes can preserve the features and continuity of the original data except in areas where scan resolution is poor. By lowering the weighting coefficients in those areas, the relatively inaccurate data can be accommodated and may not affect the quality of the retrieval.

Although we have shown, using simulated data, that the assimilation of single-Doppler radar data with a Cartesian prediction model is feasible, the truest test of this technique has to be done with real data. Application of this technique to a gust-front case observed during the Phoenix II experiment is under investigation and preliminary results have shown promise (see Sun et al., 1993). Further real data examination of the technique is being pursued.

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REFERENCES


