On the use of nonmethane hydrocarbons for the determination of age spectra in the lower stratosphere

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[1] On 9 April 2001, while approaching the West Coast of the North American continent, flight 20 of the NASA TRACE P campaign penetrated deeply into a stratospheric intrusion. From measurements aboard that flight we derive vertical profiles of nonmethane hydrocarbons (NMHCs) and other short-lived trace gases in the lower stratosphere using Dichlorofluoromethane, CF₂Cl₂, as the altitude scale. All profiles show an exponential decrease, which permits the description of their vertical distribution by a single parameter, the scale height. These scale heights are shown to be related to the respective lifetimes in a unique fashion. Using the approximation of a 1-D diffusive model with a constant eddy diffusion coefficient, K, and assuming constant lifetimes, we establish an analytical solution for this relation. By fitting this theoretical expression to the experimental data we can estimate K and thus obtain an approximate age spectrum of the form given by Hall and Plumb (1994). A much better fit to the experimental scale heights is obtained, when we allow the lifetimes to be height-dependent and calculate the theoretical scale heights numerically from a 1-D model. An optimization also suggests a constant K, but with a value of 0.46 m²/s, larger than those obtained from the fit of the analytical solution. The obtained age spectra should be valid for transit times not longer than 300 days.


1. Introduction

[2] Mean age and age spectrum of an air parcel have proven to be very useful concepts in the study of stratospheric transport [Hall and Plumb, 1994; Waugh and Hall, 2002]. They allow an efficient characterization of the action of transport, and thus provide a quantitative measure for the intercomparison of the different transport schemes used in models, as well as for the comparison between models and the transport in the real atmosphere.

[3] Mean age and age spectrum are based on the insight that any air parcel consists of a mix of fluid elements, each with its individual trajectory. The age spectrum of a stratospheric air parcel is defined as the frequency distribution of the transit times spent by the various fluid elements between their entry into the stratosphere at the tropical tropopause, the general source region of stratospheric air, and the point where the air parcel is observed. The mean age is given by the first moment of the age spectrum [Hall and Plumb, 1994].

[4] Age spectrum and mean age at any resolved location are readily computed for numerical models of the stratosphere. To obtain these quantities for the real stratosphere has proven a bit more laborious. Nevertheless, there are by now numerous empirical determinations of the mean age of air in many parts of the stratosphere. They are based on the measurement of tracers such as CO₂ or SF₆ [Bischof et al., 1980; Schmidt and Khedim, 1991; Nakazawa et al., 1995; Boering et al., 1996; Strunk et al., 2000; Andrews et al., 2001a; Maiss et al., 1996; Harnisch et al., 1996; Volk et al., 1997, respectively]. For these tracers, which are chemically inert or nearly inert in the stratosphere and have linearly increasing trends in the troposphere, the delay time after which a particular mixing ratio from the tropical tropopause arrives at a given point in the stratosphere corresponds exactly to the mean age at that point [Hall and Plumb, 1994].

[5] Empirical age spectra are more difficult to infer. Correspondingly only few such spectra have been published [Andrews et al., 1999; Johnson et al., 1999; Andrews et al., 2001b]. These studies make use of the fact that transient mixing ratio variations, such as interannual variability, or seasonal cycles are superimposed on the general trends of some of the tracers, such as CO₂ and water vapor. As a consequence, the determination of empirical age spectra demands a more detailed database than the determination of mean age: Namely time series of tracer mixing ratios in the various domains of the stratosphere in addition to a detailed history at the tropical tropopause.

[6] There is a different approach to derive stratospheric age spectra. Rather than on a temporal change in the source of one tracer, it relies on a number of tracers that have a
constant source but decay with time at different rates. Like the former, the latter also introduces a transport-dependent spatial variation in the stratospheric mixing ratios. In a recent paper, Schoeberl et al. [2005] used this approach in a 3-D chemical transport model together with satellite observations of CFCl3, CF2Cl2, CH4, and N2O to derive stratospheric age spectra.

[7] Here, we pursue two slightly different and complementary variants of this approach. The first considers the simple case, when the decay rates are constant in time and space, and thus independent of the individual trajectory of a fluid element. Then the stratospheric tracer distributions are directly linked to the age spectrum through a Laplace transformation [Schoeberl et al., 2000], which allows an analytical solution. In addition we use a numerical 1-D model with height-dependent life times to obtain age spectra. Both variants are applied to vertical profiles of NMHCs in the stratosphere. These are derived from the data of the DC-8 aircraft flight 20 of the TRACE P campaign flown on 9 April 2001, that during the latter part of the flight encountered a deep intrusion of stratospheric air. The short lifetimes of the NMHCs, a hundred days or less, limit the application to the lower stratosphere and to the section of the age spectrum at short transit times.

2. Database and Data Reduction

[8] The data presented here are part of a large file which is available electronically (http://www-gte.larc.nasa.gov/trace/TP_dat.htm). Figure 1 summarizes the geographical and meteorological situation during the last part of the flight 20. The horizontal projection of the flight track is shown in Figure 1a, and the vertical flight profile is shown in Figure 1b. The latter also includes the contour line of the 100 ppb O3 mixing ratio derived from the O3 lidar measurements aboard the DC 8 [cf. Browell et al., 2003; http://asd-www.larc.nasa.gov/lidar/trp/dial_20.html]. It serves as a surrogate for the tropopause altitude. Clearly the flight penetrated well, by about 6 km, into the stratosphere. The symbols distinguish between the samples taken while the air plane moved into the stratospheric intrusion, and samples taken when exiting the intrusion. The latter part of the flight effectively coincides with the final descent of the aircraft to the Dryden Air Base. For brevity the first part of that flight leg will be called “ascent” and the second part “descent.”

[9] Wind speed and direction at the sampling positions are also indicated (Figure 1a). The wind direction veered from west northwest at the beginning of the stratospheric part of the flight track to west south west at its end. The wind speed varied from about 75 km/h at 129°W longitude to about 150 km/h at 122°W longitude, where the maximum penetration into the stratosphere and the maximum wind speeds were reached.

[10] The backward trajectories for the two parts of the flight track are quite different. Three days earlier the air masses sampled during descent were clearly lined up along the western coastline of North America between about 46°N and 58°N latitude, whereas the air masses encountered during ascent were spread over the mid Pacific with a center at about 35°N latitude and 170°W longitude. In addition to different geographic origins of these air masses, their PV values, the - led RDF-PV (RDF, reverse domain filling), show also systematic differences. In particular, air masses observed during the descent are characterized by high and constant PV values (>4 PVU) while the air sampled during the ascent shows smaller and much more variable RDF-PV values between 2 and 6 PVU Thus we conclude that during the first part of the flight track, the composition of the observed air was influenced by mixing across the tropopause, whereas during the descent much older air masses from the lowermost stratosphere were sampled.

[11] Figure 1 also contains the mixing ratio profiles of the trace gases O3, CF2Cl2 (Figure 1c), and C2H6, C3H8 (Figure 1d), for better comparison also plotted against longitude. As to be expected for a pair of tracers with stratospheric and tropospheric origin, respectively, the mixing ratio profiles of O3 and CF2Cl2 are strongly anticorrelated. CF2Cl2 reaches a minimum mixing ratio of 468 ppt, O3 a maximum mixing ratio of 697 ppb between 122°W and 121°W longitude; they define the maximum penetration into the stratosphere. C2H6 and C3H8, two additional tracers of tropospheric origin, show a similar behavior as CF2Cl2 except that their mixing ratios decrease much faster with penetration into the stratosphere because of their short lifetimes. In fact, the C3H8 mixing ratio drops below 3 ppt, the detection limit of the gas chromatographic NMHC measurement [Colman et al., 2001].

[12] To convert the horizontal plots into vertical profiles, we need an unequivocal height scale. The complicated geometry of the intrusion (Figure 1b) makes it obvious that we cannot directly use the altitude of sample collection as the height coordinate. For the present we choose instead the CF2Cl2 mixing ratio as the vertical coordinate. Like N2O, which is often used for that purpose, it decreases on average monotonically with altitude in the stratosphere. Its advantage here is that CF2Cl2 was measured from the same samples as the NMHCs.

[13] Examples of the resulting vertical profiles are shown in Figure 2. Depending on the origin and lifetime of the trace gas considered four different categories of vertical profiles can be distinguished: Gases with a constant tropospheric source, which are nearly inert in the lower stratosphere, but have strong losses at higher altitudes, such as CH3Cl or CF2Cl2, are vertically distributed in the same manner. As a consequence CH3Cl when plotted against CF2Cl2 shows a linear decrease in that height scale. In contrast, a gas of stratospheric origin and virtually inert in the lower stratosphere, such as O3, exhibits a linear increase with CF2Cl2 as the height scale. Gases with an increasing tropospheric source and conserved everywhere in the stratosphere and above, such as CO2, have a very slight but curved decrease in the lower stratosphere. For these three types of gases the vertical distributions are dominated by mixing. Finally, for gases such as C3H8, which have a constant tropospheric source, but have lifetimes that are small compared to the timescale of vertical transport in the lower stratosphere, we observe a strong, apparently exponential decrease against CF2Cl2 as the height coordinate. For all four gas types there is reasonably good agreement between the data from ascent and descent: Apart from the tropospheric values the respective data points agree within their scatter and the resulting profiles from both data sets have the same functional forms.
2.1. Vertical Profiles of Short-Lived Trace Gases

The fact that short-lived gases decrease exponentially with \( \text{CF}_2\text{Cl}_2 \) as the height coordinate is emphasized in Figure 3. It summarizes the vertical profiles of the gases \( \text{CHCl}_3, \text{CO}, \text{C}_2\text{H}_6, \text{C}_2\text{H}_2, \text{C}_3\text{H}_8, \text{n}-\text{C}_4\text{H}_{10} \) observed during ascent and descent. All of these gases have short lifetimes, varying between about one week and two years in the lower stratosphere, and all those gases decrease exponentially as demonstrated by this arithmetic plot of their mixing ratios versus \( \text{CF}_2\text{Cl}_2 \). This holds in particular for the data collected during descent. A fit of a straight line to the latter data yields the characteristic scale height of the exponential decrease, \( \zeta_f \), where the index \( f \) indicates \( \text{CF}_2\text{Cl}_2 \) as height scale. The \( \zeta_f \) are listed in Table 1 along with their errors. For later reference the mean standard deviations of the data points from the fitted exponential decreases were also calculated, and incorporated in Table 1. As to be expected: The shorter the lifetime the faster the decrease (see Table 1).
The agreement between ascent and descent is good for the gases CO, C\textsubscript{2}H\textsubscript{6}, C\textsubscript{3}H\textsubscript{8}, apart from differences in the mixing ratios in the troposphere, which point to a tropospheric gradient away from the North American continent. It is, however, not so good for the vertical profile of CHCl\textsubscript{3} which during ascent deviates strongly from an exponential decay in the altitude range of 510 to 490 ppt CF\textsubscript{2}Cl\textsubscript{2}. A similar deviation is also found for CH\textsubscript{2}Cl\textsubscript{2} (not shown here), and is also expressed in the profile for C\textsubscript{2}H\textsubscript{2}. Thus it appears real and not an artifact of the measurement. At present there is no good explanation for it.

For reasons given below we do not include CHCl\textsubscript{3} in our further analysis. Nevertheless, the scatter in its profile during ascent serves as a warning that an individual vertical profile of a short-lived trace gas in the lower stratosphere may not necessarily follow an exponential decrease, even with CF\textsubscript{2}Cl\textsubscript{2} as the vertical scale and even when other, simultaneously measured, short-lived trace gases do show such a behavior.

The trace gas data measured during descent indicate that at least on this occasion the vertical profiles of short-lived tracers in the lower stratosphere are well approximated by an exponential decrease. Such an exponential decrease is also found for the NMHC data from the POLARIS campaigns, 1996 and 1997, albeit with a much larger scatter (F. Flocke, private communication, 2005). Thus an exponential decrease may very well represent the average pattern in the lower stratosphere. This observation is quite useful, because it provides a single parameter description of the vertical distributions of short-lived trace gases and thus a simple way of comparing them.

Finally, the CO values plotted in Figure 3 represent the directly measured mixing ratios. On the other hand stratospheric CO has another source besides the influx from the troposphere, namely the local oxidation of CH\textsubscript{4}. In the lower stratosphere, where the reaction of CH\textsubscript{4} is slow, this...
contributes about 5 ppb of CO, which in principle needs to be corrected for. If we subtract 5 ppb from the directly measured CO mixing ratios we still obtain an exponential decrease but with a scale height of 24.8 ± 1.3 ppt CF$_2$Cl$_2$ instead of 27.7 ± 1.6 ppt calculated from the directly measured data and listed in Table 1. Since the difference is relatively small, just barely within the respective 1σ errors, and the correction has an uncertainty of its own, we will proceed using the scale height for the directly measured CO values.

2.2. Relation of the CF$_2$Cl$_2$ Height Scale to Geometric Altitude

[19] For better comparison with existing vertical trace gas profiles we would like to relate the CF$_2$Cl$_2$ height scale to geometric altitude. This is not possible from the data of
flight 20 alone; it rather requires an average profile of CF$_2$Cl$_2$ in geometric altitude. Measured profiles are scarce for the years bracketing 2001. An earlier average CF$_2$Cl$_2$ profile from mid latitudes in Europe covering the period from 1988 to 1991 suggests a linear decrease in the lower stratosphere to about 19 km altitude [Fabian et al., 1996], but is likely to be influenced by the then still significant increase in tropospheric CF$_2$Cl$_2$. Two more recent individual profiles from 1996 and 1998 over New Mexico, 34°N, also indicate an approximately linear decrease of CF$_2$Cl$_2$ in the lower stratosphere up to 20 km altitude [Moore et al., 2003]. However, for further and more detailed information we have to turn to the CF$_2$Cl$_2$ distributions generated by 3-D models. Figure 5 shows the zonally and monthly averaged profiles of CF$_2$Cl$_2$ for April and the latitudes 32.9, 35.7, 38.6, 41.4, and 44.3°N. They were generated in a special run of the MOZART-3 model with WACCM 1b meteorological fields and a 106 species chemical mechanism [Kinnison et al., 2007]. Since that run did not cover the year 2001, we chose the profiles from 1999. By that time the tropospheric increase of CF$_2$Cl$_2$ had already flattened out, so that they should closely represent the distribution for 2001 as well. All the profiles in Figure 5, even those north of 40°N latitude which are slightly undulating are quite closely approximated by a linear decrease between 10 km and 18 km altitude, with slopes systematically increasing with latitude. Thus we conclude that on average the CF$_2$Cl$_2$ height scale correlates linearly with geometric altitude for the stratospheric altitude range considered here. To determine the slope we use the data from the present flight (Figure 1): At the point of deepest penetration into the stratosphere the vertical distance between flight level and tropopause was 12 km – 6 km = 6 km in geometric altitude, and 469 ppt – 538 ppt = –69 ppt in the CF$_2$Cl$_2$ height scale. (The value of 538 ppt corresponds to the CF$_2$Cl$_2$ mixing ratio observed at 100 ppb of O$_3$ when the aircraft exited the stratospheric intrusion). From these numbers we deduce a slope of –11.5 ppt/km, i.e., z$^{-}$ ≈ –11.5 z. With a precision of 3.5 ppt for the measurement of CF$_2$Cl$_2$ and an estimated uncertainty of 0.5 km in the tropopause altitude derived from O$_3$ this slope has a relative 1σ uncertainty of 11%. As is apparent from Figure 5, slopes that steep begin during descent had recently been advected from the north. Assuming an average tropopause height of 12 km the observed trace gas profiles cover the height range of 12 km to 18 km. The empirical observation that CF$_2$Cl$_2$ varies linearly with height over the altitude range considered not only provides a simple justification for the use of CF$_2$Cl$_2$ as height coordinate in the presentation of the data, it also allows an analytical determination of the age spectra. Moreover, since the slope of the linear decrease in CF$_2$Cl$_2$ is

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\zeta_0$, ppt CF$_2$Cl$_2$</th>
<th>$k_{OH}$, cm$^{-3}$/s</th>
<th>$k_{Cl}$, cm$^{-3}$/s</th>
<th>$\tau_0$, $s$</th>
<th>$\tau$, days</th>
<th>$\sigma_{\text{me}}$, ppt CF$_2$Cl$_2$</th>
<th>$\sigma_{\text{e}}$, ppt CF$_2$Cl$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHCl$_3$</td>
<td>20</td>
<td>3.1$\times 10^{-14}$</td>
<td>1.8$\times 10^{-14}$</td>
<td>745</td>
<td>745</td>
<td>4.36</td>
<td>5.23</td>
</tr>
<tr>
<td>CO</td>
<td>27.7 ± 1.6</td>
<td>1.7$\times 10^{-13}$</td>
<td>3.0$\times 10^{-13}$</td>
<td>136</td>
<td>136</td>
<td>4.36</td>
<td>3.99</td>
</tr>
<tr>
<td>C$_2$H$_6$</td>
<td>19.1 ± 0.8</td>
<td>3.8$\times 10^{-13}$</td>
<td>4.2$\times 10^{-13}$</td>
<td>127$\times e^{-7}$</td>
<td>107</td>
<td>3.81</td>
<td>2.38</td>
</tr>
<tr>
<td>C$_2$H$_4$</td>
<td>12.4 ± 0.7</td>
<td>4.7$\times 10^{-13}$</td>
<td>1.36</td>
<td>48.1$\times e^{-7}$</td>
<td>44.7</td>
<td>3.53</td>
<td>2.78</td>
</tr>
<tr>
<td>n-C$<em>4$H$</em>{10}$</td>
<td>7.9 ± 0.6</td>
<td>4.7$\times 10^{-13}$</td>
<td>1.36</td>
<td>28.3$\times e^{-7}$</td>
<td>26</td>
<td>3.04</td>
<td>2.78</td>
</tr>
<tr>
<td>n-C$<em>4$H$</em>{10}$</td>
<td>4.6 ± 0.4</td>
<td>1.7$\times 10^{-12}$</td>
<td>1.36</td>
<td>11.3$\times e^{-7}$</td>
<td>10.7</td>
<td>2.55</td>
<td>2.89</td>
</tr>
</tbody>
</table>

*Also given are the rate constant for the reactions with OH and Cl, $k_{OH}$ and $k_{Cl}$, the error factor of $k_{OH}$, $f_{OH}$ all for the temperature $T = 216$ K and the pressure 200 hPa [Sander et al., 2003; Atkinson et al., 2002], and the mean standard deviations of the measured data from the fitted exponential decreases, $\sigma_m$, and from the numerically calculated profiles, $\sigma_e$. 

The tropopause values, $\tau_{0i}$, are based on $OH = 5\times 10^5$ cm$^{-3}$ and $Cl = 1.2\times 10^5$ cm$^{-3}$ (see text). The height, z, is in km.

**Figure 4.** Correlation between the mixing ratios of C$_2$H$_6$ and n-C$_4$H$_{10}$ for the determination of the scale height in the vertical decrease of the

**Figure 5.** Zonally and monthly averaged vertical profiles of CF$_2$Cl$_2$ for April and the latitudes 32.9, 35.7, 38.6, 41.4, and 44.3°N generated by MOZART 3 (see text for details).
2.3. Lifetimes of Short-Lived Trace Gases

[20] The trace gases listed in Figure 3 are destroyed in the stratosphere by several mechanisms. Most important are the reactions with OH and Cl. All the other loss processes: Photolysis, reaction with O₃, O¹D, O³P, and Br are negligible in the height range considered here. The rate constants for the reactions with OH and Cl, k_{OH} and k_{Cl}, at the temperature T = 216 K and the pressure p = 200 hPa are also listed in Table 1.

[21] For the calculation of the lifetimes the concentration fields of OH and Cl, in particular their vertical distributions are also required. The height dependence of OH appears to be small. The measurements by Hanisco et al. [2001] showed the OH concentration in the lower stratosphere to depend essentially on the solar radiation and not on the location, i.e., to be independent of altitude. Similar results are found in 3-D model calculations. Figure 6 (top) portrays the mean meridional O for April calculated from the same MOZART-3 run as the CF₂Cl₂ profiles. It varies less than 20% in the lower stratosphere at equatorial and northern mid latitudes. We, therefore, assume the OH concentration to be constant with altitude. Since stratospheric air originates at the tropical tropopause it must spend an appreciable part of its transit time south of the latitude of the observation point. This should also be true in the present case, although the immediate trajectories of the air masses originated at around 50°N latitude. We therefore assign a value of 5 × 10⁵ cm⁻³ to the OH concentration, the average value south of 30°N latitude in the lower stratosphere.

[22] In contrast to OH the Cl concentration increases considerably with altitude (Figure 6, bottom): For example, at 30°N from about 1.2 × 10⁵ cm⁻³ at 12 km altitude to 5.6 × 10⁵ cm⁻³ at 20 km altitude (corresponding to 470 ppt CF₂Cl₂ in the model), i.e., more than a factor of 4. As a consequence the alkanes, which react significantly also with Cl, exhibit a marked decrease in lifetime with height. The lifetime of C₂H₆₅, for instance, which exhibits the largest change, decreases from 127 days at the tropopause to 37 days at 20 km altitude. The lifetimes of the gases, CO, and CHCl₃ which react mainly with OH, are virtually constant with altitude. For simplicity the height dependence of the lifetimes is approximated by an exponential function, for the alkanes with a scale height, h, of 7 km representing the common average. For C₂H₂₆, whose reactivity with Cl is lower, a scale height of 12.5 km is used. The lifetimes at the tropopause (12 km) based on the above OH and Cl concentrations and their exponential decreases with altitude are also listed in Table 1.

[23] To account for the height dependence of the lifetimes and retain the advantage of a single parameter characterization, we also calculate the lifetimes averaged over the trace gas columns in the observed altitude range:

\[ \tau_i = \frac{\int_0^{470} \rho(z_f) \cdot m_i(z_f) \cdot \tau_{i,0} \cdot \exp(-z_f/h_i) dz_f}{\int_0^{540} \rho(z_f) \cdot m_i(z_f) dz_f} \]  

[24] Here \( \rho(z_f) = \rho_0 \cdot \exp(-z_f/80) \) is the air density, \( m_i(z_f) = m_{i,0} \cdot \exp(-z_f/z_{f,0}) \) the mixing ratio, both in the CF₂Cl₂ height scale, and the index 0 refers to the tropopause values. With the slope of the CF₂Cl₂ profile given above the density scale height of 7 km transforms into \( H_f = 80 \) ppt in the CF₂Cl₂ height scale.

[25] The so averaged \( \tau_i \) are also listed in Table 1. They do not differ as much from the \( \tau_{i,0} \) at the tropopause as one might have expected from the steeply increasing Cl profile: especially the average \( \tau \) for n-C₈H₁₆, which is quite close to \( \tau_0 \) at the tropopause, because this gas does not penetrate deeply enough into the stratosphere to experience the higher Cl concentrations.

[26] Given the large uncertainty factors in the rate constants and the uncertainties in the radical concentrations, the lifetimes, \( \tau_i \) and \( \tau_{i,0} \) turn out to be much more uncertain than the scale heights \( z_{f,i} \).

2.4. Relation Between \( \zeta_{f,i} \) and \( \tau_i \)

[27] The experimental scale heights \( \zeta_{f,i} \) derived in section 2.1 are closely related to the average lifetimes, \( \tau_i \),...
Figure 7. Empirical and modeled relations between scale height $\zeta_f$ and the average lifetime $\tau_f$. The empirical data points are indicated by the squares, whose vertical extent exceeds the error in the $\zeta_f(i)$. The horizontal error bars represent the error contributed by the uncertainty in the reaction rate with OH alone. The green line represents a fit of equation (9) for $\zeta_f(\tau)$ to all five empirical data points, and the red line represents a fit to the data from CO and C$_2$H$_2$ only. The $\zeta_f(i)$ obtained numerically from a 1-D model and height-dependent lifetimes are indicated by the open triangles. The blue line, $\zeta_{el}(Z)$, is numerically calculated for tracers with an exponential decrease in $\tau$ and a scale height of 7 km.

specified in the preceding section. When plotted against each other (Figure 7) the data points indicate that there is an unequivocal functional relation, $\zeta_f = \zeta_f(\tau)$, between the scale heights and the corresponding average lifetimes in the lower stratosphere. Despite the uncertainties in $\zeta_{el}$ and $\tau_f$, $\zeta_f(\tau)$ seems to be fairly tightly characterized at least over the range covered by the present data. The error in the scale heights is contained within the vertical extent of the symbols of Figure 7. The error bars represent the error contributed by the uncertainty of $k_{OH}$ alone, obviously an underestimate of the total error in $\tau_f$. It serves, however, to demonstrate that the errors in the $\tau_f$ are much larger than those in the $\zeta_f$.

[28] Plotted in Log/Log format as in Figure 7 the data points appear to follow a straight line. Although we will make no further use of it here, for reference we mention that a fit yields:

$$\log \zeta_f = (0.0406 \pm 0.04) + (0.68 \pm 0.04) \times \log \tau, \quad (2a)$$

or $\zeta_f = (0.87 \times \tau)^{0.68}, \quad (2b)$

where $\tau$ is in units of days, $\zeta_f$ in ppt.

[29] We note that CHCl$_3$ is not included in Figure 7: A gas with a lifetime of 745 days, i.e., of more than 2 years, cannot be regarded as short-lived in the lower stratosphere. On the other hand, the $\zeta_f$ found empirically for CHCl$_3$ is only 20 ppt, about the same as that for C$_2$H$_6$, which has a much shorter lifetime of 107 days. The same applies to CH$_2$Cl$_2$ with very nearly the same theoretical lifetime and empirical scale height as CHCl$_3$. This points to an inconsistency. Either there is a significant sink missing from our calculation of the lifetimes of CHCl$_3$ and CH$_2$Cl$_2$ or the measurement of these two substances is biased by losses during collection or storage of the air samples. In either case they currently fall outside the ensemble of trace gases considered here, and we will continue to rely on the relation $\zeta_f(\tau)$ as defined by the 5 gases, CO, C$_2$H$_6$, C$_2$H$_2$, C$_3$H$_8$, n-C$_4$H$_{10}$.

[30] We also note that with an explicit expression for $\zeta_f(\tau)$ the observed exponential decrease with height of these gases can be stated as a function of $\tau$:

$$m_i(\tau_f) = m_{i,0} \cdot \exp\left(-z_f/\zeta_f(\tau_f)\right) \quad (3)$$

3. Discussion

[31] To relate the observed vertical trace gas distributions to the age spectrum we use two approaches. First, with a few simplifying assumptions and the 1-D vertical diffusion equation we derive an analytical expression for the exponential decrease. This at the same time gives an analytical expression for the dependence of the scale heights on the lifetimes, and provides some theoretical insight into what controls $\zeta_{el} = \zeta_f(\tau_f)$. Second, we solve the 1-D vertical diffusion equation numerically. This allows one to relax the assumptions and still obtain an age spectrum through a fit of the modeled trace gas profiles to the experimental ones by optimizing the profile of the vertical eddy diffusion coefficient, K.

3.1. Simplified Analytical Solution

[32] The first simplification assumes that the lifetimes are constant in time and space. Then the stratospheric steady state distribution of a tracer with a constant mixing ratio $m_{i,0}$ at the (tropical) tropopause and a lifetime $\tau_f$ can be directly expressed by the age spectrum [cf. Schoeberl et al., 2000]:

$$m_i(X, \tau_f) = m_{i,0} \cdot \int_0^\infty \exp\left(-t/\tau_f\right) \cdot G(X, t) dt \quad (4)$$

[33] $G(X, t)$ is the age spectrum at the position $X(x, y, z)$. Equation (4) can be understood intuitively. The age spectrum in the integral provides the weighting function with which transport trajectories of different transit times, $t$, contribute to the mass of the air parcel observed at $X$. The term $\exp\left(-t/\tau_f\right)$ describes the fraction of the original mixing ratio at the tropical tropopause left after traveling for the transit time. Since $\tau_f$ is constant in space and time, this fraction depends only on the transit time, but not on the actual path taken. The integral over time then represents the local steady state mixing ratio of the tracer generated by adding up the various contributions. Equation (4) also makes it clear that tracers with increasing $\tau_f$ probe increasing parts of the age spectrum and that in general it will need a number of such tracers to determine $G(X, t)$.

[34] The integral in equation (4) constitutes a Laplace transform, i.e., $m_i(X, \tau_f)$ normalized by the constant factor
$m_{t,0}^{-1}$ is the Laplace transform of $G(X, t)$. Thus, conversely, if we know the stratospheric trace gas distribution and its dependence on $r$, we should in principle be able to obtain $G(X, t)$ through an inverse Laplace transform of $m(X, r)$. Equation (4) holds generally. It can be further simplified when we adapt it to the present case, where we only consider the vertical dimension, $z$, by assuming that transport is facilitated by a 1-D vertical diffusion model with a constant eddy diffusion coefficient, $K$. In this case the age spectrum is known [Hall and Plumb, 1994]:

$$g(z, t) = \frac{z}{\sqrt{\pi \cdot 4 \cdot K \cdot t^3}} \exp \left( \frac{z^2}{2H} - \frac{K \cdot t}{4 \cdot H^2} - \frac{z^2}{4 \cdot K \cdot t} \right)$$

Equation (5) is the solution of the partial differential equation for the vertical diffusion of an inert tracer with the mixing ratio $m(z, t)$

$$\rho \cdot \frac{\partial m(z, t)}{\partial t} = \partial \frac{\partial}{\partial z} \rho \cdot K \cdot \frac{\partial m(z, t)}{\partial z}$$

for the boundary condition $m(0, t) = \delta(t)$; that is, the tracer mixing ratio is introduced as a pulse at the tropopause, $z = 0$ and the time $t = 0$. The air density $\rho = \rho_0 \cdot \exp(-z/H)$. Inserting $g(z, t)$ for the simplified 1-D case into equation (4) yields:

$$m_i(z, \tau_i) = m_{i,0} \cdot \int_0^\infty \exp(-t/\tau_i) \cdot g(z, t) dt$$

In this case the Laplace transform (7) has an analytical solution [see Schoeberl et al., 2000]. It is given by the exponential function:

$$m_i(z, \tau_i) = m_{i,0} \cdot \exp(-z/\zeta(\tau_i))$$

with

$$\zeta(\tau_i) = \frac{2H}{\left(1 + \frac{2H}{\pi \tau_i} - 1 \right)}$$

The vertical mixing ratio profile $m_i(z, \tau_i)$, equation (8) is of course also the solution of the ordinary differential equation

$$\frac{\partial}{\partial z} \rho \cdot K \frac{\partial m_i(z, \tau_i)}{\partial z} - \rho \frac{m_i(z, \tau_i)}{\tau_i} = 0$$

under the appropriate boundary conditions.

Equation (9) provides the desired analytical approximation of the functional dependence of $\zeta$ on $\tau$. It should thus explain the essential features of $\zeta(\tau_i)$ apparent in Figure 7. In addition to $\tau$, the independent variable, $\zeta(\tau)$ from equation (9) depends on two parameters, namely the scale height of the air density, $H$, and the (constant) eddy diffusion coefficient, $K$. The first parameter is known: $H = 7$ km in geometric altitude, or 80 ppt when transformed into the CF$_2$Cl$_2$ height scale using the mean vertical gradient in CF$_2$Cl$_2$ given in section 2.2. By optimizing K we can fit equation (9) to the observed data pairs $\zeta_{f,5}, \tau_i$ in Figure 7. With a fit to all 5 trace gases we obtain an average $K_{f,5} = 2.8$ ppt$^2$/day in the CF$_2$Cl$_2$ height scale. The resulting curve $\zeta_{f,5}(\tau)$ is included in Figure 7. Although hard to discern over the short data interval presented in Figure 7, it bends slightly upward for larger $\tau$. We note also, that $\zeta$ and the fitted $K_f$ depend only weakly on $H_f$. This is due to the fact that $\tau_i < 4$ H$_f$/K$_f = 9100$ days for all $\tau_i$ considered. Indeed for $\tau_i \sim 1$ day $\zeta$ approaches ($K_f \tau$)$^{0.5}$ and thus becomes independent of $H_f$ which allows a simple geometric solution for $K_f$ by extrapolating $\zeta_{f,5}(\tau)$ to $\tau_i = 1$ day in Figure 7. This also means that the error of 11% in the slope of the CF$_2$Cl$_2$ profile introduces only a small error, 2%, in the derived $K_f$.

When we restrict the set of experimental $\zeta_{f,5}, \tau_i$ to the ones for CO, and C$_2$H$_2$, for which the condition of $\tau(\tau_i) = 0$ is best met, the fit of equation (9) to the data yields a significantly larger average $K_{f,5}$ of 3.6 ppt$^2$/day. The resulting $\zeta_{f,5}(\tau)$ runs parallel and above $\zeta_{f,5}(\tau)$ (see Figure 7). Both curves agree quite well with the experimental $\zeta$ to which they were fitted. This means that the vertical profiles calculated from equation (8) with the above parameters $H_f$ and $K_f$ will match the measured height profiles of the trace gases in Figure 3 reasonably well. Moreover, when $K_{f,5}$ and $K_{f,5}$ are transformed to geometric altitude, $z$, using the slope of the average CF$_2$Cl$_2$ profile given in section 2.2, we obtain $K_2 = 0.32$ m$^2$/s and $K_5 = 0.25$ m$^2$/s. Both values agree quite well with previous estimates of K in the lower stratosphere [cf. Ehhalt et al., 2004]. In this case, however, the error of 11% in the slope of the CF$_2$Cl$_2$ decrease propagates into a large error of 21%. This is due to the fact that the slope enters squared in the conversion of $K_f$ to $K$.

In this case the derived average K together with the fixed H fully specifies the age spectrum given by equation (5) in geometric or CF$_2$Cl$_2$ altitude scales. That means that the so calculated average age spectra have the form suggested by Hall and Plumb [1994]. They will be further discussed in section 3.3 below.

3.2. Numerical Calculation of Trace Gas Profiles and $\zeta_{f,5}$ From a 1-D Model

Finally, we employ a simple 1-D model, to numerically calculate the vertical profiles of the 5 trace gases. It also calculates the vertical profile of CF$_2$Cl$_2$ with the aim to derive $\zeta_{f,5}$ from the model, and to match them to the experimental values by optimizing the $K$ profile. This implies that the model generated trace gas profiles in the CF$_2$Cl$_2$ height scale are kept as nearly exponential as possible. A numerical approach has the advantage that it does not require the approximations of constant $\tau_i$ and of constant $K$. Neither does it require the CF$_2$Cl$_2$ profile to be linear with altitude in the lower stratosphere, and indeed the modeled CF$_2$Cl$_2$ profile is slightly curved. As a first approach to height-dependent lifetimes we select the $\tau_i(z)$ derived in section 2.3 and listed in Table 1. The loss of CF$_2$Cl$_2$ is dominated by the photolysis rate, whose vertical profile is chosen to represent equinox at 30°N latitude.

As it turns out with this selection of $\tau_i(z)$ a surprisingly good fit for all 5 tracers is obtained when a vertical profile of $K_m = 0.46$ m$^2$/s, constant with altitude, is used. This agreement is first demonstrated in Figure 3, where the numerically calculated trace gas profiles are superimposed on the measured data. The modeled profiles are not exactly
exponential with CF$_2$Cl$_2$ as height coordinate. However, the
quality of the fit of the modeled profiles to the data is at
least as good as that of the linear regression lines fitted
directly to the data. This is underlined by the mean standard
deviations of the data from the numerically calculated
profiles which are given in Table 1 and compare favorably
with those for the linear regression lines. Moreover, the
modeled profiles can be well approximated by an exponen-
tial decrease. Thus the numerically calculated profiles can
also be reduced to a one parameter description. The slight
curvature in their semilog plots versus CF$_2$Cl$_2$ means,
however, that the exponential decrease fitted to the numer-
ically calculated profiles, and thus the model derived $\zeta_{i,j}$
depend slightly on the height interval of the fit. To be
comparable with the measurements, we chose the intervals
to agree with the respective height ranges over which the
data were obtained. In Figure 7 the so calculated $\zeta_{i,j}$ are
compared to the experimental ones. The agreement is
excellent.

[42] Altogether the numerically calculated trace gas pro-
files very closely match the measured ones over the full
altitude range observed. Nevertheless the result of a con-
stant K profile is not exactly curved: Slightly curved
variants of that K profile give similarly good fits. The
constant K is adopted as the simplest form of the K profile
compatible with the data. We emphasize, however, that now
the condition of a constant K is not an a priori assumption,
but rather the result of a fit. The apparent difference
between the numerically derived K and the analytically
calculated ones is exaggerated by two factors: First the
average vertical gradient of the modeled CF$_2$Cl$_2$ profile is
weaker by about 10% than the empirical one adopted in
section 2.2 and used in the analytical calculation of K. In the
conversion of K in the CF$_2$Cl$_2$ height scale to K in the
gometric height scale that gradient enters squared. This
causes a factor of 1.25 difference. The second factor results
from the averaging procedure for $\tau(z)$ by equation (1). That
weighted average favors the lower altitudes, and results in
larger $\tau_i$ than would be obtained from an unweighted
average. Since $\zeta \sim \sqrt{K(z)}$ (see section 3.1) this induces
a bias toward smaller values in the K derived analyti-
cally for a given $\zeta$. The difference in averaging amounts a factor
of 1.05 for K$_2$ and a factor of 1.1 for K$_3$. Accounting for
these two factors leaves an actual ratio of 1.1 between K$_2$
and K$_m$ and a ratio of 1.34 between K$_3$ and K$_m$ owing to
the introduction of an explicit height dependence in $\tau$.

[43] For further comparison Figure 7 also includes a
curve $\zeta_{emp}(\tau)$, blue line, numerically calculated for tracers
with an exponential decrease in $\tau$ with a scale height of 7 km
(Table 1), and K$_m = 0.46$ m$^2$/s. Its gradient is slightly steeper
than the analytically derived curves.

[44] Because of the constant K profile the age spectrum
derived from the numerical model also has the form of
equation (5). In the following section 3.3 it will be com-
pared to the analytical age spectra.

3.3. Age Spectra

[45] Figure 8 compares the model derived age spectrum
$g_m(z,t)$, with the two analytical age spectra, $g_s(z,t)$, $g_2(z,t)$, for
the altitudes 14 km, 16 km, 18 km and 20 km. These
altitudes correspond to CF$_2$Cl$_2$ mixing ratios of 545, 530,
512, 487, and 454 ppt, respectively, in the numerical 1-D
model calculation. $g_m(z,t)$ can also be obtained numerically
from the 1-D model by introducing a $\delta$ function shaped
pulse of the mixing ratio at the tropopause and following its
propagation in time at the various altitudes. The so calculated
spectra agree exactly with those derived from equation (5) for
the same altitudes.

[46] The age spectra in Figure 8 are cut off at 300 days.
Because of the short lifetimes of the trace gases considered
here the age spectrum at longer transit times does not
contribute to the integral in equation (7), and thus leaves
no signature in the vertical profiles of the mixing ratios. In
turn the measured trace gas profiles contain no information
about that part of the age spectrum. Consequently this
analysis does not allow any statement about the older part
of the age spectrum. We note, however, that the age spectra
of the form of equation (5) could be extended arbitrarily in
time. This corresponds to the implicit assumption that the
functional form of $\zeta$, equation (9), would also be applicable
at longer lifetimes.
Clearly the three age spectra differ significantly. Because it requires fewer approximations and affords the best fit to all 5 measured trace gas profiles, we consider \( g_m(z, t) \) the most realistic of the presented age spectra. Age spectrum \( g_s(z, t) \) derived from the 2 trace gases which approximate the condition of constant \( \tau_i \) is about 30% lower in maximum amplitude and, as expected from its smaller \( K \), expanded in time. The difference provides an indication of the sensitivity of the derived age spectra to the assumptions, mainly about the gradient in the vertical CF\(_2\)Cl\(_2\) profile but also about the height dependence of \( \tau_i \) and its averaging. There are other sources of uncertainty. Probably the most important are the errors in the absolute values of the \( \tau_i \). These are given by the errors of the rate constants and by the uncertainties in the OH and Cl fields experienced by the trace gases. Some of these uncertainties, e.g., that in the OH field, may result in a common scaling of the \( \tau_i \). An error of this type will translate into a linear contraction or expansion in the timescale of the age spectrum, as suggested by the inverse Laplace transform. Most errors, however, will result in displacements of the \( \tau_i \) relative to each other. Such displacement could change the form of the \( \zeta(\tau) \) curve in Figure 7, and consequently the value of the fit parameter \( K \).

A further source of error is the 1-D modeled profile of \( H \). It may even require a height-dependent \( K(z) \) for optimum fit. A much better fit to the empirical \( \zeta(z) \) is obtained, when we allow the lifetimes to be height-dependent and calculate the theoretical \( \zeta \) numerically from a 1-D model. This fit also suggests a constant \( K_m = 0.46 \text{ m/s} \), larger than that obtained from the fit of the analytical solution. The obtained age spectra should be valid for transit times to about 300 days. Because of the short lifetimes, trajectories with longer transit times do not contribute to the observed mixing ratios and thus leave no signature in the vertical trace gas profiles. A simple analysis like the present one can only account for the expression of the most salient factors influencing the derived age spectrum, namely (temporally constant) vertical transport and height-dependent lifetimes.

It appears, however, that these are also the dominant factors. The study of other influence factors, such as 3-D transport and lifetimes varying with time, will require the use of an appropriate 3-D model.

In any case, the age spectra derived here are specific for the location and time of the sampling and cannot claim to represent regional or global averages. Although there are a number of age spectra from 3-D chemical transport models in the literature [see Hall et al., 1999; Waugh and Hall, 2002], and even a few empirical ones derived from measurements of CO\(_2\) [cf. Andrews et al., 2001a], or other trace gases [Schoeberl et al., 2005], we could find none which would match the latitude and the altitude range considered here. Thus a comparison with published age spectra does not seem feasible.

We can, however, tentatively compare the age of air predicted from the present age spectra, for instance from \( g_m(z, t) \), with the mean age profiles in the literature. This obviously implies the assumption that \( g_m(z, t) \) can be extrapolated to transit times much longer than those covered by the measured trace gas profiles (see also the beginning of this section). Since \( g_m(z, t) \) is of the form given by equation (5) the vertical age profile of stratospheric air, \( \Gamma(z) \), is given by

\[
\Gamma(z) = \frac{z - H}{K} \quad (11)
\]

[Hall and Plumb, 1994], where the parameters \( K = 0.46 \text{ m/s} \) and \( H = 7 \text{ km} \) are those from \( g_m(z, t) \), and \( z \) is counted from the tropopause. With these values \( \Gamma = 6 \text{ km} \), about the highest altitude above the tropopause reached here, becomes 2, 9 y. This value is in good agreement with the corresponding literature values of \( \Gamma(6 \text{ km}) \) for 30–35°N latitude [see Andrews et al., 2001b; Hall et al., 1999], which could imply that \( g_m(z, t) \) is not too far from the zonal and annual average.

## 4. Summary and Conclusions

From trace gas measurements during the penetration of a stratospheric intrusion we derived vertical profiles of NMHC and other short-lived trace gases in the lower stratosphere. All these profiles show an exponential decrease with the mixing ratio of CF\(_2\)Cl\(_2\) which serves as the altitude scale. This allows describing their vertical distribution by one parameter, the scale height. The empirical scale heights are shown to be related to the respective lifetimes of the trace gases in a unique fashion, \( \zeta(\tau) \). Using the approximation of a 1-D diffusive model with a constant eddy diffusion constant, \( K \), and assuming constant lifetimes we can establish an analytical solution for this relation. By fitting the theoretical expression for \( \zeta(\tau) \) to the empirical data we can estimate \( K \) and thus obtain an approximate age spectrum. A much better fit to the empirical \( \zeta \) is obtained, when we allow the lifetimes to be height-dependent and calculate the theoretical \( \zeta \) numerically from a 1-D model. This fit also suggests a constant \( K_m = 0.46 \text{ m/s} \), larger than that obtained from the fit of the analytical solution. The obtained age spectra should be valid for transit times to about 300 days. Because of the short lifetimes, trajectories with longer transit times do not contribute to the observed mixing ratios and thus leave no signature in the vertical trace gas profiles. A simple analysis like the present one can only account for the expression of the most salient factors influencing the derived age spectrum, namely (temporally constant) vertical transport and height-dependent lifetimes. It appears, however, that these are also the dominant factors.

The study of other influence factors, such as 3-D transport and lifetimes varying with time, will require the use of an appropriate 3-D model.

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