The Retrieval of Asymmetric Tropical Cyclone Structures Using Doppler Radar Simulations and Observations with the Extended GBVTD Technique

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ABSTRACT

The ground-based velocity track display (GBVTD) technique is extended to two Doppler radars to retrieve the structure of a tropical cyclone's (TC's) circulation. With this extension, it is found that the asymmetric part of the TC radial wind component can be derived up to its angular wavenumber-1 structure, and the accuracy of the retrieved TC tangential wind component can be further improved. Although two radar systems are used, a comparison with the traditional dual-Doppler synthesis indicates that this extended GBVTD (EGBVTD) approach is able to estimate more of the TC circulation when there are missing data. Previous research along with this study reveals that the existence of strong asymmetric radial flows can degrade the quality of the GBVTD-derived wind fields. When a TC is observed by one radar, it is suggested that the GBVTD method be applied to TCs over a flat surface (e.g., the ocean) where the assumption of relatively smaller asymmetric radial winds than asymmetric tangential winds is more likely to be true. However, when a TC is observed by two radar systems, especially when the topographic effects are expected to be significant, the EGBVTD rather than the traditional dual-Doppler synthesis should be used.

The feasibility of the proposed EGBVTD method is demonstrated by applying it to an idealized TC circulation model as well as a real case study. Finally, the possibility of combining EGBVTD with other observational instruments, such as dropsonde or wind profilers, to recover the asymmetric TC radial flow structures with even higher wavenumbers is discussed.

1. Introduction

The capability of providing reflectivity and wind information with high temporal and spatial resolutions makes Doppler radar an important instrument for the observations of various severe weather phenomena such as tropical cyclones (TCs). Early applications of Doppler radar to the study of the vortex structure (e.g., Donaldson 1970), or the intensity and center position of an axisymmetric TC (e.g., Baynton 1979; Wood and Brown 1992), relied on the pattern recognition of the Doppler wind measurements. The results were thus usually very limited qualitative descriptions. Moreover, observations made by airborne, as well as ground-based Doppler radars (e.g., LaSeur and Hawkins 1963; Hawkins and Rubsam 1968; Marks and Houze 1987; Marks et al. 1992; Stewart and Lyons 1996; among others), frequently revealed the occurrence of asymmetric TCs. Their structures were too complicated to be resolved in detail by the simple pattern recognition method. To deduce the three-dimensional primary TC circulation in a quantitative sense, Lee et al. (1994) proposed a method called velocity track display (VTD). The VTD technique and its extension, the extended VTD

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(EVTD; Roux and Marks 1996), utilize data collected by an airborne Doppler radar system. Because of the limitations set by flight durations and operational restrictions over land, the airborne Doppler radars are often more suitable for the observations of TCs over the open ocean. Therefore, for the purpose of accommodating the deployment of the Weather Surveillance Radar-1988 Doppler (WSR-88D) network in the United States, Lee et al. (1999) and Lee and Marks (2000) reformulated the VTD equations and developed the so-called ground-based velocity track display (GBVTD) method. The successful applications of GBVTD to real case studies (e.g., Lee et al. 2000; Murillo et al. 2001; Chang et al. 2002; Harasti et al. 2004) demonstrated its validity and usefulness in terms of retrieving the three-dimensional TC circulation using only single-Doppler radar data. Lee and Wurman (2005) also extended the use of GBVTD to diagnose the small-scale structures within a tornado.

Despite these encouraging results, there are two fundamental limitations embedded in the GBVTD algorithm: 1) the information of the mean flow is aliased into the retrieved mean tangential wind and cannot be separated; 2) for the TC radial winds, only the mean values, or the axisymmetric components, can be recovered for a given analysis ring (see the next section for the definition). Consequently, the GBVTD-derived TC radial winds exhibit a somewhat unrealistic concentric pattern, with a positive (negative) value representing the outflow (inflow) motion.

The purpose of this research is to improve the technique by eliminating the limitations. The original GBVTD formulation is briefly introduced in section 2. The new approach, named the extended GBVTD (EGBVTD), is formulated in section 3. Section 4 discusses the construction of an analytical TC model from which the wind data are collected by virtual Doppler radars and applied to verify the concept of the proposed EGBVTD algorithm. Section 5 describes the results of the retrieval experiments using the idealized TC, followed by a real case study in section 6. Some possible extensions, which combine the EGBVTD approach with other instruments such as dropsonde or wind profilers, are discussed in section 7. Section 8 gives the conclusions.

2. The original GBVTD formulation

Only a brief explanation of the original GBVTD formulations is given in this section. Interested readers can refer to Lee et al. (1999) for more details regarding its assumptions and derivations. For clarity and simplicity, the same symbols and definitions as those used in Lee et al. (1999) are also adopted in this article. Figure 1 shows the GBVTD geometry and symbols. The circle with a radius $R$ is defined as the GBVTD analysis ring. After neglecting the contributions from the vertical velocity and terminal velocity, the horizontal projection of the observed Doppler velocity ($\hat{V}_d$) can be expressed by

$$\frac{\hat{V}_d}{\cos \phi} = V_M \left[ \cos(\theta_T - \theta_M) \left( \frac{1 - \cos\alpha_{\max}}{2} \cos 2\psi \right) + \frac{1 + \cos\alpha_{\max}}{2} \right] - \sin(\theta_T - \theta_M) \sin\alpha_{\max} \sin\psi + V_T \sin\psi + V_R \cos\psi, \quad (1)$$

where $\phi$ is the radar elevation angle, $\theta_T$ is the angle for the TC center viewed from the radar, and $V_M$ and $\theta_M$ are the speed and direction of the mean wind vector ($V_M$), respectively. Relative to the mean wind (not the storm motion), the tangential and radial wind components of the storm are denoted by $V_T$ and $V_R$, respectively (hereafter they will be referred to as TC tangential and radial winds). Note that in (1) an approxima-
tion sign has been replaced by an equal sign. The errors introduced by this replacement are rather small, as discussed in Lee et al. (1999). In GBVTD, the quantities \( \hat{V}_d/\cos\phi \), \( V_T \), and \( V_R \) are further decomposed by a truncated Fourier series in terms of \( \psi \):

\[
\hat{V}_d(\psi)/\cos\phi = \sum_{n=0}^{L} (A_n \cos n\psi + B_n \sin n\psi),
\]

(2)

\[
V_T(\psi) = V_T C_0 + \sum_{n=1}^{M} (V_T C_n \cos n\psi + V_T S_n \sin n\psi),
\]

(3)

\[
V_R(\psi) = V_R C_0 + \sum_{n=1}^{N} (V_R C_n \cos n\psi + V_R S_n \sin n\psi).
\]

(4)

Figure 1 indicates that \( \psi \) is defined as the angle \( \angle OET \), which starts from zero at point A, then gradually changes to \( \pi/2 \), \( \pi \), and \( 3\pi/2 \), at points B, C, and D, respectively. It should be pointed out that the values of \( \psi \) are not evenly distributed throughout this analysis ring. A comparison of \( \gamma \) and \( \psi \), which can be found in Fig. 2 of Lee et al. (1999), clearly shows the discrepancy. As revealed later, the uneven distribution of \( \psi \) causes a certain degree of distortion in the retrieved TC circulation.

The coefficients \( A_n \) and \( B_n \) in (2) can be computed through the curve fitting of the observed Doppler velocities sampled along the analysis ring. The parameters \( V_T C_n(V_R C_n) \) and \( V_T S_n(V_R S_n) \) in (3) and (4) turn out to be the unknown magnitudes of the sine and cosine waves of \( V_T(V_R) \) for the angular wavenumber \( n \) and need to be retrieved. Let \( L = 4 \) and \( M = N = 3 \), and after some mathematical manipulations, one arrives at the following formulations that bridge the computable coefficients \((A_n, B_n)\) with the unknown amplitudes \((V_T C_n, V_R C_n, V_T S_n, V_R S_n)\):

\[
V_M \cos(\theta_T - \theta_M) = A_0 + A_2 + A_4 - V_R C_1 - V_R C_3,
\]

(5)

\[
V_T C_0 = -B_1 - B_3 - V_M \sin(\theta_T - \theta_M) \times \sin\alpha_{\max} + V_R S_2,
\]

(6)

\[
V_R C_0 = A_1 + A_3 - V_R C_2,
\]

(7)

\[
V_T S_1 = A_2 - A_0 + A_4 + (A_0 + A_2 + A_4 - V_R C_1 - V_R C_3) \times \cos\alpha_{\max} + V_R S_3,
\]

(8)

\[
V_T C_1 = -2(B_2 + B_4) + V_R S_1 + V_R S_3,
\]

(9)

\[
V_T S_2 = 2A_2 - V_R C_2,
\]

(10)

\[
V_R C_2 = -2B_3 + V_R S_2,
\]

(11)

\[
V_T S_3 = 2A_4 - V_R C_3,
\]

(12)

\[
V_R C_3 = -2B_4 + V_R S_3.
\]

(13)

It can be found immediately that in (5)–(13), the number of unknown variables is actually greater than the number of equations. To solve this closure problem, Lee et al. (1999) assumed that the asymmetric \( V_R \) is much smaller than the corresponding \( V_T \). Under this assumption, only the axisymmetric component of the TC radial winds \( (V_R C_0) \) remains in (7). The terms involving \( V_R \) at higher wavenumbers (i.e., \( V_R C_n \) and \( V_R S_n, n \gtrsim 1 \)), as shown on the right-hand side of the equations, are neglected.

The above procedure describes the basic concept of the original GBVTD method. It is realized that the retrieved products of this algorithm include the TC tangential winds (angular wavenumbers 0 to 3) and the mean radial winds (wavenumber 0). The latter denotes only the axisymmetric part of the radial flows. Consequently, the GBVTD-derived radial wind at a given analysis ring is either positive (outflow) or negative (inflow). Furthermore, the mean tangential wind \( (V_T C_0) \) is contaminated by the so-called cross-beam mean flow, represented by \( V_M \sin(\theta_T - \theta_M) \) in (6). The existing information is not sufficient to make a further separation of \( V_M \) and \( \theta_M \). These are the fundamental limitations of the GBVTD method.

3. The EGBVTD approach

The proposed EGBVTD approach requires a second radar, which is capable of gathering enough data points along the analysis ring so that a GBVTD computation can also be performed using this second set of radar data. Figure 2 illustrates the applications of the GBVTD method to two radar systems, represented by radar A and radar B, respectively. The difference of the viewing angle between radar A and radar B toward the TC center is represented by \( \beta \). Note that in the previous section, Eqs. (5)–(13) are employed with the terms involving \( V_R C_n \) and \( V_R S_n \) ignored when \( n \gtrsim 1 \). In this section, the formulations are slightly modified by retaining the parameters relevant to the wavenumber-1 structure of the TC radial winds (i.e., \( V_R C_1, V_R S_1 \)).

a. Estimation of the mean flow \( V_M \) and \( \theta_M \)

We begin by using Eq. (6). It is noticed that in this equation the variable \( V_T C_0 \) on the left-hand side of the formula represents the axisymmetric portion of the tan-
gential velocity. Therefore its value should be independent of the radar-viewing angle (i.e., $\theta_T$ for radar A and $\theta'_T$ for radar B) for observations taken at the same altitude. Thus, it is reasonable to assume that the estimations of this coefficient ($V_T C_0$) by radar A and radar B ought to be the same. This assumption leads to the following expression:

$$-B_1 - B_3 - V_M \sin(\theta_T - \theta_M) \sin \alpha_{\text{max}} = -B'_1 - B'_3 - V_M \sin(\theta'_T - \theta_M) \sin \alpha'_{\text{max}}, \quad (14)$$

where the terms with a prime stand for the coefficients computed or viewed from radar B. In (14), the parameters $V_R S_2$ and $V_R S'_2$ have been neglected, but two parameters $V_M$ and $\theta_M$ remain unknown. However, noting that $V_M$ and $\theta_M$ are functions of the height only, we apply (14) to more than one analysis ring over a given horizontal plane, and formulate the following cost function, which evaluates the differences in the retrievals from the radar A and radar B data:

$$J = \sum_{i=1}^{N} \left[ \left( -B_1 - B_3 - V_M \sin(\theta_T - \theta_M) \sin \alpha_{\text{max}} \right) \right]^2 - \left[ -B'_1 - B'_3 - V_M \sin(\theta'_T - \theta_M) \sin \alpha'_{\text{max}} \right]^2. \quad (15)$$

In (15) the summation is over $N$ GBVTD analysis rings located at the same altitude; the subscript $i$ is the index of each ring. Now, we seek a set of optimal $V_M$ and $\theta_M$, which minimizes (15). The searching process is performed within a predetermined range. For example, the magnitude of $V_M$ can range from 0.0 to 15.0 m s$^{-1}$, while $\theta_M$ varies from 0 to $2\pi$. Equation (15) is applied layer by layer so that eventually the estimated mean winds can vary with height. The above description demonstrates the procedure proposed for extracting information about the mean flow by applying the GBVTD method to data from two Doppler radars.

**b. Derivation of the asymmetric TC radial wind structure**

The previous section outlines a procedure by which the mean flow $V_M$ and $\theta_M$ can be estimated. After $V_M$ and $\theta_M$ are known, Eq. (5) can be used to derive $V_R C_1$, since the coefficients $A_0$, $A_2$, and $A_4$ have already been computed using the original GBVTD, and the term $V_R C_3$ is ignored as well. Then, the following derivation starts by applying (1) at points $P_1$ and $P_2$, but using the data collected by radar B, as shown in Fig. 2. It is noted that at $P_1(P_2)$, the value of $\psi'$ (the prime denotes the angle of $\psi$ viewed from radar B) is 0 ($\pi$). Therefore, (1) can be simplified to the following forms:

$$\frac{\hat{V}_d(P_1)}{\cos \psi'} = V_M \cos(\theta'_T - \theta_M) + V_R(P_1), \quad (16)$$

$$\frac{\hat{V}_d(P_2)}{\cos \psi'} = V_M \cos(\theta'_T - \theta_M) - V_R(P_2). \quad (17)$$

The terms $\hat{V}_d(P_1)$ and $\hat{V}_d(P_2)$ are the Doppler velocities measured by radar B at points $P_1$ and $P_2$, respectively, while $V_R(P_1)$ and $V_R(P_2)$ are the total TC radial winds at these two points. Since the speed and direction of the mean flow ($V_M$ and $\theta_M$) have been estimated using the method described in the previous section, $V_R(P_1)$ and $V_R(P_2)$ can be deduced.

The next step is to expand the radial wind beyond its averaged amplitude ($V_R C_0$) to the angular wavenumber-1 structure, but using the $\psi$ angle system viewed from radar A. For example, the expansion at $P_2$ yields

$$V_R(P_2) = V_R C_0 + V_R C_1 \psi_2 + V_R S_1 \sin \psi_2, \quad (18)$$

where $\psi_2$ is the value of angle $\psi$ at $P_2$, viewed from the location of radar A. Since $V_R C_0$ and $V_R C_1$ are estimated from (7) and (5), respectively, the only unknown variable $V_R S_1$ in (18) can finally be solved via (17). This set of coefficients $V_R C_0$, $V_R C_1$, and $V_R S_1$ specifies the mean as well as the wavenumber-1 structure of the TC radial flows along the given analysis ring. Moreover, it is also found that a better retrieval of the wavenumber-1 structure of the TC tangential flow can also be achieved by inserting the newly derived $V_R C_1$ and $V_R S_1$.
into (8) and (9) to improve the accuracy of $V_T S_1$ and $V_R C_1$.

It should be emphasized that the values of $V_R C_1$ and $V_R S_1$ over a given GBVTD analysis ring depend on the viewing angles of the radars, since the zero point of the angle $\psi$ changes with look angle. In other words, the $V_R C_1$ and $V_R S_1$ estimated from radar A have different values than those estimated from radar B. However, when summed in (18), the same TC radial wind $V_R(P_2)$ is obtained from both radar A and radar B.

To summarize, the EGBVTD method is capable of recovering the asymmetric TC radial wind component up to its angular wavenumber-1 structure. Consequently, the quality of the retrieved TC tangential wind can also be improved.

Although two radar systems are needed, the EGBVTD approach still has some advantages over the traditional dual-Doppler synthesis. To estimate the mean flow so that one can isolate the TC circulation, some previous studies (e.g., Marks et al. 1992) have suggested it be performed in an area averaged over a cylindrical portion of the Doppler observational domain, with the TC located at the center. This implies that for the dual-Doppler synthesis method, the radar data coverage of both radars must comprise the storm center as well as a sufficient surrounding area. In Marks et al. (1992), this kind of data coverage is obtained by flying an airborne Doppler radar over the hurricane. However, for ground-based radar systems, this requirement may not always be easily satisfied. It is common that a ground-based radar only observes one side of the TC circulation, where the data coverage may not include the TC center. This situation often happens when a typhoon/hurricane is approaching, but is not yet close enough to the radar sites. With this type of data distribution, the spatial averaging necessary for the mean wind estimation cannot be conducted. Thus, the dual-Doppler synthesis can only derive the ground-relative wind field, in which part of the TC circulation is embedded within the mean flow. On the other hand, due to the use of curve fitting to analyze the observed Doppler winds sampled along each analysis ring, the EGBVTD method does not need full data coverage over the entire ring. Therefore, the estimation of the mean flow and the following separation of the TC circulation become possible, even if radar data are only available for one side of the storm. Moreover, the algorithm for dual-Doppler synthesis requires that both radars have Doppler wind observations covering a common area. This requirement also imposes a limit on the size of the analysis domain. By contrast, owing to the same reason mentioned above, the computation of EG-BVTD does not require a full overlap of Doppler winds measured by both radars at all points. Thus, the retrievals can be performed over a larger area than that of the traditional method.

### 4. Experimental design—The modeled analytic TC circulation and indices for verification

In this section the performance of the proposed EGBVTD method is investigated using an idealized TC circulation model. For simplicity, only a single layer of storm circulation is constructed. The height of this layer is assumed to be the same as that of the radar site, which would make the elevation angle ($\phi$) equal to zero. In addition, the position of the circulation center is also assumed to be known. The modeled TC tangential and radial winds ($V_{T}^m$ and $V_{R}^m$) are expressed by

$$V_{T}^m = V_{T,0}^m + \sum_{n=1}^{MM} (V_{T} C_n^m \cos \gamma + V_{T} S_n^m \sin \gamma),$$

(19)

$$V_{R}^m = V_{R,0}^m + \sum_{n=1}^{NN} (V_{R} C_n^m \cos \gamma + V_{R} S_n^m \sin \gamma),$$

(20)

where the superscript $m$ indicates that these variables are for the modeled winds. The axisymmetric part of $V_{T}^m$ is given by

$$V_{T} C_{0}^m = V_{T,\max} \left(\frac{R}{R_{\max}}\right)^\kappa,$$

(21)

where $R$ is the radius of the analysis ring, $V_{T,\max}$ is given the value 50 m s$^{-1}$, and $R_{\max}$, the radius of maximum wind (RMW), is specified to be 25 km. The parameter $\kappa$ is set to be 1.0 ($-0.5$) when $R \leq R_{\max} (R > R_{\max})$. Similar to Lee et al. (1999), the axisymmetric part of $V_{R}^m$ is prescribed by

$$V_{R} C_{0}^m = -0.5[(R_{\max} - R)R]^{0.5},$$

(22)

$$V_{R} S_{0}^m = 3.0(R - R_{\max})^{0.5} \frac{R_{\max}}{R},$$

(23)

The parameters relevant to the asymmetric portion of the TC circulation in (19) and (20) can be specified with different combinations. In this section, they are assigned to $MM = 3$, $NN = 1$, $V_{T} C_1^m = -10.0$ m s$^{-1}$, $V_{T} S_1^m = 10.0$ m s$^{-1}$, $V_{T} C_2^m = -5.0$ m s$^{-1}$, $V_{T} S_2^m = 5.0$ m s$^{-1}$, $V_{T} C_3^m = -2.0$ m s$^{-1}$, $V_{T} S_3^m = 2.0$ m s$^{-1}$, $V_{R} C_1^m = 10.0$ m s$^{-1}$, and $V_{R} S_1^m = 10.0$ m s$^{-1}$. For the tangential winds, the magnitudes for different wavenumbers are only approximations based on some previous studies of the hurricane structure using Doppler radar observations (e.g., Marks et al. 1992) or numerical simulations (e.g., Liu et al. 1999). Note that the amplitudes of $V_{R} C_1^m$ and $V_{R} S_1^m$ are set to be equivalent to their tangential counterparts. This design imitates the scenario where
the asymmetric component of the radial flow becomes rather significant and cannot be neglected. Comparing (19)–(20) with (3)–(4), it is noted that the modeled TC circulation is expanded in terms of \( \gamma \), while the retrieved TC structure is reconstructed based on \( \psi \). The definitions of these two angle systems depicted in Fig. 1 indicate that \( \gamma \) is linearly distributed over the analysis ring, but \( \psi \) is not. This discrepancy causes a certain degree of distortion of the retrieved wind fields, and the extent of the distortion is found to increase with the ratio \( R/R_T \), where \( R \) stands for the distance between the radar site and the circulation center (e.g., Wood and Brown 1992). Figures 3 and 4 display the modeled analytic TC tangential and radial winds in terms of their angular wavenumbers as well as the total wind fields. These plots illustrate the structure of the storm circulation alone and are the true reference solutions used for the comparisons made in the following retrieval experiments. The TC circulation is then further superimposed by the mean flow, before it is sampled by the two virtual Doppler radars to generate two sets of data for the EGBVTD analysis. The quality of the retrievals is examined by evaluating the relative root-mean-square error (rrmse) and the spatial correlation coefficient (SCC). They are defined as

\[
\text{rrmse} = \frac{\sqrt{\frac{1}{N} \sum (V_r - V_t)^2}}{\sqrt{\frac{1}{N} \sum (V_r)^2}},
\]

\[
\text{SCC} = \frac{\sum V_r V_t}{\sqrt{\sum (V_r)^2 \sum (V_t)^2}}.
\]

In (24) and (25) subscripts \( t \) and \( r \) stand for the true and retrieved wind fields, respectively. The SCC score reaches 1.0 when the pattern (phase) of the retrieved wind fields is the same as the true one. The above two indices will be applied to evaluate the accuracy of the retrieved total \( V_r \) and \( V_R \) as well as the Fourier components at different wavenumbers.

5. Retrieval results

a. Experiment 1—EGBVTD test run

To check the accuracy of the EGBVTD computations, a test run is conducted first. The deployment of radar A and radar B is similar to that shown in Fig. 2, with angle \( \beta \) equal to 60°. The radius of each analysis ring differs by 1.0 km; along a given ring the radar takes samples once every 1°. The mean flow speed \( (V_M) \) and direction \( (\theta_M) \) are somewhat arbitrarily assigned to be 10.0 m s\(^{-1}\) and 45° (southwesterly), respectively. The distances between the radar sites and the circulation center are deliberately specified to be large numbers (i.e., >1000 km), so that the distortion can be effectively removed. Assuming that the correct mean flow speed \( (V_M) \) and direction \( (\theta_M) \) are known, the retrieved TC structures (not shown) are found to be almost identical to their true counterparts, shown in Figs. 3 and 4. The rrmse index drops to as little as \( 10^{-4} \), while the SCC score can reach 1.0. The success of this test run demonstrates the correctness of the EGBVTD approach as well as the numerical computations.

b. Experiment 2—EGBVTD control run

In this experiment, the geographic locations of the dual-Doppler radar systems are the same as in the test run, but the distances from the circulation center to radars A and B are reduced to 175 and 250 km, respectively. Owing to the shorter distances, some distortion becomes inevitable. Therefore, even with exact information regarding the mean flow \( (V_M = 10.0 \text{ m s}^{-1}, \theta_M = 45°) \), it is not surprising that the retrieved results will be degraded compared to those in the previous subsection. The retrieved TC circulation structures are plotted in Figs. 5 and 6. It can be immediately recognized that the distortion is most evident near the boundary regions, where the ratio of \( R/R_T \) is largest. This is consistent with the findings of Wood and Brown (1992) and Lee et al. (1999). Nevertheless, the wave pattern and the phase of each Fourier component at a different wavenumber is generally in good agreement with the true solutions displayed in Figs. 3 and 4. It is particularly encouraging to see that the inflow (outflow) structures of the \( V_r \) have been successfully recovered (see Figs. 4c and 6c). Compared with the previous test run, the statistics listed in Table 1 do reveal an overall increase in the rrmse index and a decrease in the SCC score, apparently caused by the geometric distortion. It can also be found from the graphics and in Table 1 that the distortion appears to cause larger errors for higher wavenumbers. For example, the worst retrieval takes place for the wavenumber-3 structure of \( V_r \), where the rrmse exceeds 1.0 and the SCC score deteriorates to 0.723. Fortunately, since the magnitudes of the higher-wavenumber components are generally smaller, the influence of these relatively poor retrievals on the resulting total wind field turns out to be less destructive, as can be seen by the high degree of similarity between Figs. 3e and 5e.

c. Experiment 3—Original GBVTD run

To highlight the advantages of the EGBVTD method, it is interesting to compare the EGBVTD
FIG. 3. The modeled tangential wind fields for a tropical cyclone at wavenumber (a) 0 at intervals of 9.0 m s\(^{-1}\), (b) 1 at intervals of 2.0 m s\(^{-1}\), (c) 2 at intervals of 2.0 m s\(^{-1}\), and (d) 3 at intervals of 1.0 m s\(^{-1}\). (e) The total tangential wind at intervals of 12.0 m s\(^{-1}\). The solid (dashed) lines denote positive (negative) values. The typhoon symbol indicates the location of the TC center.
products obtained in the previous section with the TC structure retrieved by the original GBVTD algorithm. It has been emphasized earlier that in the original GBVTD approach, the mean flow cannot be separated from the storm circulation, and only the axisymmetric portion of the $V_R$ is recovered. By contrast, the EGBVTD method relaxes these limitations, and thus increases the accuracy of the retrievals. Note that the improvements only apply to $V_T$ at wavenumbers 0 and 1 and to $V_R$ at wavenumber 1. The resulting statistics reveal that, compared with the EGBVTD products (see Table 1), the rmse of the GBVTD-derived $V_T$ increases from $9.8 \times 10^{-3}$ to $6.2 \times 10^{-2}$ for wavenumber 0 and from 0.32 to 0.89 for wavenumber 1. Although the former value may not be serious, the latter is rather substantial. The retrieved total $V_T$ is illustrated in Fig. 7. It can be seen that spurious negative values do occur in the southeast corner of the domain. Compared with the true solution displayed in Fig. 3e, it is also found that there are negative and significant positive biases in the northeast and northwest side of the center near the RMW, respectively. The GBVTD-derived total $V_R$ field only contains the wavenumber-0 component of the radial winds, and is the same as the mean radial winds produced by the EGBVTD method, as presented in Fig. 6a. Since only the axisymmetric component is recoverable, it is not surprising to see that the rmse increases from 0.20 in the control run to 0.84 in the GBVTD run, and the SCC score between Fig. 6a and the true solution in Fig. 4c declines to only 0.545.

From this experiment, it can be realized that the quality of the GBVTD-derived wind fields is influenced by the existence of the asymmetric radial flows. Therefore, it is suggested that the GBVTD method be ap-

![Fig. 4. The modeled radial wind fields for a tropical cyclone at wavenumber (a) 0 at intervals of 3.0 m s$^{-1}$ and (b) 1 at intervals of 2.0 m s$^{-1}$. (c) The total radial wind at intervals of 3.0 m s$^{-1}$. The solid (dashed) lines denote positive (negative) values.](image-url)
Fig. 5. The EGBVT-D-retrieved TC tangential winds for the control run experiment at wavenumber (a) 0 at intervals of 9.0 m s\(^{-1}\), (b) 1 at intervals of 2.0 m s\(^{-1}\), (c) 2 at intervals of 2.0 m s\(^{-1}\), and (d) 3 at intervals of 1.0 m s\(^{-1}\). (e) The total tangential wind at intervals of 12.0 m s\(^{-1}\). The solid (dashed) lines are for positive (negative) values.
plied only for TCs that are over a flat surface such as the ocean where the assumption of small asymmetric radial flows is more likely to be valid than it is on land. However, for a landfalling typhoon/hurricane, since the asymmetric radial flows are expected to be significant due to the orographic effects, it is better to apply the EGBVTD technique rather than the original GBVTD method to analyze the storm structures.

d. Experiment 4—EGBVTD extensive run

In this experiment the mean flow speed \( V_M \) and direction \( \theta_M \) are assumed to be unknown. Instead, they are estimated using the method outlined in section 3a. Consequently, this experiment represents an extensive evaluation of the performance of the complete EGBVTD algorithm. The retrieval experiment is designed to take into account a large number of combinations of the mean wind speed \( V_M \) and the direction \( \theta_M \). We let \( V_M \) vary from 5.0 to 15.0 m s\(^{-1}\), at an increment of 1.0 m s\(^{-1}\), and let \( \theta_M \) change from 10° to 360°, at an increment of 10°. Thus, the total number of experiments is \( 11 \times 36 = 396 \). In each experiment, the mean flow is estimated first, followed by the TC-retrieval computations. These 396 experiments form an experiment group. Then, each experiment group is re-run with the two radar sites placed at different locations, which can be accomplished by altering the angle \( \beta \) in Fig. 2. Several experiment groups are tested in which the value of \( \beta \) is changed from 45° to 135°. An overall evaluation of the statistics resulting from these experiment groups shows that the average error of the wind direction estimation is about 15° ~ 20°, and the averaged relative error in the wind speed estimation, defined by (24), is about 0.3. Figures 8 and 9 display the
SCC scores for the retrieved $V_T$ and $V_R$ with respect to all $V_M$ and $\theta_M$ combinations when $\beta = 60^\circ$. The SCC scores computed from other experiment groups with different $\beta$ are similar to Figs. 8 and 9. Thus they are not shown here. The plots reveal a generally high similarity between the retrieved storm structures and their

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mean wind ($V_M$)</th>
<th>$\theta_M$</th>
<th>$V_T$</th>
<th>$V_R$</th>
<th>Total</th>
<th>SCC Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control run</td>
<td>10</td>
<td>45</td>
<td>9.8</td>
<td>0.22</td>
<td>0.52</td>
<td>1.10 0.52 1.0 0.27 0.84</td>
</tr>
<tr>
<td>Original GBVTD</td>
<td>10</td>
<td>45</td>
<td>9.6</td>
<td>0.22</td>
<td>0.52</td>
<td>1.10 0.52 1.0 0.27 0.84</td>
</tr>
</tbody>
</table>
true counterparts. For example, the SCC for $V_T$ remains above 0.976, while the lowest SCC for $V_R$ still reaches the level of 0.865. A closer examination indicates that the worst case of scenario occurs when $V_M = 14.0 \text{ m s}^{-1}$ and $\theta_M = 130^\circ$. With this particular set of mean flow, it is found that the estimated $V_M$ and $\theta_M$ turn out to be $3.3 \text{ m s}^{-1}$ and $155^\circ$, respectively. Although a discrepancy does exist in the estimated mean flow, it is encouraging to find that the retrieved $V_T$ field illustrated in Fig. 10a still agrees fairly well with the true solution shown in Fig. 3e. On the other hand, the $V_R$ component depicted in Fig. 10b exhibits a slight counterclockwise phase shift from the true field shown in Fig. 4c. Nevertheless, the general distributions of the inflow (outflow) in the southwest (northeast) portion of the domain still compare favorably with their true counterparts.

**e. Experiment 5—Incomplete data coverage**

When two Doppler radar systems are available for observations, the dual-Doppler synthesis is probably the most commonly used way to analyze the three-dimensional wind fields. However, as explained in section 3b, in order to estimate the mean flow and then isolate the storm circulation itself, this method requires a data overlap of both radar measurements covering a common analysis domain centered on the storm. When this data coverage requirement is not reached, the dual-Doppler synthesis can only recover a ground-relative wind field within perhaps a small portion of the storm. By contrast, with the EGBVTD approach, the TC wind fields are reconstructed by computing the wave amplitudes [i.e., $A_n$ and $B_n$ in (2)] using the method of curve fitting, which can be performed even without full data coverage covering the entire analysis ring. In other words, with limited data coverage on one side of the storm, the EGBVTD should still have the potential to infer the complete TC structure for a larger area than the traditional dual-Doppler synthesis does. To test this concept, the EGBVTD is performed with the availability of the input radar data along each analysis ring degraded systematically. By doing this the data gaps are created where the traditional dual-Doppler synthesis fails. We have investigated various types of data gaps, and Fig. 11 presents only one hypothetical example. In
this example the data-void points are continuously distributed, and located on the farther side of the TC center from the radar sites. The number of data points available for computations is gradually reduced, and a comparison between the true and the retrieved TC radial wind field is made. Our experiment shows that the quality of the EGBVTD retrieval remains satisfactory even when the length of the data gap reaches approximately 120°, or 1/3 of the complete data coverage. The SCC score reaches 0.943 between the retrieved radial wind fields, as plotted in Fig. 12, and the true field shown in Fig. 4c. This high degree of consistency demonstrates the robustness of the EGBVTD method to the incomplete data coverage. It should be pointed out that for the data distribution depicted in Fig. 11, the traditional dual-Doppler synthesis is unable to estimate the mean flow. It can only recover a ground-relative wind field within the remaining data overlap domain.


Typhoon Nari struck Taiwan on 16 September 2001 and caused extremely serious damage. Its unique track, slow-moving speed, and record-breaking precipitation made this typhoon an interesting target for further detailed studies (Sui et al. 2002). In this section we use the data collected after Nari’s landfall by two Doppler radars located in northern Taiwan to explore the feasibility of applying the EGBVTD method to real case studies. Figure 13 depicts the geographic locations of these two Doppler radar systems—the Civil Aeronautics Administration (CAA) radar and Radar Code of Wu-Fenshan (RCWF) radar, as well as the typhoon’s center at the analysis hour (1600 UTC 16 September 2001). The specifications of these two radars can be found in Table 2. Note that the terrain height in this region is greater than 2.0 km. Therefore, the asymmetric structures in the typhoon’s circulation induced by the orographic effects would be expected. The distance of the baseline is about 57.0 km. The circulation center is determined by searching for a point within an 8 km × 8 km region, with the spatial resolution of 1.0 km, in the vicinity of the eyewall that maximizes the tangential wind. This algorithm is similar to the GBVTD-simplex circulation center finding method proposed by Lee and Marks (2000). The location of the circulation center is searched layer by layer, thus allowing for a vertical variation in the center’s horizontal position. In this case the search is performed at five different altitudes: 3.0, 3.75, 4.5, 5.25, and 6.0 km. The maximum horizontal shift is only about 1.5 km. This implies that the center of the typhoon is almost vertically aligned.

a. The mean wind (\( V_M \)) in Nari’s inner core

Figure 14 depicts an averaged storm track, along with the retrieved mean winds at five different heights. The
storm track at each layer is determined by comparing the position of the circulation center 30 min after the analysis hour (e.g., 1630 UTC). The speeds of the storm’s movement at these five layers are approximately between 6.0 and 8.0 m s\(^{-1}\), and the directions vary from 235° to 250°. Note that the direction of a pure westerly (easterly) flow is 0° (180°), as defined in Fig. 1. A vertical, mass-weighted averaging of the storm movements at different layers results in a northeasterly mean storm track, with the speed and direction equal to 6.7 m s\(^{-1}\) and 244°, respectively, as revealed in Fig. 14. From 3.0 to 6.0 km, the direction of the retrieved mean wind turns counterclockwise, from approximately 191° to 291°. Below (above) 3.75 km, the mean wind tends to deviate to the right (left) of the storm track. A mass-weighted averaging throughout these five layers is also conducted for the retrieved mean wind. The resulting averaged mean wind speed is 5.0 m s\(^{-1}\), and the direction is 250°, which shows a slight leftward deviation from the average storm track.

The relationship and interaction between the surrounding mean flow and the storm motion have already been investigated rather extensively (e.g., George and Gray 1976; Holland 1983; Dong and Neumann 1986; Willoughby 1988; Fiorino and Elsberry 1989; Williams and Chan 1994). However, it should be pointed out that in the above studies, the so-called surrounding mean wind usually refers to a synoptic-scale environmental flow, with a characteristic length about 1000 km. Given this condition, the movement of the storm is investigated over a flat surface such as the open ocean, either for observed real TCs or for an idealized model vortex. By contrast, in this particular real case study, the typhoon had made landfall on Taiwan, which implies that a strong topographic effect may have a significant influence on the storm’s motions. Moreover, our analysis domain is about 50 km × 50 km, which covers only the inner-core region immediately outside the typhoon cen-

<table>
<thead>
<tr>
<th>Feature</th>
<th>CAA</th>
<th>RCWF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse repetition frequency</td>
<td>900/1200 Hz</td>
<td>318–452 Hz</td>
</tr>
<tr>
<td>Unambiguous range</td>
<td>120 km</td>
<td>230 km</td>
</tr>
<tr>
<td>Unambiguous velocity</td>
<td>±48 m s(^{-1})</td>
<td>±30 m s(^{-1})</td>
</tr>
<tr>
<td>Rotation rate</td>
<td>2, 4 rpm</td>
<td>6 rpm</td>
</tr>
<tr>
<td>Elevation angle</td>
<td>1° to 90°</td>
<td>1° to 45°</td>
</tr>
<tr>
<td>Pulse length</td>
<td>0.5 μs</td>
<td>1.57 μs, 4.5 μs</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>0.90°</td>
<td>0.95°</td>
</tr>
<tr>
<td>Wavelength</td>
<td>5.3 cm</td>
<td>10.5 cm</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>43.0 dB</td>
<td>44.5 dB</td>
</tr>
<tr>
<td>Antenna diameter</td>
<td>4.2 m</td>
<td>8.54 m</td>
</tr>
<tr>
<td>Transmitter (peak power)</td>
<td>250 kW</td>
<td>750 kW</td>
</tr>
<tr>
<td>Receiver</td>
<td>Linear</td>
<td>Linear</td>
</tr>
</tbody>
</table>

From 3.0 to 6.0 km, the direction of the retrieved mean wind turns counterclockwise, from approximately 191° to 291°. Below (above) 3.75 km, the mean wind tends to deviate to the right (left) of the storm track. A mass-weighted averaging throughout these five layers is also conducted for the retrieved mean wind. The resulting averaged mean wind speed is 5.0 m s\(^{-1}\), and the direction is 250°, which shows a slight leftward deviation from the average storm track.

The relationship and interaction between the surrounding mean flow and the storm motion have already been investigated rather extensively (e.g., George and Gray 1976; Holland 1983; Dong and Neumann 1986; Willoughby 1988; Fiorino and Elsberry 1989; Williams and Chan 1994). However, it should be pointed out that in the above studies, the so-called surrounding mean wind usually refers to a synoptic-scale environmental flow, with a characteristic length about 1000 km. Given this condition, the movement of the storm is investigated over a flat surface such as the open ocean, either for observed real TCs or for an idealized model vortex. By contrast, in this particular real case study, the typhoon had made landfall on Taiwan, which implies that a strong topographic effect may have a significant influence on the storm’s motions. Moreover, our analysis domain is about 50 km × 50 km, which covers only the inner-core region immediately outside the typhoon cen-
ter. Consequently, it may not be appropriate to apply the results and explanations of the above research to this particular case.

We, however, found that in Marks et al. (1992) a wind partitioning procedure conceptually similar to that of the EGBVTD formulation was utilized to analyze the wind fields surrounding the circulation center. In their study, which used the circulation center as the origin of a cylindrical coordinate system, the total wind field was partitioned in the following manner:

\[ V(r, \lambda, z) = V_s + V_r(z) + V^*(r, \lambda, z), \]  

where \( r \) is the radius, \( \lambda \) represents the azimuth, \( z \) denotes the height, \( V \) is the total wind, \( V_s \) stands for the horizontal storm motion, \( V_r \) is the area-averaged storm-relative wind as a function of height, and \( V^* \) and \( V^\# \) correspond to the symmetric-vortex (wavenumber 0) and asymmetric perturbation wind (wavenumber 1 and higher), respectively, of the storm circulation at each altitude. In (26), \( V_r \) is computed by making a horizontal average over a cylindrical area centered on the storm. Marks et al. (1992) used a 37.0-km radius surrounding the circulation center to calculate \( V_r \). Since \( V_r \) is calculated over an area instead of a ring, it is independent of radius. On the other hand, in the proposed EGBVTD mean wind estimation shown in (15), an area-averaged mean wind is also computed by minimizing the sum of the variations of the individual mean wind values for all the GBVTD analysis rings used in the summation. Therefore, the estimation of the mean wind proposed in EGBVTD is conceptually similar to that of Marks et al. (1992).

Marks et al. (1992) applied (26) to study the inner-core structure of Hurricane Norbert (1984) using airborne Doppler radar data. Their calculations showed that the mean flows at different layers (1.0–11.0 km) deviate substantially from the storm track. However, after subtracting a westerly bias in their airborne Doppler wind measurements, a mass-weighted deep-layer average mean flow exhibited a slight leftward deviation from the storm track. This result appears to be consistent with our findings mentioned above. However, it should be realized that in these two case studies, many factors involved in the analysis were quite different. For example, at the hour of the analysis, Nari had made its landfall and the typhoon center had already moved over Taiwan’s complex terrain, with a height over 1.0 km, but Norbert was still over the eastern Pacific Ocean. The details of the computations, such as the altitudes used for averaging the mean flow, were also different in both cases. Nevertheless, this experiment demonstrates the EGBVTD’s capability in terms of estimating the mean flow within the inner-core region in a real case study.

It can be seen that the wind partitioning procedure shown in (26) requires full radar data coverage surrounding the storm center in order to compute the area-averaged wind \( V_r \). For dual-Doppler synthesis using ground-based radar systems, this kind of data coverage may not always be easily accomplished. Thus, the products of traditional dual-Doppler synthesis often represent the ground-relative wind fields, with the mean flow and the storm circulation mixed together. However, as explained in section 3b, and as illustrated by experiment 5, the EGBVTD approach can still be used to estimate the mean winds even when the data coverage surrounding the storm center is incomplete. This gives an advantage over the traditional dual-Doppler synthesis. Another advantage of the EGBVTD approach is its ability to show a more complete picture of winds compared to traditional dual-Doppler wind syntheses when there are missing data in the Doppler velocity measurements. This issue will be discussed further in the following section.

b. The typhoon circulation

Figure 15 illustrates the radius–height distribution of the axisymmetric typhoon structures retrieved by the
EGBVTD method. Because of the blockage by the terrain, the radar data and the retrieved wind fields are only available above 3.0 km. Similar to the small horizontal variations of the typhoon center at different heights, the tangential winds shown in Fig. 15a also appear to be almost vertically aligned. This feature seems to imply that the upper and lower parts of the typhoon had not been decoupled by the mountains. Figure 15b indicates that inflow occurs below 3.5 km, and penetrates the typhoon up to 30.0 km from the center. Figures 16a–c delineate the Typhoon Nari radial winds at $Z = 6.0$ km. The results are obtained using the dual-Doppler synthesis, the EGBVTD method, as well as the original GBVTD formulation, respectively. Note that the mean wind, estimated by the EGBVTD method, has been removed from the dual-Doppler-derived wind field; therefore, the plot reveals the mean-wind relative typhoon radial flows. It can be seen from Fig. 16a that there is an inflow (outflow) taking place in the east-southeast (north-northwest) quadrant of the typhoon. This overall asymmetric structure can be successfully recovered by the proposed EGBVTD approach, as shown in Fig. 16b. In contrast, since the original GBVTD algorithm infers only the wavenumber-0 flow of the typhoon radial winds, it is not surprising that we see a totally different concentric pattern in Fig. 16c.

The typhoon tangential winds derived by the three different methods are displayed in Figs. 17a–c. The dual-Doppler results indicate that there is a strong (weak) wind region located in the northeast (southwest) portion of the typhoon. The peak wind speed in the northeast exceeds 25.0 m s$^{-1}$. Although the general pattern of the tangential flows is recovered by the EGBVTD as well as the GBVTD, it can be ascertained that the EGBVTD-derived circulation (Fig. 17b) still exhibits a much higher consistency with the true solution than the GBVTD does (Fig. 17c). Moreover, the maximum wind speed derived by the EGBVTD method can reach 30.0 m s$^{-1}$, while the GBVTD method only produces a much weaker wind speed of about 15.0 m s$^{-1}$. Apparently, the EGBVTD estimate in Fig. 17b is closer to the reality shown in Fig. 17a compared to the GBVTD estimate in Fig. 17c. Although the result of Fig. 17b is better, the EGBVTD-derived wind speed is still stronger than that of the dual-Doppler synthesis. A possible reason might be the distortion produced by the EGBVTD geometry formulation.

To further demonstrate the usefulness of the EGBVTD technique, the Nari radial and tangential winds at $Z = 3.0$ km, derived from the dual-Doppler synthesis and the EGBVTD method, are displayed in Figs. 18 and 19, respectively. Because of the limited data overlap for both radar systems, and the incomplete

Fig. 16. The mean-wind relative radial wind field of Typhoon Nari at $Z = 6.0$ km, derived by (a) dual-Doppler synthesis, (b) EGBVTD, and (c) GBVTD. The wind fields are shaded at intervals of 5.0 m s$^{-1}$. The storm center is indicated by a typhoon symbol.
data coverage surrounding the typhoon center, the mean wind cannot be estimated. Owing to this, the traditional dual-Doppler approach can only generate a ground-relative wind field in the storm’s northern quadrant within a relatively small area. However, by subtracting the mean wind, which can be estimated by EGBVTD, Figs. 18a and 19a reveal the mean-wind relative radial and tangential wind structures for the same area obtained from the dual-Doppler synthesis. Inside this small region, one can identify that the radial wind is negative (positive) in the northeast (northwest) portion of the typhoon, and that the region of maximum tangential wind with a peak speed about 25.0 m s$^{-1}$ takes place in the northeast sector of the typhoon. These features inside the northern part of the storm are well recovered by the EGBVTD method, as
shown in Figs. 18b and 19b. Furthermore, the same pictures also demonstrate that although the area of the data overlap is rather limited, the EGBVTD method is still capable of recovering the southern part of the typhoon. Thus, a complete typhoon circulation structure over a much larger domain is obtained. This is also considered to be one advantage of the EGBVTD method over the traditional dual-Doppler synthesis.

7. Other possible extensions

a. EGBVTD + dropsonde/wind profiler for \( V_R \) at higher wavenumbers

The above sections outline the procedures of the EGBVTD method for recovering the mean flow and the asymmetric structures of a TC. In the proposed new algorithm, the TC radial winds can be retrieved up to the wavenumber-1 component. However, for a given analysis ring, if an extra wind measurement is available by instruments such as dropsonde or wind profiler, then it is possible to extend the recovery of the radial wind up to its wavenumber-2 component. We begin by assuming the mean flow \((V_M, \theta_M)\) has already been estimated through the procedures explained in section 3a. The formulation of the TC radial wind expressed in (18) is modified by adding one more wave component:

\[
V_R = V_R C_0 + V_R C_1 \cos \psi + V_R S_1 \sin \psi + V_R C_2 \cos 2\psi + V_R S_2 \sin 2\psi. \tag{27}
\]

The TC radial wind \((V_R)\) at positions \(P_1\) and \(P_2\) (see Fig. 2) can be measured through the procedures described in section 3b and Eqs. (16) and (17). If the true wind \((V_{ds/wp})\), where the subscript stands for the dropsonde or wind profiler observations) is available at a third point \(P_3\), then the TC radial wind \((V_R)\) at this point can be computed by

\[
V_R(P_3) = (V_{ds/wp} - V_M) \cdot \mathbf{r}. \tag{28}
\]

where \(V_M\) is the mean flow, and \(\mathbf{r}\) is a unit vector pointing from the circulation center to the point \(P_3\). Finally, using (7) and the \(V_R\) observations at three distinct points \((P_1, P_2, \text{and } P_3)\), we arrive at a set of equations:

\[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & \sin \psi_1 & \cos 2\psi_1 & \sin 2\psi_1 \\
1 & \sin \psi_2 & \cos 2\psi_2 & \sin 2\psi_2 \\
1 & \sin \psi_3 & \cos 2\psi_3 & \sin 2\psi_3 \\
\end{bmatrix}
\begin{bmatrix}
V_R C_0 \\
V_R S_1 \\
V_R C_2 \\
V_R S_2 \\
\end{bmatrix}
= \begin{bmatrix}
A_1 + A_3 \\
V_R(P_1) - V_R C_1 \cos \psi_1 \\
V_R(P_2) - V_R C_1 \cos \psi_2 \\
V_R(P_3) - V_R C_1 \cos \psi_3 \\
\end{bmatrix}. \tag{29}
\]

Note that \(\psi_1, \psi_2, \text{and } \psi_3\) are the \(\psi\) angles at the points \(P_1, P_2, \text{and } P_3\), respectively, as viewed from radar A. From (29), the unknown coefficients \(V_R C_0, V_R S_1, V_R C_2, \text{and } V_R S_2\) can be solved. With these newly derived coefficients, one can also use \(V_R S_2\) to correct \(V_T C_0\) and \(V_T C_2\), depicted by (6) and (11), respectively, and use \(V_R C_2\) to correct \(V_T S_2\), shown in (10). Nevertheless, it should be pointed out that possible biases still remain in this extension of the method due to the wavenumber-3 amplitudes expressed in (5)–(13).

To validate the above concept, a wavenumber-2 com-
ponent is superimposed on the TC radial wind expressed in (20) with the following forms:

\[
V_{R^C_2} = 5.0 \left( \frac{R}{R_{\text{max}}} \right)^{\mu}, \quad (30)
\]

\[
V_{R^S_2} = -5.0 \left( \frac{R}{R_{\text{max}}} \right)^{\mu}, \quad (31)
\]

where \( \mu \) is set to be 0.5 (−0.5) when \( R \leq R_{\text{max}} \) (\( R > R_{\text{max}} \)). Figure 20a depicts the resulting true wavenumber-2 component of the radial winds. This high-wavenumber structure can be successfully resolved in the retrieved counterpart, except for the distortion occurring near the boundaries, as shown in Fig. 20b. Additional extension is also possible, if more dropsonde/wind profiler data are available along a given analysis ring. Theoretically, with \( N \) (\( N \) is an odd number) points of extra data points, it is possible to recover the asymmetric radial flow up to angular wavenumber \((N + 3)/2\).

b. GBVTD + dropsonde/wind profiler for \( V_R \) at wavenumber 1

In section 3a, a procedure is outlined to recover the mean flow using two Doppler radar systems. However, there are other possible methods to accomplish the above goal. For example, the scheme commonly used in typhoon bogus data (e.g., Davis and Low-Nam 2001) provides an algorithm whereby a vortex can be smoothly removed from the flow field. The wind residue, perhaps after an area averaging, might be considered the surrounding mean flow. In addition, the so-called hurricane-customized extension of the velocity azimuth display (HEVAD) technique developed by Harasti and List (2001), or the hurricane volume velocity processing (HVVP) method proposed by Harasti (2003), also offer potential alternatives to estimate the mean flow. It is worth emphasizing that both HEVAD and HVVP require only single-Doppler radar data. In other words, with the mean winds estimated by the above methods, it appears to be possible to complete the computation of the wavenumber-1 coefficients \((V_{R^C_0}, V_{R^C_1}, V_{R^S_1})\) shown in (18) using a combination of single-Doppler radar GBVTD and one set of dropsonde/wind profiler observation for a given analysis ring.

8. Conclusions and future work

In this research the usage of the GBVTD method is extended to two Doppler radar systems. The major improvement achieved by this so-called EGBVTD approach is that the asymmetric structures of the TC radial winds can be separated from the mean wind and retrieved with substantial success, and the accuracy of the retrieved tangential winds can also be improved accordingly. Although two radars are needed, a comparison with the traditional dual-Doppler synthesis indicates that the advantages of this EGBVTD method are as follows: 1) it is possible to estimate the mean wind and isolate the storm circulation when the radar data coverage surrounding the TC center is incomplete; 2) the requirement for overlapping data from two radars along a given analysis ring is less. This implies that the EGBVTD retrieval can be performed over a larger
domain than that of dual-Doppler synthesis. The results of previous studies and from our experiments also suggest that the GBVTD method is suitable only when the assumption of small asymmetric radial flow is valid. This is more likely to be true for TCs over a flat surface, such as the ocean. When a storm enters a region where two radars are available for observations, the EGBVTD method is a better approach than the GBVTD or the traditional dual-Doppler synthesis to analyze the asymmetric structures within the storm circulation.

The concept behind the proposed EGBVTD is verified using analytic TC circulation models, as well as a real case study of Typhoon Nari (2001). The authors also propose some possible extensions to combine the Doppler radar observations with other independent instruments such as dropsonde/wind profiler to recover more details of the asymmetric TC structures. Finally, it is worth mentioning that some encouraging progress has been made to eliminate the problem of distortion that appears in the retrieved wind fields using a formulation called velocity distance azimuth display (VDAD) by Jou et al. (1996). The applicability of this method will be studied and reported in a future paper.

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