Toward assimilating radio occultation data into atmospheric models

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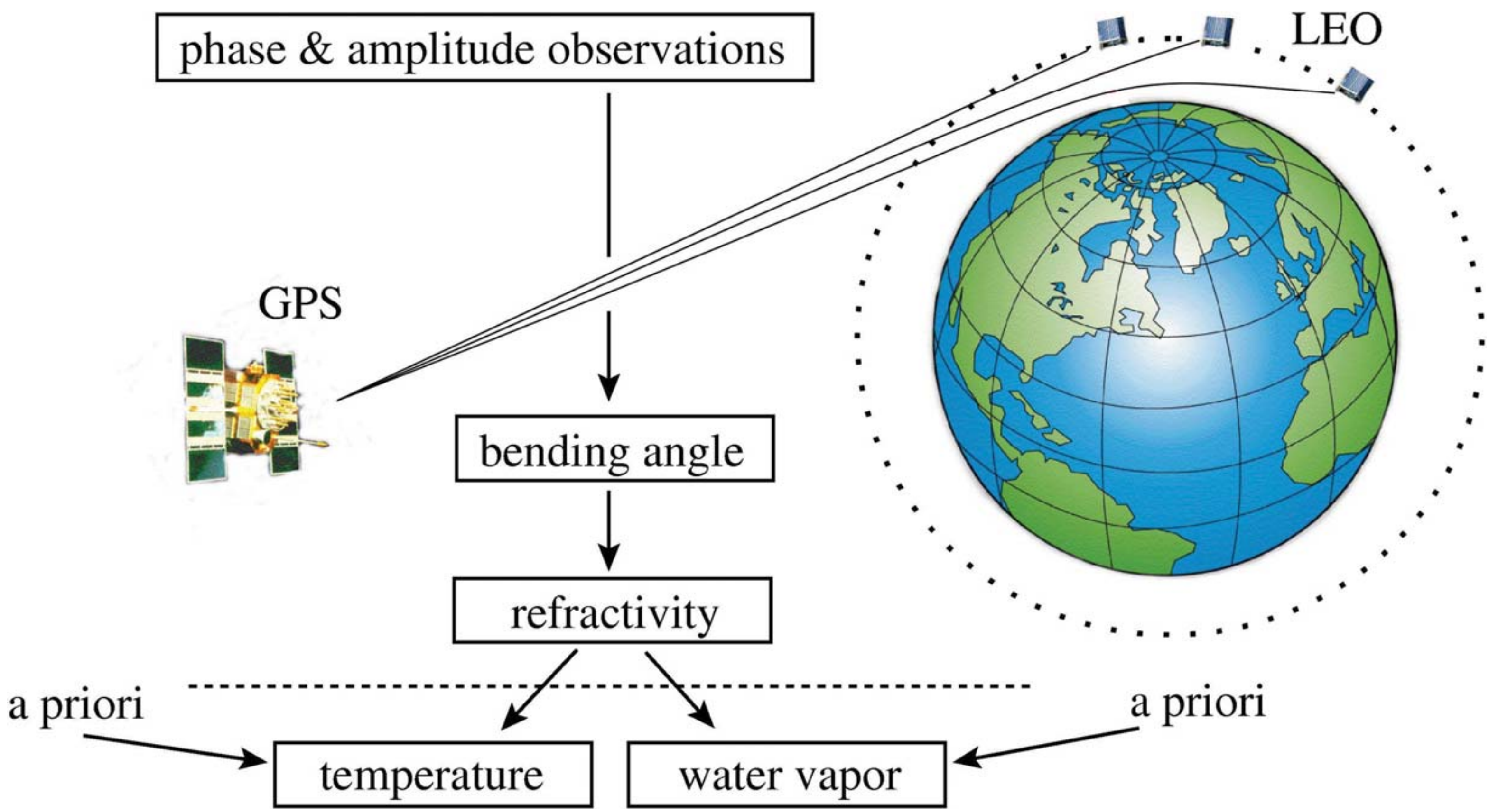
Overview

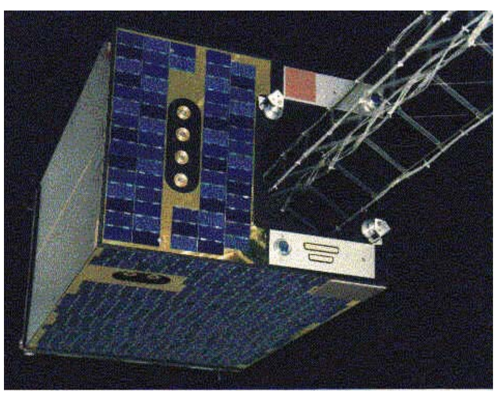
- Basic principle of GPS radio occultations
- GPS occultation data from the Danish Ørsted satellite
- Very short introduction to data assimilation
- An observation operator for the assimilation of refractivity
- Simulations with an idealized model of a weather front
- "Truth" & errors in a new perspective
- Forward-inverse mapping as a general concept

Basic principle of GPS radio occultations



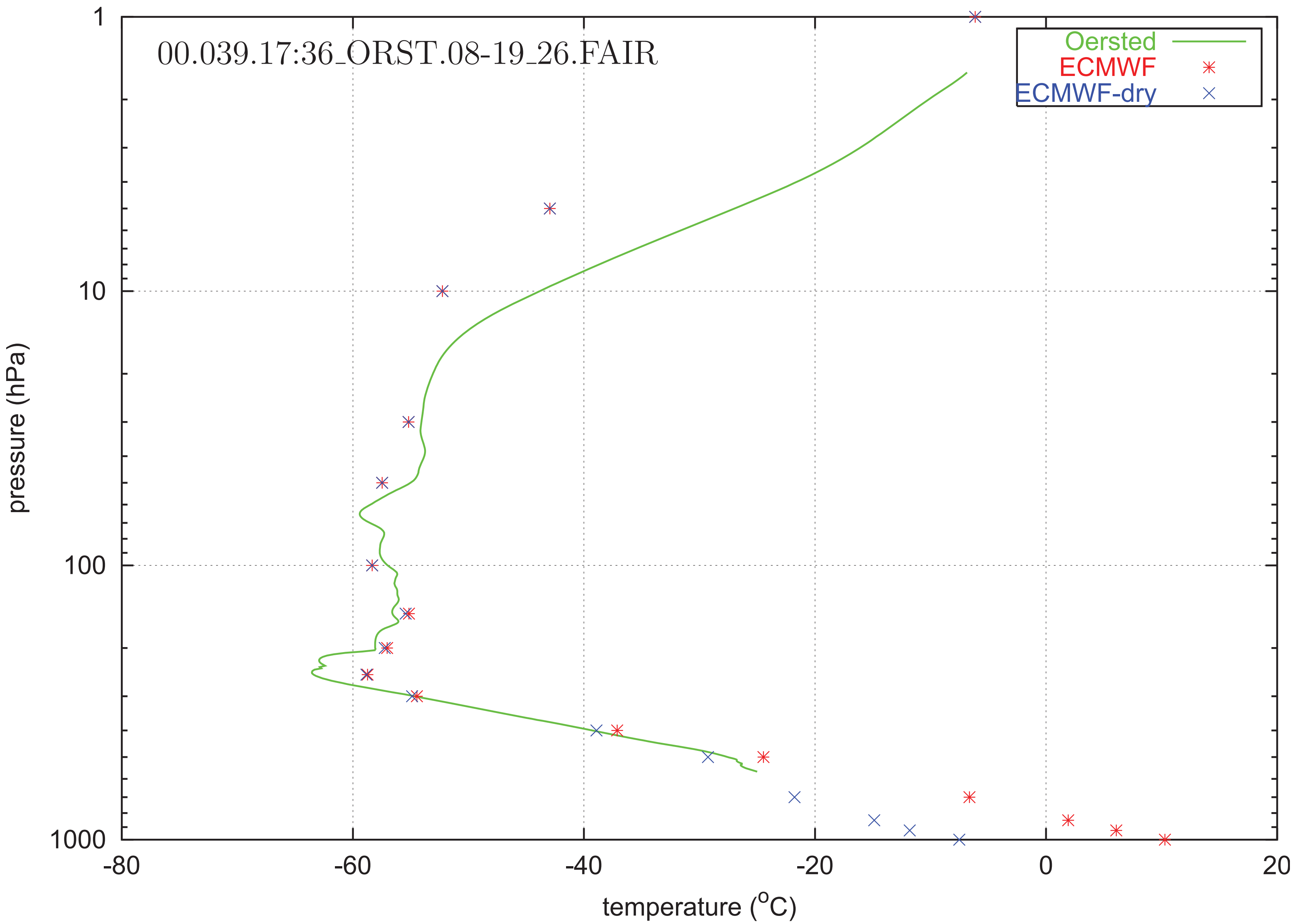
- Retrieval relies on the assumption of local spherical symmetry

The Ørsted satellite



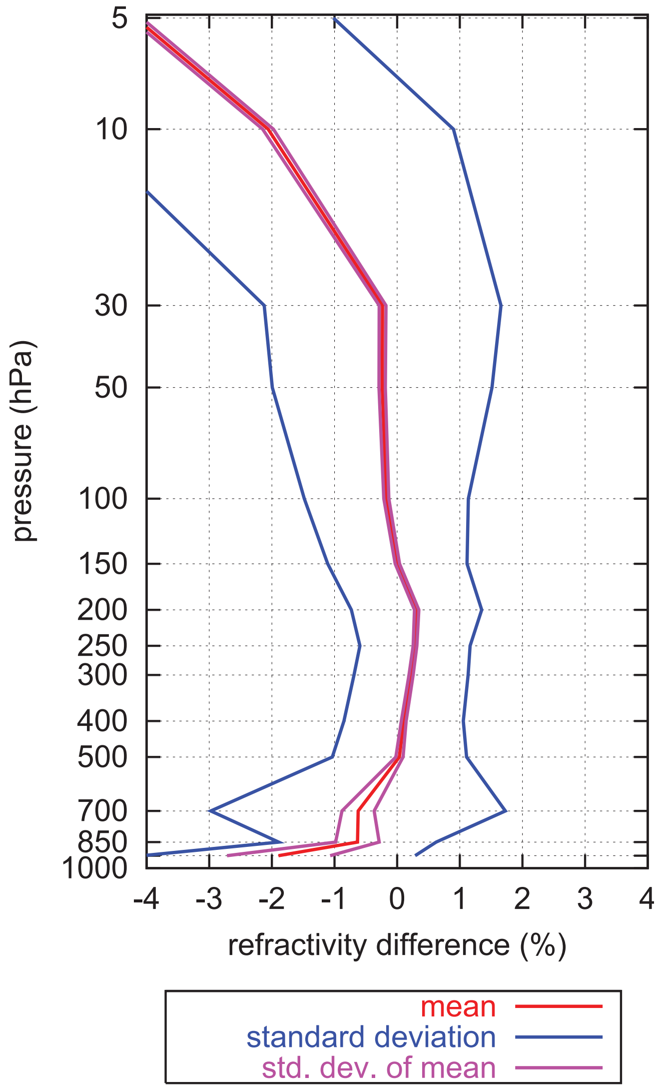
- Named after Danish physicist Hans Christian Ørsted (1777-1851)
- Launched on February 23, 1999
- Primary mission: To measure Earth's magnetic field
- Secondary mission: GPS radio occultation measurements
- Turbo-Rogue GPS receiver developed by JPL
- Occultation data (occasionally) since April 1999
- Still going strong

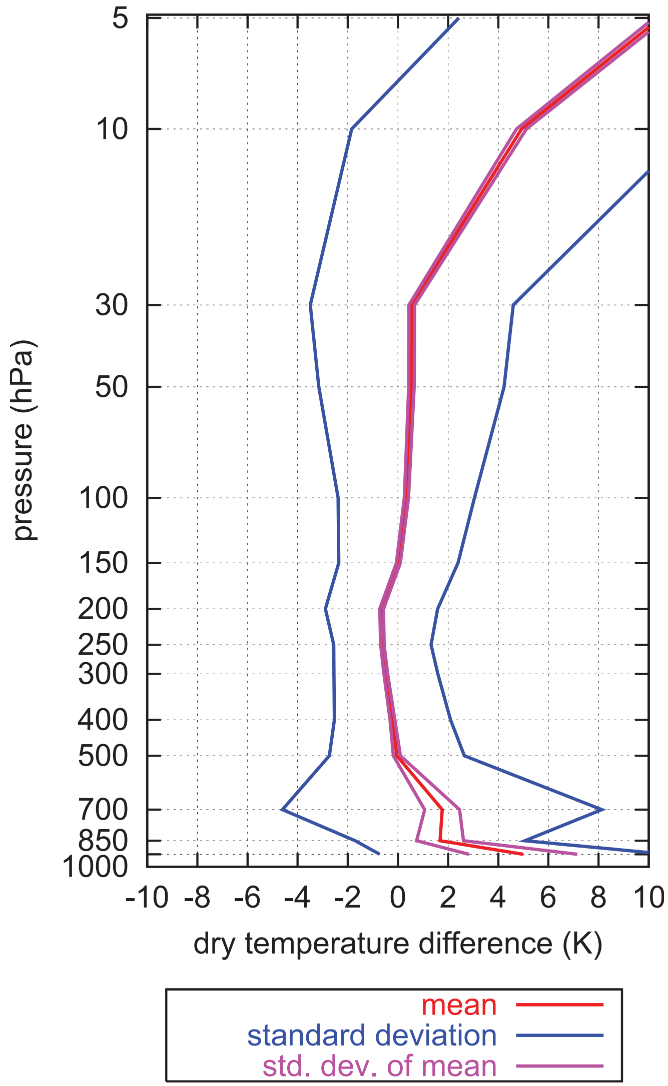
Example of retrieved temperature profile



Statistical comparisons to ECMWF

∼ 1000 profiles from 20 days in February 2000 (Ørsted−ECMWF)





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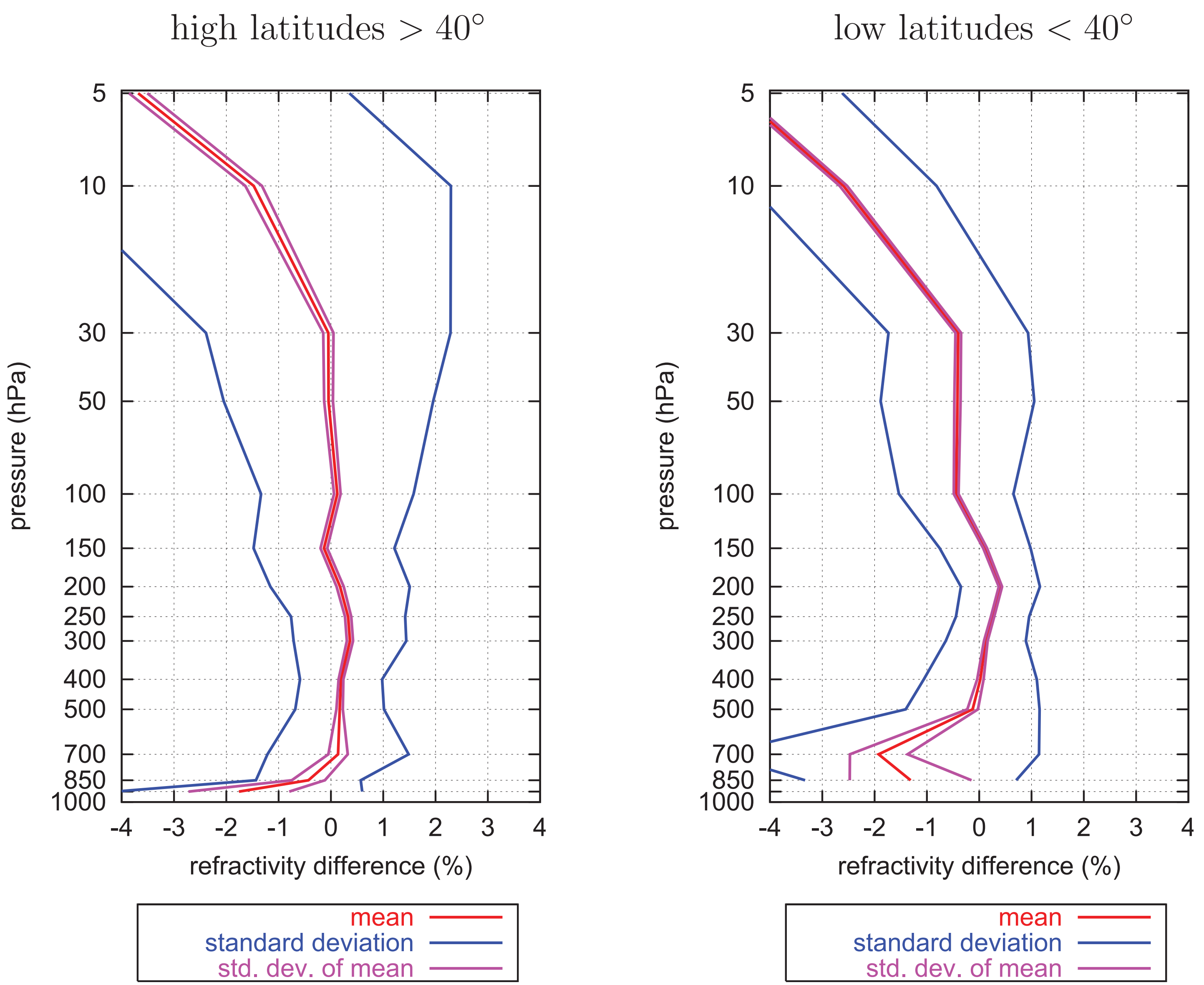
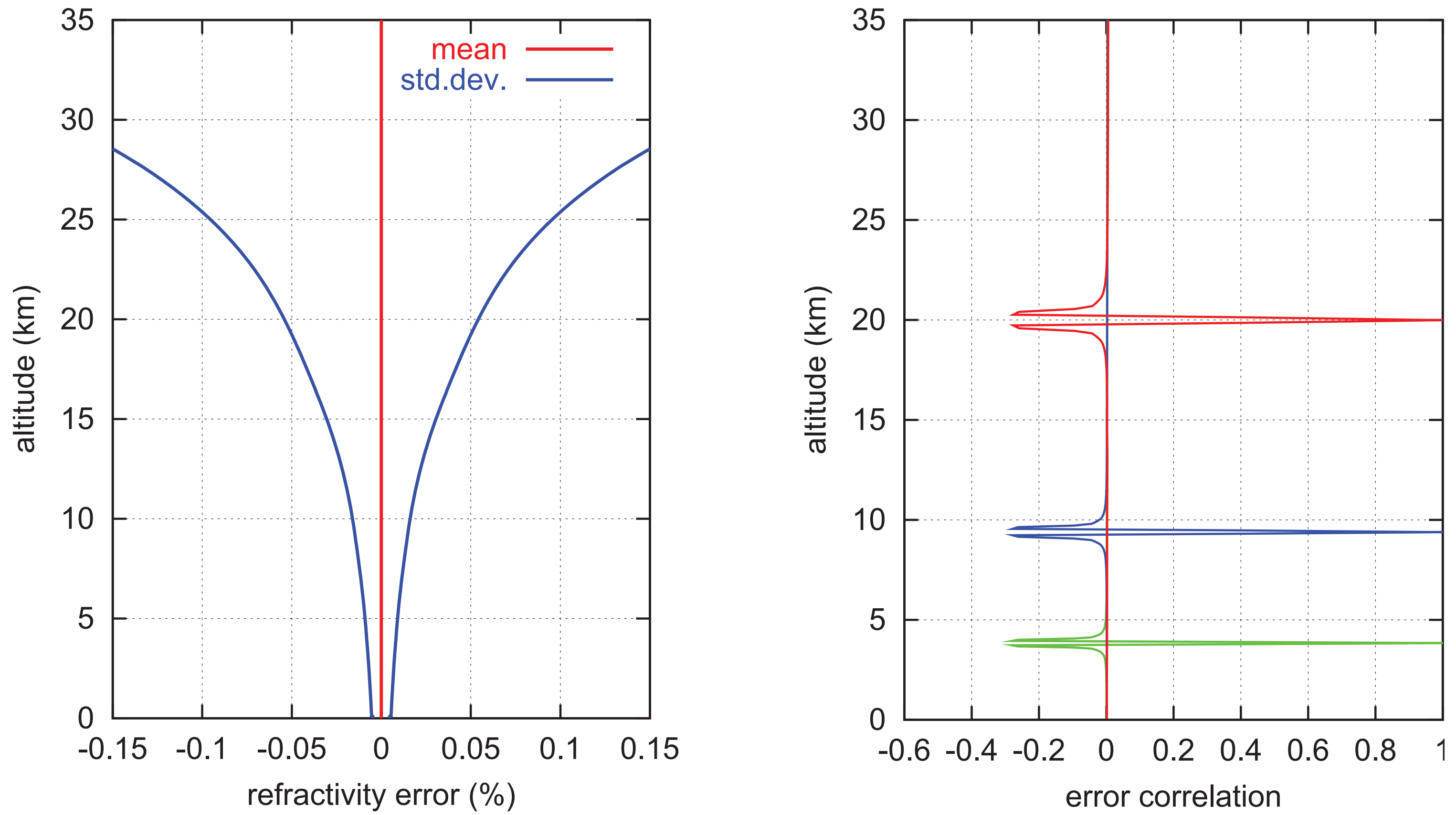


The real potential. . . perhaps

Error propagation analysis assuming 3 mm random phase noise and spherical symmetry



- Ionosphere & upper boundary uncertainty not included
- Atmospheric multipath and diffraction effects not included

Basic principle of data assimilation

Minimization of a cost function:

$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_{\mathrm{b}})^{\mathrm{T}} \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_{\mathrm{b}}) + (\mathbf{y} - \mathbf{H}(\mathbf{x}))^{\mathrm{T}} (\mathbf{O} + \mathbf{F})^{-1} (\mathbf{y} - \mathbf{H}(\mathbf{x}))$$

$\mathbf{x}$ is a vector containing the model variables to be solved for

$\mathbf{x}_{\mathrm{b}}$ is the background field provided by an NWP model

$\mathbf{y}$ is a vector containing the "observations" (e.g., refractivity)

$\mathbf{H}$ is a matrix referred to as the observation operator

$\mathbf{B}$ is the background error covariance matrix

$\mathbf{O}$ is the observation error covariance matrix

$\mathbf{F}$ is the representativeness error covariance matrix

Assimilation strategy for occultation data

Two main strategies have received the most attention

- Assimilation of bending angle [e.g., Eyre 1994; Zou et al. 1999; Healy 2001]
- Assimilation of refractivity [e.g., Zou et al. 1995; Kuo et al. 1997; Healy et al. 2003]

If refractivity: how should a derived profile be interpreted?

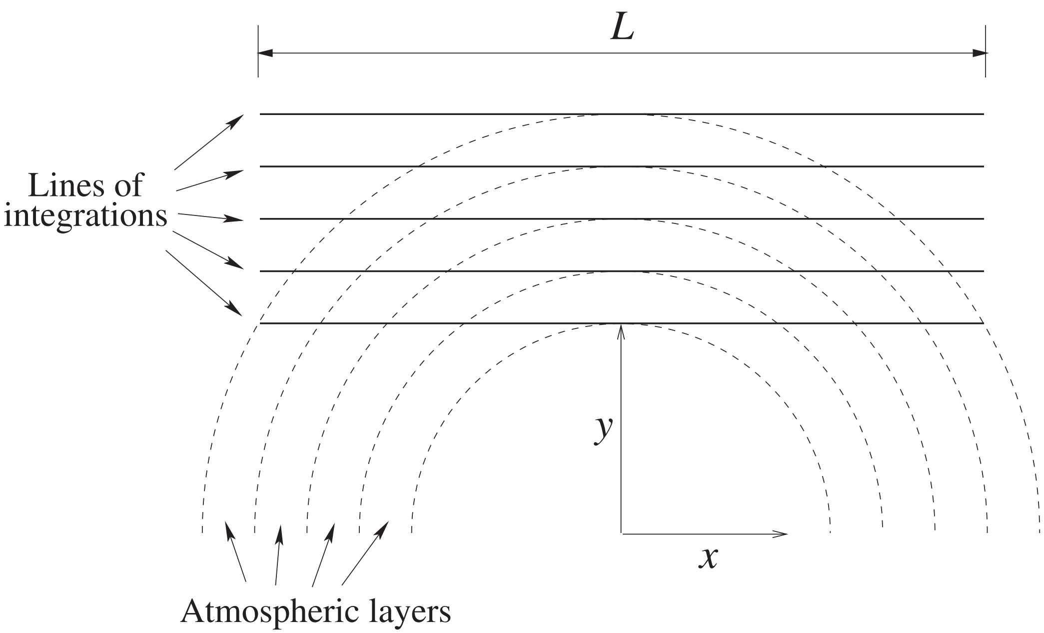
- as a vertical profile at fixed mean event location?

- along the locus of the estimated tangent points?

- some kind of 2D average of the refractivity field?

Answer: Some kind of 2D average . . .

Forward-inverse refractivity mapping



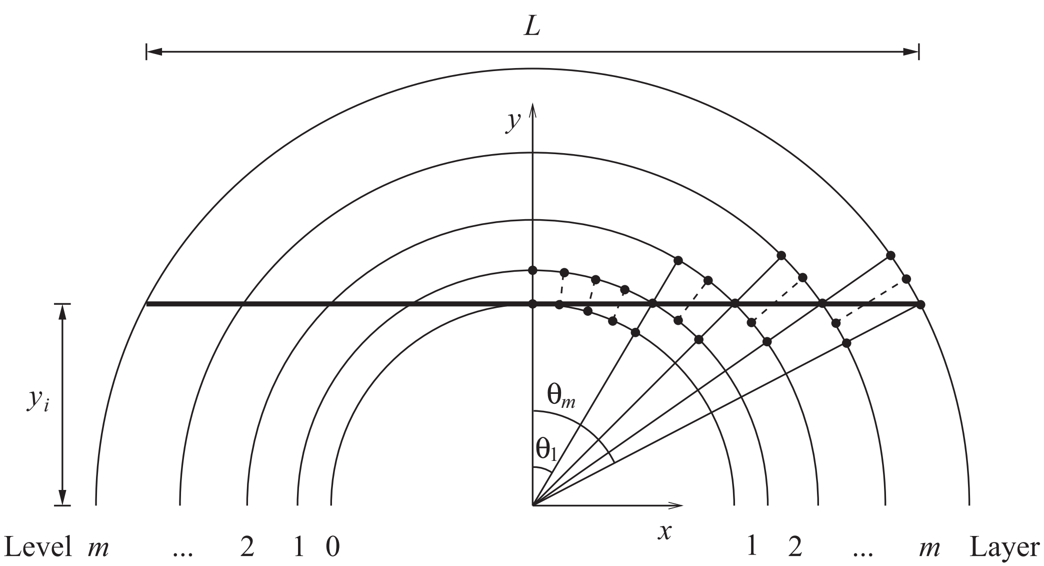
Mimicking the observations and the Abel inversion using finite straight lines

Somewhat similar to a 2D weighting function [Ahmad and Tyler 1998]

Basic requirement: $$\int_{-L/2}^{L/2} N(x,y)\mathrm{d}x = \int_{-L/2}^{L/2} N_{\mathrm{map}}(r)\mathrm{d}x$$

- Discretized and solved for $N_{\mathrm{map}}(r)$
- $N(x,y)$ evaluated at (pressure) levels of NWP model

Forward-inverse refractivity mapping



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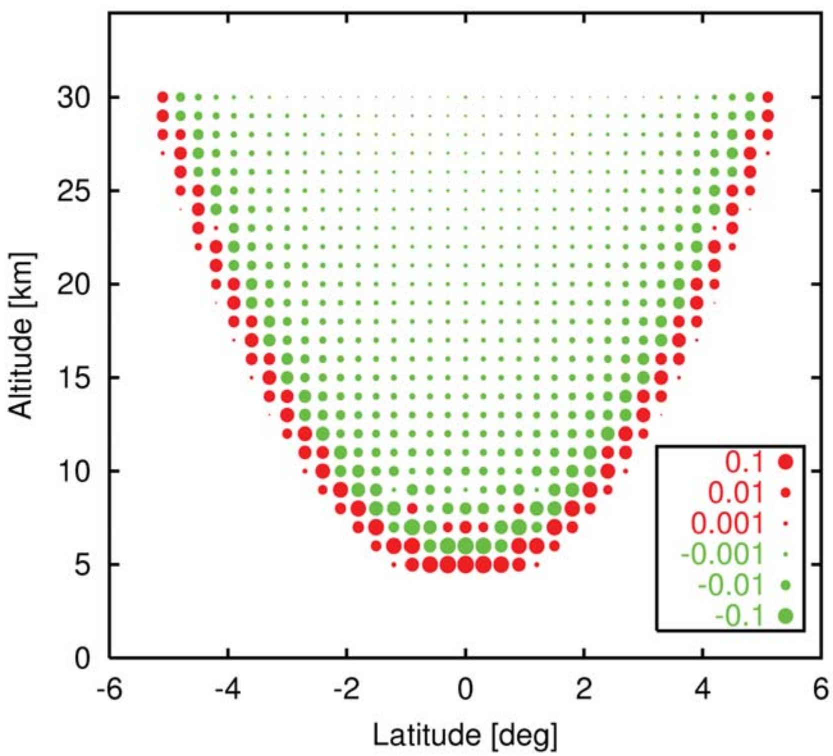
Mapping-weights on a regular grid



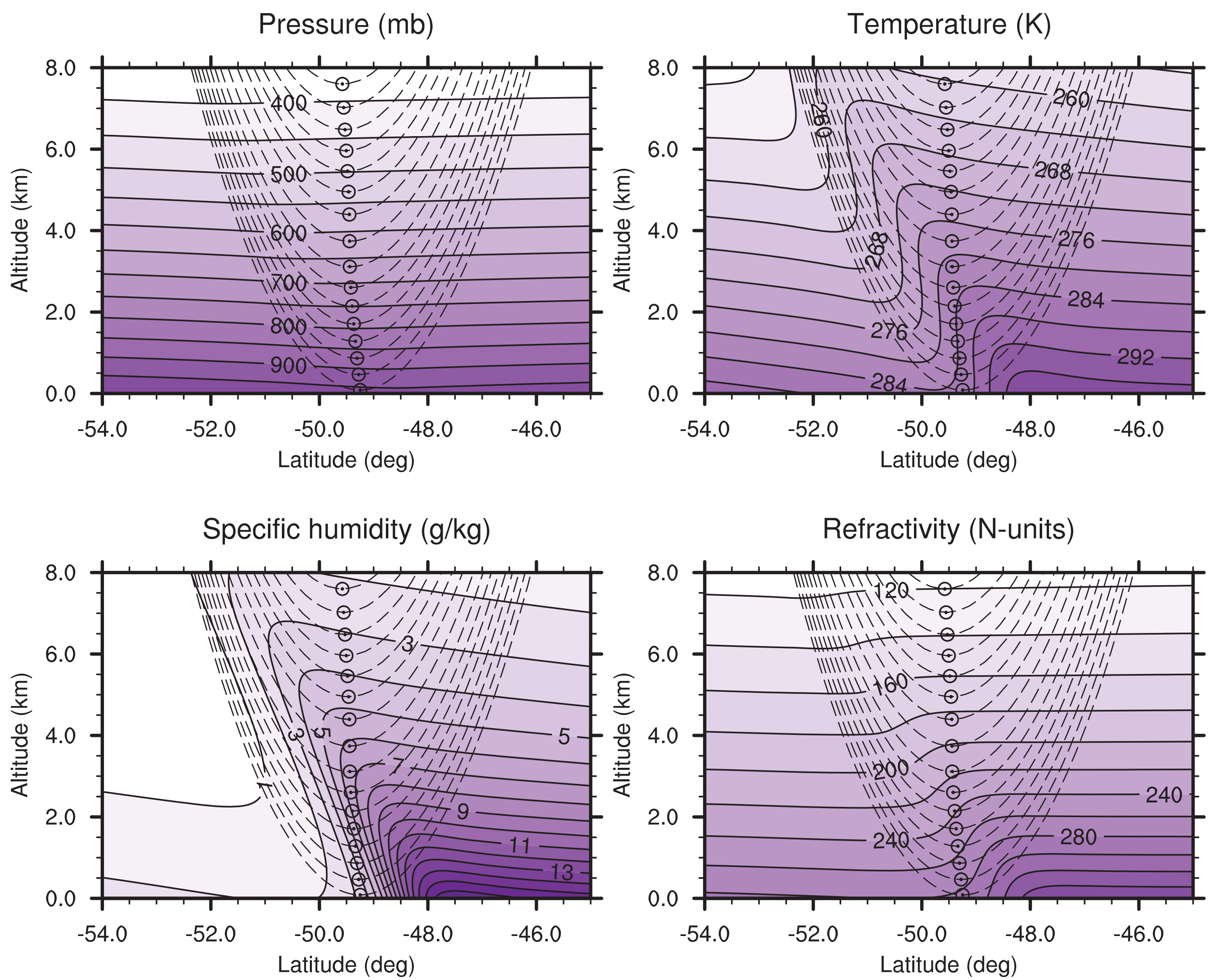
Weights for the mapping interpolated to a regular grid in altitude-latitude coordinates (tangent point at 5 km)

Mapping not restricted to fixed 2D plane; weights are in practice centered at tangent points (drifting during the occultation)

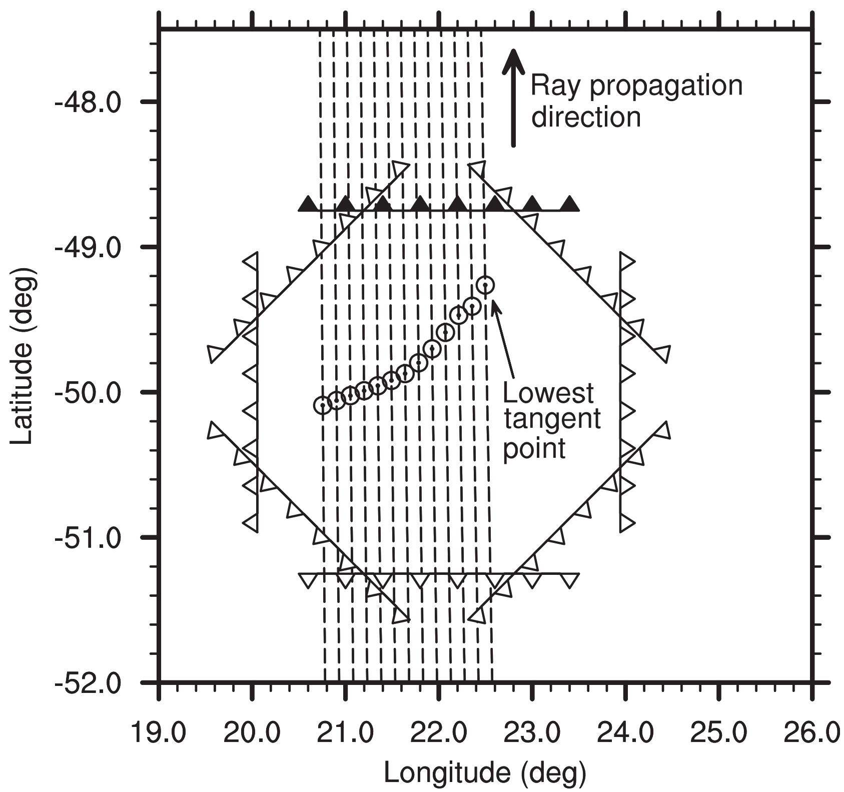
- In practice mapping consists of fast recursive formula
- Exact in (hypothetical) case of spherical symmetry
- Can be applied to model pressure levels instead of height

How well does it represent measurements with horizontal gradients?

Simulations with model of a weather front



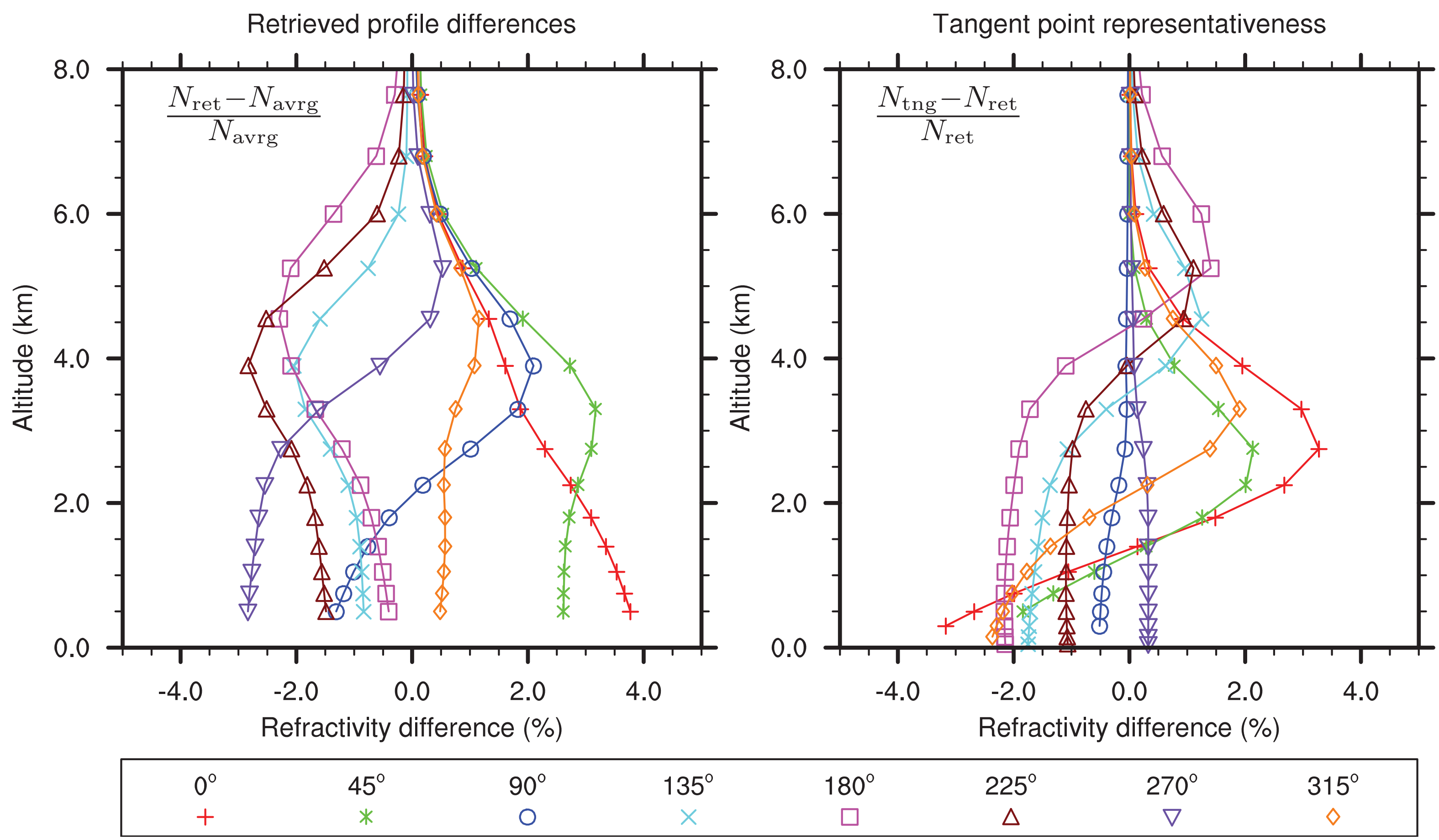
Case study setup



Eight separate simulations with different positions and orientations of the front relative to the rays

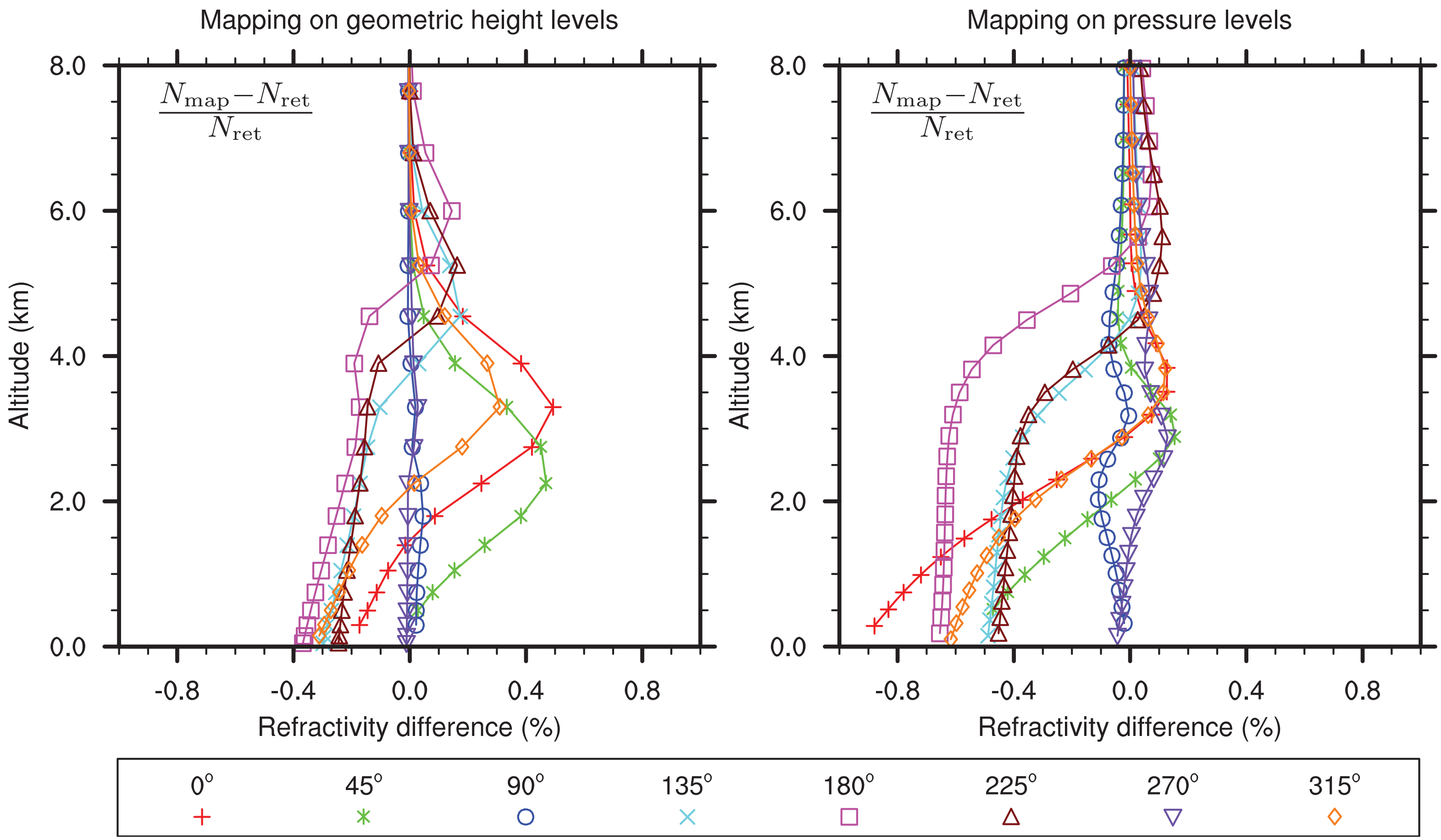
- Simulated occultation observations via accurate 3D ray-tracing
- Retrieved refractivity using spherical symmetry (Abel transform)
- Compared with the fast forward-inverse refractivity mapping

Retrieved and tangent point profiles



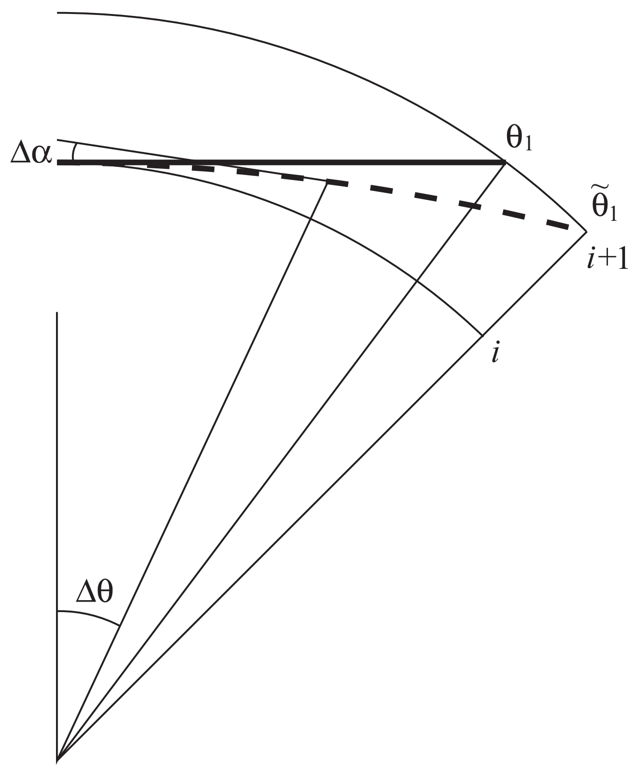
- Retrieved profiles differ from each other by a few percent
- Tangent point profiles differ from retrieved profiles by a few percent

Mapping results



- Representativeness of mapping on height levels better than 0.5%
- Mapping on pressure levels only degrades results slightly (biased?)

Taking into account the ray bending



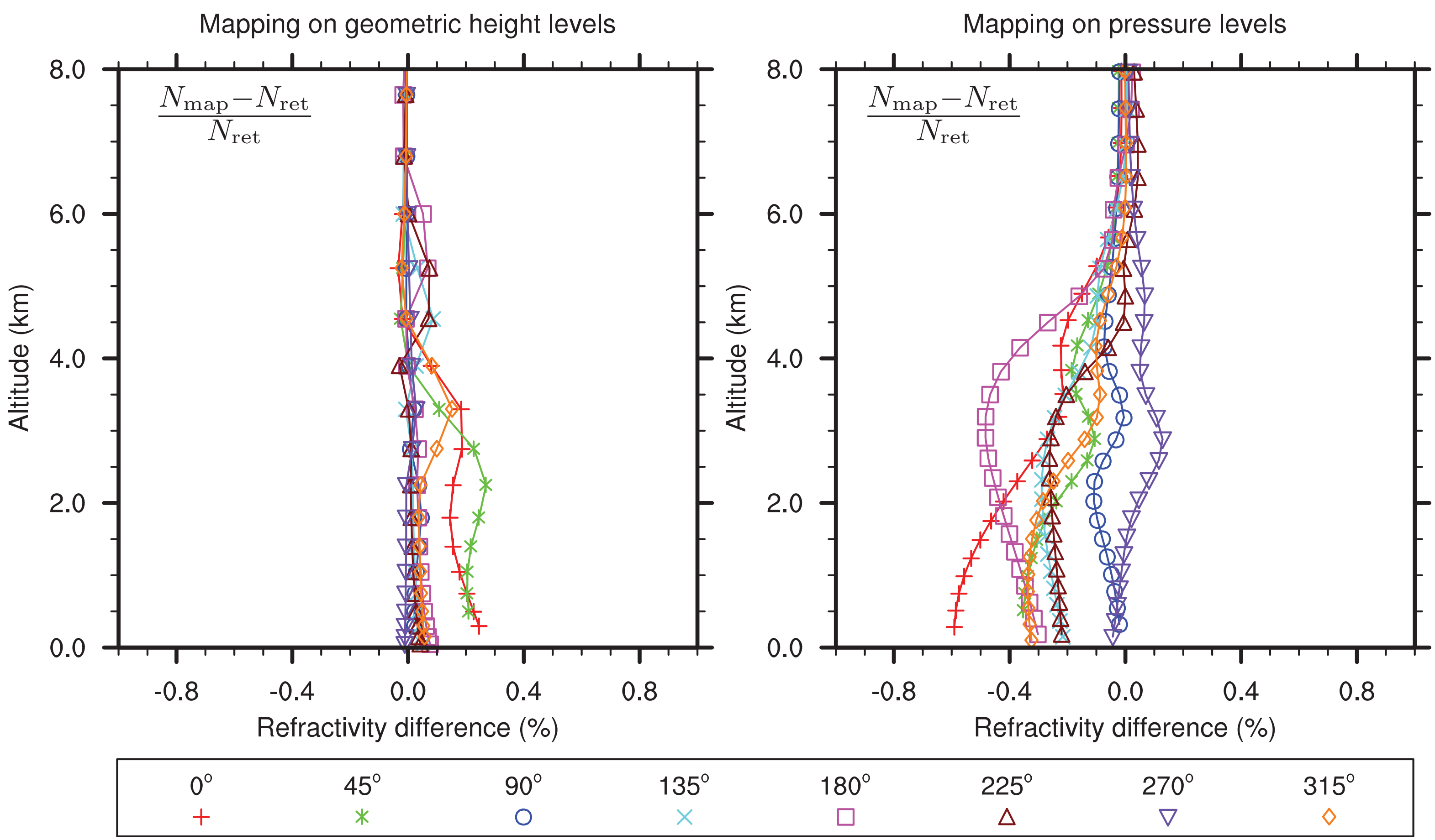
$$\frac{\mathrm{d}\alpha}{\mathrm{d}\theta} \approx 1 - \frac{\theta_1^2}{\tilde{\theta}_1^2}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\theta} \approx -R_\mathrm{e}\frac{\mathrm{d}N_\mathrm{ret}}{\mathrm{d}r}$$

Trick: Evaluate refractivity at slightly larger angles $\tilde{\theta}$

- Mapping-weights are not altered (mapping remains linear)

Mapping results with bending



- Appreciable improvement for mapping on height levels
- Small improvement for mapping on pressure levels

Example of observation operator

1. Horizontal interpolation (along pressure surfaces) of the temperature and specific humidity to the points used in the mapping
2. Evaluation of the refractivity at these points
3. Mapping the refractivity into a profile at the tangent points using the mapping operator
4. Integration of the hydrostatic equation to obtain a precise relation between pressure and geometric height at grid points near the tangent points
5. Horizontal interpolation of the geometric height to the tangent point locations
6. Vertical interpolation of the mapped refractivity to the observation points (observed tangent points)

Advantages of refractivity mapping

- Errors of representativeness are small
  - the mapping roughly represents how the observations are made and how the data are processed using the spherical symmetry constraint
- The mapping operator and its adjoint are linear and can be described by fast recursive formulas
  - weighting coefficients can be calculated once and for all for a given NWP model
- The mapping is applied within finite limits
  - extrapolation of the NWP model above its highest level is not necessary
- The mapping can be carried out on pressure levels without modification
  - subsequent integration of the hydrostatic equation is limited to a few locations near the tangent points

Truth and consequences (errors)

Definition of "truth": The horizontally homogeneous refractivity which would reproduce the actual phase observations (disregarding ionospheric effects and observation errors)

- Observation/retrieval errors
  - Orbit errors
  - Receiver related errors
  - Anti-Spoofing
  - Local multipath
  - Ionosphere correction residual
  - Ellipsoidal earth correction residual
  - Upper boundary condition (a priori)
  - Surface reflections
  - CT/FSI approximations
- Representativeness errors
  - Model interpolation errors
  - Uncertainty in refractivity formula
  - Forward-inverse mapping errors
  - Model hydrostatic integration error
- Super-refraction ambiguity?

Which of these are the most important?

Dominating errors in the troposphere

- Tracking errors (∼1–10%) [Beyerle et al. 2003; Ao et al. 2003]
  - Bias! Hopefully not a problem with open loop tracking
- Super-refraction (∼10% in worst case) [Sokolovskiy 2003]
  - Bias! Difficult to identify in practice; data still contain valuable information

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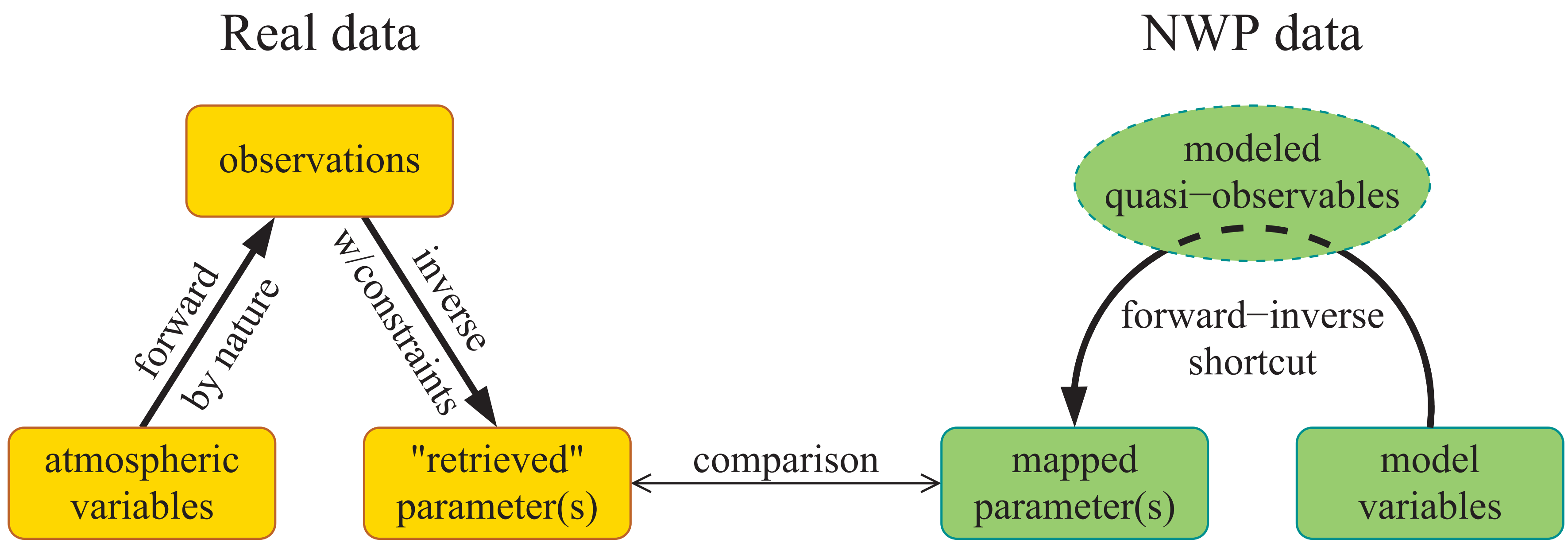
- Thermal noise; ionosphere ($< 0.1\%$ below ∼25 km)
- CT/FSI approximations (?) . . . random error propagation (?)
- Errors when GPS Anti-Spoofing is on (?)

- Forward-inverse mapping errors (∼0.5% in worst case)
- Other representativeness errors (?)

Refractivity error covariances needed

Forward-inverse mapping in general



Strength: Near cancellation of otherwise crude approximations $\Rightarrow$ fast, but still reasonably accurate

- Useful for all kinds of occultation measurements (absorption too)
- Could perhaps be adapted for assimilation of radiances, etc...

Forward-inverse mapping in general

- Identification of constraint(s) such that…
  - the inversion of the remotely sensed observations is a well-posed problem
  - the inversion can be viewed merely as a transformation of the observables into another parameter space without significant loss of information

- Identification of approximations in a forward-inverse mapping which…
  - to some extent mimics the observation process
  - adheres to the constraint(s) in the inverse part

The approximations should allow for a fast implementation of the forward-inverse mapping, while their near cancelation should ensure a small error of representativeness.

Conclusion—what should we assimilate?

- Linear refractivity mapping is fast (how fast?) and reasonably accurate

- Simple (tangent point) refractivity profile assimilation would be faster but less accurate ($\sim$ factor of five)

- Bending angle assimilation is potentially the most accurate but too slow $\Rightarrow$ trade-off between accuracy and speed $\Rightarrow$ corresponding error probably not smaller than linear refractivity mapping $\Rightarrow$ good argument for operational assimilation of bending angles seems hard to find