Radio-holographic inversions of tropospheric RO signals

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What are the radio-holographic (RH) methods?

Why do we need RH methods for radio occultations?

Back propagation method.

Sliding spectral (radio-optics) method.

Methods transforming complex wave function from coordinate to impact parameter representation
(i) principle of the phase matching
(ii) Full Spectrum Inversion
(iii) Canonical Transform

The goal is to determine impact parameters and bending angles for all rays arriving at receiver during RO.

When only one ray arrives at each point, the arrival angle is determined from the derivative of phase (Doppler).

This is not possible when several rays are arriving at one point.

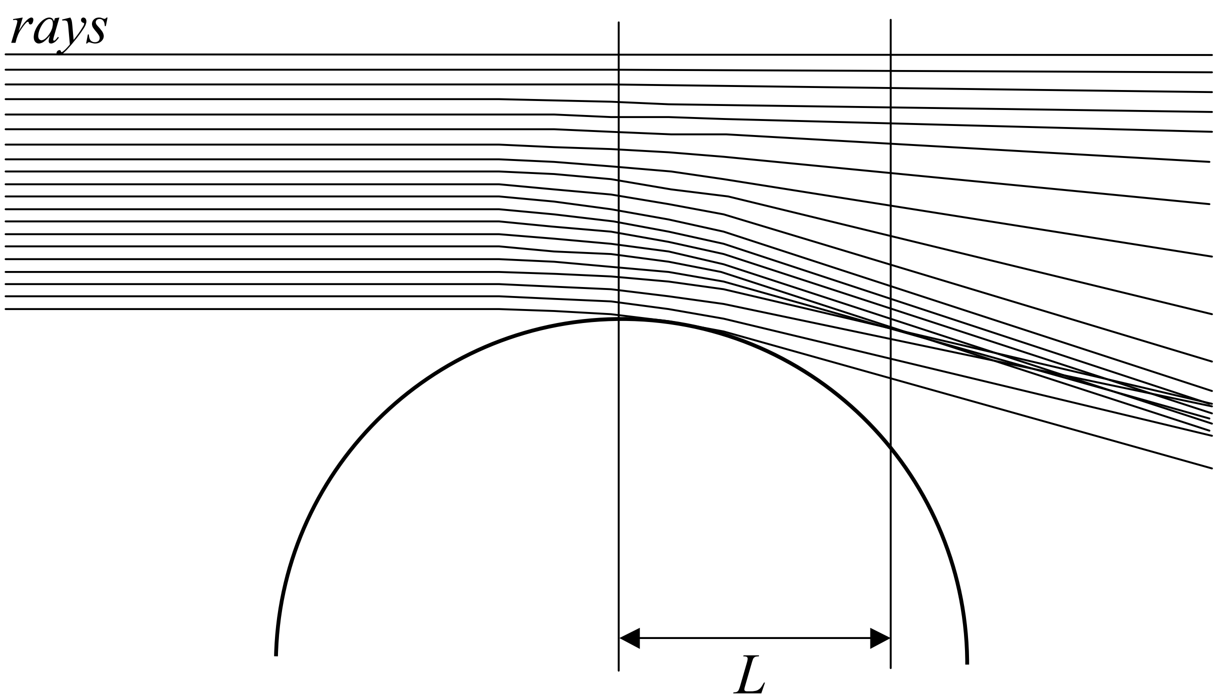
Multi-path propagation almost always occurs the moist troposphere.

RH methods allow to find arrival angles for individual rays under multi-path propagation.

RH methods use both phase and amplitude of RO signal.

RH methods are non-local. Arrival angle of each ray is determined from RO signal inside some finite time interval, or from the whole RO signal.

Multi-path(ray) propagation



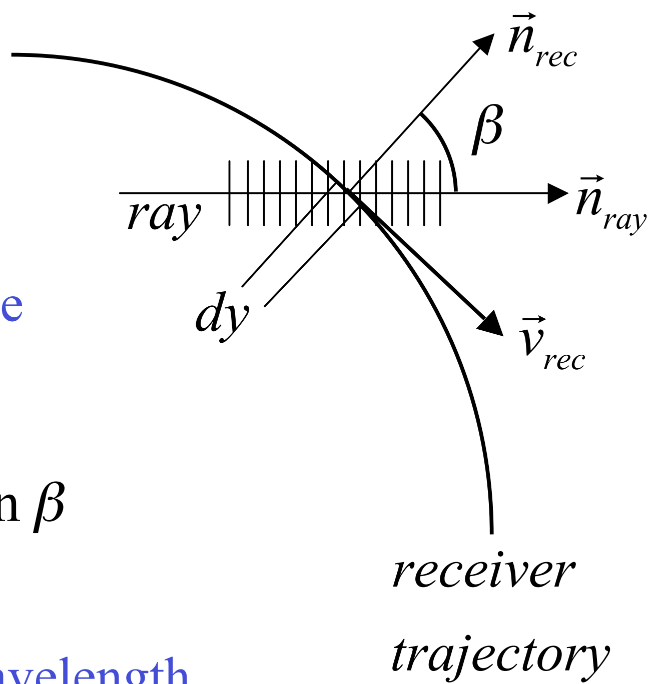
$$\frac{d\alpha}{da} > 0$$ condition for multi-path

$$L \approx \left(\frac{d\alpha}{da}\right)^{-1}$$ distance to caustic

Single-ray propagation

In spherically-symmetric atmosphere, a ray is the curve in the RO plain: transmitter-receiver-center of sphericity. Only motion across wave fronts (in the RO plain) contributes to the change of the measured phase ϕ

$$u(y) = a(y)\exp[i\phi(y)]$$



When the transmitter is stationary, the derivative of phase (Doppler frequency) of RO signal allows to determine the angle between the ray and receiver trajectory.

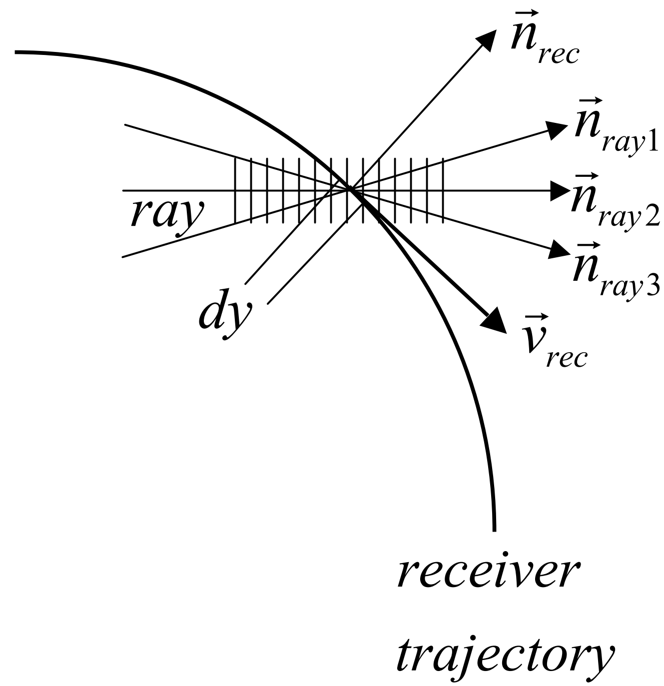
$$\frac{d\phi}{dy} = k\sin\beta \qquad \text{or} \qquad f_d = \frac{d\phi}{dt} = kv_{rec}\sin\beta$$

where $k = \dfrac{2\pi}{\lambda}$ wave-number λ wavelength

$\vec{v}_{rec}$ receiver velocity $\vec{n}_{rec}$ normal to receiver trajectory

Multi-ray propagation

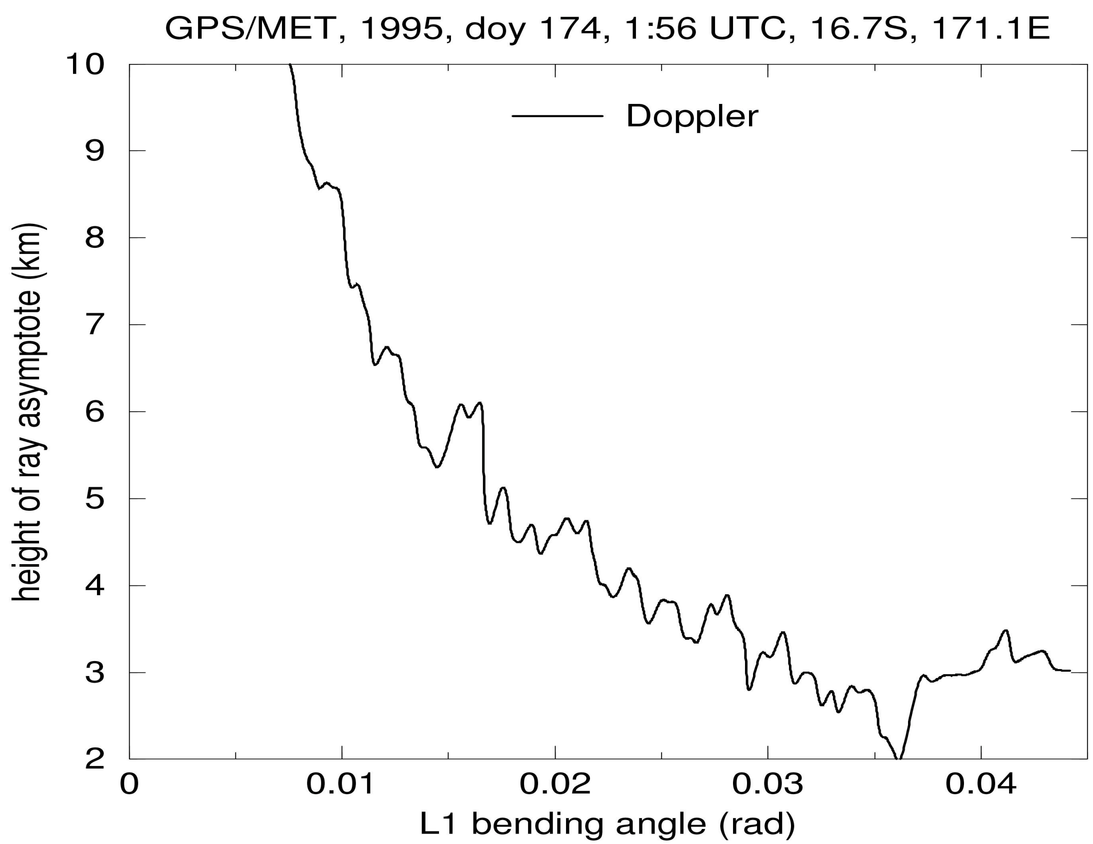
When more than one ray
are arriving at one point



$$u(y) = A(y)\exp[i\phi(y)] =$$
$$= A_1(y)\exp[i\phi_1(y)] +$$
$$+ A_2(y)\exp[i\phi_2(y)] +$$
$$+ A_3(y)\exp[i\phi_3(y)] +$$
$$+ \ldots\ldots$$

it is not possible to determine the arriving angle
of any of the rays from the derivative of phase

Bending angle as the function of height of ray asymptote reconstructed from Doppler for GPS/MET tropical occultation



Back propagation method

Let's temporary forget about rays

Let's assume that the transmitter is stationary

When N=N(r), the EM field does not depend on the coordinate transverse to RO plane

EM field in a vacuum is described by Helmholtz equation

$$\Delta u(\vec{r}) + k^2 u(\vec{r}) = 0, \quad \vec{r} = \{x, z\}$$

The boundary condition: the complex EM field $u(\vec{y})$ on receiver trajectory

The solution (at large distances from receiver trajectory):

$$u(\vec{r}) = \sqrt{\frac{k}{2\pi}} \int u(\vec{y}) \cos\varphi_{ry} \frac{\exp(-ik\,|\,\vec{r} - \vec{y}\,| + i\pi/4)}{\sqrt{|\,\vec{r} - \vec{y}\,|}} dy$$

Back propagation method

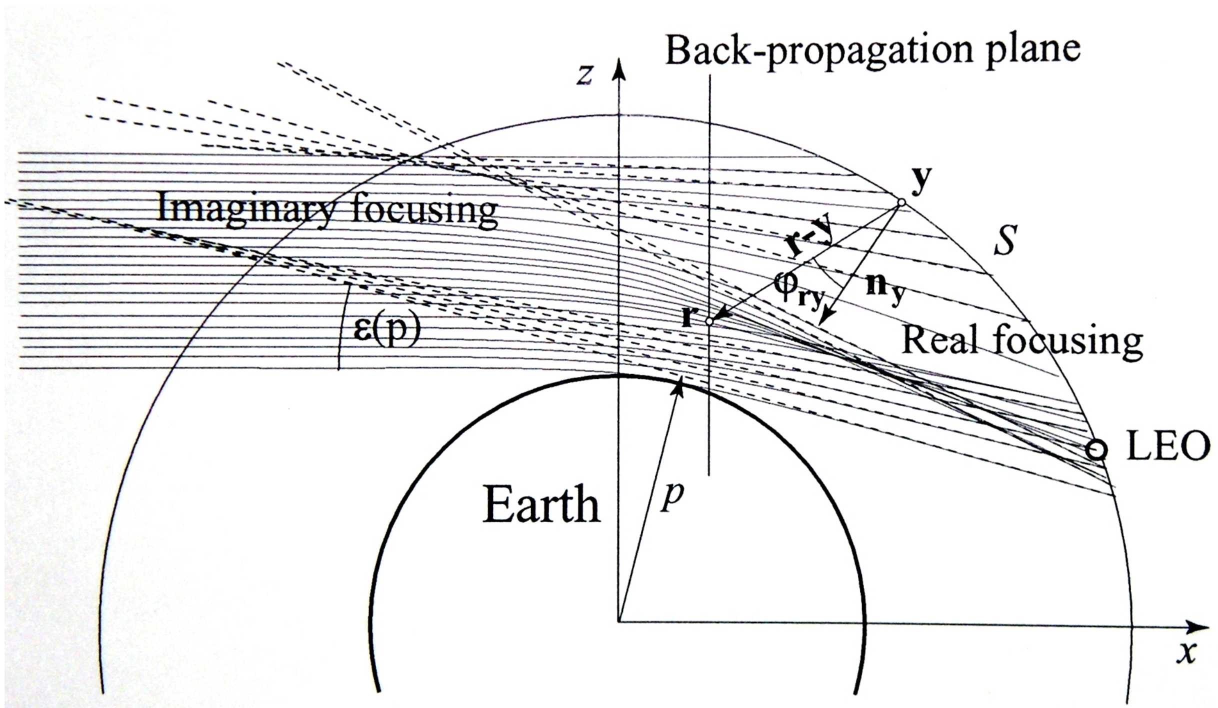


Figure by M.Gorbunov, 2000

$$u(\vec{r}) = \sqrt{\frac{k}{2\pi}} \int A(\vec{y}) \cos\varphi_{ry} \frac{\exp[ikS(\vec{y}) - ik\,|\vec{r}-\vec{y}| + i\pi/4]}{\sqrt{|\vec{r}-\vec{y}|}} dy$$

Back propagation method

What is the physical sense of the back-propagated (BP) EM field?

The BP EM field is the coherent sum of partial waves arriving from different points on the receiver trajectory

The main input is introduced at the phase stationary points:

$$\frac{dS}{dy} - \frac{\partial |\vec{r} - \vec{y}|}{\partial y} = 0$$

For each **r**, each stationary point corresponds to separate virtual ray propagating from receiver trajectory back to **r**

In practice, it is sufficient to calculate the integral in some interval around the main stationary point (which covers all other local stationary points) :

$$S(\vec{y}) - |\vec{r} - \vec{y}| = abs.\min$$

Back propagation method

The BP EM field in a vacuum is different from real EM field in the atmosphere

EM wave propagation in a vacuum is equivalent to straight-line rays
in geometric optics

BP is equivalent to straight-line continuation of rays back to the atmosphere

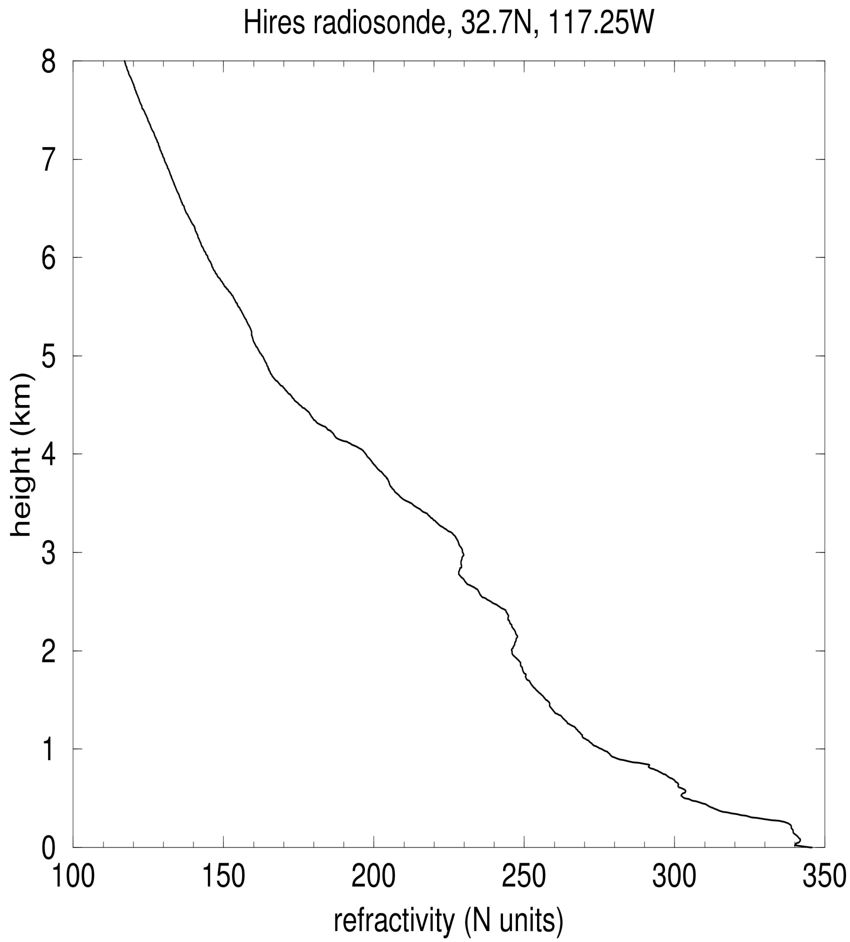
The straight-line continuation does not change impact parameters of the rays

Arrival angles of rays can be calculated from the derivative of the phase
on a virtual trajectory located in a single-ray region (closer to atmosphere)

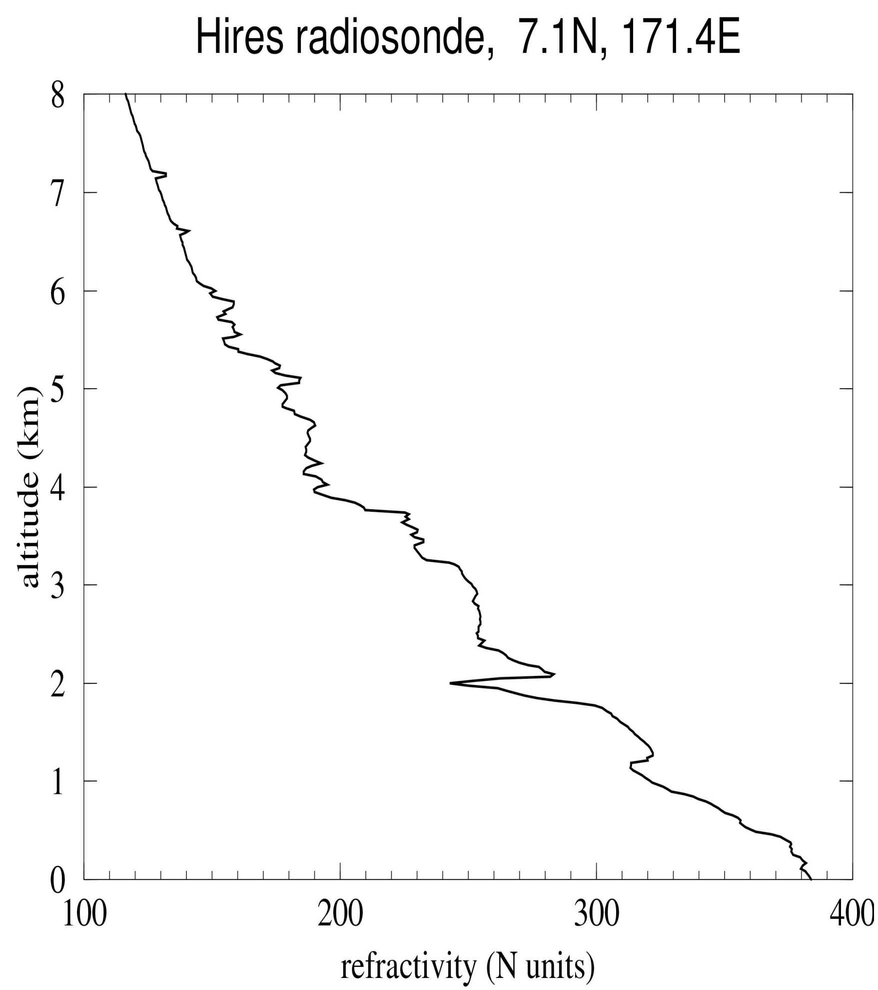
The single-ray region (between real and imaginary caustics) always exists,
except for the case of the super-refraction

High-resolution radiosonde N-profiles

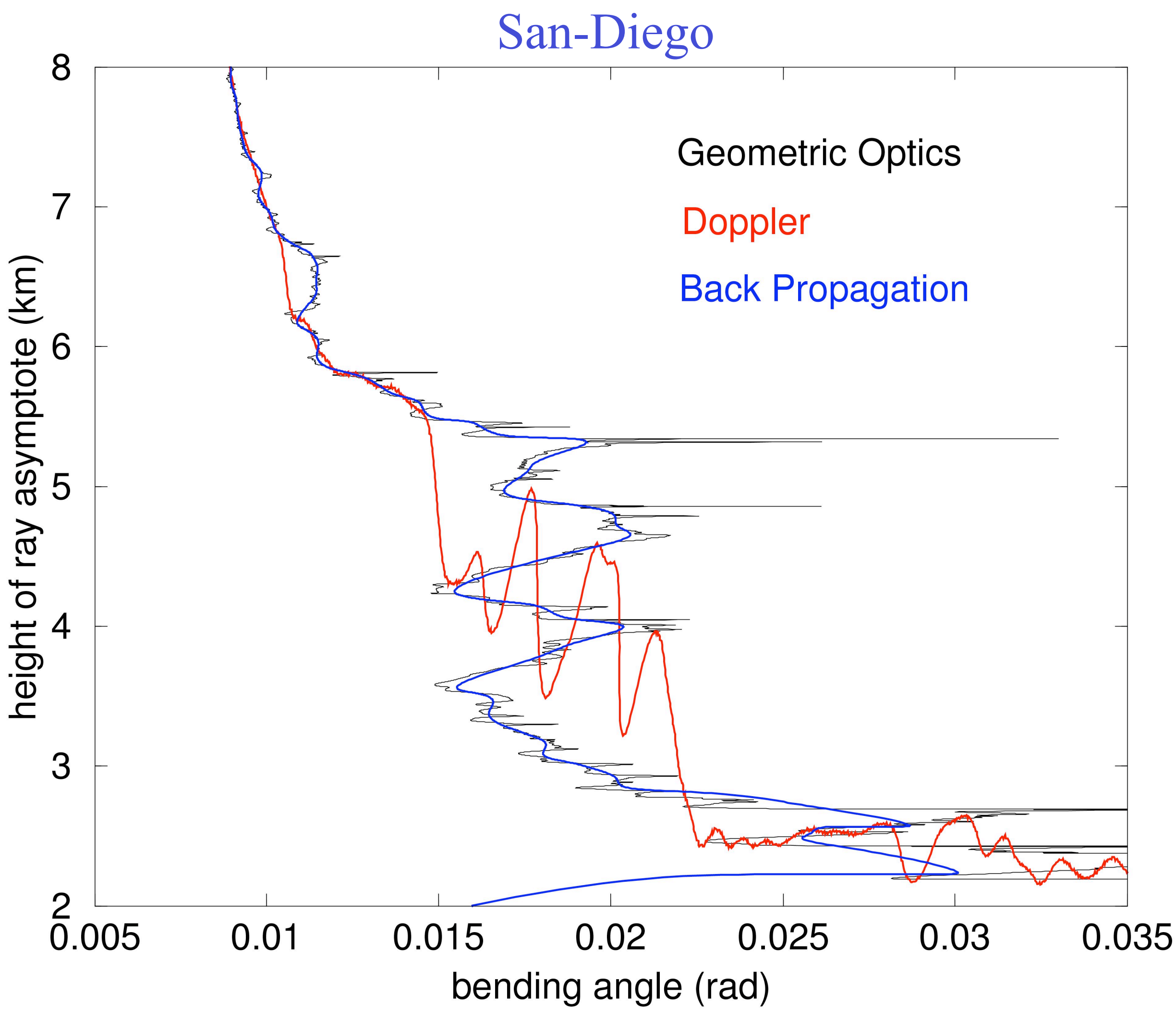
San-Diego, CA



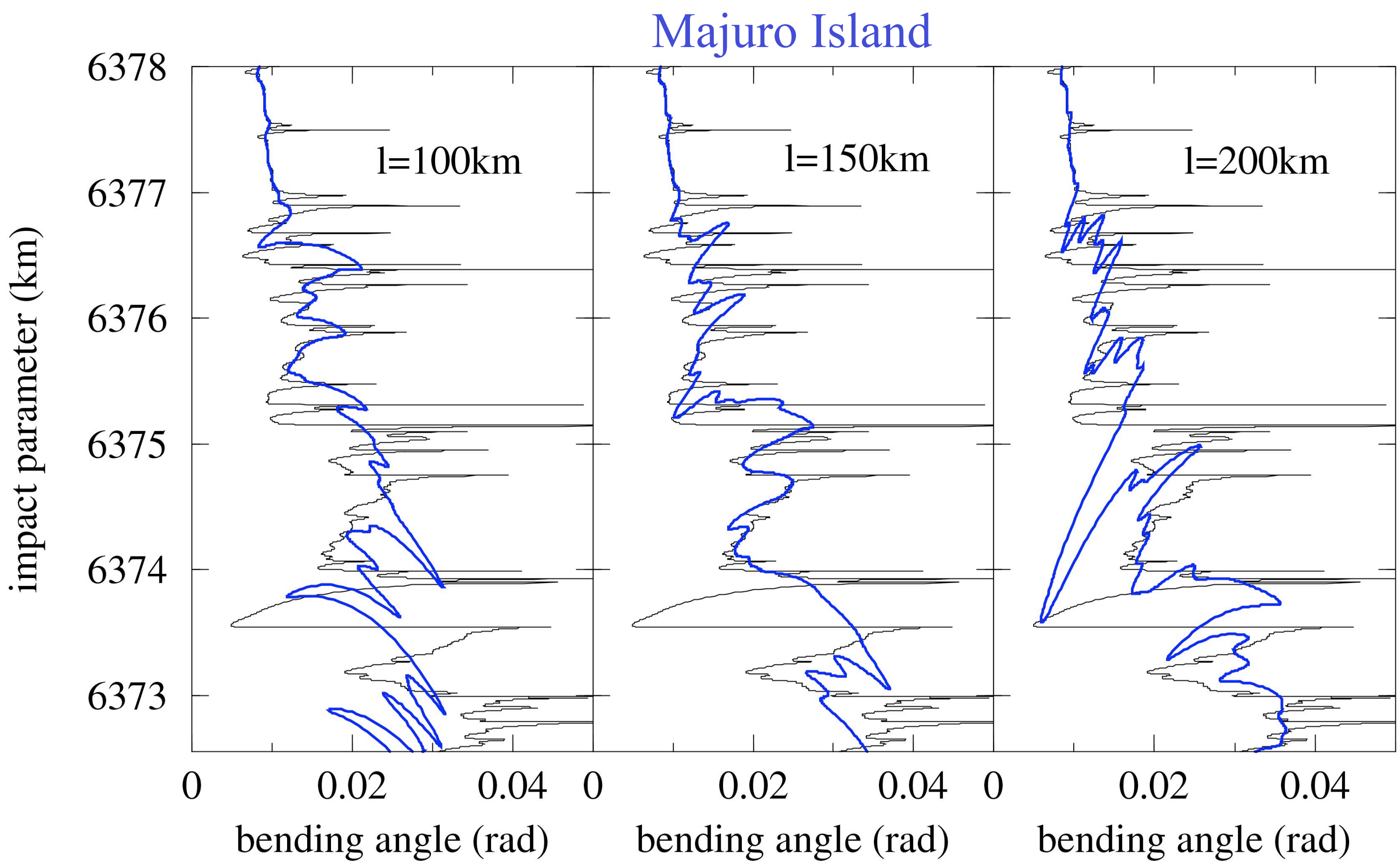
Majuro Island



Reconstruction of bending angles from Doppler and by back-propagation

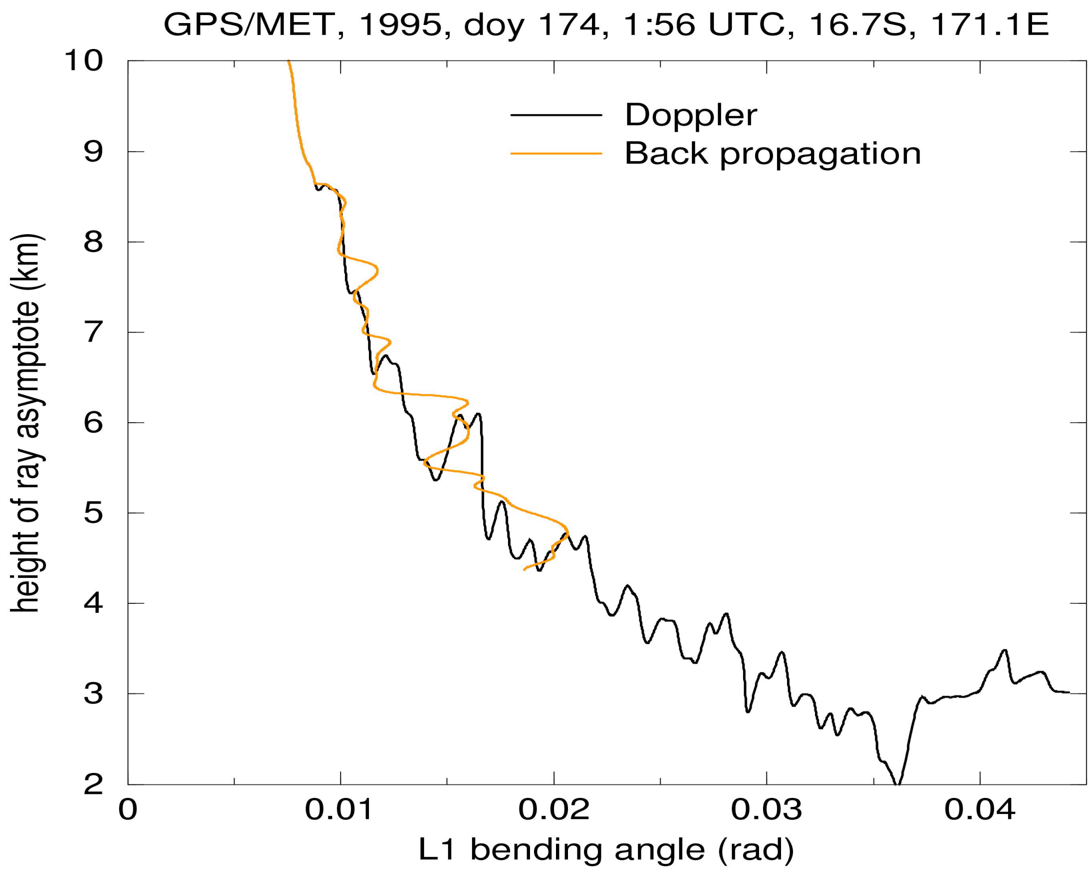


Reconstruction of bending angles from Doppler and by back-propagation



In case of super-refraction layers there is no space between real and imaginary caustics. Any location of the auxiliary BP trajectory results in multi-path.

Reconstruction of L1 bending angle by back-propagation method for GPS/MET tropical occultation



Sliding spectral (radio optics) method

Instead of differentiation of the phase (infinitely small aperture) the method uses spectral analysis of RO signal in sliding window (aperture) of finite size

The frequencies that correspond to maxima of the spectral amplitude are associated with separate rays arriving at the center of the aperture

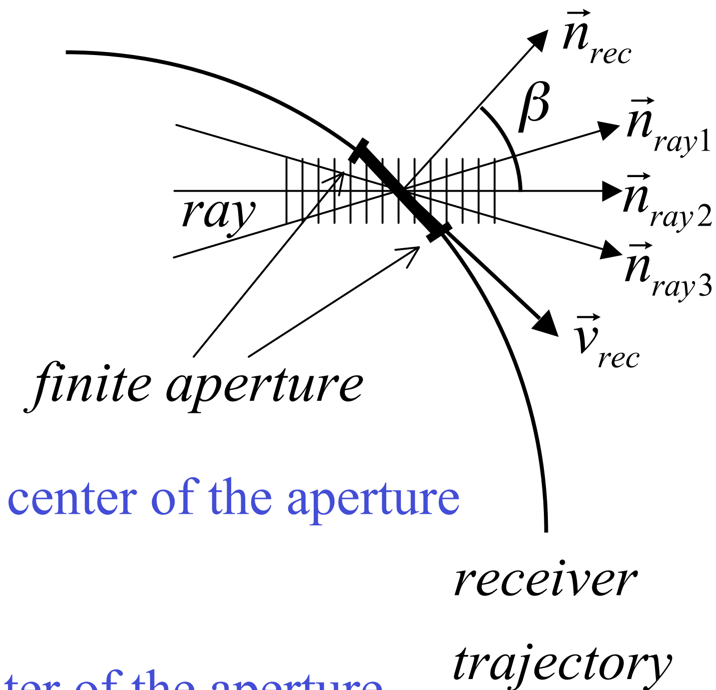
The frequency defines the arrival angle of the ray

The arrival angle and the position of the center of the aperture define the impact parameter and the bending angle

The size of the aperture must be large enough to allow sufficient spectral resolution, but small enough in order to neglect the change of the angular spectrum of arriving EM waves within the aperture

Sliding spectral (radio optics) method

$w(f)$ is the Fourier spectrum of RO signal in a finite aperture



Each frequency f is associated with ray arrival angle:

$$\beta = \arcsin(f / kv_{rec})$$

All rays are assumed to be arriving at the center of the aperture

impact parameter: $a = r_{rec} \sin\phi$

where: $\vec{r}_{rec}$ is the radius-vector to the center of the aperture

$\phi = \beta + \gamma$ is the zenith angle of ray

γ is the angle between $\vec{r}_{rec}$ and $\vec{n}_{rec}$

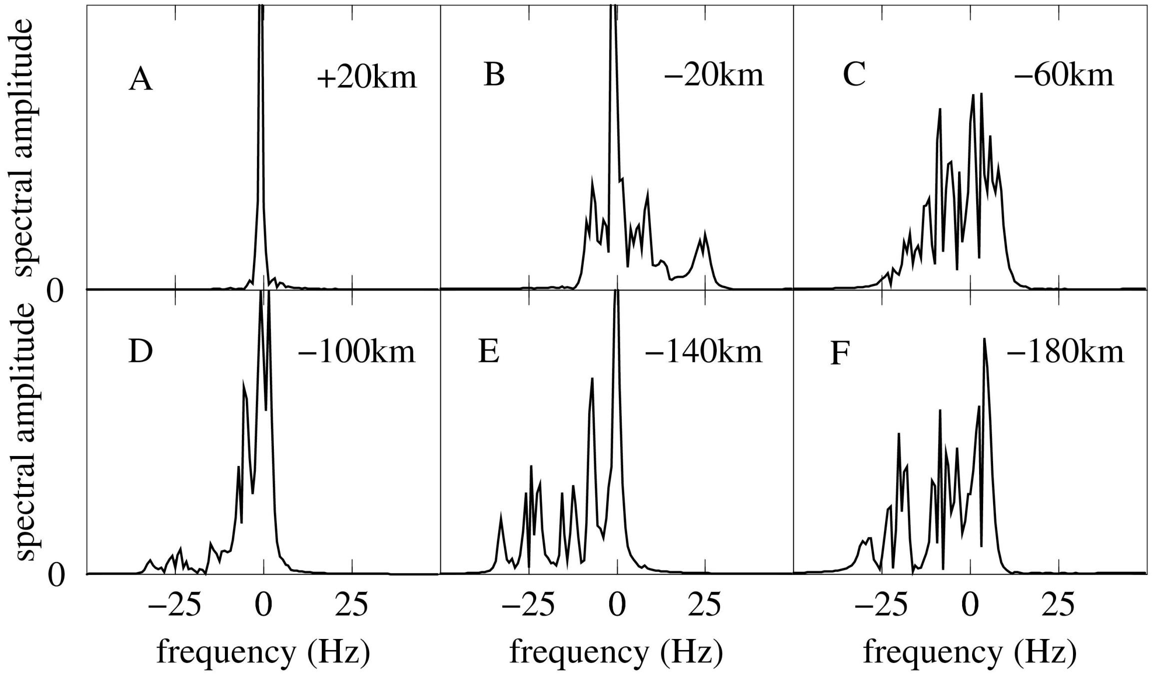
bending angle: $\alpha = \arcsin(a / r_{trans}) + \arcsin(a / r_{rec}) + \theta - \pi$

where: θ is the central angle between transmitter and receiver

Sliding spectral (radio optics) method

When local spectra of RO signal have well resolved maxima, those maxima can be associated with rays

What to do when the spectral maxima are not well resolved?



Sliding spectral (radio optics) method

Ad hoc approach: instead of identifying local spectral maxima, to utilize the whole spectral content in all apertures by its weighting with spectral power.

Each aperture provides the discrete set: a_k, α_k, w_k
(impact parameter values overlap in adjacent apertures)

1) the sets from all apertures are combined in one set
2) the sets are ranked according to increasing impact parameter
3) the sets of bending angle and impact parameter are subject to sliding averaging, weighted according to the spectral power

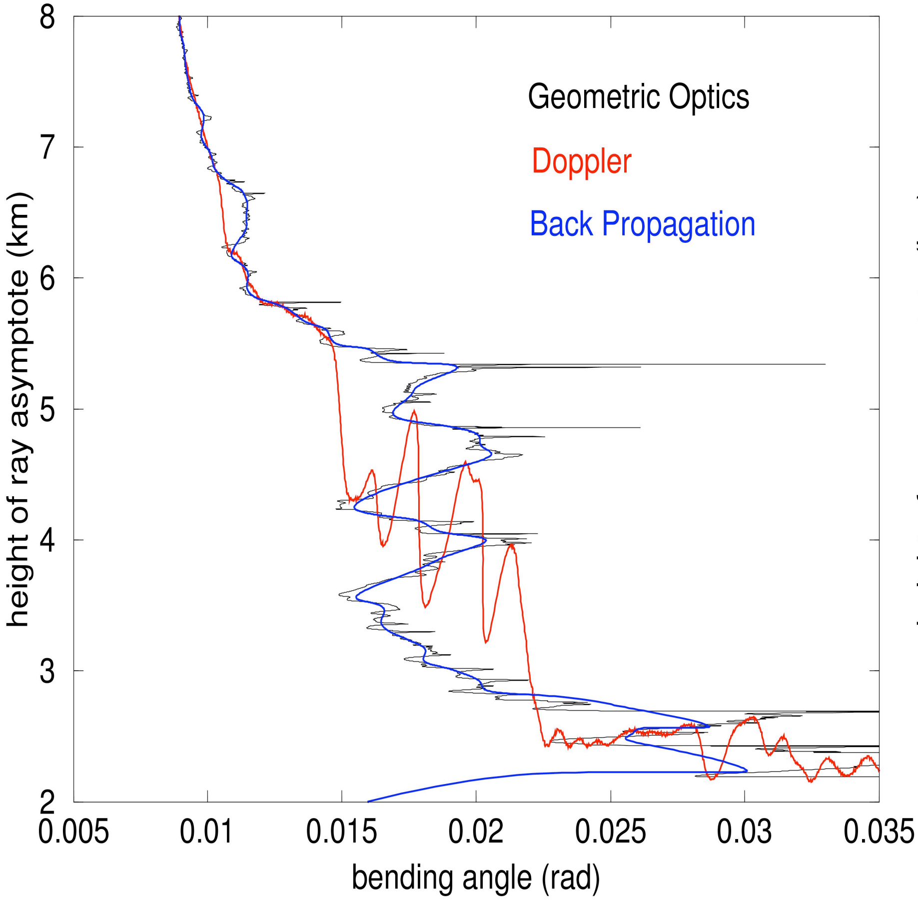
$$\alpha_k = \frac{1}{c_k} \sum_{j=k-K/2}^{k+K/2} \alpha_j \, | w_j |^2$$

$$a_k = \frac{1}{c_k} \sum_{j=k-K/2}^{k+K/2} a_j \, | w_j |^2$$

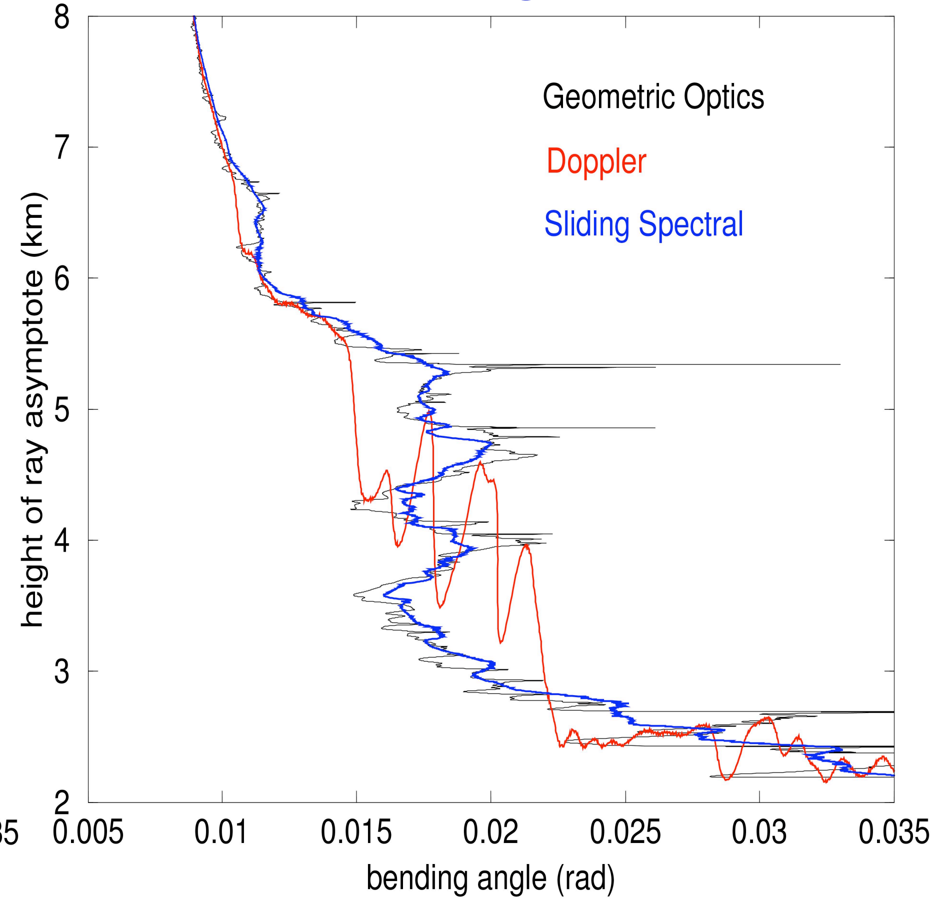
$$c_k = \sum_{j=k-K/2}^{k+K/2} | w_j |^2$$

Reconstruction of bending angles by back-propagation and by sliding spectral methods

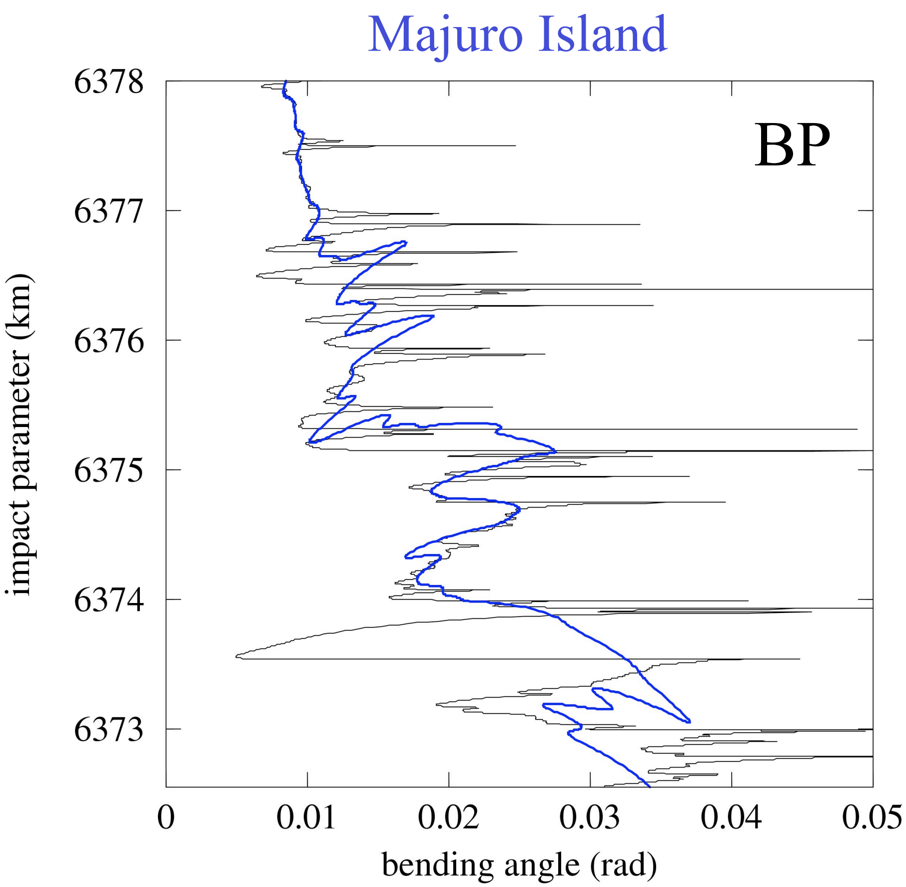
San-Diego

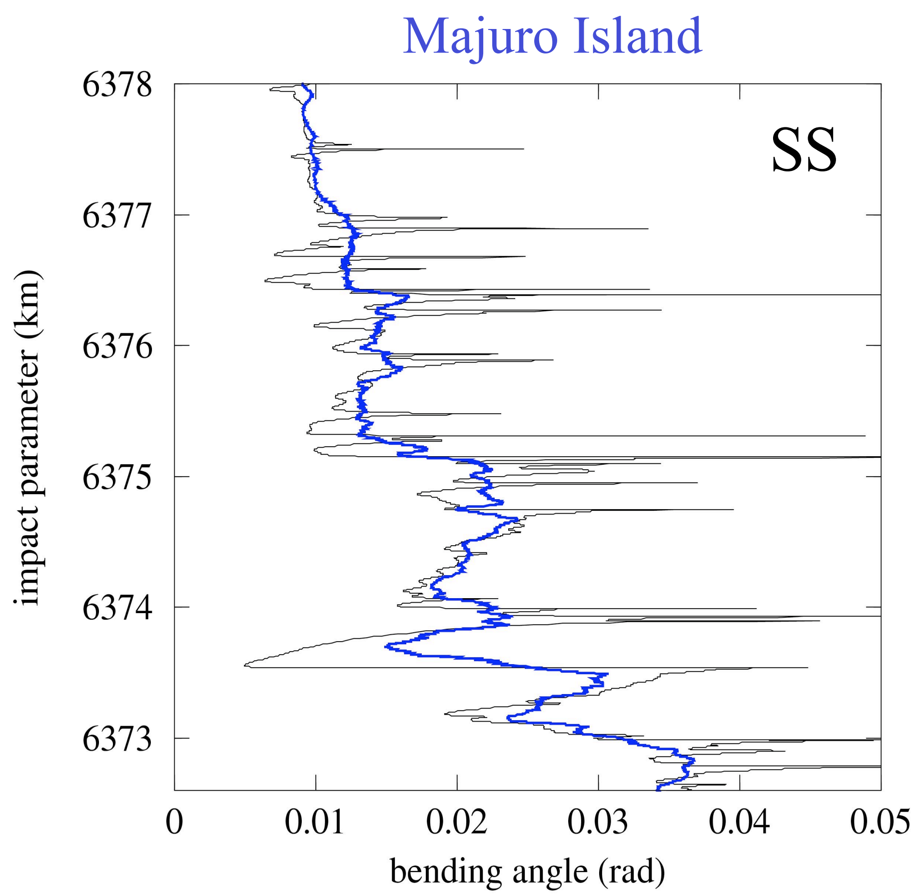


San-Diego

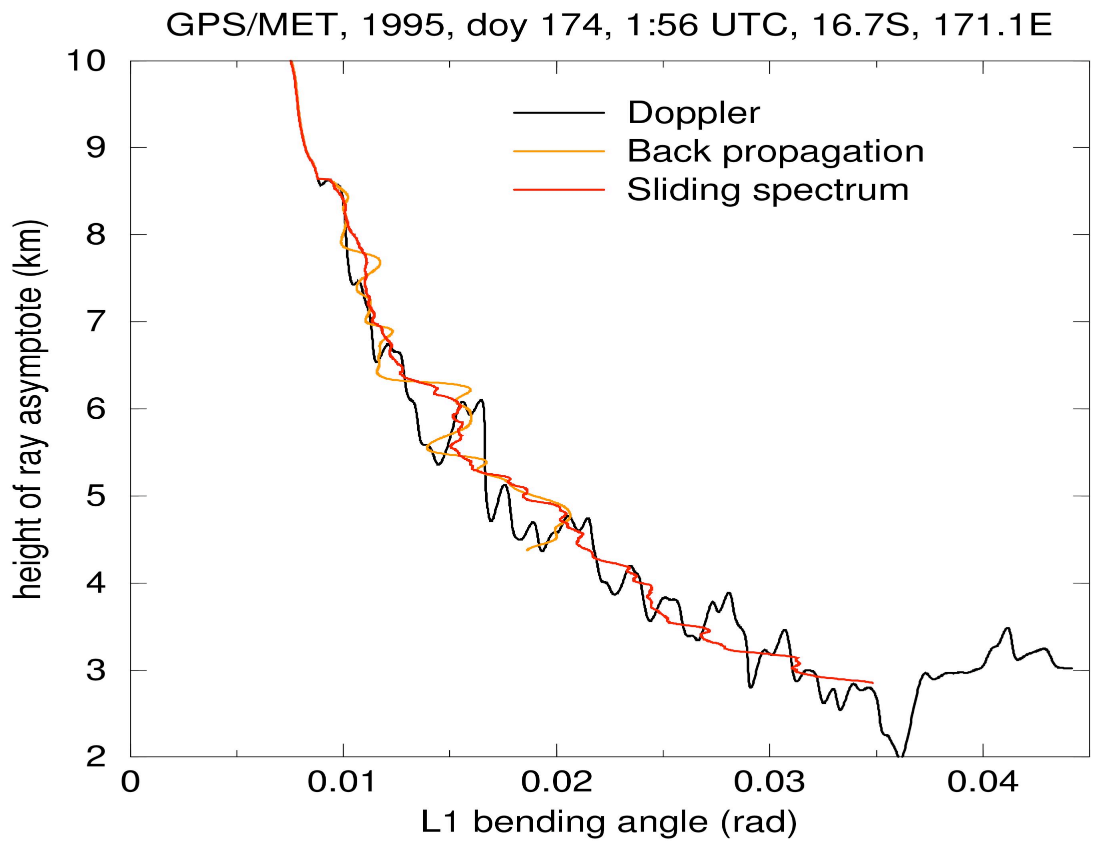


Reconstruction of bending angles by back-propagation and by sliding spectral methods for "worst case" RO signal





Reconstruction of L1 bending angle by sliding-spectral method for GPS/MET occultation in tropics.



Aperture size and resolution of the sliding spectral method

the uncertainty relation: $\Delta f \Delta t \sim 1$ or $\Delta\eta\Delta y \sim \lambda$

uncertainty of the impact parameter related to the uncertainty of coordinate:

$$\Delta a_1 \sim \frac{da}{dy}\Delta y \sim K\Delta y \quad \text{where:} \quad K = \left|1 - x\frac{d\alpha}{da}\right|^{-1}$$

uncertainty of the impact parameter related to the uncertainty of arrival angle:

$$\Delta a_2 \sim x\Delta\eta \sim x\frac{\lambda}{\Delta y}$$

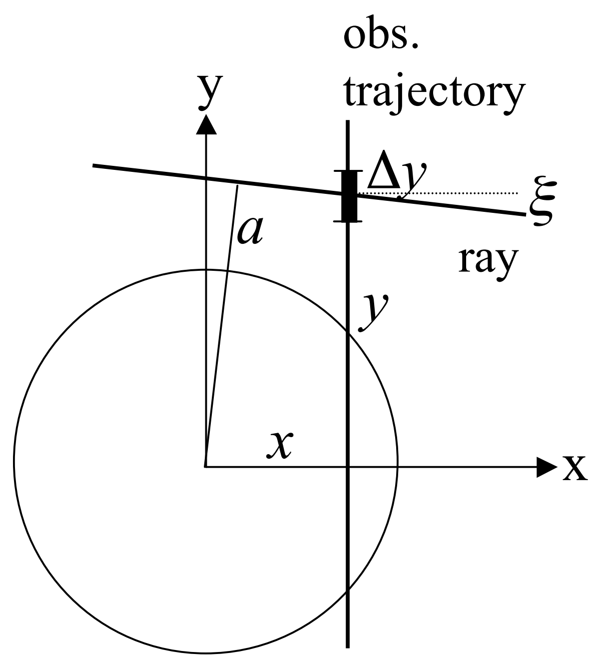
$$\Delta a^2 = \Delta a_1^2 + \Delta a_2^2 = K^2\Delta y^2 + \frac{x^2\lambda^2}{\Delta y^2} = \min$$

$$\Delta y \sim \sqrt{\frac{\lambda x}{K}}$$

this aperture size minimizes the uncertainty of impact parameter:

$$\Delta a \sim \sqrt{\lambda x K}$$

this is the uncertainty of impact parameter, i.e. the resolution of the sliding spectral method



$$\xi = \arcsin\eta$$

Methods transforming complex RO signal from coordinate/time to impact parameter representation

These methods are most accurate

They do not use any approximations/assumptions except GO and spherical symmetry N(r)

They do not depend on tunable parameters, such as the position of BP plane or the size of the sliding aperture

Based on:

(i) principle of phase matching (Jensen et al., 2004)

(ii) canonical formalism (Gorbunov, 2002)

Phase matching in coordinate/time domain

Under multipath, the complex RO signal is the sum of the sub-signals:

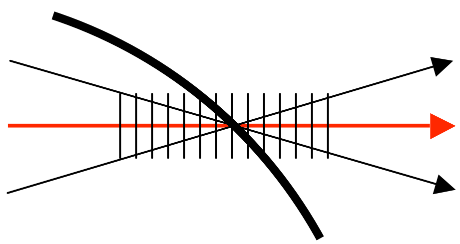
$u(y) = A(y)\exp[i\Phi(y)]$ where y is coordinate/time

Convolving of RO signal with the GO model which depends on a parameter:

$$F(a) = \int u(y)\exp[-i\Phi_m(a,y)]dy = \int A(y)\exp[i\Phi(y) - i\Phi_m(a,y)]dy$$

where: $\Phi_m(a,y)$ is GO phase model; a is impact parameter

For any given a, the main input in F(a) is introduced around the stationary point y_s where the frequency of any of the sub-signals is equal (matches) the frequency of the model



$$\frac{d\Phi}{dy_s} - \frac{\partial\Phi_m(a,y_s)}{\partial y_s} = 0$$

The input of other sub-signals arriving at the same point is close to zero because their frequency does not match the model.

Phase matching in coordinate/time domain

But how we know where/when is the stationary point $\mathbf{y_s}$?

Let’s estimate the convolved RO signal by Method of Stationary Phase:

$$F(a) = cA(y_s)\exp[\Phi(y_s) - i\Phi_m(a, y_s)] = B(a)\exp[i\Psi(a)]$$

where: $y_s = y_s(a)$

Let’s differentiate the phase of the convolution:

$$\frac{d\Psi}{da} = \cancel{\left(\frac{d\Phi}{dy_s} - \frac{\partial\Phi_m}{\partial y_s}\right)}\frac{dy_s}{da} - \frac{\partial\Phi_m(a, y_s)}{\partial a} = -\frac{\partial\Phi_m(a, y_s)}{\partial a}$$

This equation allows to determine the stationary point y_s from the derivative of the phase of RO signal convolved with the GO model

The stationary point is the coordinate (time) where (when) the ray with a given impact parameter is observed during a radio occultation

GO excess frequency model of RO signal in time domain

$$\omega(a,t) = ka\frac{d\theta}{dt} + k\frac{dr_1}{dt}\sqrt{1-\frac{a^2}{r_1^2}} + k\frac{dr_2}{dt}\sqrt{1-\frac{a^2}{r_2^2}}$$

GO excess phase model of RO signal in time domain

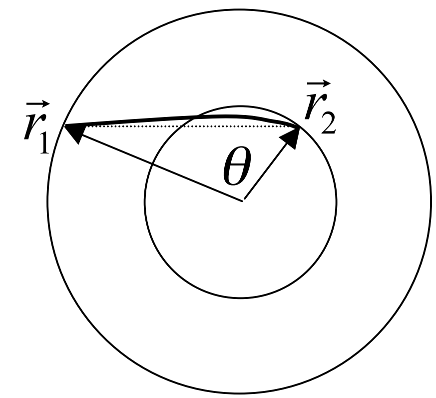
$$\boxed{\begin{aligned}&\Phi_m(a,t) = \int \omega(a,t)\,dt\\ &\Phi_m(a,t) = ka\theta + k\left(\sqrt{r_1^2-a^2} + \sqrt{r_2^2-a^2} - \arccos\frac{a}{r_1} - \arccos\frac{a}{r_2}\right)\end{aligned}}$$

r_1 transmitter radius r_2 receiver radius

a impact parameter θ central angle between transmitter and receiver

The problem is solved! But, convolution of RO signal with its GO model on arbitrary trajectory is computationally inefficient.

Full Spectrum Inversion (Jensen et al., 2003)



Special case: $r_1 = const \quad r_2 = const$

Consider central angle instead of time

The RO signal: $u(\theta) = A(\theta)\exp[i\Phi(\theta)]$

The phase model: $\Phi_m(a,\theta) = ka\theta + const$

The convolution is simply the Fourier transform of complex RO signal:

$$F(a) = \int u(\theta)\exp(-ika\theta)d\theta = B(a)\exp[i\Psi(a)]$$

The derivative of the phase of the convolved RO signal by impact parameter yields the central angle at which that impact parameter is observed:

$$\frac{d\Psi}{da} = -k\theta_s$$

The bending angle:

$$\alpha = \arcsin(a/r_1) + \arcsin(a/r_2) + \theta_s - \pi$$

In practice:

Before FSI, RO signal must be up-sampled (to prevent from the aliasing of the Fourier spectrum) and interpolated on θ grid

To reduce the up-sampling frequency, RO signal must be down-converted by removing mean vacuum Doppler frequency during RO:

$$u'(\theta) = u(\theta)\exp(-ika_0\theta)$$

The down-converted RO signal subject to FSI:

$$F(a') = \int u'(\theta)\exp(-ika'\theta)d\theta$$

Then the full impact parameter is equal:

$$a = a_0 + a'$$

In practice:

Application of the phase matching in coordinate/time domain for RO signal on arbitrary trajectory is computationally inefficient

The value of the FSI is that it reduces the convolution of RO signal to Fourier transform (FFT is computationally efficient).

Are there any other trajectories where the problem can be reduced to Fourier transform?

Yes. Straight line. But the signal must be convolved with the GO model not in the coordinate (time), but in the frequency domain:

$$u(y) = \int \tilde{u}(\eta)\exp(iky\eta)d\eta$$

representation of RO signal as function of coordinate

$$\tilde{u}(\eta) = \frac{k}{2\pi}\int u(y)\exp(-iky\eta)dy$$

representation of RO signal as function of spatial frequency

$$\boxed{\eta = \eta(a, y)}$$

GO model of RO signal (for a given receiver trajectory)

$$\eta = \eta(a, y) \quad \Phi_m(a, y) = \int \eta(a, y)dy$$

phase matching function for the coordinate representation

Similarly:

$$y = y(a,\eta) \quad \tilde{\Phi}_m(a,\eta) = -k\int y(a,\eta)d\eta$$

phase matching function for the frequency representation

$$\tilde{F}(a) = \int \tilde{u}(\eta)\exp[-i\tilde{\Phi}_m(a,\eta)]d\eta$$

convolving RO signal with phase model

$$\frac{d\tilde{\Psi}}{da} = -\frac{\partial\tilde{\Phi}_m(a,\eta_s)}{\partial a}$$

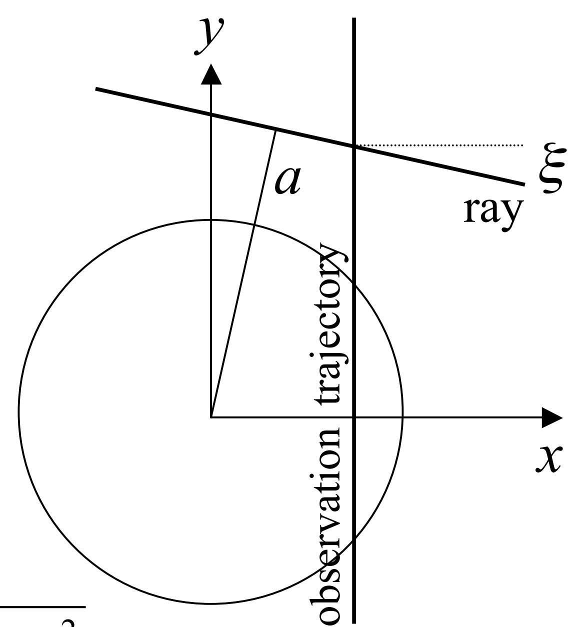
stationary point defines the frequency at which the ray with impact parameter a crosses the receiver trajectory

Phase matching in frequency domain (RO signal is defined on straight line)

Equation of straight ray with impact parameter a:

$$a = -x\sin\xi + y\cos\xi$$

Let's use it for the



GO model of RO signal on straight line $x=const$:

$$a = -x\eta + y\sqrt{1-\eta^2}, \quad \eta = \sin\xi$$

Frequency matching function in η representation:

$$y(a,\eta) = (a + x\eta)/\sqrt{1-\eta^2}$$

Phase matching function in η representation:

$$\tilde{\Phi}_m(a,\eta) = -k\int y(a,\eta)d\eta = -ka\arcsin\eta + x\sqrt{1-\eta^2}$$

Convolution of RO signal, known as the "canonical transform" (CT)*

$$\tilde{F}(a) = \int\tilde{u}(\eta)\exp[ik(a\arcsin\eta - x\sqrt{1-\eta^2})]d\eta = \tilde{B}(a)\exp[i\tilde{\Psi}(a)]$$

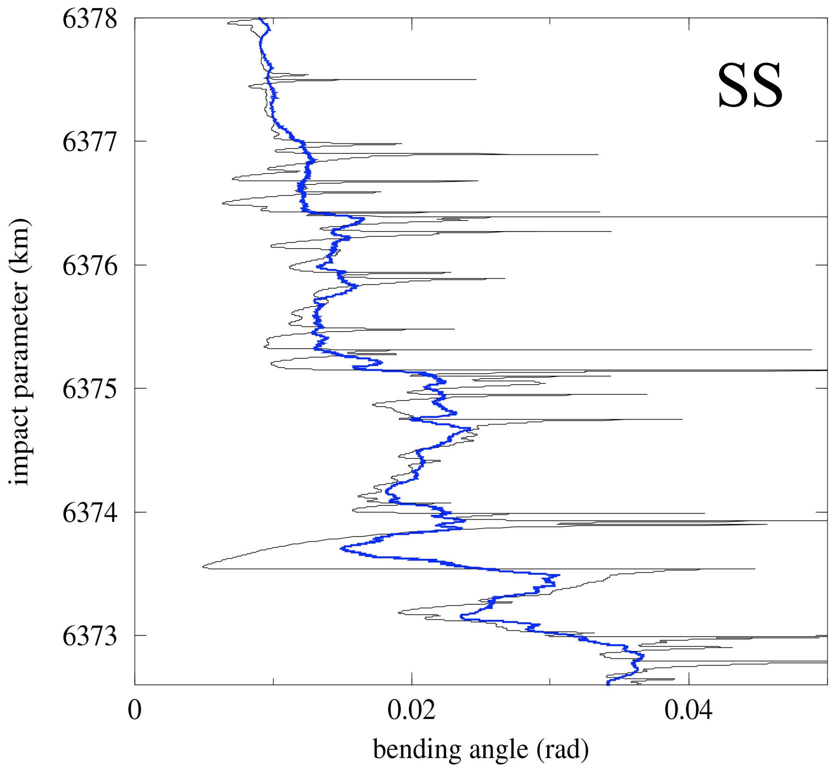
The derivative of the phase yields the ray arrival angle for a given a

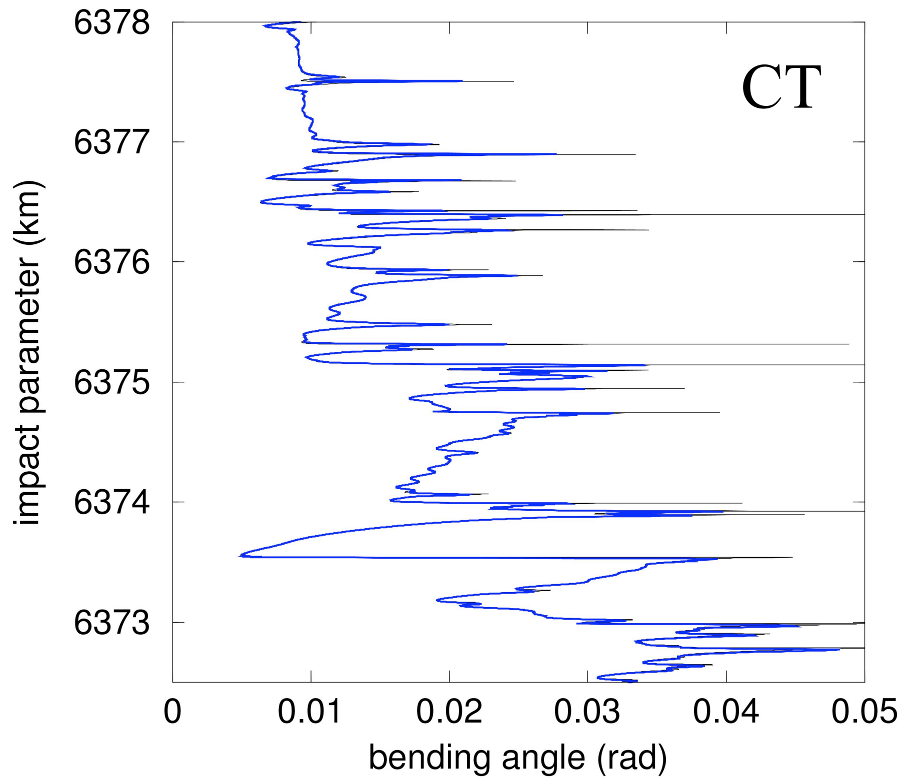
$$\frac{d\tilde{\Psi}}{da} = -\frac{\partial\tilde{\Phi}_m(a,\eta_s)}{\partial a} = k\arcsin\eta = k\xi$$

* Derived by canonical formalism, by Gorbunov, 2002

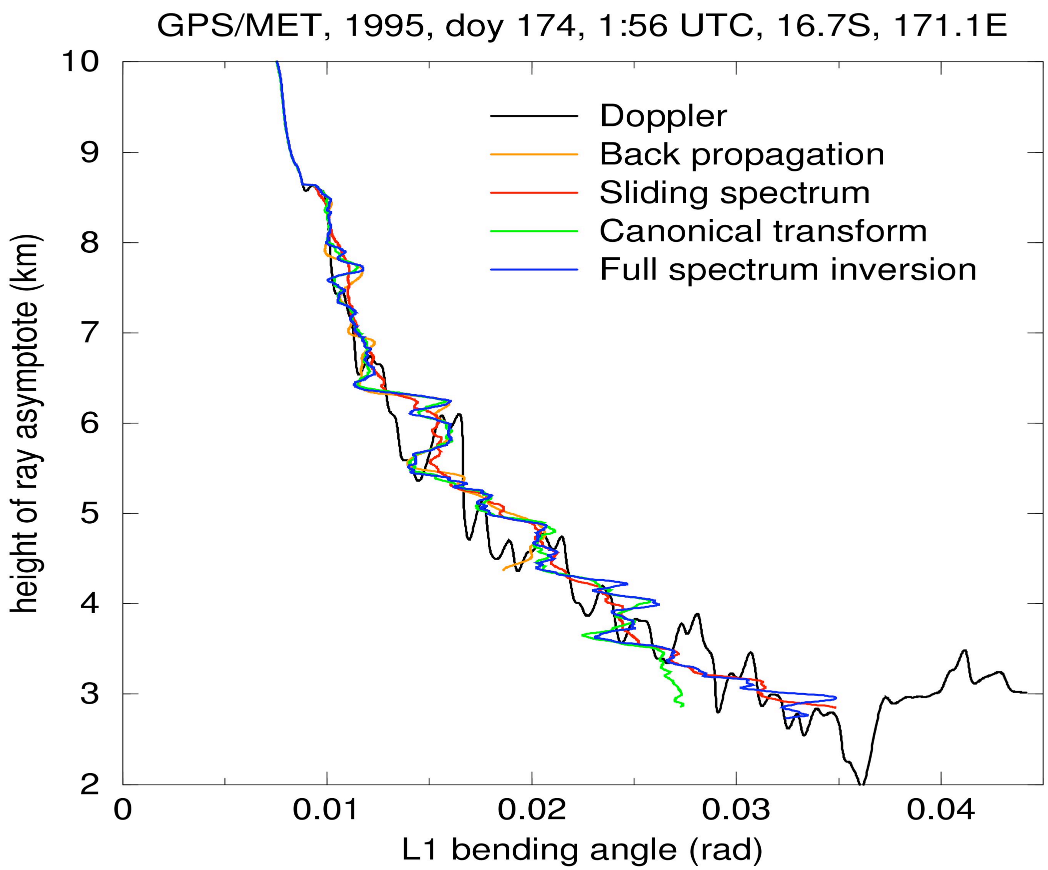
Reconstruction of bending angles by sliding spectral and by canonical transform methods

Majuro Island



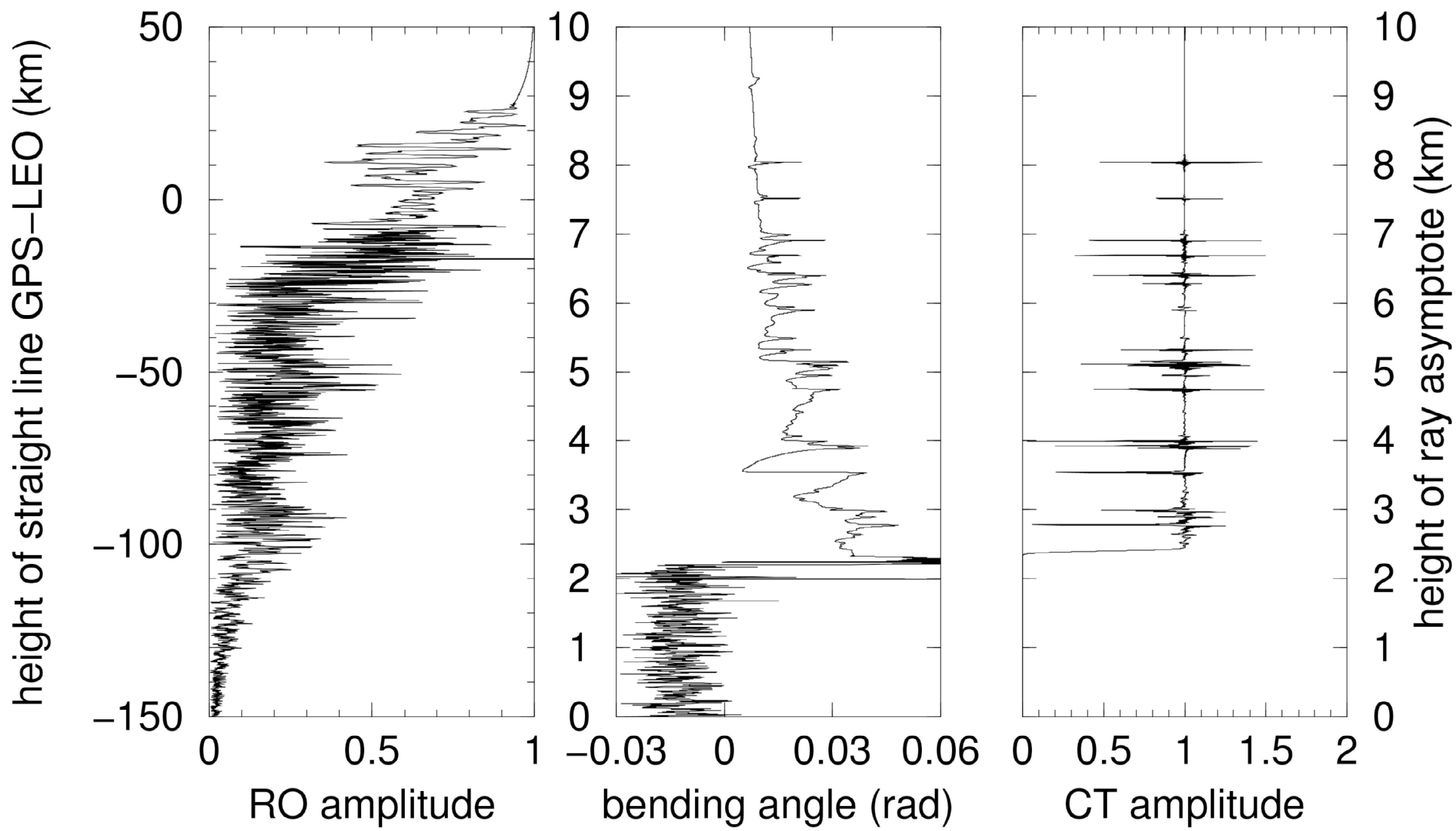


Reconstruction of L1 bending angle by all radio-holographic methods for GPS/MET occultation in tropics.



The disagreement between radio-holographic methods is much smaller than between any of them and the Doppler method.

Amplitude of RO signal, transformed to impact parameter representation (CT, FSI), allows to determine the cut-off of the retrieved bending angle profile



Propagation of RO signal to circular orbits

For the BP and SS methods, it is necessary: r_1=const

For the FSI method, it is necessary: r_1=const, r_2=const

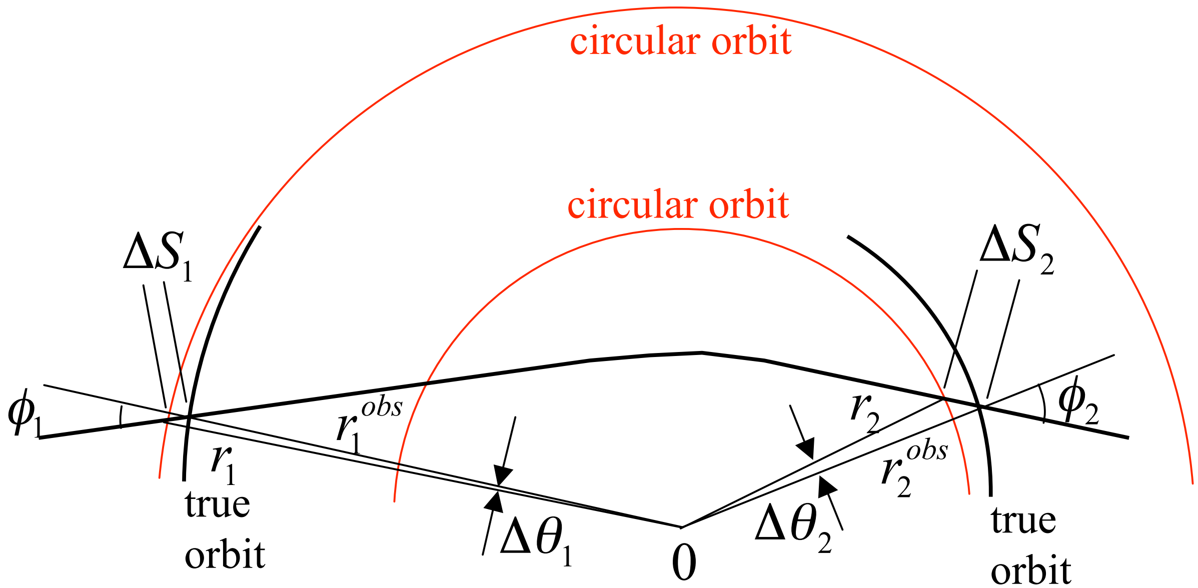
When real orbits are not circular, but have small excentricity, the EM field can be propagated to close circular orbits, in GO approximation, by straight-line continuation of rays.

The GO propagation is valid until continued rays overlap (hit caustic).

In practice, for raw 50 Hz RO signals, GO propagation is applicable at distances 10-20 km. This is sufficient for GPS and LEO orbits with small excentricity (GPS/MET, CHAMP, SAC-C, COSMIC …)

Can be extended to larger distances by smoothing RO observation phase (used for determination of ray directions) by still providing fairly accurate results.

Propagation of RO signal to circular orbits



(i) impact parameter is determined from the observation Doppler given $r_1^{obs}, \quad r_2^{obs}$

(ii) zenith angles are determined from the impact parameter $\phi_{1,2} = \arcsin(a / r_{1,2}^{obs})$

(iii) angular corrections are determined $\Delta\theta_{1,2} = \phi_{1,2} - \arcsin[(r_{1,2} / r_{1,2}^{obs}) \sin\phi_{1,2}]$

(iv) corrections of the phase path are determined $\Delta S_{1,2} = r_{1,2} \sin\Delta\theta_{1,2} / \sin\phi_{1,2}$

Summary

Methods based on transforming received complex RO signal from the time (coordinate) to the impact parameter representation (FSI, CT) are most accurate RH methods for calculation of bending angle as the function of impact parameter. They precisely solve for multi-path propagation (in spherically-symmetric atmosphere) and for diffraction effects in a vacuum, thus resulting in sub-Fresnel resolution (the resolution is limited by only diffraction effects inside the atmosphere). They do not depend on tunable parameters.

The FSI is the best method for orbits with small excentricity (all present, current and most planned RO missions, incl. COSMIC)

In case of strongly excentric orbit(s), BP + CT method is appropriate.