# Statistical optimization of radio occultation data with dynamical estimation of error covariances

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## Abstract

The dominant error source in radio occultation soundings of the upper stratosphere and the mesosphere is residual ionospheric noise. This noise can be significantly reduced by using statistical optimization, in which measured bending angle profiles are combined with a priori or First Guess bending angle profiles in a statistical optimal manner. First Guess profiles are normally derived from a climate model. In order for this technique to work optimally, the error covariance of the observations and the error covariance of the model must be known, which is generally not the case. It is common practice to assume that each of these errors is uncorrelated. In this study it is shown that if this assumption is applied together with dynamical error estimation, it is important to account for the fact that the First Guess and the observational bending angle errors are not damped equally when refractivity profiles are computed through the Abel transform. It is demonstrated, that the difference in noise damping can be accounted for by simply scaling the ratio of the observational to the First Guess error variances. It is shown that the scaling factor can be related to the ratio between the error correlation lengths of the observational errors and the model errors. We present a simple procedure where variances are estimated dynamically and scaled as described above. This approach is applied to one month of CHAMP occultations and the retrieved refractivity profiles are compared with corresponding profiles derived from ECMWF operational analysis. It is found that in the height range from 30 to 40 km the relative refractivity errors is reduced by approximately 0.25-1 percentage point compared to a ‘standard’’ statistical optimization scheme where the relative error in the First Guess bending angle profile is assumed to be 20%.

Keywords: Refractivity retrieval，Statistical optimization，Dynamical error estimation

## 1. Introduction

A common approach when using statistical optimization (SO) for radio occultaion (RO) data is to assume that bending angle errors are vertically uncorrelated. In this case the optimal combination of First Guess bending angles, $\alpha_{guess}$, and observational bending angles, $\alpha_{obs}$, can be expressed as:

$$\alpha_{opt}(a) = \frac{\sigma^2_{guess}(a)\alpha_{guess}(a) + \sigma^2_{obs}(a)\alpha_{obs}(a)}{\sigma^2_{guess}(a) + \sigma^2_{obs}(a)} \qquad (1)$$

where $a$ is the impact parameter, whereas $\sigma_{guess}$ and $\sigma_{obs}$ are the standard deviations of the model (First Guess) bending angles and the observed bending angles, respectively.

This approach has been applied in a number of studies see e.g., [*Gobiet et al.*, 2002; *Gorbunov*, 2002; *Gorbunov and Gurvich*, 1998; *Hajj et al.*, 2002; *Hocke*, 1997; *Kuo et al.*, 2004; *Rocken et al.*, 1997; *Sokolovskiy and Hunt*, 1996; *Steiner et al.*, 1999].

When applying this method, observational errors are normally estimated dynamically from the upper part of the occultation and the standard deviation of the First Guess errors are assumed to correspond to a fixed fraction of the First Guess bending angles, typically 5% to 20%, as originally suggested by [*Sokolovskiy and Hunt*, 1996]. The popularity of this approach is mainly due to its simplicity and the fact that this technique has been demonstrated to work well in a number of studies (see the references above). However, this method is not optimal mainly for two reasons. First, a fixed relative First Guess error is assumed though the accuracy of climate models varies with both season and geographical location. Second, this approach does not account for vertical correlation of the observational and model errors.

[*Gorbunov*, 2002] presented a combined algorithm of the ionospheric correction and noise reduction based on SO, neglecting vertical correlation but applying dynamic estimates for the magnitude of both the observational and the First Guess errors.

In this study we present an approach where the First Guess error covariances and the observational errors covariances of the ionospherically corrected bending angle profiles are estimated dynamically. The combined profile is computed from (1) with a modified ratio between $\sigma_{obs}$ and $\sigma_{guess}$ based on the estimated error correlation lengths.

## 2. Methodology

When a bending angle profile that includes additive noise is subject to Abel transform, the characteristics of the noise are changed according to the frequency response, *F*, of the Abel transform. By linearizing the Abel transform, *F* can be approximated as:

$$F(k) = 10^6 \left( \frac{1}{\pi} \int_{a_0}^{a_o+\Delta a} \frac{\exp(ika')}{\sqrt{a'^2 - a_0^2}} da' \right) \qquad (2)$$

where *k* is the wave number.

Figure 1 shows the magnitude, $|F(k)|$, of the frequency response of the Abel transform for $a_0$ = 6400 km and $\Delta a$ = 10 km, 30 km, 100 km, and ∞.

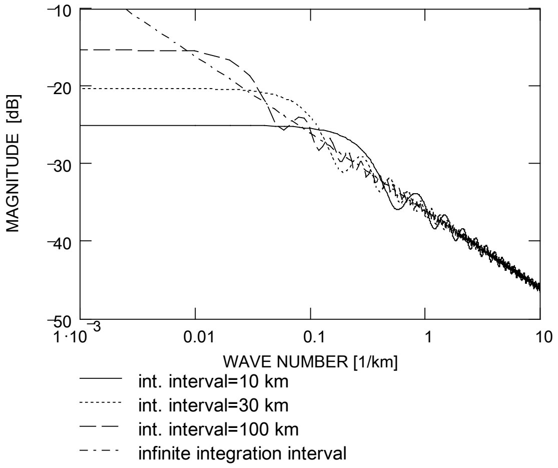

**Figure 1:** *Magnitude response of Abel transform for different length of the integration interval.*

As it follows from Figure 1, the Abel transform acts like a low-pass filter with a cut-off frequency approximately corresponding to the inverse length, $(\Delta a)^{-1}$, of the integration interval.

In order to investigate how First Guess errors and observational errors propagate through the Abel transform we assume that both types of errors have a Gaussian covariance function and thus a Gaussian power spectrum. By applying (2) we can compute the ratio, $d$, between input noise power and the power of the Abel transformed noise, for the two error types.

Meassured bending angles will generally contain more high frequency noise than the First Guess bending angles. Correlation lengths of the observational errors are approximately in the range from 0 to 2 km depending on the filter applied for smoothing of the observational phases. Correlation lengths of climate model errors are generally not known and may be in the range from a few kilometers to more than 20 kilometers depending on the climate model used as the First Guess. Figure 2 depicts the ratio between the damping of the observational errors, $d_{obs}$, and the First Guess errors, $d_{guess}$, for different correlation lengths.

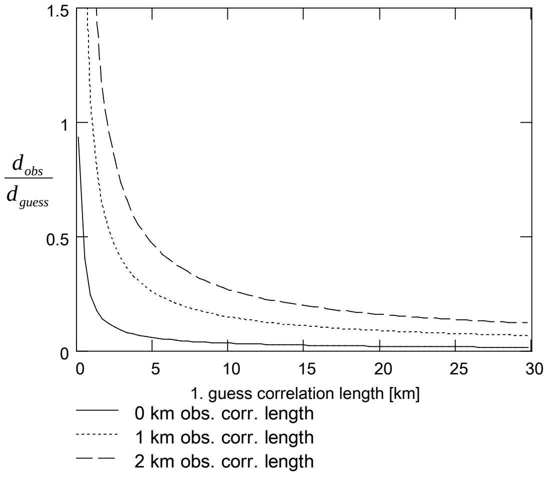

Figure 2: *Ratio of damping factors for the observational noise power and First Guess noise power as a function of the First Guess correlation length. The three different curves correspond to different correlation length of the observational noise.*

Figure 2 shows that a few kilometers difference between the correlation lengths of the First Guess and the correlation length the observational errors can lead to significantly different damping of these two noises. This clearly illustrates the necessity for accounting for the difference in correlation length when dynamic errors estimation is applied. Without proper account, the First Guess can be overweighted after applying Abel inversion for the optimized bending angle (1). In this study a new statistical optimization scheme is therefore suggested in which both the observational error covariance and the First Guess error covariance are estimated for individual occultations. Error correlation lengths are computed by applying a Gaussian fit to the estimated error covariances which allows for computation of $d_{obs}$ and $d_{guess}$. Finally, the ratio between model variances and observational variances are adjusted according to the ratio between $d_{obs}$ and $d_{guess}$ before (1) is applied.

The observational error correlation function, $r_{obs},(\tau)$, is simply estimated from the differences, $\Delta\alpha$, between model and observations at 60-80 km heights.

Estimation of the First Guess error covariances is more complicated because the magnitude of the First Guess errors, in general, varies with height. Thus, some assumption about the structure of these errors must be made. Here we apply the assumption that the magnitude of the First Guess error standard deviation is equal to a fixed fraction, $K$, of the First Guess bending angle profile. Then, as the observational errors and First Guess errors are expected to be uncorrelated the following expression must be valid at any height:

$$K^2 = \frac{\sum_i \left[ \left( \Delta\alpha(a_i) \right)^2 - \sigma_{obs}^2 \right]}{\sum_i \alpha_{guess}^2(a_i)} \qquad (3)$$

Similarly the error correlation function, $r_{guess}(\tau)$, can be estimated as:

$$\frac{r_{guess}(\tau)}{r_{guess}(0)} = \frac{\sum_i [\Delta\alpha(a_i)\Delta\alpha(a_i+\tau) - r_{obs}(\tau)]}{K^2 \sum_i \alpha_{guess}(a_i)\alpha_{guess}(a_i+\tau)} \qquad (4)$$

In this study (3) and (4) are applied at 20-60 km heights. Correlation lengths are estimated by approximating the computed correlation functions with a Gaussian using a least square fit.

Finally, we can summarize the approach described in this section as follows:

1) Estimate noise covariance from height range 60-80 km.

2) Estimate model covariance from height range 20-60 km using (3) and (4).

3) Multiply the estimated observational variance with $d_{obs}/d_{guess}$ *or* multiply the estimated First Guess variance with $d_{guess}/d_{obs}$.

4) Perform statistical optimization using (1).

5) Use Abel transform to compute refractivity profile from $\alpha_{opt}$.

## 3. Results and Discussion

We now apply this new statistical optimization (SO) scheme to one month of CHAMP occultations from August 2002 and compare the retrieved refractivity to the ECMWF analysis. For comparison similar calculations are performed using a widely used 'standard' statistical optimization scheme where only the observational noise is estimated dynamically whereas the standard deviation of the First Guess errors is assumed to be equal 20 % of the First Guess bending angles.

The ionospheric free bending angle profiles used as input to the two SO schemes are based on geometrical optics as implemented in the UCAR RO processing chain, CDAAC. As the First Guess we apply a NCAR (National Center for Atmosphric and Climate Research) climate model that is based on the results from the recent SPARC study [*Randel et al.*, 2002]. In order to improve the agreement between First Guess and observations, the model bending angle profiles are scaled providing a least square fit of the observations and model in the interval from 40 to 60 km, as suggested by [*Gorbunov*, 2002].

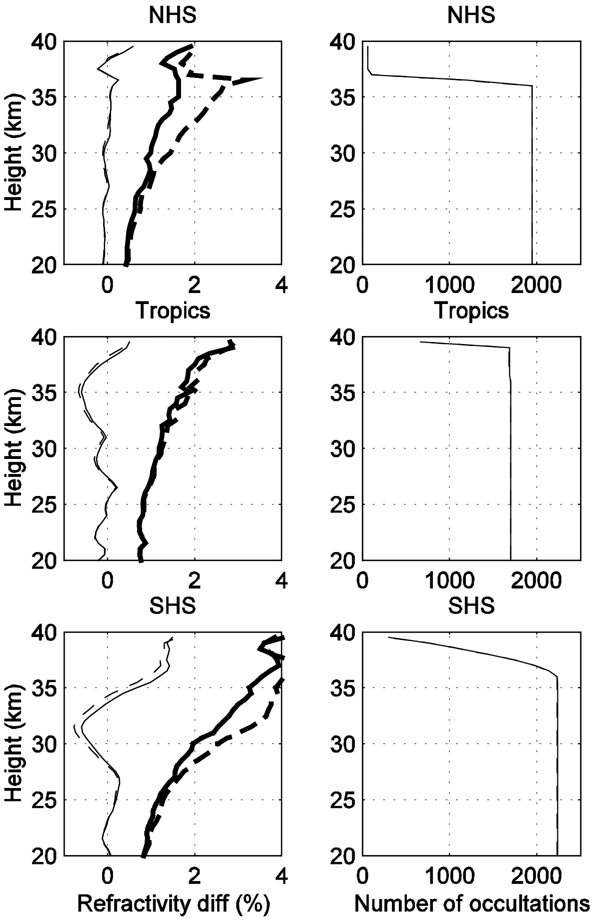

***Figure 3:*** *Fractional refractivity difference relative to ECMWF, for 30°N-90°N(NHS), 30°S-30°N(Tropics), and 30°S-90°S(SHS) for CHAMP occultations from August 2002. Solid lines: new SO scheme; dashed lines the old*

*SO scheme. Thin lines/dashes represent the mean difference and thick lines/dashes represent the standard deviation.*

Figure 3 depicts the mean and the standard deviation of the fractional refractivity difference from ECMWF for the two SO schemes for the height range 20-40 km.

It is seen that in general fractional refractivity differences relative to ECMWF are in the interval from 1 to 4 percent whereas biases are in the interval from -1 to 2 percent for the considered height interval. The performance of both SO schemes is significantly lower in the southern hemisphere than in the tropics and the northern hemisphere.

The largest differences between the two SO schemes are in the northern and southern hemisphere where the fractional refractivity difference of the new scheme is about one percent point smaller than for the standard scheme above approximately 30 km. This clearly shows the advantage of the new SO scheme.

It is also useful to explorer the distributions of the estimated First Guess relative errors and correlation lengths for the scaled climate model profile. Figures 4 and 5 depict scatter diagrams of, respectively, the estimated relative errors of the model and the model error correlation lengths as functions of latitude.

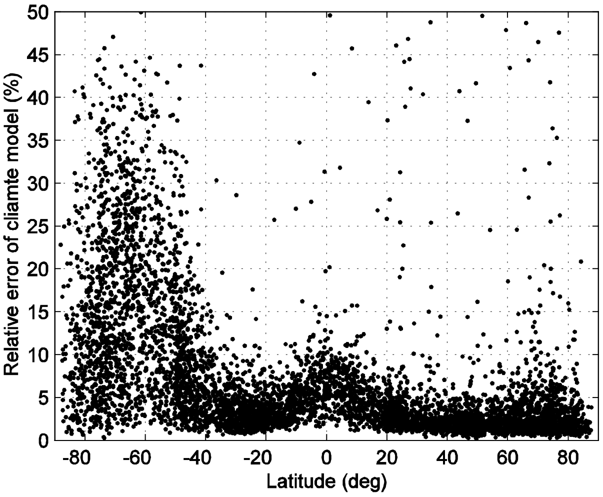

***Figure 4:*** *Relative error of climate model as function of latitude, estimated for the considered CHAMP occultations (August 2002).*

These figures shows that the estimated relative errors and correlation lengths of the First Guess profile are found to be mainly in the ranges from 1-50 percent and 0.5 km-25 km, respectively. The largest relative errors and correlation lengths are found in the southern hemisphere. Observational error correlation lengths were found to be mainly in the interval 0.5-1 km and evenly distributed with latitude. This is in agreement with the fact that a low-pass filter with a bandwidth of 2 Hz was applied to the L1 and L2 phases. The larger relative errors of the First Guess and larger fractional errors compared to ECMWF, observed in the southern hemisphere, are believed to be caused by significant day to day variations which are common for the stratospheric polar winter [*Kuo et al.*, 2004]. Such variations increase the error of the First Guess and result in larger errors in the retrieved refractivity.

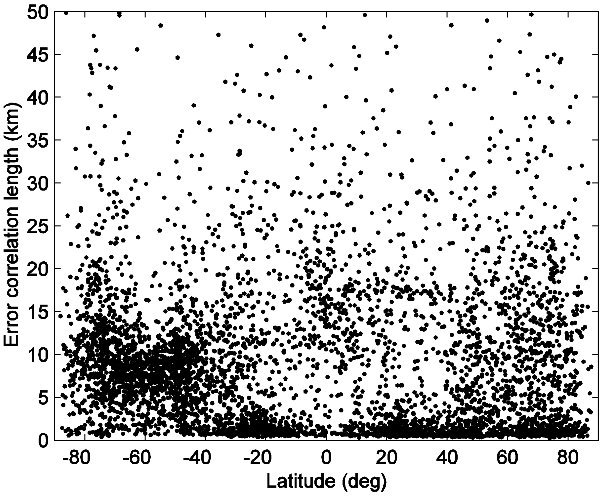

***Figure 5:*** *Estimated error correlation lengths of First Guess errors, estimated for the considered CHAMP occultations (August 2002).*

## 4. Summary

This study demonstrated that Abel inversion of the bending angles, statistical optimized under the assumption of vertical uncorrelated errors, requires adjustment (scaling) of the estimated error magnitudes according to their estimated vertical correlation lengths.

A new statistical optimization scheme was therefore presented with dynamical error estimation and error adjustment. We applied this new statistical optimization (SO) scheme to one month of CHAMP occultations from August 2002 and compared the retrieved refractivity to the ECMWF analysis. For comparison similar calculations were performed by using a ‘standard’ statistical optimization scheme where only the observational noise is estimated dynamically whereas the standard deviation of the First Guess errors is assumed to be equal 20 percent of the First Guess bending angles.

The largest differences between the two schemes were found in the northern and southern hemisphere where the fractional errors of the new scheme were about one percent point smaller than the fractional error of the standard scheme above approximately 30 km.

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