A conditional distribution for severe storm environments

Eric Gilleland
http://www.rap.ucar.edu/staff/ericg

Barbara G. Brown and Caspar M. Ammann

CESM UQ Interest Group Meeting, 20 February 2013

Weather and Climate Impacts Assessment Science Project
http://www.assessment.ucar.edu/

Research Applications Laboratory
National Center for Atmospheric Research
Boulder, Colorado
Background
Background

NCAR/NCEP reanalysis
CCSM3 Global Climate Model
Background

CAPE × Shear
(J/kg² × m/s)

\[ W_{\text{max}} \times \text{Shear} \]
(m²/s²)

\[ W_{\text{max}} = \sqrt{2 \times \text{CAPE}} \]
(WmSh)
Background

Pr\{Winning \geq 10,000 \text{ USD in one drawing}\} \approx 1.3 \times 10^{-6}

In ten years, playing one ticket everyday,

Pr\{Winning \geq 10,000 \text{ USD once in one drawing}\} \approx 4.8 \times 10^{-3}

In 100 years \approx 0.05

In 1000 years \approx 0.77
“Il est impossible que l’improbable n’arrive jamais”
--Emil Gumbel
Background

Extreme Value Analysis: Generalized Extreme Value (GEV) Distribution

\[
\Pr\{M_n \leq z\} = \exp\left\{-\left[1 + \frac{\xi}{\sigma} \left(\frac{z - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}
\]

- \(\mu \in \mathbb{R}\) (location parameter)
- \(\sigma > 0\) (scale parameter)
- \(\xi \in \mathbb{R}\) (shape parameter)

\[M_n = \max\{X_1, \ldots, X_n\}\]
Background

Similar theory holds for excesses over a threshold instead of the maximum. In this case, the generalized Pareto (GP) df is the limiting df, but it also encompasses three types.

Three types are now: Exponential (shape = 0), Pareto (shape > 0) and Beta (shape < 0).

Model is for the excesses over a high threshold. Specifically:

$$
\Pr\left\{X - u \leq x \mid X > u\right\}
$$
Background

Multivariate Extreme Value Analysis

Without loss of generality, assume a standard max-stable df (can always transform to one). For simplicity, consider the bivariate case for variables $X$ and $Y$.

Model is for:

$$M_n = \left( \max \left\{ X_1, \ldots, X_n \right\} / n, \max \left\{ Y_1, \ldots, Y_n \right\} / n \right)$$
Background

Multivariate Extreme Value Analysis

Model is for:

\[ M_n = \left( \max \{ X_1, \ldots, X_n \} / n, \max \{ Y_1, \ldots, Y_n \} / n \right) \]

Analogous to the univariate setting, if there is a non-degenerate limiting df, \( G \), then it must have the form:

\[ G(x, y) = \exp\{-V(x, y)\}, x > 0, y > 0 \]

where

\[ V(x, y) = 2 \int_0^1 \max \left( \frac{w}{x}, \frac{1-w}{y} \right) dH(w), \text{ with} \]

\[ \int_0^1 wdH(w) = 1 / 2 \]
Background

\[
\Pr[\mathbf{T}_A < 1, \mathbf{T}_B < 1] = \Phi_2(\Phi^{-1}(F_A(1)), \Phi^{-1}(F_B(1)), \gamma)
\]

Recipe for Disaster: The formula that Killed Wall Street

Wired Magazine, 2/23/2009, by Feliz Salmon
Conditional EVA Model

Heffernan and Tawn (2004, JRSS B, 66, 497 – 546) found that for a wide class of coupla models, the form of the sequences of functions $a$ and $b$ fell into a simple class for positively associated variables (and a slightly more complicated form for negatively associated random variables). Namely,

$$a(y) = \alpha y \text{ and } b(y) = y^\beta$$

$$\alpha, \beta \in [0,1] \times (-\infty, 1)$$

$$\alpha, \beta \in [-1,1] \times (-\infty, 1)$$

Using Laplace marginals!
(Obviates the added complexity for negatively associated r.v.'s)
Conditional EVA Model

A consequence of the result that leads to a way to estimate the model (semi-parametrically) is that:

\[ X_{Y > u} = \alpha Y + Y^\beta Z_{Y > u} \]

\[ Z \perp Y, Z \sim G \]

No simple closed-form expression for \( G \).
\[\text{WmSh}_1, \ldots, \text{WmSh}_n | \text{High field energy}\]

Field Energy

We take the upper quartile over space to obtain a univariate time series that gives a measure of high WmSh over, possibly a small area of, space. Call it q75.
Winter

1958 - 1978

1979 - 1992

1993 - 1999

WmSh (m/s)^2

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Spring

1958 - 1978

1979 - 1992

1993 - 1999

WmSh (m/s)^2

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Conclusions/Discussion

• Conditional df provides a framework for investigating spatial patterns of severe storm environments (WmSh) when a lot of energy is present in the field.
• Estimating the model is difficult, but results are physically meaningful. New estimation methods being developed.
• Allows for modeling all values of WmSh, whether they are extreme or not.
• WmSh does not appear to be changing considerably in the extremes, but perhaps smaller changes exist that still have serious consequences.
• Provides a mechanism for potentially evaluating climate models for the current period.
• Difficult to assess changes in [WmSh | High energy] because intensity changes could simply be location shifts, but that is still important information.
• R software for performing EVA (extRemes):
  http://www.assessment.ucar.edu/toolkit
• Lookout for extRemes version 2.0-0

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