An Introduction to Ground Based GPS Meteorology

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Outline

Motivation for ground-based GPS meteorology

Tropospheric refraction

Tropospheric modeling and estimation

Mapping functions

Delay to water vapor conversion

Validation and examples
Motivation (1)

With the wide variety of atmospheric sensing techniques what can (ground based) GPS offer in addition?

• All weather
• Continuous operation
• High temporal resolution
• High accuracy - sufficiently accurate to provide useful data for improving weather forecasting
• Long-term stability - data set that is independent of radiosondes and that can establish a climate record
• GPS observes the “travel time” of the signal from the transmitters to the receiving antenna
• It is possible to determine that part of the travel time due to the atmosphere: “atmospheric delay”
• From the “atmospheric delay” of the GPS signal the “zenith tropospheric delay” and “zenith precipitable water vapor” can be determined
The GPS Observation Equation

\[ L_r^s = \rho_r^s + c \cdot \delta t_r + c \cdot \delta t_{r,sys} - c \cdot \delta t^s - c \cdot \delta t_{sys}^s + \delta \rho_{trp} + \delta \rho_{ion} + \delta \rho_{rel} + \delta \rho_{mul} + \lambda \cdot N_r^s + \ldots + \epsilon \]

\[ \rho_r^s \] Geometrical distance between satellite and receiver

\[ c \] Speed of light in vacuum

\[ \delta t_r \] Station clock correction: \textit{receiver clocks} (time and frequency transfer)

\[ \delta t_{r,sys} \] Delays in receiver and its antenna (cables, electronics, \ldots)

\[ \delta t^s \] Satellite clock correction: \textit{satellite clocks}

\[ \delta t_{s,sys} \] Delays in satellite and its antenna (cables, electronics, \ldots)

\[ \delta \rho_{trp} \] Tropospheric delay: \textit{troposphere parameters} (meteorology, climatology)

\[ \delta \rho_{ion} \] Ionospheric delay: \textit{ionosphere parameters} (atmosphere physics)

\[ \delta \rho_{rel} \] Relativistic corrections (Special and General Relativity)

\[ \delta \rho_{mul} \] Multipath, scattering, bending effects

\[ \lambda \] Wavelength of the GPS signal (\( L_1 \) or \( L_2 \))

\[ N_r^s \] Phase ambiguity: \textit{ambiguity parameters} (ambiguity resolution)

\[ \epsilon \] Measurement error
There are several ways to obtain $\delta \rho_{trp}$ from the GPS observations

$$L_r^s = \rho_r^s + c \cdot \delta t_r + c \cdot \delta t_{r,sys} - c \cdot \delta t^s - c \cdot \delta t_{sys} + \delta \rho_{trp} + \delta \rho_{ion} + \delta \rho_{rel} + \delta \rho_{mul} + \lambda \cdot N_r^s + \ldots + \epsilon$$

(1) Remove all other components from $L_r^s$. This is done for estimating the “atmospheric delay for radio occultation observations where all other components must be known from separate processing steps.

(2) Model it and estimate as a parameter. This is done for ground based GPS and will be explained in more detail in this lecture.
\[ L_E = \int_L n(s) \, ds \]

\( L_E \) is the path length along the path \( L \) and \( n(s) \) is the index of refraction which is a function of position \( s \) along the path \( L \).

\[ \delta \rho_{trp} = L_E - G = \int_L n(s) \, ds - G = \int_L (n(s) - 1) \, ds + (S - G) \]

Bending effect is \( (S - G) \) and refractivity is defined as \( N = (n-1) \times 10^6 \).

Figure 2.2: Tropospheric refraction (Elgered, 1993)
\[ \delta \rho_{trp} = L_E - G = \int_L n(s) \, ds - G = \int_L (n(s) - 1) \, ds + (S - G) \]

“(S-G)” is the effect of bending

**Example size of tropospheric delay from CIRA+Q model atmosphere**

- Total zenith delay ~2.2 m
- Delay at 5 degrees elevation is ~ 25 m
- GPS phase measurement is precise to ~0.001 m
Assuming bending effect can be ignored then the GPS tropospheric delay is:

\[ \delta \rho_{trp} = \int_L (n(s) - 1) \, ds = 10^{-6} \int_L N(s) \, ds \]

The refractivity \( N \) is defined as \( N = (n-1) \times 10^6 \).

The relationship between refractivity and temperature, pressure, humidity is often given as:

\[ N = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \]

\[ N = k_1 \frac{P}{T} + (k_2 - k_1) \frac{e}{T} + k_3 \frac{e}{T^2} \]

Where: \( k_1, k_2, k_3 \) are constants (77.607 K mb\(^{-1}\); 64.8 K mb\(^{-1}\); 377,600 K\(^2\) mb\(^{-1}\)) and \( P_d \) is the partial pressure of dry air, \( P = P_d + e \) is the hydrostatic pressure of dry air [mb], \( e \) is the partial pressure or water vapor and \( T \) is the temperature [K].
Some times the expression for N is further simplified to:

$$N = 77.6 \frac{P}{T} + 373000 \frac{e}{T^2}$$

The partial pressure of water vapor $e$ is related to relative humidity the equation below.

$$e = \frac{H}{100} e^{-37.2465+0.213166T-0.000256908T^{-2}}$$

Example size of tropospheric delay from CIRA+Q model atmosphere

Typical wet delay values

Zenith delay due to water vapor varies between 0 - 50 cm

At 5 degrees wet delay can be several meters.

Wet delay is only ~10% of total tropospheric delay but it is the most interesting part of the tropospheric delay for meteorology.
Estimation of the zenith tropospheric delay

Tropospheric delay can be modeled as a function of zenith delay:

\[ \delta \rho_{\text{trp}}(z) = \delta \rho_{\text{dry}}(0) \cdot m_{f,\text{dry}}(z) + \delta \rho_{\text{wet}}(0) \cdot m_{f,\text{wet}}(z) \]

Where the tropospheric delay at the zenith angle \( z \) is modeled as the sum of the zenith dry delay \( \delta \rho_{\text{dry}}(0) \) mapped to the angle \( z \) with the mapping function \( m_{f,\text{dry}}(z) \) plus the zenith wet delay \( \delta \rho_{\text{dry}}(0) \) mapped with the wet mapping function \( m_{f,\text{wet}}(z) \)

We now have expressed the tropospheric delay as a parameter in a function of the zenith angle of the GPS satellite.

The simplest mapping function is simply the geometric relation given here for zenith angle \( z \) and elevation angle \( \theta \):

\[ m_{f,\text{dry}} = m_{f,\text{wet}} = \frac{1}{\cos(z)} = \frac{1}{\sin(\theta)} \]
\[
\hat{\ell} = \psi(\hat{x}) \\
= \ell' + v \\
= \psi(x_0) + A x \\
\text{ (linearization)}
\]

and therefore
\[
v = A x - (\ell' - \psi(x_0)) \\
= A x - \ell
\]

with
\[
\hat{x} = x_0 + x \\
\ell = \ell' - \psi(x_0)
\]

where
\[
\psi \quad \text{Model function (mathematical relationship between observations and parameters; observation equation)} \\
\hat{\ell} \quad \text{Column matrix of adjusted observations} \\
\ell' \quad \text{Column matrix of observations} \\
\ell \quad \text{Column matrix of “observed–computed” (O–C)} \\
\hat{x} \quad \text{Estimated parameters} \\
x_0 \quad \text{A priori values of parameters} \\
x \quad \text{Improvements to the a priori parameter values} \\
v \quad \text{Residuals} \\
A \quad \text{Jacobi matrix of partial derivatives}
\]

The Jacobi matrix \( A \) is defined by
\[
A = \left( \frac{\partial \psi(\hat{x})}{\partial (\hat{x})} \right)
\]  \hspace{1cm} (1.13)
\[
A = \begin{bmatrix}
\frac{\partial \psi_1(\hat{x}_o)}{\partial \hat{x}_1} & \frac{\partial \psi_1(\hat{x}_o)}{\partial \hat{x}_2} & \cdots & \frac{\partial \psi_1(\hat{x}_o)}{\partial \hat{x}_m} \\
\frac{\partial \psi_2(\hat{x}_o)}{\partial \hat{x}_1} & \cdots & & \\
\vdots & \ddots & \ddots & \\
\frac{\partial \psi_n(\hat{x}_o)}{\partial \hat{x}_1} & & & \frac{\partial \psi_n(\hat{x}_o)}{\partial \hat{x}_m}
\end{bmatrix}_{n \times m}
\]

\( n \) observations

\( m \) parameters
The stochastic model of the observations is given by the covariance matrix

\[ \mathbf{K}_{\ell \ell} = \sigma_0^2 \mathbf{Q}_{\ell \ell} = \sigma_0^2 \mathbf{P}^{-1} \quad (1.14) \]

Here \( \sigma_0^2 \) denotes the a priori variance of the observations, \( \mathbf{Q}_{\ell \ell} \) the cofactor matrix of the observations and \( \mathbf{P} \) the weight matrix of the observations.

The solution of the linear system of equations system (1.10) follows from the least squares condition:

\[ \mathbf{v}^T \mathbf{P} \mathbf{v} = \text{minimal} \quad (1.15) \]

and we obtain

\[ \mathbf{x} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P} \ell = \mathbf{Q}_{xx} \mathbf{b} \quad (1.16) \]

where

\[ \mathbf{Q}_{xx}^{-1} = \mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A} \quad (1.17) \]

\[ \mathbf{b} = \mathbf{A}^T \mathbf{P} \ell \quad (1.18) \]
As an example for the numerical estimation of the zenith tropospheric delay we want to show the very simple case where we have the following observation equation:

\[
\delta \rho_{\text{trp}}(0) \frac{1}{\sin(\theta)} - (\rho - \rho_0) = v
\]

Here the only parameter that has to be estimated is the zenith tropospheric delay which is modeled using the simple mapping function from earlier. \(\rho\) denotes the range to the satellite \(\rho_0\) the a priori guess for this range.

The zenith tropospheric delay is the previously defined “improvement” parameter \(x\) that we need to compute with the LSA. In this example case the ”\(l\)” is the “O minus C” caused by the tropospheric delay in the direction of the GPS satellites which we generate using the mapping function and adding some random value \(v\) which could be caused by inhomogeneities in the atmosphere.

\[
x = (A^T PA)^{-1} A^T l = \delta \rho_{\text{trp}}(0)
\]

\[
l = Ax + v = \left[ \frac{1}{\sin(\theta)} \right] x + v
\]

\[
\begin{bmatrix}
\text{swd}_1 \\
\text{swd}_2 \\
\text{swd}_3 \\
\text{swd}_4
\end{bmatrix} =
\begin{bmatrix}
1210 \\
462 \\
286 \\
239
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
1/\sin(\theta_1) & 1/\sin(10) & 5.76 \\
1/\sin(\theta_2) & 1/\sin(25) & 2.37 \\
1/\sin(\theta_3) & 1/\sin(45) & 1.41 \\
1/\sin(\theta_4) & 1/\sin(60) & 1.15
\end{bmatrix}
\]

\[
I = \begin{bmatrix}
\text{swd}_1 \\
\text{swd}_2 \\
\text{swd}_3 \\
\text{swd}_4
\end{bmatrix}
\]

\[
\text{swd}_1 = 1210, \quad \text{swd}_2 = 462, \quad \text{swd}_3 = 286, \quad \text{swd}_4 = 239
\]
Now we can solve for the zenith tropospheric delay $\delta \rho_{trop}(0)$

\[ x = (A^T PA)^{-1} A^T Pl \]

\[
A^T PA = A^T IA = \begin{bmatrix}
5.76 & 2.37 & 1.41 & 1.15 \\
2.37 & 1.41 & 1.15 & 1.15 \\
1.41 & 1.15 & 1.15 & 1.15 \\
1.15 & 1.15 & 1.15 & 1.15 \\
\end{bmatrix} = 42.11
\]

\[
A^T Pl = A^T Il = \begin{bmatrix}
1210 \\
462 \\
286 \\
239 \\
\end{bmatrix} = 8742.65
\]

\[ x = (A^T PA)^{-1} A^T Pl = \left(42.11\right)^{-1} 8742.65 = 207.61 \]

Thus in this greatly simplified numerical example we estimated the tropospheric delay to be $\sim 207.6$
The previous example was meant as an illustration of the LSA and not realistic for several reasons:

(1) We only processed data from one station - in general multiple stations are processed simultaneously in a network. Network solutions are needed

(2) We assumed that only the tropospheric delay had to be estimated and that all other parameters in the observation equation were known or cancelled. In general additional parameters have to be estimated.

(3) We only estimated one constant tropospheric delay parameter - in general time changing parameters have to be estimated. Also sometimes gradient parameters have to be estimated.

(4) We used a very simple mapping function, that did not account for signal bending and did not distinguish between dry and wet mapping. In real GPS data analysis more sophisticated mapping is needed.

(5) We used equal weighting of the observations - in the real case this is not done

(6) We assumed “perfect” station coordinates and orbits

Next we will discuss the implications of these assumptions
Single station versus network solution in GPS Meteorology

Multiple stations are needed to cancel satellite clock errors. Unless satellite clock corrections are provided by a service like the IGS (International GPS Service) at least two stations are needed to cancel (estimate) satellite clocks.

At least two stations are needed for so-called carrier phase ambiguity resolution (described in previous talk). Ambiguity resolution (at the double difference level) is important because of the correlation between ambiguity parameters and tropospheric delay parameters.

Problem for short baselines. If sites A and B are not far separated then the zenith angles of observed satellites are almost the same $z_A \sim z_B$

$$\Delta \delta \rho_{AB, trp}(z) = \frac{\delta \rho_{A, trp}(0)}{\sin(\theta_A)} - \frac{\delta \rho_{B, trp}(0)}{\sin(\theta_B)} \approx \frac{\delta \rho_{A, trp}(0) - \delta \rho_{B, trp}(0)}{\sin(\theta_A)}$$

This is the reason that one can only estimate the difference in tropospheric delay with small networks. Only if the network (baseline) is spaced over $>500$ km can one estimate the absolute tropospheric delay at all sites.
Estimation of tropospheric parameter - constraints through pseudo observations

Instead of estimating a new and independent tropospheric parameter for every time interval (i.e. every 30 minutes) GPS processing software constraints the change in these parameters through the introduction of pseudo observations.

\[ p_{i+1} - p_i = v_{i,i+1} \]

\[ w_{i,i+1} = \frac{\sigma_0^2}{\sigma_{i,i+1}^2} \]

Where \( p_i (i=1,2 \ldots,n_t) \) is the ith tropospheric parameter for a station, and \( w_{i,i+1} \) is the weight of the pseudo observations. \( \sigma_0^2 \) is the a prior variance of the observations and \( \sigma_{i,i+1}^2 \) the a priori variance of the parameter differences.

\[
A = \begin{bmatrix}
1 & -1 & 0 & \ldots & 0 \\
0 & 1 & -1 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 1 & -1 \\
\end{bmatrix}
\]

\[ n_t \text{ obs.} \]

\[ \Delta N_{rel} = A^T P A = \begin{bmatrix}
\frac{\sigma_0^2}{\sigma_{1,2}^2} & -\frac{\sigma_0^2}{\sigma_{1,2}^2} & 0 & \ldots & 0 \\
-\frac{\sigma_0^2}{\sigma_{1,2}^2} & \frac{\sigma_0^2}{\sigma_{1,2}^2} + \frac{\sigma_0^2}{\sigma_{2,3}^2} & -\frac{\sigma_0^2}{\sigma_{2,3}^2} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \frac{\sigma_0^2}{\sigma_{n_t-1,n_t}^2} \\
\end{bmatrix}
\]

Large weights \( w_{i,i+1} \) result in reduced variability of the tropospheric estimates.
Estimation of tropospheric parameter - constraints through pseudo observations (cont.)

The delta normal equations matrix of the pseudo-observations from the previous slide is added to the normal equation matrix of the observations to estimate constrained tropospheric delay parameters.

This figure shows an example of the effect of the tropospheric constraints on the estimated tropospheric delay parameter for 1 day of station BRUS. In this case a new tropospheric delay parameter was estimated every 15 minutes and several constraints of the change in delay during the 15 minutes were applied.

Constraints help suppress noise in the solution but when they are too tight they can also result in missing of real atmospheric features.

Constrained should be tailored to station location (high in polar regions, lower in mid-latitude summers)
Mapping functions

Mapping functions have two important functions in GPS analysis for meteorology. (a) they are used to determine the a priori tropospheric delay (typically the dry delay) in the direction of the GPS satellites, and (b) they are used (as the partial derivative wrt the tropospheric delay) to estimate the tropospheric parameters (typically the wet delay).

The “1/sin(elevation angle)” mapping function that we used in the example above is not sufficient for accurate GPS meteorology because it does not account for signal bending and it does not consider the Earth curvature.

![Graph showing bending effect](image)

Bending effect “(S-G)”
Model atmosphere

Note that the effect of bending can be ~ 20 cm at 5 degrees elevation angle already as shown in this example where “S-G” was computed for a model “CIRA+Q” atmosphere

State-of-the-art mapping function do not have errors due to the bending term as in the figure above and try to account for bending.
Mapping functions (continued)
The most widely used mapping function presently is the “Niell mapping function” which maps the zenith tropospheric to lower angles and requires information about the station latitude, height, and month of the year.

Recent studies have shown that additional improvement over the Niell mapping function can be achieved if the mapping is computed by ray-tracing through numerical weather models for specific locations and at the time of the GPS observations.

(1) Use radiosondes as truth raytrace through radiosonde profile

compute radiosonde mapping as \( m_{RS} \text{(ele)} = \frac{D_{Hz}}{D_{hele}} \)

(2) Use Neill mapping function to compute \( m_{Neill} \text{(ele)} \)

(3) Use Direct mapping function to compute \( m_{direct} \text{(ele)} \) for cases:
    a) NCEP numerical model
    b) CIRA_Q climatology
    c) CIRA_Q + surface Met.
# Mapping Function Comparison Example

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<th>Elevation [deg]</th>
<th>Radiosonde mapping</th>
<th>Neill mapping</th>
<th>Delta</th>
<th>Delta % cm</th>
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<td>5.548</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>56.6</td>
</tr>
</tbody>
</table>
For mapping function comparison all possible radiosonde stations from 1997 were used. (Every 5th day of the year)
Mapping Errors at 5 degrees elevation / Winter
day 001, 1997, all global radiosondes

Niell mapping has significant scatter in the northern winter
Bias up to -4% (~9 cm delay) in southern hemisphere

Direct mapping with NCEP shows reduced scatter and no bias in southern hemisphere
Effect of using different dry mapping (direct mapping with AVN model and Niell mapping) on comparison with water vapor radiometer.

The radiometer is believed to be biased by ~3%.

The change in mapping makes a several percent difference in the PWV estimate.

The AVN mapping results are in better agreement with the WVR.

The diurnal signal will be discussed later but is at least in part caused by the way we convert from wet delay to PWV.
Weighting of GPS phase observations

There are many advantages to include low elevation angle observations into the tropospheric estimation - in general it is suggested to track down as low as possible and to use observations to down to 5 degrees elevation.

One problem with low elevation observations is that they tend to be more affected by noise (mostly multipath) and by remaining errors in the mapping functions. In order to reduce the sensitivity of the GPS solution to these low elevation errors the observations can be weighted according to elevation angle. A commonly used de-weighting function is \( w = \cos^2(z) \) where \( z \) is the the zenith angle.

Estimation of azimuthal gradients

Especially when low elevation observations are available the GPS phase measurements can be used to estimate gradients with azimuthal direction in the tropospheric delay. These gradients can be parameterized as:

\[
\delta \rho_{\text{trp}}(\alpha, z) = \delta \rho_{\text{trp}}(0) \cdot m_f(z) + \frac{dm_f(z)}{dz} \cos \alpha \cdot \delta \rho_n + \frac{dm_f(z)}{dz} \sin \alpha \cdot \delta \rho_e
\]

Where \( \alpha \) is the azimuth angle (relative to “north) and \( \delta \rho_n, \delta \rho_e \) are the north and east gradient parameters respectively.

Estimation of gradients usually requires longer data sets. Estimation of tropospheric gradients has been shown to significantly improve horizontal positioning with GPS (factor of 2) and vertical positioning by about 20%. Horizontal gradients have not yet been used / tested with weather models.
Coordinate errors

In our numerical example we assumed that the geometric range from the GPS antenna to the satellite was modeled without any error. However for GPS meteorology it is possible to assume that the station coordinates are known perfectly if (a) the coordinates are re-computed and (b) temporal coordinate changes are modeled.

A vertical coordinate error of 3 cm causes an error in the zenith tropospheric delay of ~1 cm (~2mm in zenith water vapor).

It is thus important to ensure accuracy of vertical coordinate at the sub-cm level. Horizontal coordinates are less critical and usually also better know.

Rothacher, 2002

Figure 1.2: Global velocities from GPS (7-year solution of CODE)

Ocean Loading effect at Forteleza, Brasil

Atmospheric loading effect: ~0.35 - 0.55 mm/mbar

Rothacher, 2002
Orbits

GPS meteorology places some special requirements on the GPS orbits:

(a) Orbits need to be accurate (~10 cm orbit of IGU orbits causes only small tropospheric error)
(b) Orbits need to be available in real-time
(c) IGU orbits need to be tested for outliers. Orbit maneuvers cannot be predicted.

Estimation of how much the orbit error contributes to the tropospheric estimation error has not been done (to my knowledge and could be an interesting project.

Using recent 6-hour IGU orbits there is very little difference in the PWV estimates compared to IGS
Conversion of wet delay to precipitable water vapor (1): total delay, dry delay, wet delay

\[ \delta \rho_{\text{trp}}(z) = \delta \rho_{\text{dry}}(0) \cdot m_{f,\text{dry}}(z) + \delta \rho_{\text{wet}}(0) \cdot m_{f,\text{wet}}(z) \]

\[ \delta \rho_{\text{wet}}(0) = \delta \rho_{\text{trp}}(0) - \delta \rho_{\text{dry}}(0) \]

\[ \delta \rho_{\text{dry}}(0) = 2.2779 \cdot 10^{-3} \frac{P_s}{f(lat, height)} \]

\[ f(lat, height) = 1 - 0.00266 \cos(2 \cdot lat) - 0.00028 \cdot \text{height} \]

Where: \( z \) is the zenith angle, \( \delta \rho_{\text{trp}}(z) \) is total tropospheric delay, \( \delta \rho_{\text{dry}}(z) \) the dry or hydrostatic delay, \( \delta \rho_{\text{wet}}(z) \) the wet delay due to water vapor, \( m_{f,\text{dry}}(z) \) is the dry mapping function, \( m_{f,\text{wet}}(z) \) is the wet mapping function, \( P_s \) is the station pressure in mbar, \( lat \) is geographic latitude and \( height \) is station height in km.

A 1-mb error in the surface pressure measurement results in an error of 2 mm in the zenith wet delay \( \delta \rho_{\text{wet}}(0) \).
Conversion of wet delay to precipitable water vapor

(2) The “π-factor”

$$\pi = \frac{IWV}{\delta\rho_{wet}(0)} = \frac{\int L \rho_d dz}{10^{-6} \int L N_v dz}$$

$$\rho_d = \frac{1}{R_d} \left( \frac{P_d}{T} \right)$$

$$N_v = 3.73 \times 10^5 \frac{P_v}{T^2} = k \frac{P_v}{T^2}$$  
(Gas law)

$$N_v \approx \frac{1}{R_v} \left( \frac{P_v}{T} \right) dz$$

$$\pi = \frac{\int L \frac{P_v}{T} dz}{10^{-6} k \int L \frac{P_v}{T^2} dz} = \frac{10^6}{R_v k} T_m$$

(Refractivity of water vapor approximate)
$T_m$ is called the “mean temperature of the atmosphere”

$$T_m = \frac{\int P_v dz}{\int L \frac{P_v}{T^2} dz}$$

Based on linear regression of radiosonde observation a linear relation between $T_m$ and the surface temperature $T_s$ was derived by Bevis et al. 1994. $T_m \sim 70.2 + 0.72 T_s$

Alternatively one can compute $T_m$ from numerical weather models (similar to mapping functions). Recent (Teresa VanHove, not yet peer reviewed) work showed an improvement in the comparison between GPS and water vapor radiometers when $T_m$ was computed from by integration through weather models rather than from the approximate expression given above.

This figure shows the difference between GPS PWV and WVR for 2003 (red using the linear function and blue using a numerical model for computation of $T_m$). There is a pronounced diurnal effect in the difference which is reduced for the blue curve.
Validation

GPS PWV can be compared to measurements from Water Vapor Radiometer measurements and to integrated radiosonde measurements. WVR observations are completely independent from GPS and provide a good calibration. GPS and WVR agreement is typically at the 1.5 mm rms level. Part of this error is due the WVR and GPS maybe at the 1-mm level or better.
GPS Meteorology at Sea

Explorer August 2003 PWV

C. Rocken

Ground based GPS Meteorology

NCAR GPS Meteorology Colloquium, June 20 - July 2, 2004, Boulder, CO
Summary and Conclusions

• Ground based GPS meteorology is a well established and proven technique.
• Recent results at NOAA showed positive impact on weather forecasting.
• Large networks are established world-wide for multipurpose applications including meteorology.
• Near-real time solutions and real-time solutions are almost as good as post-processed results.
• There were few recent improvements in accuracy because the meteorology community did not ask for them.
• Areas for improvements are: (1) mapping; (2) conversion delay-> PWV; (3) site multipath calibration; (4) moving platform(ocean) observations.
• Meteorological research into assimilation of ground-based and slant observations ongoing.
• Studies of use of these data for climate monitoring and model improvement just beginning.

C Rocken “Ground based GPS Meteorology” NCAR GPS Meteorology Colloquium, June 20 - July 2, 2004, Boulder, CO