ON NUMERICAL REALIZABILITY OF THERMAL CONVECTION

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1. ABSTRACT

Astounded at the regularity of convective structures observed in simulations of mesoscale flow past realistic topography, we take a deeper look into numerics of a classical problem of flow over a heated plate [1]. We find that solutions are sensitive to viscosity, which is either incorporated a priori or effectively realized in numerical models. In particular, anisotropic viscosity can lead to regular convective structures [2,3,4] that mimic naturally realizable Rayleigh-Benard cells that are, however, unphysical for the specified external parameter range. The details of the viscosity appear to play secondary role; that is, similar structures can occur for prescribed constant viscosities, explicit subgrid-scale turbulence models, ad-hoc numerical filters, or implicit dissipation of numerical schemes. This calls for careful selection of numerical tools suitable for cloud-resolving simulations of atmospheric circulations [5].

2. LINEAR THEORY

Linear model equations that account for anisotropy of the viscosity in Rayleigh-Benard convection can be written as

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} &= -\nabla \phi + g\alpha \theta \nabla z + \nu_h \Delta_h \mathbf{u} + \nu_v \Delta_z \mathbf{u} \\
\frac{\partial \theta}{\partial t} &= \beta w + \kappa_h \Delta_h \theta + \kappa_v \Delta_z \theta \\
\nabla \cdot \mathbf{u} &= 0.
\end{align*}
\]

Here \( \mathbf{u} \) is velocity vector, and \( w \) its vertical component; \( \Phi \) denotes normalized pressure perturbation; \( \alpha \) is the volume expansion coefficient; \( \theta \) is the potential temperature deviation from a linear profile with adverse gradient \( \beta \); \( g \) is the acceleration of gravity, and subscripts \( h \) and \( v \) refer to the horizontal and vertical values of viscosity and diffusivity, \( \nu \) and \( \kappa \) respectively. The resulting marginal stability relation demarcating regime transition is

\[
Ra_h = \frac{H^4}{k^2} \left( n \left( \frac{\pi}{\sqrt{r}} \right)^2 + k^2 \right) \left( \frac{n \left( \frac{\pi}{\sqrt{r}} \right)^2 + k^2}{n \left( \frac{\pi}{\sqrt{r}} \right)^2 + r^2} \right). 
\]

That is, for any given \( Ra_h \) all horizontal modes with wave number \( k \) such that the rhs of the marginal stability relation exceeds \( Ra_h \) are unstable. \( Ra_h \) denotes Rayleigh number \( Ra = -N^2 \Delta H / \kappa \); \( r := \nu_v / \nu_h = \kappa_v / \kappa_h \) is the anisotropy ratio; \( N \) denotes the buoyancy frequency (imaginary), and \( H \) is the layer depth. The asymptotics of marginal stability relation in Figure 1 indicate that decreasing \( \nu_v \) at constant \( \nu_h \) accentuates instability of long horizontal wavelengths, whereas decreasing \( \nu_h \) at constant \( \nu_v \) enhances instability of short modes.

![Figure 1. Asymptotic marginal stability relations for a finite Prandtl number and \( \nu_v = \nu_h \) (solid), \( \nu_v = 0 \) (circles) and \( \nu_h = 0 \) (squares). Respective Rayleigh numbers \( Ra_h \), \( Ra \) and \( Ra_v \) are shown in function of the squared horizontal wave number. Stability region is below the curves.](image)

Because the squared aspect ratio of dominant convective cells is predicted as

\[
\left( \frac{2H}{\lambda} \right)^2 = \frac{1}{4} \left( \sqrt{8r + 1} - 1 \right),
\]

the simulated convective structures may vary dramatically with the effective anisotropy of viscosity departing from unity.
3. NUMERICAL MODEL

Numerical model Eulag [6] adopted in this study solves thermally forced, viscous, nonhydrostatic anelastic equations of Lipps and Hemler, which can be compactly written as

\[
\begin{align*}
\frac{D\mathbf{u}}{Dt} &= -\text{Grad} (\pi') - g \frac{\partial \theta}{\partial \theta_b} + \mathcal{D}_m(E, \nabla \mathbf{u}) \\
\frac{D\theta}{Dt} &= -\mathbf{u} \cdot \text{Grad} \theta_e + \mathcal{D}_h(E, \nabla \theta) \\
\text{Div}(\rho_b \mathbf{u}) &= 0
\end{align*}
\]

Here the operators $D/Dt$, $\text{Grad}$ and $\text{Div}$ symbolize the material derivative, gradient and divergence; $\mathbf{u}$ denotes the velocity vector; $\theta, \rho,$ and $\pi$ denote potential temperature, density, and a density-normalized pressure; and $\mathbf{g}$ symbolizes the vector of gravitational acceleration. Subscripts $b$ and $e$ refer to the basic and ambient states, respectively, and primes denote deviations from the environmental state. The “$D$” terms on the rhs of the momentum and entropy equations symbolize explicit viscous forcings, which depend on the derivatives of dependent variables and, eventually, on the turbulent kinetic energy $E$ predicted with subgrid-scale models. The prognostic equations of the model are supplemented with diagnostic anelastic mass continuity constraint, implying the formulation of the elliptic equation for pressure. For integrating the prognostic equations, Eulag uses a second-order-accurate, nonoscillatory forward-in-time MPDATA approach [7]

4. SIMULATION RESULTS

The effects of incorporating effective anisotropic viscosity manifest themselves both in idealized and realistic (numerical) experiments. Figure 2, shows the structure of Rayleigh-Benard convection over a heated plate, after 6h of simulation with horizontal resolution 500 m and constant heat flux 200 W/m² imposed at the lower boundary. Cellular structure is apparent for the case with larger horizontal viscosity. In turn, Figure 3 shows the result of simulation of moist convection forming over heated terrain in southern Poland. The routine hydrostatic mesoscale predictions at 17 km resolution, using the Unified Model for Poland Area (UMPL), continuously supplied the initial, boundary, and ambient conditions for high-resolution simulations using EULAG. The EULAG domain of 240×200 km squared, embedded in the UMPL Central European domain (2000×2400 km squared) was covered with 1 km horizontal grid intervals; while keeping the vertical resolution double of UMPL. Similar as in the idealized case, the increase of $\nu_h$ in the effective stress tensor manifests as cell broadening, consistent with the linear-theory predictions of section 2.
motivated with arguments of subgrid-scale modeling --- may force convection to group into structures closely resembling Rayleigh-Benard cells observed in the maritime conditions, with geometric characteristics of natural cells, as in Figure 3. Looking forward toward petascale computing, we advocate careful selection of numerical filters for cloud resolving NWP, to avoid a leakage of uncontrolled viscous effects into the models' physics.

6. BIBLIOGRAPHY

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7. ACKNOWLEDGEMENTS

The National Center for Atmospheric Research (NCAR) is sponsored by the National Science Foundation. This work was supported in part by the USA Department of Energy CCPP and SciDAC research programs, and by the Polish Ministry of Science and Higher Education grant No. N307059034. Imagery produced by VAPOR www.vapor.ucar.edu