COMPARISON BETWEEN WIND WAVES AT SEA
AND IN THE LABORATORY

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Correlations between laboratory and geophysical data are presented for certain statistical properties of wind waves. The parameters chosen include (a) Relations between mean wave height, and the height of the highest one-third or one-tenth waves, as given by a Rayleigh probability distribution, and (b) Amplitude spectra for waves, as given by Phillips' equilibrium theory. The correlation between laboratory results and geophysical data is very satisfactory over a wide range of wave size.
Introduction

The processes of interaction between the atmosphere and the sea are important because they determine the fluxes of momentum, heat and moisture in the atmospheric boundary layer over oceans, and the fluxes of momentum, heat and salinity in the surface layers of the water. This interaction also controls the characteristics of the air-sea interface through the presence of wind generated waves and wind-driven currents. At present the mechanisms of air-sea interaction are only partially understood. Part of the reason for this limitation lies in the inadequate observation data available for testing the applicability of modern dynamical theories. It is often exceedingly difficult to obtain good experimental data in the field; hence, some investigators have retreated to the laboratory where precision reproducible measurements can be made on analogous systems to the geophysical case. Because of the difficulties in scaling laboratory data to the much larger atmosphere-sea interaction, most investigators have considered the laboratory results as having only the most meager applicability to significantly similar geophysical phenomena.

The purpose of this note is to present a comparison between some typical statistical properties of water waves in a wind-water tunnel, and at sea. Data are included for relations between certain scale heights for waves, as well as for amplitude spectra. The results amply demonstrate that such results from the laboratory and the field can be correlated uniformly well in terms of known methods.

Experimental

The experiments were performed in the wind-water tunnel at Colorado State University. This facility has been described in
Near the inlet of the tunnel, the water surface was covered by an aluminum plate supported by legs from the floor of the tunnel. The plate was placed at effectively the same height as the mean water depth to ensure as smooth a transition as possible for the adjoining air and water flow. Also the presence of the plate allowed definition of the flow conditions in the air before the air began passing over the water.

Measurements of the mean air flow and water motion have been made in the CSU tunnel; the methods used and results obtained are described in detail elsewhere (Hess, et al, 1968; Plate, et al 1968).

The displacement of the water surface resulting from the air motion was measured with a capacitance gauge described by Plate, et al (1968). This device has a sensitivity to surface disturbance to $\sim 1 \mu m$ amplitude over a frequency range 0 - 30 Hz. Strip chart recordings of the time variation in elevation of the water surface were recorded on a Brush recorder. These traces were later digitized using a Gerber reduction system. The digitized data were analyzed on the NCAR-CDC 6600 computer using programs for statistical processing following methods described by Plate, et al (1968), and Colonell (1966). Calculations presented here include ones for (a) relation between the mean wave height, and the r.m.s. wave height, the highest one third waves, and the highest one-tenth waves, and (b) the frequency spectra for wave amplitudes $\hat{\phi} (n)$.

**Results and Discussion**

The conditions of mean air flow as measured in the CSU tunnel were found to be correlated well by the well known logarithmic
law of the wall. The air motion reflected a transition from flow over a smooth surface to flow over a fully rough surface over the length of the tunnel test section. The structure of air motion has been described elsewhere in more detail by Hess, et al (1968). But the important point for purposes of this note is that the wind developed in the CSU tunnel is essentially similar to that observed in air motion near ocean waves for corresponding conditions of surface roughness.

Under the urging of the wind, waves are generated in the laboratory channel with wave lengths ranging from about 1 cm to 25 cm, and with standard deviations of the water surface displacement of ~ 0.01 cm to 2 cm. Probability distributions calculated from the capacitance gauge data for sequences of 100 waves or more showed the amplitudes of the wind waves to be nearly Gaussian, with principal deviation from such a distribution far downstream from the tunnel inlet.

Longuet-Higgins (1952), and Cartwright and Longuet-Higgins (1956) have derived a theoretical expression for the probability distribution of maxima in the water surface displacement record (see also the review of Colonell, 1966). By assuming that the wave spectrum is confined to a single narrow band, their probability distribution reduces to the Rayleigh probability distribution. That is, the probability density of maxima is given by

\[
Pr \left[ \frac{\eta_{\text{max}}}{\sigma_{\eta}} \right] = \frac{\eta_{\text{max}}}{\sigma_{\eta}} \exp \left( -\frac{\eta_{\text{max}}^2}{2\sigma_{\eta}^2} \right) \quad (1)
\]

where \( \eta_{\text{max}} \) is the maximum water surface displacement and \( \sigma_{\eta} \) is the standard deviation of the water surface displacement.

The wave height, \( H \), is defined as the vertical distance between the crest and the trough, or as \( 2\eta_{\text{max}} \). By using Eq. (1) the relationship of the
standard deviation of wave heights, the mean wave height of the highest one-third of the waves, the mean wave height of the highest one-tenth of the waves, and so forth, to the mean wave height of all the waves can be calculated.

In Fig. (1) the geophysical studies of Goodknight and Russell (1963) and Putz (1952) and the wind-water tunnel studies of Sibul (1955) and Colonell (1966) are compared with the present study. One sees that the results of the geophysical cases and those from the laboratory are consistent, and are quite well represented by the theoretical Rayleigh distribution relations. There is, however, a small but systematic deviation from the Rayleigh curve for the laboratory results for the highest one-tenth waves, where the laboratory results are lower than the Rayleigh curve. This deviation may be due to the laboratory conditions where waves are generated over very limited fetch. However, there does not seem to be any reason on physical grounds why this should be the case.

Perhaps more interesting to the oceanographer than the relations for the mean wave heights are the amplitude spectra of waves. Spectra for wave data taken at three successively longer fetches measured from the upstream edge of the water are shown in Fig. 2. For comparison, several cases of geophysical data are included in the figure, ranging from wave spectra taken by Longuet-Higgins, et al. (1963) for waves coming from a 500 km fetch, and newer observations for waves observed 17.9 km from Hurricane Dora (Collins, 1966) to data taken over a limited fetch such as those of Kinsman (1960). Laboratory measurements taken previously in the CSU tunnel also are shown for comparison. A solid line is drawn in with a -5 slope. This corresponds to Phillips' theoretical relation for the equilibrium spectrum:
Analysis of a number of oceanographic observations indicates that $\beta \approx 1.17 \times 10^{-2}$ (Phillips, 1966). We find this value agrees satisfactorily with the laboratory data. However, our value of $\beta$, is likely to be more like $2.0 \times 10^{-2}$ for the wind-water tunnel results. Even with the difference in $\beta$, it is indeed remarkable that Phillips' predicted results correlate very satisfactorily. Spectra in the gravity wave regime taken both in the laboratory and at sea over a range of more than nine decades in spectral density.

It is of interest to note that the data taken at limited fetch of less than 100 km, including all the results except for Pierson (1962) and Longuet-Higgins, et al. (1963), tend to be systematically higher in spectral density than Phillips' curve using $\beta \approx 1.17 \times 10^{-2}$. These accumulated data, along with the laboratory results may indicate that $\beta$ itself is a weak function of fetch. On the other hand, the range of scatter between results shown in Fig. 2 may just reflect the range of error in wave gauge measurements taken with various instruments and experimental conditions. In the case of the wind tunnel results, the drift current in the water might be responsible for the apparent systematic positive deviation from Phillips' curve. The data taken in the tunnel involve measurement of frequency by a fixed probe. Thus in these cases reflects a drift velocity in addition to the celerity of the waves. Hidy and Plate (1966) found, for example, that the celerity of waves in the tunnel, measured relative to a fixed point, exceeded those predicted by gravity wave theory by 10 - 20% depending on water depth. If the results shown in Fig. 2 were corrected in frequency for the effect of surface drift,
agreement between Phillips' curve and our results could be improved.

In connection with the correlation shown in Fig. 2, it is of interest to note that Plate, et al. (1968) have found that Phillips' equilibrium spectrum, as given by Eq. (2), can be found to apply best to the envelope curve for the spectral peaks of growing wave trains in the limited fetch data, but does not hold over a range of frequencies in individual spectra. Careful examination of such spectra indicates that their shape is steeper than $n^{-5}$ over much of the range of frequencies just beyond the spectral peak. Plate, et al. (1968) have shown that the $n^{-5}$ rule must apply to the spectral peaks if all spectra are similar in shape. Provided that (a) the limiting acceleration of water particles at the wave crests is proportional to the acceleration of gravity, and (b) the wave heights related to the variance of the water surface displacement by the Rayleigh distribution, one can derive a value of $\beta$ equivalent to Phillips' (1966) empirical value. For the latter assumption, virtually all of the data taken from the CSU tunnel, and the empirical similarity spectrum of Hidy and Plate (1965) lend justification. To derive $\beta = 1.17 \times 10^{-5}$, the proportionality factor between $g$ and the maximum in water particle acceleration was taken as 0.3.

The data shown in this note indicate that wind waves tend to behave in a uniform way regardless of the size or origin. Statistical relations such as those for mean wave heights and amplitude spectra show a pattern which self consistent between the laboratory and the field. The remarkable ability to correlate such results by known theory points towards the fact that one can construct useful direct analogies between laboratory studies and
the geophysical counterpart. In particular, the similarity in shape of the wave spectra generated by wind in the laboratory suggests that relatively simple experimental procedures could be developed, for example, in constructing dynamic models for wave forces on structures in wind-water tunnels.

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Fig. 1. A comparison of theoretical relations for the mean wave height with data from wind-water tunnels (this study and Colonell, 1966) and geophysical data of Goodknight and Russell (1913). Here $\mu$ is equal to $\sqrt{2\pi \sigma_\eta}$.

Fig. 2. Comparison between wave spectra taken in a wind-water tunnel with ocean wave spectra.
Figure 1
Figure 2