Confidence intervals for trend estimates with autocorrelated observations

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1 Introduction

In many cases the statistical analysis of geophysical data collected over time hinges on the estimate of a trend. One area where trend analysis is used extensively is for climate series such as global temperature averages. (e.g [3]). Besides the trend estimate itself, attaching a reliable measure of uncertainty is an important component and facilitates drawing objective scientific conclusions. Unfortunately because such data typically exhibit serial correlation, simple statistical inference using least squares fitting and the T distribution can be misleading. Part of the confusion is that although least squares can give good trend estimates the naive standard errors derived from least squares are underestimated and can be misleading. (e.g [1]). This problem is well known both in the statistical literature and subject matter literature such as climate analysis. However, to date a correct treatment using maximum likelihood or Bayesian approaches requires specialized statistical software. This work provides an alternative to more advanced statistical methods by simply adjusting the least squares standard error using a simple formula for effective sample size. By itself, adjusting the sample size for correlation is an old idea. New in this work is the development of a formula that has rigorous justification for small and moderate size data sets through extensive Monte Carlo simulations. By comparing this approach to the more sophisticated maximum likelihood estimators we find negligible differences. Thus a sample size correction approach has the added benefit of matching a method with good theoretical properties.

2 Statistical Model

Related work on this problem is the comparison of two means based on serially correlated data ([4]). In their paper, Zwiers and VonStorch (1995) provide recommendations for use when the effective sample size is small. In one sense this paper is an extension of those authors' work to the more general case of linear trends. Assume that observations Y_t are made at equally spaced time intervals $t = 1, 2, ..., n$ and follow the statistical model

$$Y_t = \mu + \beta t + U_t \tag{1}$$

where U_t is a random error following the model

$$U_t = \rho U_{t-1} + e_t.$$

$|\rho|$ is assumed to less than one and $\{e_t\}$ are mean zero independent random variables with variance σ^2. If $\rho = 0$ then $U_t = e_t$ and one would have the usual trend model with independent errors. In general the random errors will be serially correlated with the correlation of U_t with U_{t-1} being ρ. This model for serial correlation is referred to an autoregressive model of order one (AR(1)) because the present value of U_t only depends on the previous value. Although the AR(1) model used here is a special case of more general autoregressive models, it very useful for approximating serial dependence and can be applied to data sets with moderate sample sizes.

3 Trend Estimates and standard errors

If one assumes that the errors in model (1) are uncorrelated (i.e. $\rho = 0$) then the least squares estimate of trend is

$$\hat{\beta} = \frac{\sum_{t=1}^{n}(Y_t - \bar{Y})(t - \bar{t})}{\sum_{t=1}^{n}(t - \bar{t})^2} \tag{2}$$

with $\bar{Y}$ the sample mean of the observations and $\bar{t} = (n+1)/2$. Also $\hat{\mu} = \bar{Y} - \hat{\beta}\bar{t}$ and finally we define residuals

$$r_t = Y_t - \hat{\mu} - \hat{\beta}t$$

Under the assumption that the errors are uncorrelated the standard error associated with $\hat{\beta}$ is

$$SE(n) = \sqrt{\frac{\sum_{t=1}^{n} r_t^2/(n-2)}{\sum_{t=1}^{n}(t - \bar{t})^2}} \tag{3}$$

and under the assumption of a Gaussian distribution gives a $(1-\alpha)100$ percent confidence interval

$$\hat{\beta} \pm t_{\alpha/2,n-2}SE(n) \tag{4}$$

with $t_{\alpha/2,n-2}$ a T value at tail probability $\alpha/2$ and degrees of freedom $n-2$.

The effect of strong positive serial correlation causes this interval (hereafter referred to as 'NAIVE') to cover the true trend less often than the nominal frequency based on α. This is not a minor statistical point that can be safely ignored. For example from Figure 2 we see that for $n = 40$ and $\rho = .5$, a NAIVE "95 % confidence interval" will only cover the true slope approximately 70 % of the time.

The strategy in this paper is to retain the simplicity, and we will see also the accuracy, of the least squares trend but adjust the formula for the confidence interval so that has at the right level. (By the right level we mean that an $(1-\alpha)100$ confidence interval actually contains the true parameter this percent of the time.) To bring the confidence to the correct level we developed a simple modification based on the sample size. This method will depend in turn on an estimate of ρ based on the sample autocorrelation of the least squares residuals. Let $\hat{\rho}$ denote the lag one autocorrelation in the residuals from the least squares fit.

$$\hat{\rho} = \frac{\sum_{t=1}^{n-1}(r_t - \bar{r})((r_{t+1} - \bar{r}^*)}{\sqrt{\sum_{t=1}^{n-1}(r_t - \bar{r})^2 \sum_{t=2}^{n}(r_t - \bar{r}^*)^2}}$$

In this formula $\bar{r}$ is the sample mean omitting the last residual (r_n) and $\bar{r}^*$ is the mean omitting the first residual (r_1). With the intercept term in the regression model the full sample mean will be identically zero and so we note that both $\bar{r}$ and $\bar{r}^*$ will have small departures from zero. The estimate of the serial correlation is of value in its own right as a description of the data and its properties will be discussed in the next section. Here we use it to derive an effective sample size for small values of n.

$$n'_e = n\left(\frac{1 - \hat{\rho} - .68/\sqrt{n}}{1 + \hat{\rho} + .68/\sqrt{n}}\right) \tag{5}$$

To adjust the confidence interval to be approximately correct over a wide range of serial correlations this effective sample size should replace n in the two places in 4 giving the adjusted interval

$$\hat{\beta} \pm t_{\alpha/2, n'_e - 2} SE(n'_e) \tag{6}$$

The interpretation is that n serially correlated observations are equivalent to approximately n_e independent ones. Here equivalence means having the same width confidence interval for the trend parameter. The formula at 5 is a slight modification of the classical adjustment (henceforth 'CLASSICAL') for equivalent sample size derived Mitchell (1966). Our addition is the inflation factor of $.68/\sqrt{n}$ appearing in the numerator and denominator. Although in the next section we give some statistical justification for this term the main point is that the confidence intervals with this adjustment have the right frequency coverage for small sample sizes. Another advantage is quantifying the effective sample size induced by the serial correlation. Because we just change the sample size in the standard confidence interval formula,

n_e is a useful measure of how the effective size of the data is reduced with autocorrelation. Such a reduction can be a sobering index of the limitations of serial correlated data for trend analysis. For example, from Table 1 if $n = 40$ and $\hat{\rho} = .5$ the ADJUSTED effective sample size is equivalent to only about 10 observations from uncorrelated errors.

4 Back ground for sample size adjustment

The classical formula for sample size adjustment that is the basis for this work was derived by Mitchell (1966) under the assumption of n being large and ρ known and can be approximated by

$$n\left(\frac{1-\rho}{1+\rho}.\right) \tag{7}$$

We found that using this expression to determine n_e when using ρ is known gives confidence intervals that are conservative. In practice ρ is not known and the usual approach is to substitute an estimate for ρ in equation 7. Figure 1 illustrates how the distribution of the estimate of ρ changes as a function of sample size and the value of ρ. The estimate is biased, on the average, underestimating the true value. Also, for large amounts of correlation the variance of the estimate increases. Both of these properties have an impact on the behavior of the sample size adjustment. The result is that n_e, now a random quantity, tends to give effective sample sizes that are too large. So the switch from the true value for ρ to an estimate has shifted the confidence intervals from being too conservative to too optimistic. This discussion serves as motivation for the formula at 5. To bring the coverage of the confidence intervals closer to the correct level the effective sample size is decreased by a small factor depending on n. Although $\hat{\rho} + .68/\sqrt{n}$ can be interpreted as an approximate 50% upper confidence bound on ρ, we should emphasize that it's primary justification is through simulation results described in the Section 6.

5 Maximum likelihood estimates

Before presenting simulation results we review an alternative trend estimate that has a better theoretical justification. At the outset one might expect that the maximum likelihood estimate (MLE) would be more accurate and the confidence intervals shorter. In this case it would be preferable to use it instead of the simpler least squares trend estimate.

Under the assumption that the errors $\{e_t\}$ are normally distributed the joint probability density function for the observed data is

$$L(\mathbf{Y}, \mu, \beta, \sigma, \rho) = \frac{\sqrt{1-\rho^2}}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\left((1-\rho^2)(Y_1-\mu-\beta)^2+\sum_{t=2}^{n}[\{Y_t-\mu-\beta t\}-\rho\{Y_{t-1}-\mu-\beta(t-1)\}]^2\right)}$$

Fixing the data ($\{Y_t\}$) and varying the parameters of this expression gives us a likelihood function. Informally, for specific values of the parameters one can interpret the value of L, the likelihood, as proportional to the probability of observing a particular set of data. Given this perspective, values for the parameters are found by maximizing the likelihood. Although a closed form expression does not exist for such estimates the likelihood function can be found numerically by standard optimization algorithms. One simplification that helps the calculation is that for fixed ρ the likelihood can be maximized in closed form over the other parameters. Thus, the nonlinear optimization is only over the single variable ρ. Likelihood theory also provides guidance in constructing a confidence interval for the parameters, allowing us to make comparisons with the least squares methods. [1]

6 Simulation results

The Monte Carlo simulations to study the statistical procedures have two factors: sample size ($n = 40, 60, 120, 240$) and correlation parameter ($\rho = 0, .05, .1, \ldots, .90$). For each of these cases, 10000 simulated time series were generated and confidence

[1]The code used to determine MLE estimates and confidence intervals was written by Richard H. Jones, 1999 and is available at the following web address: http://www.cgd.ucar.edu.

intervals were calculated using four methods: NAIVE, CLASSICAL, ADJUSTED, and MLE. Of course the first three methods are based on the same least squares trend and the maximum likelihood estimate using a different form of estimator and confidence interval. To reduce the size of this study only confidence intervals at the 95 and 99 % level were evaluated. Because of the linear relationship between the trend estimate and the data it can be shown that the performance of the confidence intervals is invariant to the actual values of β, σ and μ. So for convenience we choose the normalization $\beta = 0$, $\sigma = 1$ and $\mu = 0$. [2]

Figures 2 and 3 summarize the results of this study for the two smallest sample sizes, ($n = 40$, and $n = 60$). The first plot in each figure gives the actual coverage of 95 % confidence intervals as a function of ρ. As expected the methods agreed in the absence of serial dependence ($\rho = 0$) but the NAIVE method has substantially lower coverage as ρ increases. The ADJUSTED method holds its level the best with the CLASSICAL and MLE based intervals being lower especially for higher values of ρ. The bottom plots indicate the ADJUSTED method also performs well at the 99 % level. Tables 3 through 7 report the numerical results for this study across all sample sizes and the two confidence levels.

Although the ADJUSTED method performs substantially better than the other methods for small and moderate sample sizes, it is important to note that even it has some limitations. For example, when $n = 40$ and $\rho = .8$, the actual coverage falls off to be about 92 % at a nominal level of 95 % . However, note that from Table 1 when $n = 40$ and $\rho = .8$, $\hat{\rho} = .63$ which gives an ADJUSTED effective sample size of only 6. The true effective sample size would be even smaller. Detecting a trend

[2] Since the sampling variability of ρ can give us unrealistic values for n_e we used the following constraints suggested by Zwiers and VonStorch (1995).

$$n'_e = \begin{cases} 2, & \text{if } n_e \leq 2; \\ n_e, & \text{if } 2 < n_e \leq n; \\ n, & \text{otherwise.} \end{cases} \tag{8}$$

with a sample size this small will be difficult in any situation.

To compare the MLE to the least squares trend estimates we considered the paired differences for these with the least squares trend estimates. Statistics for these comparisons are summarized in Figure 6 and Table 7. The top plot in Figure 6 compares the distribution of the MLE and least squares trend estimates for several values of ρ when $n = 40$. The bottom plot in Figure 6 illustrates the distribution of the differences between the MLE and least squares estimates. The differences were normalized by the standard deviation of the MLE. Although the variance tends to increase, the differences remain centered around 0. The distributions appear to be similar and a paired T test indicates no significant difference in the estimates between the two methods. Table 7 indicates that the standard deviation of the estimates are similar as well. From these results we can conclude that there are negligible differences between the two methods.

Recommmendations

The simulation results summarized in Figures 2 through 5 and Tables 3 through 7 lead us to the following suggestions based on the calculated value of n_e for a data set.

- Use the procedure explained in Equations 5 and 6 to bring the coverages to their nominal levels when the ADJUSTED effective sample size is small to moderate.

- Notice that when the ADJUSTED effective sample size is large, the coverages may become somewhat conservative (e.g Figure 5). However, large sample sizes tend to give more accurate estimates, leading to smaller confidence intervals.

- One should realize that when $n_e < 6$ serial correlation makes it difficult to make inferences and the confidence intervals may not be reliable.

7 Discussion

Using a simple modification of the sample size based on the sample autocorrelation we have established a reliable formula for confidence intervals of the trend. The justification of this method is by extensive simulations that verify the level of the proposed confidence intervals. In addition they indicate there is little difference between the least squares trend estimate and that derived from maximum likelihood. This suggests that there is little advantage using this more complicated procedure. It will yield the same inference.

It is important to realize any statistical method is limited to the context under which it is derived. In this case we have assumed that the trend is linear in shape, the errors are normally distributed and finally, that the serial dependence follows an AR (1) model. If the data exhibits other types of serial dependence then this procedure will not be valid. For example, if the dependence was a long memory process or a higher order autoregressive model the amount of correlation could be underestimated and we conjecture that the resulting confidence intervals would be too short. Despite these qualifications, adjusting the sample size is an important aspect of trend analysis and the simplicity and interpretability of the adjusted sample size method make it a valuable statistical tool.

References

[1] Jones, R.H., 1975: Estimating the variance of time averages. *Journal of Applied Meteorology*, 14:159-163.

[2] Mitchell, J.M., B. Dzerdzeevskii, H. Flohn, W.L. Hofmeyr, H.H. Lamb, K.N. Rao and C.C. Wallen, 1966: Climatic change. *WMO Tech. Note 79.*

[3] Santer, B.D., T.M.L. Wigley, J.S. Boyle, D. Gaffen, J.J. Hnilo, D. Nychka, D.E. Parker and K.E. Taylor, 1999: Statistical significance of trend differences

in layer-average temperature time series. *In Review.*

[4] Zweirs, F.W. and H. Von Storch, 1995: Taking serial correlation into account in tests of the mean. *Journal of Climate*, 8:336-351.

Table 1: Effective Sample Sizes using ADJUSTED Formula

$\hat{\rho}$	N=40	N=60	N=120	N=240
0.1	26	41	87	180
0.2	21	33	70	146
0.3	17	26	56	117
0.4	13	21	44	92
0.5	10	16	34	71
0.6	7	11	24	52
0.7	4	7	16	35
0.8	2	4	9	20
0.9	0	0	2	7

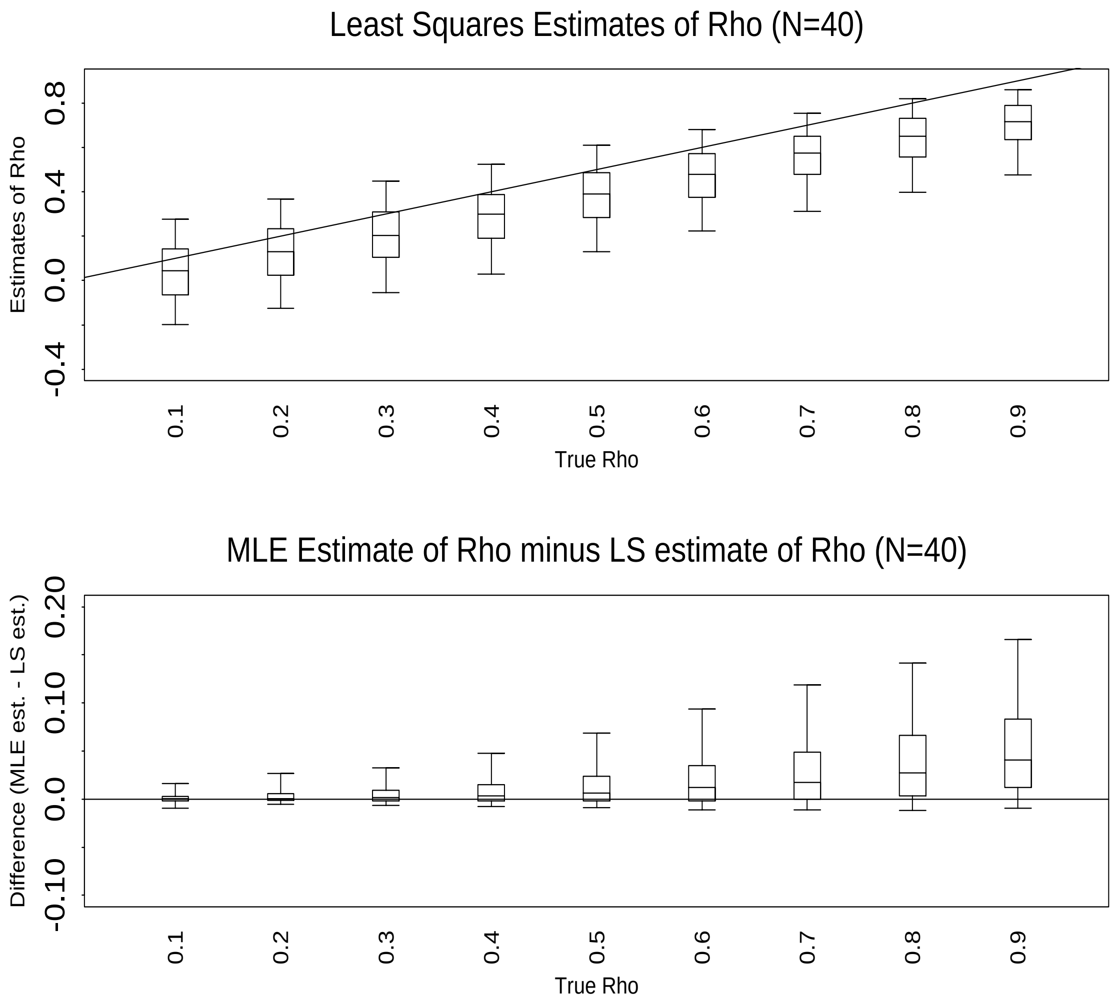
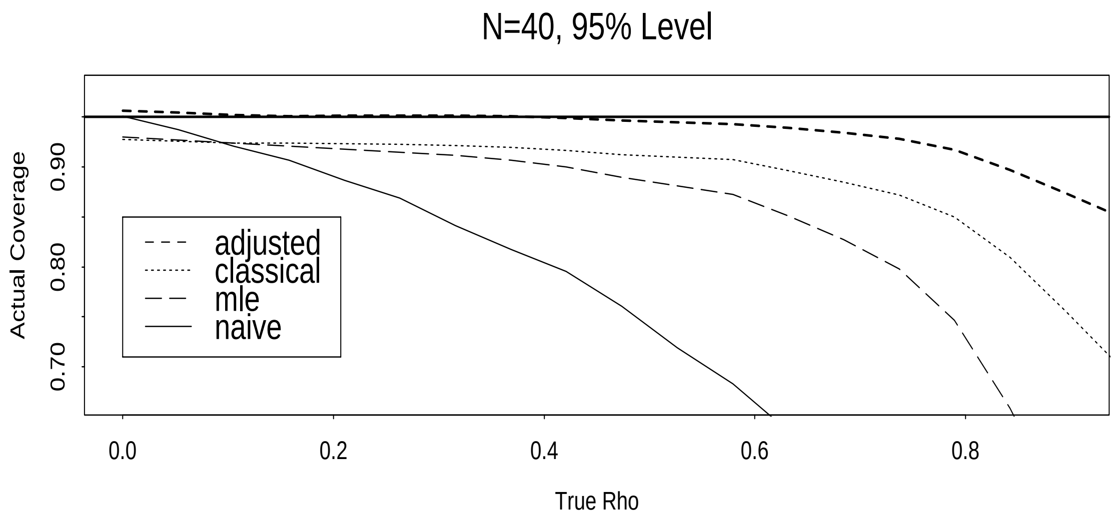


Figure 1: Distribution of the estimates of ρ from least squares residuals and from a maximum likelihood estimate. The top plot contains boxplots illustrating the distribution of estimates of ρ based on the sample autocorrelation of the least squares residuals. In this case the sample size is 40 and the distributions do not depend on the values of the trend parameter or the error variance (σ^2). The 45^o line indicates the bias in the estimator. The bottom plots gives results for the maximum likelihood estimate of ρ minus the least squares estimate of ρ. Although both the LS and MLE estimates of ρ are biased, the MLE estimates are slightly improved.



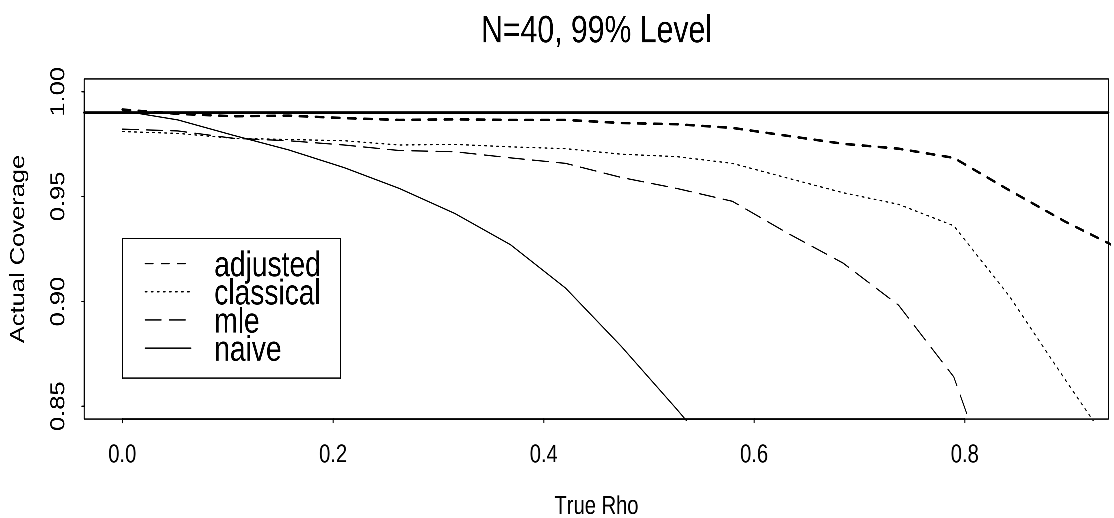


Figure 2: Coverage probabilities for confidence intervals based on sample size of $n = 40$ and as a function of ρ. The top plot is for intervals constructed at a nominal level of 95 % and the bottom plot is for level 99 %. The NAIVE line is the coverage under the assumption of uncorrelated errors. The MLE line represents the coverage using Maximum Likelihood estimates. The CLASSICAL line represents the coverage using the effective sample size adjustment given in 7. The ADJUSTED line represents the coverage using the inflated sample size adjustment given in 5.

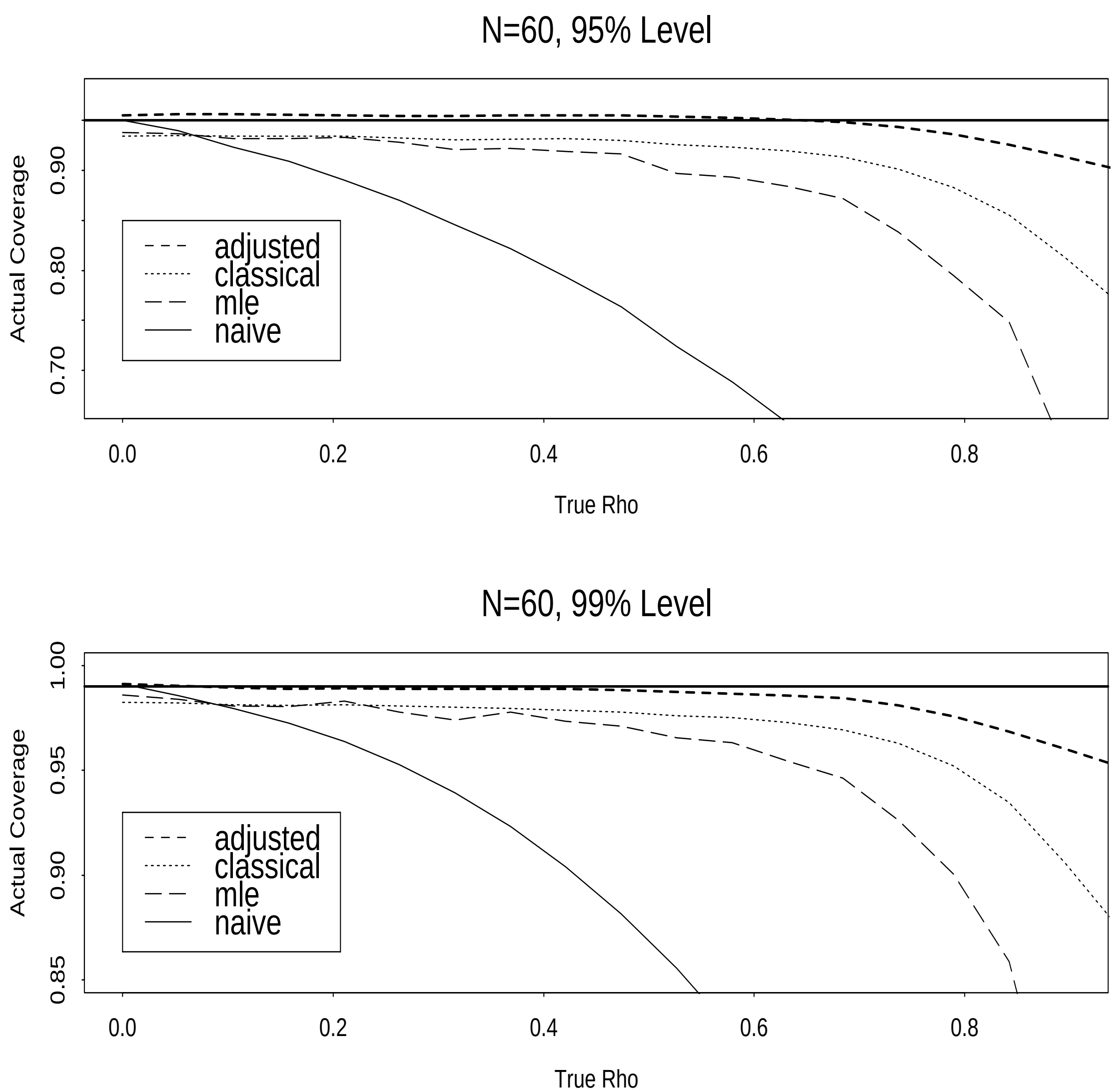


Figure 3: Coverage probabilities for confidence intervals based on sample size of $n = 60$ and as a function of ρ. The top plot is for intervals constructed at a nominal level of 95 % and the bottom plot is for level 99 %. See Figure 2 for descriptions of methods included.

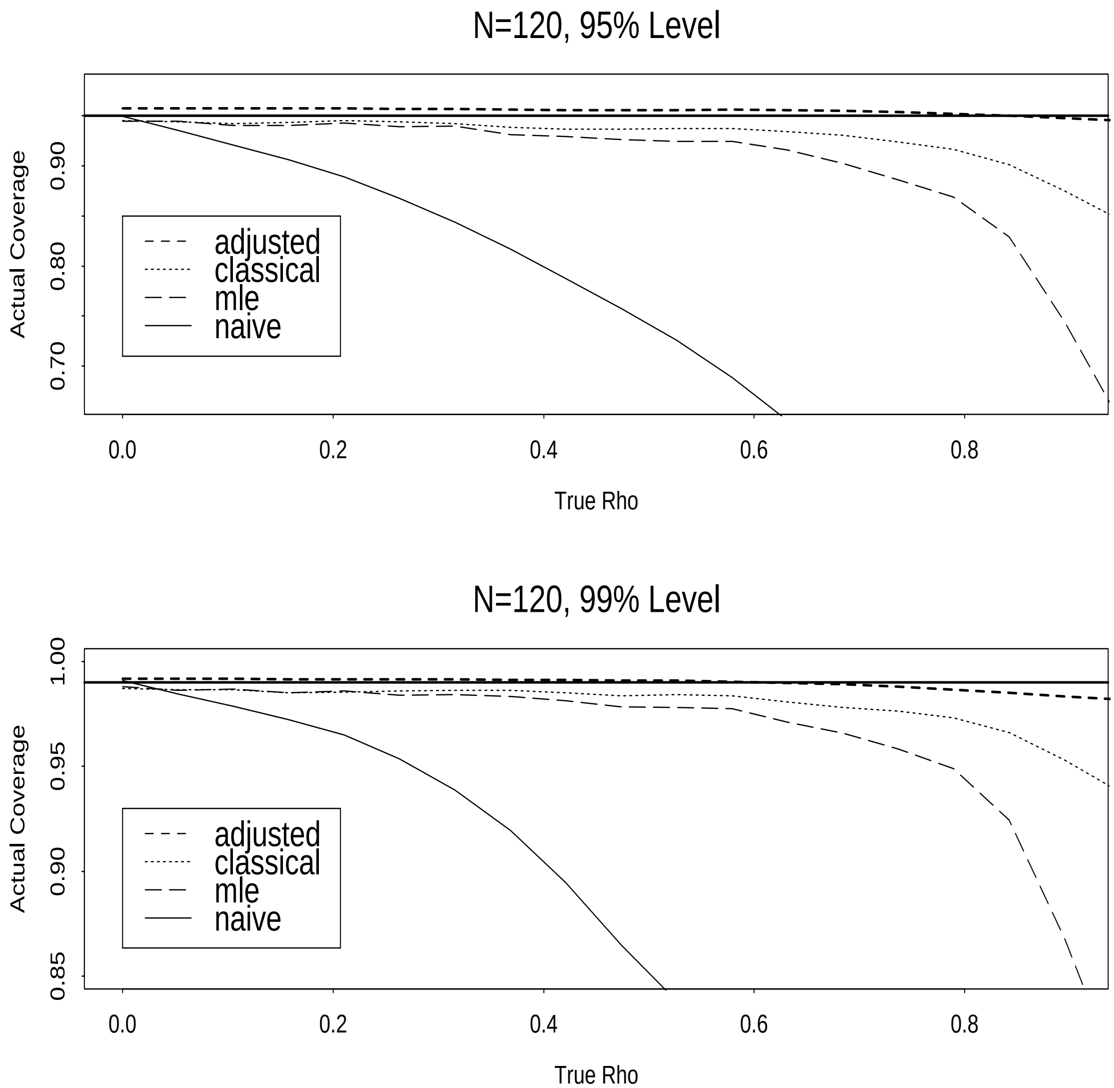


Figure 4: Coverage probabilities for confidence intervals based on sample size of $n = 120$ and as a function of ρ. The top plot is for intervals constructed at a nominal level of 95 % and the bottom plot is for level 99 %.

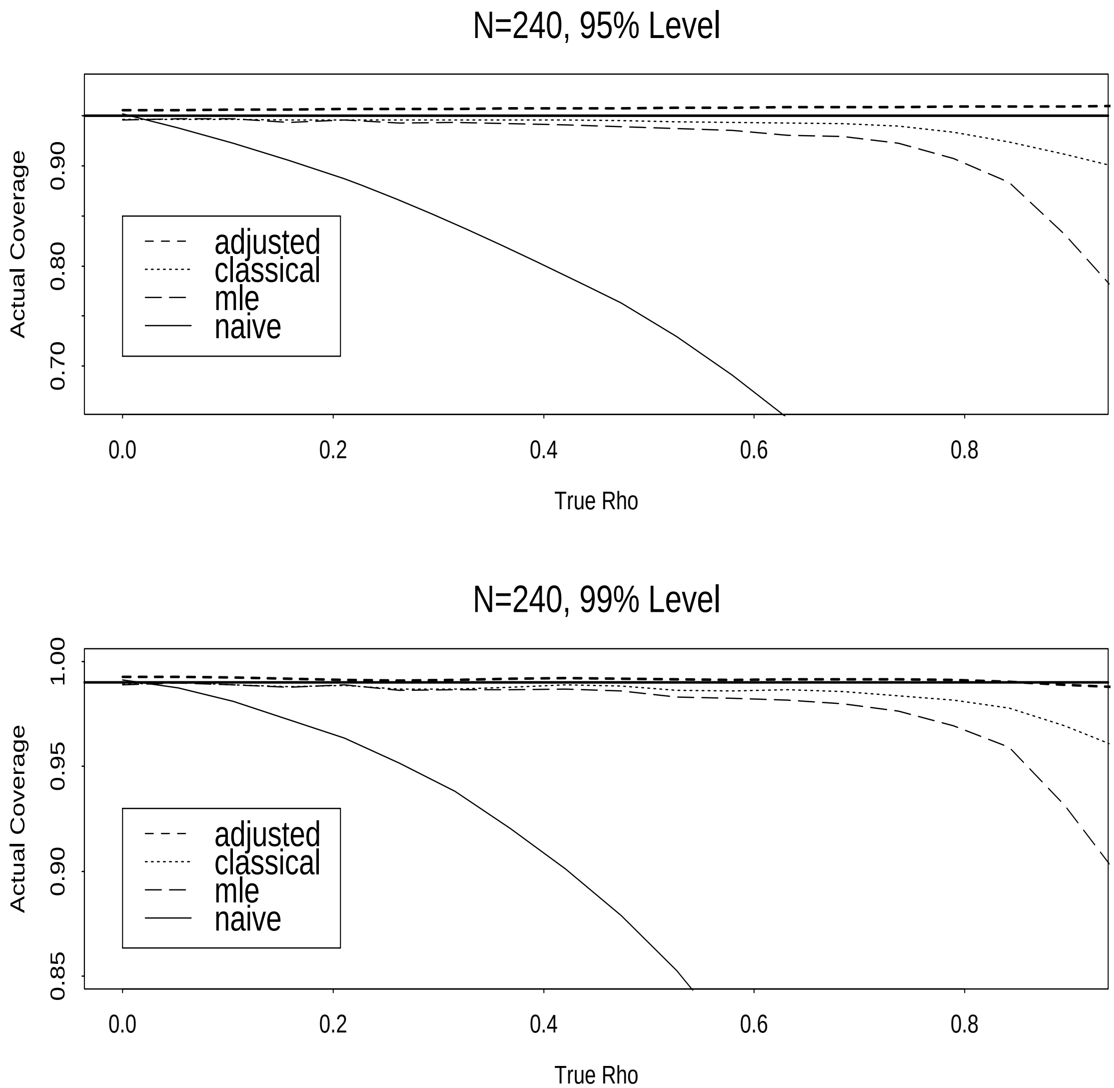


Figure 5: Coverage probabilities for confidence intervals based on sample size of $n = 240$ and as a function of ρ. The top plot is for intervals constructed at a nominal level of 95 % and the bottom plot is for level 99 %.

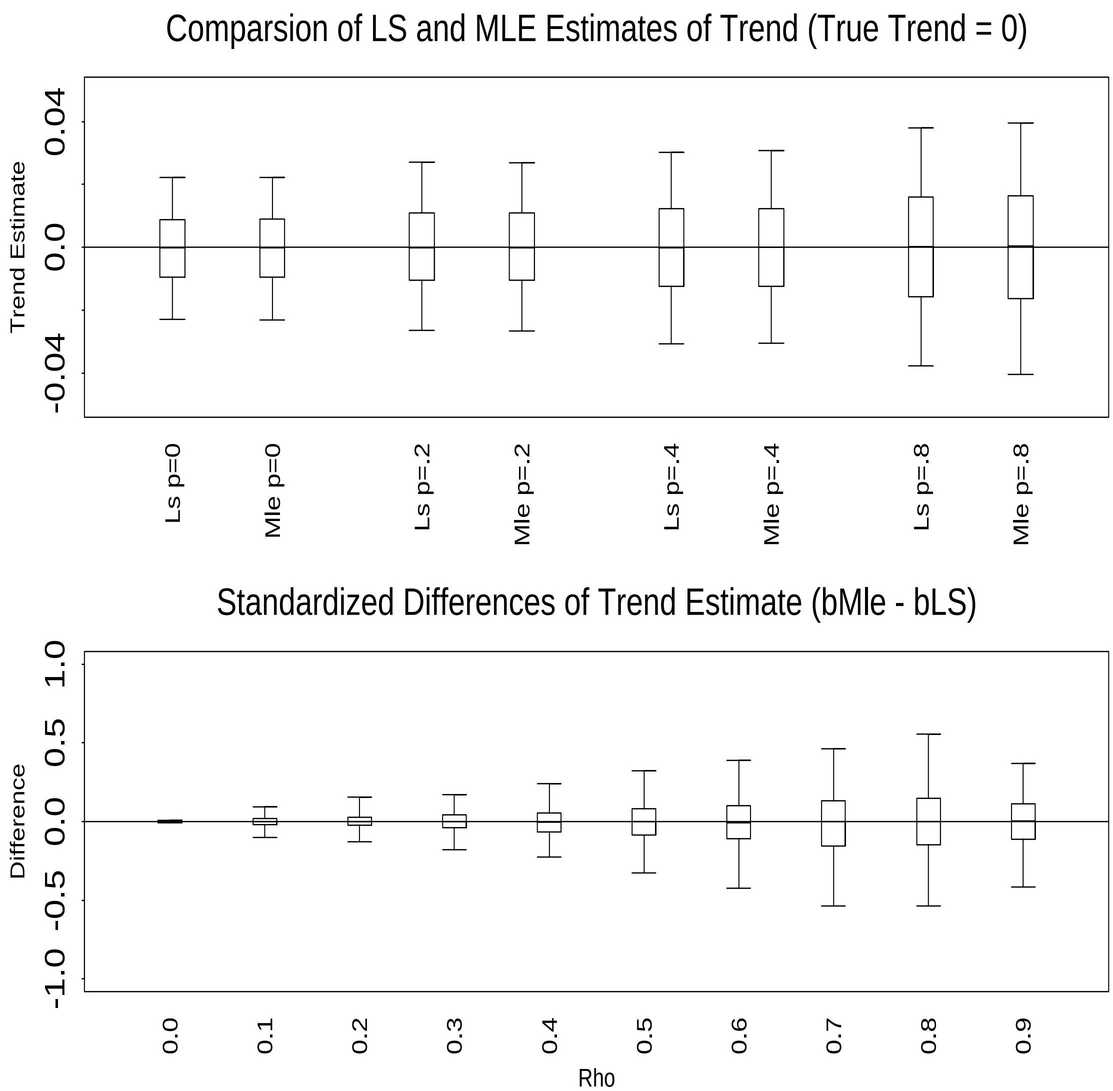


Figure 6: The top plot compares the least squares and maximum likelihood estimates of trend for selected values of ρ (N=40). The bottom plot illustrates the distribution for the differences between the least squares and MLE estimates of trend. These differences have been standardized by the sample standard deviation of the MLE for each value of ρ.

Table 2: Least Squares and MLE Estimates of Autocorrelation

	N=40		N=60		N=120		N=240	
True Phi	LS	MLE	LS	MLE	LS	MLE	LS	MLE
0.00	-0.049	-0.050	-0.032	-0.033	-0.016	0.017	-0.008	-0.008
0.05	-0.006	-0.006	0.014	0.014	0.032	0.032	0.041	0.041
0.10	0.037	0.038	0.057	0.058	0.079	0.080	0.090	0.090
0.15	0.081	0.084	0.104	0.106	0.127	0.128	0.138	0.139
0.20	0.124	0.128	0.150	0.153	0.176	0.178	0.189	0.190
0.25	0.168	0.173	0.195	0.199	0.223	0.225	0.236	0.238
0.30	0.211	0.218	0.240	0.246	0.271	0.273	0.285	0.286
0.35	0.256	0.265	0.288	0.294	0.319	0.323	0.335	0.336
0.40	0.295	0.305	0.330	0.338	0.365	0.369	0.382	0.384
0.45	0.339	0.351	0.377	0.387	0.414	0.419	0.431	0.434
0.50	0.384	0.400	0.423	0.434	0.462	0.468	0.481	0.484
0.55	0.427	0.445	0.469	0.482	0.510	0.516	0.530	0.533
0.60	0.470	0.491	0.514	0.529	0.558	0.566	0.579	0.582
0.65	0.509	0.535	0.559	0.577	0.604	0.614	0.628	0.632
0.70	0.552	0.583	0.605	0.626	0.653	0.664	0.677	0.683
0.75	0.592	0.627	0.646	0.671	0.700	0.714	0.725	0.732
0.80	0.631	0.672	0.690	0.721	0.748	0.764	0.774	0.783
0.85	0.666	0.714	0.732	0.768	0.794	0.815	0.823	0.835
0.90	0.701	0.755	0.774	0.816	0.842	0.867	0.872	0.887

Table 3: Actual Coverages for the Five Methods at N=40

	Level of Sig = .95					Level of Sig = .99				
TruePhi	Naive	MLE	Adj.	Infl.	PhiKn	Naive	MLE	Adj.	Infl.	PhiKn
0.00	.951	.930	.930	.956	.951	.991	.982	.981	.991	.991
0.05	.934	.928	.926	.956	.954	.987	.981	.980	.990	.992
0.10	.922	.924	.923	.952	.950	.979	.978	.978	.988	.990
0.15	.911	.922	.925	.950	.955	.974	.977	.977	.989	.992
0.20	.889	.918	.923	.952	.955	.965	.975	.977	.988	.992
0.25	.876	.917	.923	.952	.958	.957	.972	.975	.987	.993
0.30	.850	.912	.921	.951	.960	.945	.972	.975	.987	.995
0.35	.824	.911	.921	.951	.965	.935	.970	.974	.986	.995
0.40	.806	.901	.917	.949	.965	.913	.967	.973	.987	.995
0.45	.779	.898	.917	.950	.968	.895	.963	.972	.986	.996
0.50	.739	.880	.910	.943	.971	.860	.955	.969	.984	.998
0.55	.702	.882	.912	.945	.975	.837	.953	.969	.984	.998
0.60	.668	.866	.905	.943	.978	.801	.942	.963	.981	.998
0.65	.615	.841	.890	.935	.983	.753	.927	.956	.977	1.00
0.70	.566	.820	.881	.933	.989	.701	.914	.950	.974	1.00
0.75	.512	.788	.868	.962	.996	.644	.891	.945	.972	1.00
0.80	.438	.735	.846	.918	1.00	.562	.854	.932	.966	1.00
0.85	.346	.642	.802	.892	1.00	.444	.774	.896	.950	1.00
0.90	.252	.537	.749	.871	1.00	.335	.669	.859	.937	1.00

Table 4: Actual Coverages for the Five Methods at N=60

	Level of Sig = .95					Level of Sig = .99				
True Phi	Naive	MLE	Adj.	Infl.	PhiKn	Naive	MLE	Adj.	Infl.	PhiKn
0.00	.950	.938	.934	.954	.950	.990	.986	.982	.991	.990
0.05	.941	.937	.936	.958	.954	.987	.984	.983	.991	.991
0.10	.924	.932	.934	.957	.953	.978	.981	.981	.989	.990
0.15	.913	.932	.934	.955	.954	.974	.980	.980	.988	.991
0.20	.893	.933	.935	.956	.954	.967	.983	.982	.990	.993
0.25	.877	.931	.935	.956	.955	.956	.980	.981	.989	.993
0.30	.851	.921	.928	.950	.954	.941	.976	.979	.988	.992
0.35	.833	.923	.933	.957	.956	.933	.978	.981	.989	.993
0.40	.803	.919	.930	.954	.960	.908	.975	.978	.989	.994
0.45	.780	.919	.934	.958	.964	.895	.972	.979	.989	.994
0.50	.744	.909	.926	.954	.965	.868	.969	.977	.988	.995
0.55	.706	.890	.924	.951	.965	.838	.963	.974	.986	.995
0.60	.675	.895	.924	.954	.971	.809	.962	.977	.986	.997
0.65	.629	.877	.915	.948	.970	.760	.950	.969	.985	.997
0.70	.581	.867	.914	.950	.977	.767	.943	.970	.985	.998
0.75	.505	.826	.894	.941	.981	.641	.919	.959	.979	.999
0.80	.445	.787	.879	.935	.989	.569	.895	.949	.975	1.00
0.85	.377	.736	.855	.927	.997	.484	.848	.935	.967	1.00
0.90	.264	.602	.805	.909	1.00	.346	.742	.900	.959	1.00

Table 5: Actual Coverages for the Five Methods at N=120

	Level of Sig = .95					Level of Sig = .99				
True Phi	Naive	MLE	Adj.	Infl.	PhiKn	Naive	MLE	Adj.	Infl.	PhiKn
0.00	.950	.945	.945	.957	.950	.991	.988	.987	.992	.991
0.05	.936	.945	.945	.959	.951	.984	.986	.986	.991	.990
0.10	.922	.941	.942	.956	.950	.980	.987	.987	.992	.990
0.15	.908	.940	.942	.956	.949	.971	.985	.985	.991	.990
0.20	.895	.944	.947	.960	.956	.969	.986	.987	.992	.992
0.25	.871	.939	.943	.956	.954	.956	.984	.986	.991	.991
0.30	.853	.941	.944	.958	.955	.943	.984	.986	.992	.992
0.35	.826	.934	.940	.958	.955	.927	.984	.986	.992	.993
0.40	.800	.929	.936	.954	.953	.906	.982	.986	.990	.991
0.45	.769	.929	.938	.956	.956	.885	.980	.984	.991	.993
0.50	.743	.924	.936	.953	.953	.864	.977	.983	.989	.992
0.55	.711	.926	.939	.959	.959	.839	.979	.985	.992	.994
0.60	.671	.922	.936	.958	.959	.798	.975	.982	.991	.994
0.65	.630	.912	.933	.955	.959	.759	.969	.973	.990	.994
0.70	.579	.898	.930	.957	.959	.711	.964	.977	.988	.993
0.75	.530	.882	.920	.952	.962	.652	.956	.976	.988	.996
0.80	.458	.864	.916	.954	.966	.583	.946	.972	.986	.999
0.85	.387	.819	.899	.950	.975	.492	.918	.965	.987	.999
0.90	.291	.734	.871	.946	.989	.374	.861	.951	.981	1.00

Table 6: Actual Coverages for the Five Methods at N=240

	Level of Sig = .95					Level of Sig = .99				
True Phi	Naive	MLE	Adj.	Infl.	PhiKn	Naive	MLE	Adj.	Infl.	PhiKn
0.00	.951	.946	.946	.957	.951	.991	.990	.989	.993	.991
0.05	.939	.947	.947	.957	.951	.988	.990	.990	.992	.991
0.10	.926	.948	.949	.959	.953	.983	.989	.989	.993	.992
0.15	.905	.942	.943	.952	.948	.972	.987	.988	.991	.990
0.20	.894	.947	.948	.959	.955	.967	.989	.989	.992	.991
0.25	.870	.943	.946	.956	.949	.953	.987	.987	.990	.990
0.30	.851	.944	.946	.957	.954	.944	.986	.987	.991	.990
0.35	.824	.943	.945	.956	.951	.927	.986	.987	.992	.991
0.40	.798	.941	.945	.958	.952	.908	.987	.988	.992	.991
0.45	.778	.941	.948	.960	.956	.890	.987	.989	.993	.991
0.50	.749	.938	.944	.956	.953	.865	.985	.987	.991	.992
0.55	.712	.937	.943	.955	.953	.840	.982	.986	.991	.990
0.60	.675	.935	.944	.959	.955	.806	.983	.986	.991	.990
0.65	.625	.928	.939	.957	.955	.763	.981	.986	.992	.992
0.70	.597	.930	.946	.963	.960	.724	.980	.985	.991	.992
0.75	.534	.919	.938	.961	.958	.664	.975	.983	.992	.993
0.80	.481	.904	.933	.960	.958	.064	.968	.981	.991	.994
0.85	.400	.879	.925	.959	.960	.512	.957	.977	.990	.994
0.90	.313	.825	.908	.958	.962	.404	.928	.968	.988	.995

Table 7: Standard Deviations of LS vs MLE Trend Estimates (N=40)

	St.Dev. of Trend Estimates(10000 reps)	
True Phi	LeastSqrs st. dev	MLE st. dev
0.00	.01365	.01372
0.05	.01434	.01440
0.10	.01513	.01517
0.15	.01569	.01573
0.20	.01621	.01623
0.25	.01674	.01678
0.30	.01737	.01739
0.35	.01801	.01802
0.40	.01851	.01850
0.45	.01900	.01896
0.50	.01989	.01988
0.55	.02032	.02032
0.60	.02091	.02105
0.65	.02155	.02251
0.70	.02212	.02329
0.75	.02262	.02329
0.80	.02316	.02411
0.85	.02469	.02577
0.90	.02506	.02601