Large-scale Tropical Circulations Induced by Heat Sources
in Two Simple Models

Alex O. Gonzalez

Academic Affiliation, Fall 2007: Junior, The Pennsylvania State University

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Science Research Mentors: Nedjeljka Žagar, Wayne Schubert
Writing and Communication Mentors: Lesley Smith, Tim Barnes
Community Mentor: Andrea Sealy
Peer Mentor: Luna Rodriguez

ABSTRACT

Transient and steady-state model response due to diabatic heating in the equatorial region is studied to gain insight of tropical circulations using two simple nonlinear numerical models. In the first part of this study, different heat sources are used to represent the continental areas of South America, Africa and Indonesia. The results show that the most dominant wave response arises from the Kelvin wave originating in the Indonesian region as the low-level eastward flow over the Pacific Ocean. This is a consequence of the Indonesian islands comprising the largest area of heating. In the second part of this study, different convective heat sources are placed in the eastern Pacific Intertropical convergence zone (ITCZ) and Panama bight. In these experiments, the model also includes realistic topography at a higher resolution than the first model. It is found that topography has only localized effects on the winds and geopotential height. Thus, the Caribbean low-level jet, which partially motivated these experiments, is yet to be explained. Also, when the position or intensity of the ITCZ changes, the large-scale winds in the Panama bight and the surrounding region change significantly, directly affecting large-scale circulations.

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1. Introduction

Atmospheric waves are the mechanism by which a localized forcing sends information to the rest of the atmosphere. Waves in the atmosphere are anisotropic in that their response is not the same in all directions, producing different types of wave structures. For waves in the tropics, the forcing is represented by diabatic heating, due mainly to latent heat release. Latent heat is released when water vapor condenses into liquid in deep tropical convective clouds.

There have been numerous significant investigations made of disturbances near the equator to understand circulations in the tropical atmosphere. Understanding tropical circulations is vital to improving predictions for global circulations because systems near the equator constantly interact with large-scale mid-latitude systems. Circulations near the equator are unique in comparison to other latitudinal circulations in that they contain heat-induced large-scale waves trapped close to the equator.

The tropical atmosphere contains not only Rossby and inertia-gravity waves present in the mid-latitudes, but also Kelvin and mixed Rossby-gravity waves (Matsuno 1966). These waves are unique to the tropics because their existence is due to the change of the sign of Coriolis force at the equator (Wheeler 2002). Another reason why the tropics are unique is that they have a smaller day to day weather change when compared to the mid-latitudes (Zagar 2004).

The first scientist who presented solutions for tropical waves was Matsuno (1966), deriving and solving linearized shallow-water equations on the equatorial $\beta$-plane. His findings proved waves such as Kelvin waves and mixed Rossby-gravity waves actually do exist in the equations – and thus could exist in the real atmosphere. The equatorial wave solutions obtained by Matsuno are excited by diabatic heating in the tropics. As they propagate horizontally along the equatorial region, they communicate effects of the heat sources far from the equator; through their vertical energy and momentum transport, they play a vital role for global circulations (Holton 1992).

Gill (1980) initially evaluated the steady-state response of tropical circulations to prescribed heat sources by using a linear analytical model based on shallow-water equations. He extended this previous research, taking nonlinear effects into consideration (Gill and Phillips 1986). Gill and Phillips found out that even though nonlinearity is important, the flow pattern is not greatly affected by nonlinear terms. Their research had significant results, but also brought up uncertainties in nonlinear modeling. “Even though significant advances have been obtained on equatorial waves using numerical methods, nonlinear wave behavior is far from completely understood” (Rapp and Silva Dias 2005). Since the features of the tropical atmosphere are based on nonlinear equations, and progress has been made using numerical modeling to solve these equations, it is vital to approximate the solutions of these equations by continuing to use numerical modeling.

In this study, two simple numerical models are used to solve nonlinear equations for response of tropical circulations to prescribed heat sources, thus allowing for wave interactions. The first model used for main experiments of this research was inspired by the original tropical
model of Gill (1980). Gill’s work illustrated that even simple models can represent some of the large-scale features in the tropics, such as Kelvin wave response to heating resembling the Walker circulation, and the asymmetrical Rossby wave response to heating resembling the Hadley circulation. Hadley circulation is a general symmetric circulation where diabatically heated equatorial air rises and flows towards the poles, it cools, sinks, and returns back towards the equatorial region (Holton 1992). Walker circulation is an asymmetric circulation in which overturning cells dominate over Hadley cells due to strong longitudinal variations in sea surface temperatures (SSTs) driven by ocean currents (Holton 1992). While Gill’s monumental paper describes analytical solutions for steady-state tropical circulations, the model used here solves full nonlinear equations numerically (Zagar et al. 2007).

The majority of the heating that drives tropical circulation is concentrated over the continental regions of South America, Africa and Indonesia. As one can infer from Fig. 1, the maximum diabatic heating, on average, occurs in these three localities due to the amount of longwave radiation absorbed. The longwave radiation is low in these areas in comparison to the surrounding oceans and the subtropics because of the deep convective clouds that engulf these regions.

Figure 1: Average longwave radiation for March 2000 measured by National Aeronautics and Space Administration (NASA) Clouds and Earth Radiant Energy System, from Zagar (2004).

In the first part of this study, these three large heating areas are represented by analytical heating sources and the model response is studied. The goal is to use a simple model to gain knowledge of dynamical aspects in the tropics, then to modify solutions of Gill (1980), by prescribing multiple and moving sources.

The second part extends experiments done at Colorado State University (CSU), where different convective heat sources are placed in the Intertropical Convergence Zone (ITCZ), a zonally stretched region of deep convective clouds extending from the Pacific to the Atlantic Oceans, and the Panama bight (W. Schubert, pers. comm.). The ITCZ and Panama bight are
chosen to investigate circulation in an area just north of South America where significant winds form a westward Caribbean low-level jet (Fig. 2a).

Figure 2: (a) Observational diagram of mean winds for the month of February (2000-2006), where the maximum winds form a Caribbean westward jet. (b) The mean daily rainfall for the ITCZ, Panama bight, and surrounding region (Figures courtesy of W. Schubert).

The area near the ITCZ and Panama bight, just off the eastern Pacific coast, receives a daily average of 8-10 mm of precipitation, as shown in Fig. 2b. These two areas are vital components of the region’s convection and circulation, so they are taken into account in these experiments. The westward low-level jet in the southwestern Caribbean is a feature that is yet to be explained by scientists; some think that it may be associated with topographically enhanced flow around the northwestern Andes. One example of a topographic feature in this region that may fuel flow convergence is a geographical gap in the northwestern Andes, the Mistrató Pass, because velocity increases and pressure decreases (Poveda and Mesa 2000). The CSU model neglected to include topographical features, so these experiments take realistic topography into consideration, and is performed at a much higher resolution than the experiments in the first part of this study.

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2. Methodology

a. Numerical model

The first model (žagar et al. 2007) solves a system of three nonlinear equations which describe the horizontal structure of potential temperature in the mid-troposphere and two horizontal wind components in the lower troposphere. These equations correspond to the vertical structure of the first baroclinic mode (Fig. 3). They are of the form:

\[
\begin{align*}
\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} - \frac{\theta_0 N^2 h_0}{g} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) &= Q + Q_{LIJ} - \varepsilon \theta, \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v &= \frac{g h_0}{\theta_0} \frac{\partial \theta}{\partial x} - \varepsilon u, \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u &= \frac{g h_0}{\theta_0} \frac{\partial \theta}{\partial y} - \varepsilon v.
\end{align*}
\] (1)

![Diagram](image)

Figure 3: Tropical atmosphere in regions of deep convection, as described by the model. \( Q \) corresponds to \( Q + Q_{LIJ} \) in Eq. (1), from Žagar (2004).

Here, \( \theta \) is the potential temperature perturbation from the mean potential temperature \( (\theta_0) \). \( V=(u, v) \) is the wind field aligned with the horizontal \( (x, y) \) axes, \( N^2 \) is a representative buoyancy frequency, \( h_0 \) is the depth of the lower layer (lower troposphere), \( f=2 \Omega \sin(\theta_0) \) represents the Coriolis parameter, \( \varepsilon \) is friction, \( g \) is gravity, and \( Q \) represents the spatially and temporally varying heat source. Total latent heating is denoted by \( Q_{LIJ} \).

The second experiment series utilized nonlinear shallow-water equations on the equatorial \( \beta \)-plane (žagar et al. 2004). They are of the form:
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \beta_y \frac{\partial}{\partial x} (u h) + \frac{\partial}{\partial y} (v h) = -\frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial h}{\partial y} \right)
\]

where \( \beta_y = f \) represents the Coriolis parameter using the \( \beta \)-plane approximation and \( h \) is the height of the free fluid surface, which has a mean depth \( h_0 \) (Fig. 4). All other variables are the same as in Eq. (1). This is a one layer fluid with a constant density. This model is chosen because of its ability to incorporate realistic topography. Both models are spectral and apply Fourier series in both horizontal directions. For further details about the numerical model, see Žagar et al. (2004).

The linearized form of both systems of equations leads to the same dispersion equations for wave solutions. This is described in the Appendix, following Matsuno (1966).

b. Setup of the experiments

In the first part of this study, the domain for the simulations is 33°S to 33°N in the meridional direction, and 180° W to 180° E in the zonal direction, with a horizontal resolution of 1° latitude and 1° longitude. The boundary conditions are constant potential temperatures and westward wind velocities.

The domain for the second part, based on the shallow-water model, is 31°S to 31°N in the meridional direction, and 171° W to 51° E in the zonal direction, with a horizontal resolution of 0.2° latitude and 0.2° longitude. The boundary conditions are constant geopotential height and a motionless fluid.

Both models use realistic rates for friction, which is dependent on land and sea surface, and is different for the mass and wind fields. The models use the same formula for diabatic heat sources in specified regions. The horizontal structure of the heat sources are of the form (Schubert and Masarik 2006):

\[
Q(x, y, t) = 0.5 Q_0 \exp \left[ -\left( \frac{y - y_0}{b_0} \right)^2 \right] \begin{cases} 1 + \cos \left( \frac{\pi}{a_0} (x - ct) \right) & |x - ct| \leq a_0, \\ 0 & |x - ct| \geq a_0, \end{cases}
\]

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where $Q_0$ represents initial diabatic heat rate, $b_0$ is the meridional extent, $a_0$ is the zonal extent, $x_0$ is the initial longitudinal position, $y_0$ is the initial latitudinal position of the heat source (Fig. 5). Also, $c$ is the horizontal heat source velocity, $(x, y)$ is the position, and $t$ is the time.

![Diagram](image)

Figure 5: Horizontal structure of diabatic heat sources including variables that are manipulated.

**c. Experiments with three continental heat sources**

These experiments study wave response due to heat sources in South America, Africa, and Indonesia (Fig. 6). Wave response to heat sources with the same diabatic heating rate and with different diabatic heating rates are then examined. The heating rates are manipulated because in reality diabatic heating is not the same for each region. The model response to heat sources given a constant velocity of the source, and sources that have different diabatic heating rates are also compared. By prescribing these sources to move in different directions, wave response between the Pacific and Atlantic oceans can be analyzed. Also, evaluating the response to these experiments confirms the validity of the model against other studies. For specific variable values used in each model run, refer to Table 1.

**Table 1: Variables and their designated values for three heat sources in South America, Africa, and Indonesia.**

<table>
<thead>
<tr>
<th>Run</th>
<th>South America</th>
<th>Africa</th>
<th>Indonesia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_0$ (K/day)</td>
<td>$a_0$ (km)</td>
<td>$b_0$ (km)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>800</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>800</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>800</td>
<td>2000</td>
</tr>
</tbody>
</table>

![Map](image)

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Figure 6: Model setup for the experiments with three continental heat sources. Model runs include stationary heat sources containing the same diabatic heating rate, stationary heat sources with different diabatic heating rates, and moving heat sources with different diabatic heating rates.

d. **Experiments with the ITCZ and Panama bight heat sources**

These experiments compare flow in the surrounding region due to heat sources in the ITCZ and Panama bight. In the model, the mean depth of the fluid is 6 km, with the maximum height of the topographical features about 5.8 km. The first run contains a stationary heat source only in the ITCZ, centered at 8° N. The next run consists of a stationary heat source only in the Panama bight, at 5.5° N, with the same diabatic heating rate as the ITCZ heating rate. Evaluating the response when there is only one source allows one to compare this to the response when both sources are considered in the same model run. Both sources, the ITCZ and the Panama bight, are considered in Run 3 (Fig. 7). Next, the heating rate in the ITCZ is lowered to half its original value. Since the size of the heating source in the ITCZ is much larger than that of the Panama bight, this run allows more response in the Panama bight region. Since the ITCZ varies between about 2°-10° N depending on the time of the year, the final experiment moves the ITCZ so that it is centered at 2° N (Fig. 8). This experiment takes more specific dynamical events and explores the relationships between them. Solutions from this experiment are compared to observations (i.e. Fig. 19 and 20). Refer to Table 2 below for specified values given to each variable.

<table>
<thead>
<tr>
<th>Run</th>
<th>$Q_0$ (m/s)</th>
<th>$a_0$ (km)</th>
<th>$b_0$ (km)</th>
<th>$(x_0, y_0)$ ($^\circ$N, $^\circ$W)</th>
<th>$Q_0$ (m/s)</th>
<th>$a_0$ (km)</th>
<th>$b_0$ (km)</th>
<th>$(x_0, y_0)$ ($^\circ$N, $^\circ$W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.5</td>
<td>600</td>
<td>75</td>
<td>(8,120)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.5</td>
<td>150</td>
<td>100</td>
<td>(5.5,84)</td>
</tr>
<tr>
<td>3</td>
<td>.5</td>
<td>600</td>
<td>75</td>
<td>(8,120)</td>
<td>.5</td>
<td>150</td>
<td>100</td>
<td>(5.5,84)</td>
</tr>
<tr>
<td>4</td>
<td>.25</td>
<td>600</td>
<td>75</td>
<td>(8,120)</td>
<td>.5</td>
<td>150</td>
<td>100</td>
<td>(5.5,84)</td>
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<tr>
<td>5</td>
<td>.5</td>
<td>600</td>
<td>75</td>
<td>(2,120)</td>
<td>.5</td>
<td>150</td>
<td>100</td>
<td>(5.5,84)</td>
</tr>
</tbody>
</table>

Table 2: Variables and their designated values for heat sources in the ITCZ and Panama bight.
Figure 7: Model setup for Run 3 and 4. In Run 3 there are heat sources in the ITCZ (8° N) and Panama bight (5.5° N) with the same diabatic heating rate, in Run 4 the diabatic heating rate of the heat source in the ITCZ is lowered.

Figure 8: Model setup for Run 5, where there are heat sources in the ITCZ (2° N) and Panama bight (5.5° N) with the same diabatic heating rate.
3. Results and Discussion

a. Experiments with three continental heat sources

The advantage of having a model that can produce transient solutions and not just steady-state solutions is one can observe when each wave response meets, the amount of time it takes for a specific wave response to travel across a desired distance, and changes in wave structure due to multiple sources. In Fig. 9(a), after 12.5 hours, the most organized wave response is in the Indonesian region, with the greatest potential temperature perturbations (1-1.25 K). After 25 hours (Fig. 9(b)), the model response in each region gets closer to each other, eventually becoming global after about 2.25 days.

The feature of these experiments that is most important to note is the overall wave response over the Indonesian region. Since the area of the Indonesian source is more than twice as large as the other regions, it dominates the wave response in all of the experiments (refer to Table 1). Also, this Kelvin wave travels across the Pacific Ocean (in about 2 days in Fig. 10), 2.25 days in Fig. 11), the largest body of water, where it receives negligible changes in friction. The sources over Africa and South America contain westward Rossby waves, with different composition due in part to the initial meridional position of the heat source not being at the equator. The further the heat source is from the equator, the larger the Coriolis force, therefore the greater the wave response.

The solution in Fig. 12 also has only two centers of maximum potential temperature contours: in the South American and African regions, compared to three centers of maximum potential temperature in Fig. 11. One can infer the Kelvin wave response in Indonesia has merged with the Rossby wave that originated in South America. The wave response in South America and Africa moves further west, while the wave response in Indonesia moves farther east. This response corresponds to the direction of heat source velocity given in Fig. 12. Also, the Kelvin wave response originating in Indonesia has made its way across the globe faster, allowing for the steady state to be reached sooner, after about seven days, instead of ten (Fig. 12).
Figure 9: (a) The transient solution of Run 1 after 25 hours. Red contours represent potential temperature perturbations every 0.25 K, black vectors represent wind field. (b) Same as Figure (a) except after 25 hours.
Figure 10: The steady-state solution of Run 1 (10 days, 10 hours). Red contours represent potential temperature perturbations every 0.5 K (first contour is 1 K), black vectors represent wind field.

Figure 11: The steady-state solution of Run 2 (10 days, 10 hours). Blue contours represent potential temperature perturbations every 0.5 K (first contour is 1 K), black vectors represent wind field.

Figure 12: The steady-state solution of Run 3 (10 days, 10 hours). Red contours represent potential temperature perturbations every 0.5 K (first contour is 1 K), black vectors represent wind field.

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b. Experiments with the ITCZ and Panama bight heat sources

When there is only one heat source in the ITCZ, the solution contains very strong winds near the ITCZ in comparison to the winds near the Panama bight, in Fig. 14. The solutions in Fig. 13 and 14 are compared to the solution in Fig. 15. The solutions are evaluated numerically to find that the change between the experiments is approximately linear. This is an interesting observation since a nonlinear model is used, and it is isolated to the simple background model state with only two stationary sources involved.

Difference in winds and geopotential height with and without topography near the Caribbean jet and the surrounding area are calculated in Run 3 (Fig. 15 and 16). The difference between geopotential height and winds are localized near the Andes mountain ranges, not supporting the speculated relationship between topographically enhanced flow and the Caribbean jet. In order to further investigate the Caribbean jet in another study, a multi-level model is needed. Since the jet has been recorded in very low levels (about 3-4 km), a numerical model that has vertical structure in lower levels would be preferable.

The response when lowering the ITCZ’s diabatic heating rate is also interpreted, because the ITCZ has a dominating response in Run 3 that affects our results. When we lower the diabatic heating rate to half its previous value, the wind field is significantly weaker near the ITCZ, with stronger winds pointing toward the Panama bight region, as seen in Fig. 17. Also, there are weak winds in the area around 88-92° W, which move closer to the ITCZ when the diabatic heating rate in the ITCZ is decreased. When the diabatic heating rate in the ITCZ changes, the large-scale winds in the surrounding region change.

Another alteration that would make the experiments more realistic is to move the ITCZ’s meridional position, since it varies depending on the time of the year. Thus, the position of the ITCZ is changed to 2° N. Taking the difference between Run 3, with the ITCZ at 8° N, and with the ITCZ at 2° N, the winds significantly change between 10° S and 20° N, as seen in Fig. 18. It is evident that when the position of the ITCZ changes, the large-scale winds in the surrounding region change.
Figure 17: The steady-state solution of Run 4 (24 hours). Blue contours represent geopotential heights every 5 m, black vectors represent wind field.

Figure 18: The difference steady-state solution between Run 5 and Run 3 (24 hours). Blue contours represent negative geopotential heights every 5 m, red contours represent positive geopotential heights every 5 m, black vectors represent wind field.

Figure 19: The steady-state solution of Run 5 (24 hours). Black vectors represent wind field.

Figure 20: Observational solution of mean winds for the month of March (2000-2006), when the ITCZ is approximately at 2° N (Figure courtesy of W. Schubert).
4. Conclusion

The most dominant wave response in the first experiment is the eastward flow over the Pacific Ocean in the form of a Kelvin wave originating from the Indonesian region. This is because the Indonesian islands comprise the greatest heat source when compared to the other continental regions. These solutions are compared with past theoretical solutions of Gill (1980).

In the second part of this study, topographical effects on the winds and geopotential height in the region northwest of South America are only localized near the Andes mountain range, and do not affect the Caribbean westward low-level jet in this model. Since the jet has been recorded in very low levels, a numerical model that has vertical structure in lower levels would be preferable. Also, when the diabatic heating rate or the position of the ITCZ changes, the large-scale winds in the surrounding region change significantly. These solutions are partially verified by comparing them to observational solutions.

In conclusion, from these two experiments, simple models, such as the ones used in this research, can show some realistic features of the tropical atmosphere, but they cannot show complex phenomena such as the Caribbean westward low-level jet. The disadvantage of more sophisticated models is the time it takes to write and run them. In complex models, it is also difficult to isolate a specific phenomenon. Looking ahead to future research in this area, simple models should be used to learn more about the underlying dynamics in the tropics, in order to figure out what type of complex models to use. By doing this, predictions of not only tropical circulations will be improved, but global circulations as well.
REFERENCES


5. Appendix

Tropical waves are derived here following Matsuno (1966), beginning with a system of linearized shallow water equations on the equatorial $\beta$-plane:

\[
\frac{\partial u}{\partial t} - \beta y \frac{\partial v}{\partial x} + g \frac{\partial h}{\partial x} = 0
\]
\[
\frac{\partial v}{\partial t} + \beta y \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial y} = 0
\]
\[
\frac{\partial h}{\partial t} - h_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0
\]

(A1)

After non-dimensionalization of Eqs. (A1), with the phase speed squared, $c^2 = gh_0$, the form changes to:

\[
\frac{\partial u}{\partial t} - y v + \frac{\partial \phi}{\partial x} = 0
\]
\[
\frac{\partial v}{\partial t} + y u + \frac{\partial \phi}{\partial y} = 0
\]
\[
\frac{\partial \phi}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(A2)

where the geopotential height, $\phi = gh$, is substituted. Assume all quantities have the factor $e^{i(\omega t + k_0 y)}$, and eliminate $u$ and $\phi$. Eq. (A2) simplifies to:

\[
\frac{d^2 v}{dy^2} + (\omega^2 - k^2 + \frac{k}{\omega} - y^2)v = 0
\]

(A3)

where frequency, $\omega$, is a function of zonal wave number, $k$. Assuming wave motions close to the equator, $y$ is sufficiently close to 0. But (in Eq. (A3)) $v \to 0$, $y \to \pm\infty$ is only satisfied when

\[
\omega^2 - k^2 + \frac{k}{\omega} = 2n + 1 \quad (n=0, 1, 2, \ldots)
\]

(A4)

equals an odd integer. The solution to (A4) becomes:

\[
v(y) = C e^{-s_+ y} H_n(y),
\]

(A5)

where $H_n(y)$ is the Hermite polynomial of the $n^{th}$ order. There are three roots when solving for $\omega$ ($n=0$), using Eqs. (A4) and (A5):
\[ \omega_{EIG} = -0.5k - \sqrt{(0.5k)^2} \]

\[ \omega_{WIG} = \begin{cases} \sqrt{(0.5k)^2 + 1 - 0.5k} & k \leq \frac{1}{\sqrt{2}} \\ k & k \geq \frac{1}{\sqrt{2}} \end{cases} \]

\[ \omega_{RSB} = \begin{cases} k & k \leq \frac{1}{\sqrt{2}} \\ \sqrt{(0.5k)^2 + 1 - 0.5k} & k \geq \frac{1}{\sqrt{2}} \end{cases} \] (A6)

The first equation in the system (A6) corresponds to an eastward equatorial inertiogravity (EIG) wave, the second to a westward equatorial inertiogravity (WIG) wave, and the third to an equatorial Rossby (RSB) wave. There is a special case, when \( n = -1 \), in which a Kelvin wave is produced. The mixed Rossby-gravity wave occurs when the westward inertiogravity wave coincides with the Rossby wave \( (n=0) \) at \( k = \frac{1}{\sqrt{2}} \) (Fig. A1).

The group velocity, \( \frac{\partial \omega}{\partial k} \), is the slope of the frequency curves shown in Fig. A1. For long waves (small \( k \)), the slope of the Kelvin wave curve is -1 (a), and the slope of the Rossby wave curve is approximately 1/3 (b). A negative slope indicates an eastward group velocity, whereas a positive slope value indicates a westward group velocity.

![Figure A1: Dispersion diagram for equatorial waves, from Matsuno (1966).](image-url)