Wavelet Transforms of Chromospheric Timeseries Data

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Abstract

We applied wavelet transforms to a remarkable chromospheric time-series dataset, obtained on 25 April 1997 by the SUMER spectrograph on board the SOHO spacecraft, which shows a strong oscillatory signal. This signal is attributed by many as belonging to upward propagating waves generated in the photosphere. The dataset consists of one continuum band (C I - 1043 Å) and two emission lines (C II - 1037.0 Å and O VI - 1037.6 Å) observed simultaneously for some 2 hours.

Previous thorough analysis of this dataset by Wikstøl et al. (2000, ApJ, V. 531, p 1150) has detailed many temporal characteristics of these waves. However, application of wavelet transforms has allowed us to gain information about, not only the temporal nature of the waves, but frequency behavior also. The addition of frequency shifting of the waves with altitude allows us to further probe the atmosphere and compile a picture of the Sun’s atmosphere during these observations.
1 Introduction

The chromospheric and coronal heating problem is one of the many puzzles in solar physics. We wish to understand the physical processes in the chromosphere and the transition region in order to gain insight on this problem.

Through the use of wavelet transforms, we derive time and frequency information of the wave packets as they propagate through the outer regions of the solar atmosphere. Recent observation by Viktos et al. (2000) on the same data set give numbers that relate the time lags, but no frequency information is given. We reproduce time lags similar to these, but we also include frequency shifts. Frequency shift information is important: if we assume this is the same wave propagating upward, then we should see a shift in frequency (Mihalas & Mihalas 1984).

Section 2 deals with background information pertinent to the comprehension of the solar atmosphere and the tools utilized in this paper. This section provides information on the solar atmosphere, wavelets and wavelet transforms and the behavior of waves in stellar atmospheres. In section 3 we focus on the data taken on 25 April 1997 by the SUMER spectrograph on board the SOHO spacecraft. Section 4 focuses on the processes involved in analyzing the data, including correlations made in wavelet power, space, frequency and time over the different atmospheric levels. Section 5 concentrates on the results, their physical interpretation, and their implication on solar physics.

2 Background

The Sun is the paramount entity of our solar system, and the Sun's radiation is the main energy source for all life on Earth. The electromagnetic radiation emitted by the Sun is the proxy by which we infer its physical properties, such as temperature, density structure, and chemical composition. However, inference of physical structure requires that we gain an understanding of the physical mechanisms present that dynamically move the Sun's hot gas, and heat the outer regions of the atmosphere. Such mechanisms are not well understood; we still ponder the question of how the solar corona is heated. Therefore, it is imperative to understand dynamic motions and their influence on the solar atmosphere if we are ever to understand the complexity and variability of the Sun.

2.1 Solar Atmosphere

For the purposes of this paper, we will consider the atmosphere of the Sun as a plane-parallel model generated by data from Vernazza et al. (1981). In the region called the photosphere, at a height below 600 kilometers, (see Figure 1), the average temperature, $T_e(z)$, decreases monotonically as the height, $z$, from the center of the Sun, increases. The photosphere is the region where the bulk of the optical radiation is emitted. Higher up in the atmosphere, between 600 and 2,000 kilometers, is the region called the chromosphere. The chromosphere is a region of near constant temperature between 4,500 K and 10,000 K. In this region we see a decrease in the electron and gas density by several orders of magnitude with increasing height. Still higher up in the atmosphere, above 2,500 kilometers, is the corona. This region has an average temperature around 1,000,000 K. The region between the chromosphere and the corona, called the transition region, displays a discontinuous jump in temperature.

The second law of thermodynamics states that heat cannot flow from cooler to hotter material by thermal processes. Therefore there must be some external driving mechanism heating the chromosphere and the corona and producing the discontinuous jump in temperature in the transition region. One of the big puzzles in solar physics is this coronal and chromospheric heating problem. We want to see if we can gain understanding of the
Figure 1: Average temperature (solid line) and density (dashed line) structure of the quiet solar photosphere, chromosphere, transition region (TR) and corona. From the photosphere \(z = 0\) to a temperature of \(4.5 \times 10^5\) K the values plotted are given in Vernazza et al. (1981) and values into the corona from a quiet "network" model are given in Mariska (1992). (McIntosh 1998)

...dynamical processes in these atmospheric regions through the use of wavelet transforms.

2.2 Wavelets and Wavelet Transforms

A wavelet is defined as a small wave that has a finite duration and a specific frequency. This definition requires that two properties be satisfied simultaneously. A function is only a wavelet if it has an oscillatory signal with zero mean; i.e., the areas above and below the \(y = 0\) axis are equal, and if the function tends to zero in a reasonable number of periods. In other words, a function can be seen as a wavelet if it is localized in both space and time.

A wavelet transform is a mathematical tool that allows us to use a "mother" wavelet to construct a "family" of wavelets. All the wavelets belonging to a particular mother wavelet have the same basic family traits. This family of wavelets is then used to characterize the time-series data. Using this method we are able to derive information about frequency and time intervals present in the data. An example of a mother wavelet is given in Figure 2.

In constructing a family of wavelets, we look to see if there are a set of frequencies and amplitudes that, when summed, can explain the signal in the time-series observations. This is done through the use of scaling and translating of the mother wavelet. Because a wavelet has an oscillatory signal of fixed frequency, it has a scale that we can vary by compressing or stretching to match any frequency that may be in the signal. The scale of the wavelet is its spatial extent, or its width. Since a wavelet is also localized in time, we can move, or translate, the mother wavelet along the time axis in order to estimate where an event happened. A wavelet
transform can be constructed, in principle, by choosing a scale, shifting that scaled wavelet along the time-series signal, noting how strong the similarity between the two is for that value of shift, and repeating the process using different values for the scale parameter.

In mathematical terms, given a time-series of N measurements \( x_n, (n=1,\ldots,N) \) the wavelet transform, \( W_n(S) \), of the signal for any scale \( s \) and translation \( n \) is given by

\[
W_n(S) = \sum_{n'=0}^{N-1} x_{n'} \psi^* \left[ \frac{(n'-n)\delta t}{s} \right]
\]

where * indicates the complex conjugate of \( \psi(n) \). This equation represents the scaling and translating of the mother wavelet to find how well a measure of each scale and translation member of the family fits the time-series signal. The computation of this mathematical statement is very complex. In order to make it numerically efficient we must reduce the number of calculations by forming the wavelet transform in frequency space. This allows us to compute all the possible scalings simultaneously. We begin by expressing the signal as a sum of sinusoidal terms. The wavelet transform in frequency space is

\[
W_n(S) = \sum_{k=0}^{N-1} \hat{x}_k \hat{\psi}^* (sw_k) e^{-i\omega_n n \delta t}
\]

where \( w_k = 2\pi k/(N\delta t) \) if \( k \leq N/2 \) and \( w_k = -2\pi k/(N\delta t) \) if \( k > N/2 \).

It is important to normalize the wavelets so that if we decide to use another form of mother wavelet to analyze the data, the properties derived from the signal are consistent. This normalization ensures that the wavelet transform is weighted only by the amplitudes of the Fourier coefficients, \( \hat{x}_k \), and not by the wavelet function itself. To do this, we always have at each scale \( s \)

\[
\hat{\psi}(sw_k) = \left[ \frac{2\pi s}{\delta t} \right]^{1/2} \hat{\psi}_0(sw_k)
\]

so that the wavelet now has unit energy. Since all the unscaled mother wavelets are frequency normalized, i.e.,

\[
\int_{-\infty}^{\infty} |\hat{\psi}_0(w')| dw' = 1
\]
we can show that
\[ \sum_{k=0}^{N-1} |\tilde{w}(sw_k)| = N \] (5)

2.2.1 Wavelet Power Spectra

In general, wavelet functions are complex, therefore the resulting wavelet transform, \( W_n(s) \), will also be complex. Information can be derived from both the real and imaginary parts. The wavelet power spectrum, \( |W_n(s)|^2 \), is the quantity we will explore in depth.

Figure 4 shows the wavelet power spectrum and its properties for data taken on April 25, 1997 with the SUMER spectrograph (see section 3.2). We will use this figure as a template to explain some features of the wavelet power spectrum. In Figure 4b we see the normalized wavelet power spectrum for the time-averaged normalized time series in Figure 4a. We can see from this figure that much of the power is concentrated in the 3-10 mHz scales over all times. This 3-10 mHz frequency band is characteristic of the ultraviolet regime of the solar spectrum. Figure 3 shows a fast Fourier Transform of the C I continuum. It is clear to see the characteristic concentration of power from the 3-10 mHz band.

![Fourier Power Spectrum of C I Continuum Intensity](image)

Figure 3: Fourier power spectrum of the C I continuum intensity. Notice the concentration of power in the 3-10 mHz band, characteristic of the solar spectrum.

One of the most noticeable features of Figure 4b is the cross-hatched area at the bottom of the graph. This area is called the cone of influence, or C-o-I. The C-o-I is needed because the time-series signal is “padded.” This padding is done so that the number of terms is a power of 2, which makes the wavelet transform significantly faster in the computation. This padding, though, causes edge effects and aliasing for certain frequencies, affecting each one differently, but in a predictable manner. As a result of this predictability we are able to draw the C-o-I and state that any wavelet power lying within the C-o-I is not to be considered the result of real variability in the signal.

The idea of the C-o-I is related to the idea of time and frequency resolution. Since the mother wavelet is not a point function (it covers a finite area of frequency and time space) it has a finite spread. This means that we cannot exactly specify the time at which a burst occurred or the exact frequency of this burst. Hence, resolution in the wavelet transform is important.

Most mother wavelets are known as “constant-Q” wavelets. The mother has a characteristic value, called Q,
Figure 4: This plot shows a typical wavelet power spectrum for the C I continuum emission formed in the middle part of the chromosphere at a wavelength of 1043 Å. Notice the concentration of wavelet power in the 3-10 mHz band, very typical of the solar spectrum. a) The mean-subtracted time series we analyzed. Removing the mean makes the transform “unbiased.” We did this to transform the variations from the mean. b) The wavelet power spectrum of the time series in (a). c) The global wavelet power, determined by finding the mean of all frequency bins and the scale-average time series. This allows us to see where the bulk of the fluctuation exists in “good” power (i.e., inside the cone of influence and confidence intervals).

that is some measure of resolution. Q can be thought of as the area of a rectangle which has a width according to the envelope (scale) and height according to the frequency of the mother. At high frequencies the rectangles are tall and narrow, at low frequencies the rectangles are wide and short, but all the rectangles have the same area. Figure 5 shows the resolution space of the Morlet mother wavelet.
Another feature in Figure 4b are the contours around some of the wavelet power spectrum “blobs.” These contours are known as significance levels and are statistical in nature. For any time-series signal, it is assumed that a mean power spectrum exists if a peak in the wavelet power spectrum is significantly above this background spectrum contour. It is then assumed to be a true feature with a certain percent confidence, called the confidence interval. The confidence interval is the probability that the true wavelet power, at a certain time and frequency, lies within a certain interval about the estimated wavelet power. Simply stated, if the wavelet power spectrum “blob” lies within the 95% confidence interval, then we can be 95% confident that the “blob” is real. Figure 4b is plotted with a 95% confidence interval.

2.3 Wave Propagation in Stellar Atmospheres

In an atmosphere such as that of the Sun the interaction between gravitational, thermodynamic and magnetic forces can produce several restoring forces. Each of these forces is capable of creating and influencing plasma motions as they propagate through the atmosphere. Simply put, thermodynamic forces depend on pressure perturbations and the generated waves are essentially sound waves (longitudinal mode comprising of subsequent compression and rarefaction of the plasma). Magnetic waves are transverse in nature and travel along magnetic field lines and are driven by perturbations in the “tension” of the field (cf. waves on a plucked guitar string). Gravity waves occur as a consequence of an unbalance between the density of layers (or shells) of the plasma. In the outer reaches of the solar atmosphere (i.e. above the photosphere) we can neglect the gravitational restoring forces. The following discussion will treat only thermodynamic and magnetic influences on the plasma.
5 Discussion

Figure 11 shows the frequency shifts and time lags as a function of height. As the wave propagates upward, the frequency changes. We can also see that it takes the wave a finite amount of time to travel upwards through the different layers. The time lags we derived are 48 seconds from the C I continuum to the C II velocity, and 11 seconds from the C II velocity to the O VI velocity. These are consistent with those derived by Wikstol et al. (2000) on the same data set. In addition, because we have used wavelet transforms, we were also able to derive frequency information.

It is interesting to see from Figure 11 that as we move from the C I continuum to the C II velocity, the frequency shifts in the negative direction. In other words, the frequency decreases from a peak of 5.59 mHz in the C I to a peak of 5.40 mHz in the C II velocity. Yet from the C I continuum to the O VI velocity we see a net frequency increase. The frequency shifts from a peak of 5.59 mHz in the C I continuum to a peak of 6.20 mHz in the O VI velocity. In Table 1 we give the values of both time lags and frequency shifts for the various atmospheric layers.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Time Lags (s)</th>
<th>Frequency Shifts (mHz)</th>
<th>WHCJ* Time Lags (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C I continuum to C II velocity</td>
<td>48</td>
<td>-0.19</td>
<td>50</td>
</tr>
<tr>
<td>C I continuum to O VI velocity</td>
<td>83</td>
<td>0.40</td>
<td>-</td>
</tr>
<tr>
<td>C II velocity to O VI velocity</td>
<td>11</td>
<td>0.19</td>
<td>4</td>
</tr>
</tbody>
</table>

Over-plotted in Figure 11 on the top panel is a plot of the cut-off frequency as a function of height. Due to the temperature gradient change in the upper region of the chromosphere, the cut-off frequency reduces (see section 1.3).

As the wave moves from the C I continuum formation region to the C II formation region, the cut-off frequency decreases, allowing for a drop in frequency, i.e., a negative frequency shift. As the wave continues to propagate up, the cut-off frequency increases. The lowest frequency range this wave can propagate through is higher, causing the positive frequency shift from the C II velocity to the O VI velocity. This variation in the cut-off frequency with altitude is illustrated in Figure 54.2 of Mihalas & Mihalas (1984).

Figure 12 contains useful information as to how the wave behaves as it propagates through the solar atmosphere. To create this figure, we took as a sample pixel 52 and rastered over the neighboring pixels to see over how many pixels this signal was consistent. In other words, we looked at the coherence of the signal to test the spatial extent of the wave. In the bottom panel of Figure 12 we see the coherence of the C I continuum (red), the C II velocity (blue) and the O VI velocity (green). Using the full-width-half-max (FWHM) value, the C I continuum is coherent over 4 pixels, the C II is coherent over 6 pixels, and the O VI is coherent over 10 pixels. These are important and interesting results. We’re seeing that the spatial extent of the wave is expanding as it moves up through the solar atmosphere. Remember that as we move up the influence of the magnetic field increases as the gas pressure decreases. Figure 12 shows us that as the wave propagates, the wave front expands, and as it expands, the frequency of the wave changes.

It is also important to note the rough asymmetry between the north (left) and south sides of the detector. The north side seems to have a different drop-off rate for all three measurements. This asymmetry implies a slightly inclined magnetic field. This is plotted in the upper panel of Figure 12. As the wave propagates up this incline it should expand preferentially to the north side, causing this asymmetry.

*WHCJ refers to Wikstol et al. (2000)
Figure 11: Frequency shifts (above) and time lags (below) as a function of height. The top panel describes the changes in frequency the wave undergoes as it propagates through the solar atmosphere. The bottom panel shows the wave packet travel times from one layer to the next.
Figure 12: Coherence of the spatial extent of the wave. Notice how the coherence increases as we move up through the solar atmosphere. Also notice the asymmetrical drop-off on the North side of the detector. This implies a slightly inclined magnetic field indicated in the top panel.

Figure 13 shows an exaggerated depiction of this physical process. The wave starts as a source point in the photosphere. As the wave propagates through the solar atmosphere it expands and shifts in frequency. It is important to note that in all three measurements (C I continuum, C II velocity, and O VI velocity) the spatial extent and peak frequency of the wave is different. This accounts for the derived frequency shifts and the increase in the coherency of the signal over the different layers. Also note that as the height increases, the gas density decreases and the influence of the magnetic field increases.

In summary we have found that:

* As the wave propagates the wave fronts expand with the relative decay in gas density.

* The peak frequency of each oscillation changes with altitude.

* The wave takes a finite amount of time to travel to each level and doesn’t appear to be reflected.

In addition, we have also confirmed results of McIntosh et al. (2000) that this is the same wave moving through the solar atmosphere but changing characteristics as it propagates. We have also reproduced time lags consistent with those of Wikstel et al. (2000). The use of wavelet transforms has allowed us to look at the frequency behavior of the waves also, a result not determined previously for any dataset in this way.

We have demonstrated that wavelet transforms are potentially important tools for analyzing datasets such as this.
Figure 13: Illustration of wave expansion and frequency shifts. The wave starts as a point source and expands and increases in frequency as it moves up through the solar atmosphere.

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