The Dragon's Footprints:
Microwave Radar Remote Sensing of Hydrometeors
With the NEXRAD/WSR-88D and
NCAR CP-2 Radar Systems

by
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ABSTRACT

The National Weather Service is in the process of establishing the Weather Surveillance Radar-1988, Doppler system (WSR-88D), also known as the NEXRAD (Next Generation Radar) system. The WSR-88D offers improved and expanded capabilities over the current non-Doppler weather radar system, WSR-57. The NCAR CP-2 dual-wavelength polarimetric radar possesses all the functions of the WSR-88D, as well as additional capabilities that allow differentiation between different types of hydrometeors. This paper will examine both systems, with emphasis on hydrometeor detection, and explore possible methods of upgrading the NEXRAD system to polarimetric capability.

INTRODUCTION

Pulsed-Doppler radar is used in meteorology to collect information about the state of the atmosphere. This paper is concerned mainly with the detection of various types of hydrometeors, which are water particles in solid or liquid state. Pulsed-Doppler radar measures three meteorologically relevant quantities: reflectivity, mean radial velocity, and spectrum width. Reflectivity is a measure of particle backscattering cross section density. Backscattering cross section is a function of the target size, thermodynamic phase state of the hydrometeor (solid, liquid, or mixed), and wavelength of radar used [wavelengths commonly used by weather radar are 10 cm (known as S-band), 5.6 cm (C-band), and 3.21 cm (X-band)]. Reflectivity factor, measured in logarithmic units known as dBZ, is related to the number and characteristics of hydrometeors in the observed volume; non-precipitating cloud structures producing values from near -20 dBZ to thunderstorms containing heavy rain and hail measuring more than 60 dBZ. Mean radial velocity is the component of a particle’s velocity towards or away from the radar. Spectrum width is the measure of the variance of the radial velocity vectors within the observation volume. Measurement of reflectivity is the most useful measurement for detecting and classifying hydrometeors, and is therefore the focus of this paper.
THE NEXRAD SYSTEM

The Nexrad radar system comprises 137 WSR-88D radars strategically spaced to provide virtually complete coverage of the conterminous United States. Each WSR-88D radar has an effective range of 50 kilometers with variable range resolution from 150 to 300 meters, and an azimuthal resolution of 1°. Each of the three Doppler radar quantities measured by the Nexrad system provide several products useful to meteorologists. Precipitation rates calculated from reflectivity factors are used in river forecasting models to predict floods. Three-dimensional reflectivity measurements map interior storm structure. These radar scans, taken over time, show interior storm dynamics, and provide storm movement tracks which can be extrapolated into storm position forecasts. Measurements of vertically integrated liquid, the total amount of water contained in a storm above any given area on the ground, allow us to calculate of the probability of severe weather over that area.

Radial velocity data are used to identify rotating three-dimensional regions in storms, called mesocyclones. When intense wind shears develop in mesocyclone structures, tornadoes may form. The WSR-88D radar can identify tornadic conditions and issue the necessary tornado warnings. The radar's Velocity Azimuth Display gives a vertical display of horizontal wind velocity vectors at a given range, showing wind speed and direction at any given point. As radial velocity measurements are taken at various distances and azimuths, we find the combined wind shear contour. With the combined wind shear contour and measurements of the spectrum width, we can estimate atmospheric turbulence.

THE PROBLEM

While the WSR-88D system provides a wealth of useful and accurate information, it cannot tell us everything we may wish to know about cloud structures. In particular, it is not able to reliably differentiate between different types of hydrometeors; rain, sleet, hail, and ice crystals.
This is because different types of hydrometeors may give identical reflectivity readings. It is analogous to finding a footprint of some large, vaguely reptilian creature (fig.1). Being seasoned travelers in dragon country, we suspect that a dragon is in the vicinity and we can make rough guesses as to how large and heavy it may be. However, we don’t know exactly what it looks like and we cannot guess as to what type of dragon it is. What color is it? Is it fire-breathing or non-fire-breathing? Is it of the airborne variety and if so, what is it’s wingspan and maximum airspeed? Most importantly, what does it like to eat? We need more information than just the tracks our dragon leaves. Similar ambiguities are present in the meteorological realm. We see a large thunderstorm, but where in the storm will it hail, and to what extent? Where will it snow? Are there dangerous icing conditions that require the issuing of warnings to airplane pilots? It is here that the additional capabilities of the NCAR CP-2 radar come into play.

THE NCAR CP-2 RADAR SYSTEM

While conventional (non-polarimetric) Doppler radars transmit and receive in one polarization form--horizontal, vertical, or circular; polarization diversity radars such as the NCAR CP-2 employ polarimetric techniques to observe size, shape, orientation (a measure of how far off the axis of polarization a nonspherical particle is aligned, also known as canting angle), and phase state of particles in the observation volume. The unique feature of the CP-2 radar is its ability to measure reflectivity and mean velocity with dual-polarization at both S- and X-band wavelengths, which allows us to form a Dual-Wavelength Hail Signal. The CP-2 radar system produces four sets of data to show the contents of any observed cloud structure; horizontal polarization Reflectivity Factor ($Z_H$), Differential Reflectivity ($Z_{DR}$), Linear Depolarization Ratio (LDR), and Dual Frequency Ratio(DFR). DFR is also known as Hail Signal(HS). Mathematical theory behind these parameters is found in the Appendix, while the parameters and the hydrometeors that produce them are given in Table 1.
The CP-2 radar measures reflectivity factor $Z$ in horizontal polarization ($Z_H$) because large water droplets (diameter $> 1$ mm) tend to assume a flattened ellipsoidal, or oblate, shape during their fall through the atmosphere (fig. 2). A backscattered electromagnetic wave is thus easier to detect when the transmitted wave is polarized parallel to the raindrops larger horizontal axis. $Z_H$ is defined by the equation (Doviak & Zrnic, 1984)

$$Z_{H,V} = \left(\frac{\lambda^4}{\pi^5}\right) K^2 \eta_{H,V}$$

where $\lambda$ is the radar wavelength, $K$ is a constant associated with the complex index of refraction of the target hydrometeor, and $\eta_{H,V}$ is the backscattering cross section from a horizontally or vertically polarized transmitted wave.

Differential reflectivity $Z_{DR}$ involves the measurement of echo signals from two incident waves emitted consecutively in two orthogonal polarization planes, horizontal ($Z_H$) and vertical ($Z_V$). The ratio of horizontal backscattered radiation to vertical backscattered radiation is measured in logarithmic scale and is called $Z_{DR}$ (Doviak & Zrnic, 1984):

$$Z_{DR} = 10 \log \left(\frac{\eta_H}{\eta_V}\right)$$

$Z_{DR}$ is not a measure of the size of the scatterers, but rather a measure of the mean shape of the particles, ranging from 0 dB for spherical particles to 5 dB for highly ellipsoidal shapes.

Differential reflectivity is used in conjunction with horizontal reflectivity $Z_H$ to detect hail and ice crystals. Ice has a much smaller index of refraction compared to liquid water, resulting in reduced vertical and horizontal polarized backscattering measurements. Large solid particles also have a larger range of canting angles due to tumbling in the atmosphere. Because Doppler radar measures the mean of the sampled reflectivity data taken over a period of time, tumbling ice particles “look spherical” to the observing radar, even though they often physically assume nonspherical shapes (fig. 3). This is in contrast to large (diameter $> 1$ mm) water droplets which
develop a flattened ellipsoidal, or oblate, shape (fig. 2). When observing particles of the same volume, the combination of low reflectivity and spherical shape makes $Z_{DR}$ measurements of ice much smaller than those of water. Where low $Z_{DR}$ values occur, there exist either hail or small water droplets. Therefore, when areas of low $Z_{DR}$ (about zero dB) are detected inside areas of substantial $Z_H$ (about 40 to 50 dB), hail is very likely to be present.

Linear Depolarization Ratio is similar to $Z_{DR}$ in that it takes advantage of the wide range of canting angles exhibited by falling hail; however where $Z_{DR}$ is dependent upon of the spherical appearance of tumbling hail, LDR measurements involve the actual non-spherical shape of hail particles, as well as the canting angles of the targets. LDR delineates tumbling hail and a region known as the melting band. The melting band is a region of marked increase in reflectivity factor in the regions of a cloud where ice particles begin to melt and develop a thin coating of water. This band is a result of the fact that a thin shell of water around an ice particle results in a very large increase in reflectivity. For example, when one-tenth of the diameter of a 4mm ice sphere melts, the resulting particle backscatters 90% of the radiation of a water droplet of the same size.

To measure LDR, a horizontally polarized wave is transmitted and intercepts a target particle, from which cross-polar and copolar echo signals are backscattered. The cross- and copolar signals arise from the fact that when a horizontally polarized wave is incident upon a target which is canted off the horizontal polarization axis, the backscattered radiation that results is depolarized. The depolarized radiation is received by the radar and then resolved into its component parts; a cross-polar wave orthogonal to the original incident polarization, and a copolar wave with the same polarization as the original transmitted wave. LDR is defined as the logarithmic ratio of cross-polarized backscattered radiation to copolarized backscattered radiation (Jameson & Johnson, 1990):

$$LDR = 10 \log \left( \frac{X_{HV}}{X_{HH}} \right)$$
where $X_{HV}$ is the cross-polar and $X_{HH}$ the copolar returned signal of the horizontally polarized transmitted wave. Since small water droplets are nearly spherical and large oblate water droplets are highly oriented, rain is associated with very small values of LDR (from $-\infty$ to -25 dB) while highly canting ice and hail give LDR of about -15 to -5 dB.

The dual-wavelength hail signal (HS) is formed by taking reflectivity factor $Z_H$ measurements at two wavelengths, the CP-2 radars aforementioned 10 cm S-band and 3 cm X-band. Reflectivity measurements are calculated using Rayleigh scattering theory and Mie scattering theory, both dependent on the illuminating wavelengths relative to the target size. When the observed particles are small compared to the radar wavelength, the Rayleigh scattering and Mie scattering produce nearly the same reflectivity factor. Dual Frequency Ratio DFR is defined as the logarithmic ratio of $Z_H$ from the long wavelength to $Z_{\text{hf}}$ of the shorter wavelength (Srivastava and Jameson, 1977):

$$HS = 10 \log \left( \frac{Z_L}{Z_S} \right)$$

where $Z_L$ and $Z_S$ are the reflectivity factors as measured by the long and short wavelengths, respectively. Particles with a diameter of eight millimeters or less, which are small compared to both 3cm and 10cm radar and thus will give nearly identical $Z_H$ from both Rayleigh and Mie scattering ($Z_L \approx Z_S$), and HS will be approximately zero. When observed particles are larger (diameter > 10 mm), the calculations from Mie scattering theory result in the short wavelength giving a lower value of $Z_H$ than that of the long wavelength ($Z_L > Z_S$). Thus large particles give higher (HS) measurements. Rainfall rate estimated in this manner is obviously dependent on the drop size distribution in the observed volume, HS is 0 dB for small particles and greater than 3 dB for larger particles. Water droplets rarely reach sizes greater than 0.8 cm diameter due to atmospheric turbulence and other effects of terminal velocity fall, so any substantial hail signal observed will be due to large scatterers, i.e. hail and ice.
From plots of actual radar data, one can see the differences between the various methods of hydrometeor detection. Horizontal reflectivity $Z_H$ (fig.4) shows structure throughout the entire storm, indicating the general intensity and location of hydrometeors. Large dBZ values show the areas of more and larger scatterers, though ambiguous in differentiating between the scattering hydrometer types. Differential reflectivity $Z_{DR}$ (fig. 5) shows the regions of ice particles with measurements around 0 dB. The 3 dB region at 20 km shows the marked reflectivity increase that accompanies melting ice turning to water. At about 15 km there are low dB measurements at ground level and negative readings at higher altitudes. This leads us to believe that the negative dB area denotes large ice particles, since the large ice hydrometeors take longer to melt and would therefore produce the lower reflectivity readings near ground at 15 km. The relationship between the ambiguous $Z_H$ and the more precise $Z_{DR}$ readings are shown in the overlay plot (fig. 6) and the graph from Sirmans, Zrnic & Balakrishnan (1988), (fig.7). The dual frequency ratio/hail signal DFR/HS (fig. 8) shows regions of large ice particles with measurements of 3 dB or greater. There is general agreement between areas of large particles given by hail signal and the conclusions drawn from $Z_H$ and $Z_{DR}$; large ice particles may be found at 15 km from the radar. The linear depolarization ratio (fig. 9) shows tumbling hail and water-coated ice particles as regions measuring -12 dB or greater. Again, large ice particles are found at 15 km.

CONCLUSIONS

From the CP-2 radar parameters $Z_H$, $Z_{DR}$, LDR and HS we can infer a great deal about the characteristics of any given cloud structure. We can differentiate between types of hydrometeors, rain, hail, ice crystals, and mixed particles. This increased capability leads to a better understanding of cloud microphysics and storm evolution and behavior. Some useful products given by the CP-2 radar include improved estimates and forecasting of rainfall, hail and icing conditions, which pose a potentially lethal threat to aircraft. The CP-2 radar is an extremely
capable remote sensing device; a much more useful tool than the WSR-88D radar.

Upgrading the NEXRAD system to dual-wavelength polarimetric capability would require modifications to the radar’s microwave circuitry, signal processing, and data analysis engineering. To generate the $Z_{DR}$ parameter, the existing radar’s transmitter/receiver requires a high-speed latching ferrite circulator to switch between horizontal/vertical transmission, and an orthomode coupler/antenna feed to generate and illuminate the reflector with the horizontally or vertically polarized wave. Signal processing capabilities would need to be expanded to compute the differential measurements. The raw data received would require the addition of a second clutter filter as the horizontal and vertical polarization channels need to be filtered separately.

Production of the LDR parameter may be done in several ways, the most efficient of which would use the existing S-band antenna/receiver and employ a dual-mode feed and ferrite switch to receive both cross-polarized $Z_{HV}$ and copolarized $Z_{HH}$ components. The receiver would need to be extremely sensitive to detect the cross-polar component of the backscattered wave, as the orthogonal component may be 20 to 30 dB below the copolar component and must be “heard” through electromagnetic noise and other interference. The cross-polar data must be processed and filtered separately from the copolar, requiring further expansion of signal processing and data analysis facilities. The dual frequency ratio parameter requires the addition of an entirely separate radar to transmit and receive in 3cm wavelength. The X-band data must then be processed and analyzed with the S-band data to generate the hail signal product.

The modifications necessary to upgrade each of the 137 WSR-88D radars in the nationwide NEXRAD system to match the CP-2s dual wavelength polarimetric capability would be prohibitively expensive at this time. Thus the CP-2 radar is presently only available as a research tool. However, with future advances in radar engineering and signal processing technology, we hope to make the NCAR CP-2 the next next generation weather service radar system.
ACKNOWLEDGEMENTS

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REFERENCES


Horizontal / Vertical Polarization Reflectivity $Z_{H/V}$:

$$Z_{H,V} = \left(\frac{\lambda^4}{\pi^5}\right)|K|^2\eta_{H,V}$$

Differential Reflectivity $Z_{DR}$:

$$Z_{DR} = 10\log\left(\frac{\eta_H}{\eta_V}\right)$$

Linear Depolarization Ratio $LDR$:

$$LDR = 10\log\left(\frac{X_{HV}}{X_{HH}}\right)$$

Dual Frequency Ratio $DFR$ / Hail Signal $HS$:

$$DFR.HS = 10\log\left(\frac{Z_L}{Z_S}\right)$$

### Hydrometeor Types and Corresponding Backscattered Signals

<table>
<thead>
<tr>
<th>Hydrometeor</th>
<th>$Z_H$</th>
<th>$Z_{DR}$</th>
<th>LDR</th>
<th>HS</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>Includes large oblate drops</td>
</tr>
<tr>
<td>Drizzle, fog</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>Small spherical drops of liquid or ice</td>
</tr>
<tr>
<td>Dry snow</td>
<td>Med-Low</td>
<td>Med-Low</td>
<td>Low</td>
<td>Low</td>
<td>Large horizontal low-density aggregates</td>
</tr>
<tr>
<td>Sleet/Wet snow</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>Large oblate particles</td>
</tr>
<tr>
<td>Wet hail</td>
<td>High</td>
<td>Variable</td>
<td>High</td>
<td>High</td>
<td>Large non-spherical particles</td>
</tr>
<tr>
<td>Dry hail</td>
<td>Medium</td>
<td>Low</td>
<td>Low</td>
<td>High</td>
<td>High-density ice</td>
</tr>
</tbody>
</table>
Drop Diameter (mm)

8.00  7.35  5.80  5.30  3.45  2.70

(From Doviak & Zrnic, 1984, pg. 180)
Hail Size (cm)

(From Gokhale, 1975, pg. 92)
(a) The precipitation type/intensity model from Lipshutz et al. (1986). Bold line is rain/hail boundary given by Leitao and Watson (1984).
Backscattering $H/V$ vs Axial Ratio $b/a$

Backscattering Ratio $H/V$
Zdr vs Rainfall Rate, max Deq=10cm

Zdr(dB) & NHR[10log(Zh/No)]
Backscattering, Horizontal Polarization

Backscattering Cross Section (mm sqrd)

- 10.00000 cm - Water
- 5.60000 cm - Water
- 3.21000 cm - Water
- 0.85700 cm - Water
- 10.00000 cm - Ice
- 5.60000 cm - Ice
- 3.21000 cm - Ice
- 0.85700 cm - Ice

Deq(mm)
Equivalent-volume diameter

axis ratio

green
empirical 1.03
empirical 1.00

diameter (mm)
APPENDIX

A. Reflectivity

Reflectivity is defined as the sixth power of the drop diameter summed over all drops in the unit volume. Reflectivity $\eta$ is a measure of the of particle backscattering cross section $\sigma$ per cubic meter. Backscattering cross section of a non-spherical particle (water particles and hail often assuming non-spherical shapes-see figs. 2 and 3) is related to drop size, radar wavelength $\lambda$, and complex refractive index $m$. For horizontal or vertical polarization, $\sigma$ is given by the equation

$$\sigma_{H,V} = \left( \frac{16\pi^7 D_{eq}^6}{9\lambda^4} \right) \frac{m^2 - 1}{4\pi + (m^2 - 1)P_{H,V}}$$

where $D_{eq}$ is the equivalent diameter of the particle, defined as the diameter of a spherical raindrop of equivalent volume (Doviak & Zrnic, 1984). $D_{eq}$ is related to the axis ratio $b/a$ (b=major axis, a=minor axis) of the prolate spheroid by the equation

$$D_{eq} = 2 \sqrt{\left\{ \left( \frac{T_s}{g \rho_w} \right) \left[ \left( \frac{b}{a} \right)^2 - 2 \left( \frac{b}{a} \right)^{1/3} + 1 \right] \left( \frac{b}{a} \right)^{1/3} \right\}}$$

where $T_s=0.07275 \text{ J/m}^2$ is the surface tension of water, $g$=acceleration due to gravity, and $\rho_w$=density of water (Doviak & Zrnic, 1984). An empirically derived equation may be used to calculate $P_H$ and $P_V$ for drops ranging in size from about 0.5mm to 6mm, (Pruppacher & Pitter, 1971)

$$\frac{a}{b} = 1.03 - 0.062 D_{eq}$$

where $a/b$ is used to calculate eccentricity of the ellipsoid,

$$e^2 = 1 - \left( \frac{a}{b} \right)^2$$
for use in the equation to calculate $P_{V,H}$ (Doviak & Zrnic, 1984)

$$P_V = \left( \frac{4\pi}{e^2} \right) \left[ 1 - \left( \frac{1 - e^2}{e^2} \right) \sin e \right] = 4\pi - 2 P_H$$

The complex coefficient of refraction $m$ used in the $\sigma$ equation is a function of $\lambda$ and temperature. The real and imaginary components of coefficient $m = n - ik$ were given by my advisor, Vivek and may also be found in Battan, 1973. For water, $m$ is given by

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Temperature(°C)</th>
<th>Wavelength(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>5.6</td>
</tr>
<tr>
<td>$n$</td>
<td>20</td>
<td>8.8577</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>8.9832</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>9.0353</td>
</tr>
<tr>
<td>$k$</td>
<td>20</td>
<td>0.74711</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.0144</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1.3944</td>
</tr>
</tbody>
</table>

while, for ice, $m = 1.79$ for all wavelengths and temperatures in question.

As reflectivity $\eta$ is a function of drop size distribution, $\eta_{H,V}$ may be expressed in terms of $D_{eq}$:

$$\eta_{H,V} = \int \sigma_{H,V}(D_{eq}) N(D_{eq}) dD_{eq}$$

where $N(D_{eq})$ is the exponential drop size distribution

$$N(D_{eq}) = N_0 e^{-AD_{eq}}$$

Using a truncated exponential drop size distribution, the reflectivity may be represented by the equation

$$\eta_{H,V} = \frac{16N_0 \pi^7}{9\lambda^4} \int_{D_{min}}^{D_{max}} D^6 e^{-\frac{m^2 - 1}{4\pi + m^2 - 1} P_{H,V}} e^{-AD_{eq}} dD_{eq}$$

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where \( N_0 \) is the number of particles per cubic meter and \( \Lambda = 4.1R^{-0.21} \text{mm}^{-1} \), where \( R \) is the rainfall rate in millimeters per hour (Doviak & Zrnic, 1984).

B. Polarimetric Methods

1. Horizontal, Vertical, and Differential Reflectivity

When the horizontally polarized transmitted wave is incident upon a nonspherical particle, there results a depolarized backscattered wave. Upon reception, the horizontally polarized component \( \eta_H \) is extracted from the depolarized wave. The same is done to obtain \( \eta_V \) from the signal backscattered from the vertically polarized transmitted wave. With horizontal polarized reflectivity \( \eta_H \) and vertical polarized reflectivity \( \eta_V \), we define differential reflectivity \( Z_{DR} \) as the logarithm of the reflectivity ratio (Doviak & Zrnic, 1984):

\[
Z_{DR} = 10\log \left( \frac{\eta_H}{\eta_V} \right)
\]

For a spherical target, \( \eta_H = \eta_V \) and \( Z_{DR} = 0 \). When the hydrometeor in question is nonspherical and horizontally oriented, as is the case for large raindrops, the \( \eta_H \) return will be greater than the \( \eta_V \) return and \( Z_{DR} \) will be positive, its magnitude depending on how extreme the nonsphericity of the particle.

Reflectivity factors for horizontal and vertical polarization \( Z_H \) and \( Z_V \) are given in terms of \( \eta_{H,V} \) (Doviak & Zrnic, 1984):

\[
Z_{H,V} = \left( \frac{\lambda^4}{\pi^5} \right) K |2\eta_{H,V}|
\]
where $K$ is used to designate (Battan, 1973)

$$\frac{m^2 - 1}{m^2 + 2}$$

2. Linear Depolarization Ratio

Linear Depolarization Ratio (LDR) is defined as the power received in the cross-polarized channel divided by the power received in the copolarized channel. X-band radar;

$$LDR = 10 \log \left( \frac{X_{HV}}{X_{HH}} \right)$$

in units of dB, where $X_{HV}$ is the cross-polar and $X_{HH}$ the copolar returned signal of a 10cm horizontally polarized transmitted wave (Jameson and Johnson, 1990). Since there is no depolarization from spherical particles or from extremely ellipsoidal particles (oblate “plates” or prolate “needles”) oriented parallel or orthogonal to the polarization axis, $X_{HV}$ for such particles will be zero, resulting in LDR of $-\infty$. Stable, non-canted oblate shapes characteristic of large raindrops give LDR from $-\infty$ to -25dB. Oblate or prolate ice particles exhibit tumbling and large canting angles which give a cross-polar return $X_{HV}$ and thus the LDR of such particles ranges from -15 to -5 dB.

3. Dual Frequency Ratio (DFR) / Hail Signal (HS)

Drop size parameters are found from the differences in measured reflectivity factors due to Rayleigh and Mie scattering characteristics. Hail Signal (HS) is the ratio of reflectivities measured at two wavelengths, one long and one short, with respect to the illuminated particle size. The long wavelength radar calculates backscattering and resulting reflectivity factors from Rayleigh theory, the short wavelength from Mie theory. On logarithmic scale, HS is defined by the equation

$$HS = 10 \log \left( \frac{Z_L}{Z_S} \right)$$
resulting in \( \text{HS} = 0.0 \). When the particle diameter approaches that of the short wavelength, (hail particle diameters of 1 - 2 cm are not uncommon) \( Z_s \) is influenced by Mie theory, \( Z_s < Z_L \), resulting in \( \text{HS} > 0 \).

C. Computer Programs and Graphs of Various Relevant Hydrometeor Characteristics

To illustrate the relationships between hydrometeor properties and various radar measurements, I wrote several programs in FORTRAN to perform the necessary numerical calculations and reproduce figs. 8.18 and 8.19 from the Doviak & Zrnic (1984) text, and fig. 4.2 from Battan (1973). I then restructured the graphs to a form more easily interpreted by those without a meteorological background (figs. 10, 11 and 12, respectively), and combined all of my separate programs into one program which I present here. The program “Deq” was written by Wayne Adams, to see where the empirically derived relationship between axial ratio and equivalent diameter (Pruppacher & Pitter, 1971)

\[
\frac{a}{b} = 1.03 - 0.062 D_{eq}
\]

is a valid approximation of the equation (Doviak & Zrnic, 1984):

\[
D_{eq} = 2 \sqrt{\frac{T_s}{g \rho_w}} \left[ \left( \frac{b}{a} \right)^2 - 2 \left( \frac{b}{a} \right)^{1/3} + 1 \right] \left( \frac{b}{a} \right)^{1/3}
\]

originally taken from A.W. Green (1975). The empirical approximation is valid for \( D_{eq} \) values from 0.5 mm to 6 mm. To approximate with \( D_{eq} < 0.5 \) mm, the equation

\[
\frac{a}{b} = 1.00 - 0.062 D_{eq}
\]

may be used with slightly greater error (fig. 13)
- main program to compute
ZDR and normalized Zh vs rainfall rate, to reproduce
fig. 8.19, pg. 217 Doviak & Zrnic text
using equations 8.39-8.41, 8.44a pgs. 215-216
"Doppler Radar & Weather Observation" (Doviak & Zrnic, 1984)
Program contains algorithms to compute backscattering from
spherical particles, using equations from "Radar Observation
Of The Atmosphere" (Battan, 1973). Program contains algorithms
to compute backscattering cross section H/V vs. axial ratio
b/a, to reproduce fig. 8.18, pg 215 Doviak & Zrnic.
Output in Sun Sparcstation XGraph compatible form.

variables used-
wave/wvlnth=radar wavelength (mm/cm)
Deq=oblate spheroid's equivalent diameter
n=real part of complex index of refraction
j=imaginary part of complex index of refraction
m=n-j
rainrate=rainfall rate (mm/hr)
deltx=integration increment on x axis
y,h=V/H polarized equation value for corresponding deltx
evenv/oddv=even/odd values for vertical polarization of y/h
evenh/oddh=even and odd values for horizontal polarization of y/h
areaV/areaH=value of integral for V/H polarization
areae=area ratio of H/V polarization reflectivities
NHR=Normalized Horizontal Reflectivity
ratio=Ratio of major axis to minor axis of oblate spheroid
Ecc=eccentricity of oblate spheroid
Ph/Pv=components of Ph and P V geometric factors
PV=vertical polarization geometric factor
PH=horizontal polarization geometric factor
modsqV=modulus squared term of backscatter equation, vertical
modsqH=modulus squared term of backscatter equation, horizontal
expower=exponential term of reflectivity integral
No=number of particles in observation volume (8000)

define variables
parameter (Q=4, Z=101)
dimension wave(q), y(z), h(z)
external eqV, eqH
include "common.xf"
real wvlnth,j, wave(Q), coeff, y(z), h(z), x, degmax, degmin,
+ Kw, Zh, norm, NHR, areae, areaH, areaV, evenv, oddv,
+ evenh, oddh, deltx, No

open output file
open(unit=2, file="ZDR.out")
open(unit=4, file="ZH.out")
open(unit=3, file="water.data")
write(2, *), "TitleText: Zdr vs Rainfall Rate"
write(2, *), "XUnitText: Rainfall Rate (mm/hr)"
write(2, *), "YUnitText: Zdr (dB) & NHR [10log(Zh/No)]"
write(2, *), "LogX: true"

start wavelength loop
  do 10 w=1.4, 1
    read(3, *), wave(w), n, j
    wvlnth=wave(w)/10.

  min and max values for Deq
  Degmin=-1
  Degmax=10.
  write(2, 3), wvlnth, Degmax
  format (', , f5.2, ', cm', Dmax=', ', f4.1, ', mm, ZDR')
  write(4, 6), wvlnth, Degmax
  format (', , f5.2, ', cm', Dmax=', ', f4.1, ', mm, NHR')
  k=j*(-1)
  pi=4.*atan(1.)
  No=8000.
  coeff=16.*No*(pi**7)/(9.*(wave(w)**4))
start rainrate loop
rainrate=801
  do 60 r=1,80,1
    rainrate=rainrate-10
  end

Set increment of integration
  delx=(Degmax-Degmin)/100.
  x=Degmin
  f=Degmin
integration - Simpson's rule
define array of y values
  do 30 p=1,101,1
    y(p)=eqV(x)
    h(p)=eqH(x)
    x=x+delx
  continue
accumulate majority of even subscripted Y elements
  evenv=0.
  evenh=0.
  do 40 s=2,100,2
    evenv=evenv+y(s)
    evenh=evenh+h(s)
  continue
accumulate majority of odd subscripted Y elements
  oddv=0.
  oddh=0.
  do 50 s=3,99,2
    oddv=oddv+y(s)
    oddh=oddh+h(s)
  continue
add first and last terms (w/ weighting)
  areaV=coeff*delx/3.*((y(1)+(4.*evenv)+(2.*oddv)+y(101)))
  areaH=coeff*delx/3.*((h(1)+(4.*evenh)+(2.*oddh)+h(101)))
calculate Zdr
  Zdr=areaH/areaV
  Zdr=10.*a8log10(areaat)
write(2,*),rainrate,Zdr
calculate normalized horizontal reflectivity
  Kw=.92
  Zh=((wavelength(w)**4)/(pi**5))*Kw*areaH
  norm=Zh/8000.
  NHR=10.*a8log10(norm)
write(4,*),rainrate,NHR
continue
write(2,*),"
write(4,*),"
continue
stop
end

subroutine to calculate backscattering cross section
subroutine backscat(x)
include "common.reflect";
real ratio,Ecc,Pva,Pvb,PvV,PvH,x
complex m,modnum,moddenomV,moddenomH,modV,modH
-calculate Pv
c
  ratio=1.-(.062*x)
  Ecc=1.-((ratio)**2)
  Pva=(4.*pi)/(Ecc)
  Pvb=1-((1-(Ecc))/(Ecc)**.5)*((asin(sqrt(Ecc))))
  PvV=Pva*Pvb
  PvH=(4.*pi)-PvV/2
-calculate modulus squared
m=cmplx(n,k)
modnum=(m**2)-1.
moddenomV=4.*pi+((m**2)-1.)*(PvV))
moddenomH=4.*pi+((m**2)-1.)*(PvH))
modV=(modnum)/(moddenomV)
modH=(modnum)/(moddenomH)
modsqV=(cabs(modV))**2
modsqH=(cabs(modH))**2

-calculate exponential
Lambda=4.1*(rainrate**)(-.21))
expower=EXP((-1.)*Lambda*x)
return
end

-c subprogram with vertical polarized function
function eqV(x)
include "common.reflect"
real eqV,x
call backscat(x)
modsqV=(x**6)*modsqV*expower
return
end

-c subprogram with horizontal polarized function
function eqH(x)
include "common.reflect"
real eqH,x
call backscat(x)
modsqH=(x**6)*modsqH*expower
return
end
Common file and data files used.

Common file

common pi,n,k,rainrate,modsqV,modsqH,expower,Lambda
real pi,n,k,rainrate,modsqV,modsqH,expower,Lambda

Data file with wavelengths and complex indices of refraction for water

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>n</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9.0353</td>
<td>1.399440</td>
</tr>
<tr>
<td>56</td>
<td>8.4158</td>
<td>2.18169</td>
</tr>
<tr>
<td>32.1</td>
<td>7.2621</td>
<td>2.85382</td>
</tr>
<tr>
<td>8.57</td>
<td>4.0320</td>
<td>2.44963</td>
</tr>
</tbody>
</table>

Data file with wavelengths for ice

<table>
<thead>
<tr>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
</tr>
<tr>
<td>56</td>
</tr>
<tr>
<td>32.1</td>
</tr>
<tr>
<td>8.57</td>
</tr>
</tbody>
</table>
program Deq

   calculate equivalent-volume diameter of raindrops
   by Green (J. Appl. Met. 14, 1578-1583) and by the
   empirical expression (b/a) = 1.03 - 0.062*Deq.
   the "dz" refers to Doviak & Zrnic's version of Green's
   equation (p.213).
   dump output in xgraph-compatible format
   Program written by Wayne Adams, 1992

integer i, imax
real dzr, gr, rstep, dzrmax, dzratio(500), gratio(500)
real Ts, grav, dens
real rat1, rat2, diaml(500), diam2(500), eratio

Ts is surface tension of water (J/m^2)
grav is acceleration of gravity (m/sec^2)
dens is density of water (kg/m^3)
   Ts = 7.275e-02
   grav = 9.81
   dens = 998.0

fix step size in axis ratio (rstep), maximum axis ratio (dzrmax),
and start both Green's and Doviak & Zrnic's axis ratios at 1.00
(one will increase, the other decrease, in their respective
notations)
rstep = 0.01
dzrmax = 2.0
dzr = 1.0
gr = 1.0

step through axis ratio values, calculating equivalent volume
diameters
   i = 1
   repeat
      rat1 = (dzr*dzr - 2.0*(dzr**((1.0/3.0)) + 1.0)*dzr**((1.0/3.0))
      rat2 = (1.0/(gr*gr) - 2.0/(gr**((1.0/3.0)) + 1.0)/(gr**((1.0/3.0))
      dzratio(i) = dzr
      gratio(i) = gr
      diaml(i) = 2*sqrt((Ts/(grav*dens))*rat1)*1000.0
      diam2(i) = 2*sqrt((Ts/(grav*dens))*rat2)*1000.0
      dzr = dzr + rstep
      gr = gr - rstep
      if (dzr .le. dzrmax) then
         i = i + 1
      go to 10
   endif
   until (r > rmax)
   imax = i

write (*,*) 'TitleText: Equivalent-volume diameter'
write (*,*) 'XUnitText: diameter (mm)'
write (*,*) 'YUnitText: axis ratio'
write (*,*) 'XLowLimit: 0.0'
write (*,*) 'XHighLimit: 10.0'
write (*,*) 'YLowLimit: 0.3'
write (*,*) 'YHighLimit: 1.0'
write (*,*) 'BoundBox: true'
write (*,*) 'Ticks: false'
write (*,*) 'Foreground: black'
write (*,*) 'Background: grey'

dump data from Doviak & Zrnic's version (reciprocal
axis ratio of Green)
write (*,*) ''
write (*,*) '""doviak/zrnic""
   do 20 i = 1, imax
      write (*,*) diaml(i), dzratio(i)
   20 continue
C 20 continue

C dump data from Green's equation; truncate at large diameter
write (*,*)
write (*,*) """"green"""
DO 30 I = 1, IMAX
   IF (DIAM2(I) .LE. 10.0) WRITE (*,*) DIAM2(I), GRATIO(I)
30 CONTINUE

C dump data from empirical equation; truncate
write (*,*) ' ',
write (*,*) """"empirical 1.03"""
DO 40 I = 1, IMAX
   ERATIO = 1.03 - 0.062*DIAM2(I)
   IF (DIAM2(I) .LE. 10.0) WRITE (*,*) DIAM2(I), ERATIO
40 CONTINUE

C now calculate with modified empirical equation; truncate
write (*,*) ' ',
write (*,*) """"empirical 1.00"""
DO 50 I = 1, IMAX
   ERATIO = 1.00 - 0.062*DIAM2(I)
   IF (DIAM2(I) .LE. 10.0) WRITE (*,*) DIAM2(I), ERATIO
50 CONTINUE

STOP
END
D. CP-2 Radar Plots

The CP-2 radar plots (figs. 4-6, 8, 9) are data taken from the Convection and Precipitation/Electrification Experiment (CaPE) project performed in Florida from June 8th to August 18th, 1991. The readings were taken on the 12th of August; "22 27 36-22 30 29" refers to the universal time (UTC) of one radar scan, the time elapsed for these frames to be taken was 2 minutes and 53 seconds. A Cartesian coordinate system is used, the radar is located at (0.0, 0.0). The negative X distances on the horizontal axis denote increasing distances in the left half of the coordinate plane, the "Y=37.50 KM" defines a plane parallel to the X-Z plane intersecting the Y axis at 37.5 kilometers. The "Z KM" marking the vertical axis refers to a height scale. The plots of $Z_{II}, Z_{DR}, LDR$ and HS are therefore a vertical "slice" of a storm taken in the second quadrant of the CP-2 coordinate system: