Sensitivity of a Short Range Forecast to Initial and Boundary Conditions

Christopher L. Castro

SOARS Protégé
The Pennsylvania State University
University Park, Pennsylvania

Research Mentor:

Dr. Tomislava Vukicevic
Climate and Global Dynamics Division
The National Center for Atmospheric Research
Boulder, Colorado

SOARS

Significant Opportunities in Atmospheric Research and Science

Sponsored by the National Science Foundation

August 9, 1996
Abstract

Modern numerical weather prediction models must be necessarily specified initial and boundary conditions as basic state vectors. Errors in these conditions affect the model forecast at verification time. An adjoint model efficiently determines where these errors have the greatest impact on the forecast, or, in other words, where the model is most spatially and temporally sensitive. Such a model calculates the sensitivity gradient and total sensitivity of a forecast aspect, or difference in a specified meteorological variable, with respect to the basic state vector. This study investigates the sensitivity of a 48-h forecast on 4-6 April 1982 using the Mesoscale Adjoint Modeling System (MAMS) developed at the National Center for Atmospheric Research (NCAR). We determine which basic state variables the nonlinear forecast model (NLM) within MAMS is most sensitive to, show the sensitivity gradient plots that indicate the synoptic prerequisites for cyclogenesis, and prove that the NLM is more sensitive to initial conditions. This research is intended as a model for future sensitivity studies on existing operational regional forecast models.

Introduction

Modern numerical weather prediction models, such as the Nested Grid Model (NGM), must necessarily be initialized and bounded. Errors in the initial and lateral boundary conditions (ICs and LBCs) can affect a model forecast at verification time. There are two ways to determine what the effect of errors will be. A seemingly straightforward approach is an impact study in which a control forecast is compared to an experimental one with altered LBCs or ICs. This might be appropriate in an instance where the number of input parameters is small. However, when the number of input parameters is literally huge, for example, in LBCs, ICs, or physical parameterizations, an impact study is impractical. It could only show the net result of one perturbation or series of perturbations, and exclude perturbations that generate instability. Any conclusion about forecast sensitivity from such a study would be flawed. To correctly diagnose forecast sensitivity, an inordinate number of perturbation experiments must be generated. In terms of supercomputing time, this is costly and wasteful.

A more efficient means to determine forecast model sensitivity is the adjoint method. In contrast to the impact study, which uses a numerical weather model to integrate change forward in time, the adjoint, a linear operator derived from the model, works backward in time. Each adjoint operator reveals the sensitivity of a particular forecast aspect, such as surface pressure, winds, temperature and vorticity, to all input parameters. This information can be displayed as a gradient field on maps to illustrate the portions of selected aspects most responsible for affecting the forecast at verification time. Not only does such an expedient method aid the forecaster in recognizing the strengths and weaknesses of a particular numerical weather model; it also provides insights into what pre-existing conditions are necessary for evolution of specific synoptic
phenomena, such as an extratropical cyclone. The adjoint model is very cost-effective because it conserves computer resources. An equivalent impact study that would yield the same result as computing the derivative of forecast aspects using the adjoint model would require more than a million experimental iterations.

Adjoint modeling has two main drawbacks. First, because adjoint model is linear, the sensitivity of nonlinear processes in the atmosphere may be inaccurate, especially as integration time increases. However, for a short-term 48-hour forecast, such as the NGM provides, Vukicevic and Errico (1993) concluded that the model forecast incorporating moist physics is fairly accurate. Second, the adjoint method requires that the forecast aspect be defined, and this method represents the sensitivity to the pre-defined forecast aspect only. Whereas the impact study considers the entire model using iterative solutions, we must evaluate additional adjoint solutions for each new aspect.

This paper investigates the sensitivity of a 48 hour forecast for a case of East Coast cyclogenesis on 4-6 April 1982 using the adjoint method. The model employed in this study is the Mesoscale Adjoint Modeling System (MAMS) developed at the National Center for Atmospheric Research (NCAR) by Ronald Errico, Tomislava Vukicevic, and Kevin Raeder. The specific questions this paper investigates are:

1. To which basic state variables is the nonlinear forecast model within MAMS most sensitive?
2. By analysis of the gradient fields generated for several forecast aspects, can any conclusions be made for the necessary pre-existing conditions for cyclogenesis in this sample case?
3. Is the forecast more sensitive to initial or boundary conditions?

In developing the answers to these questions, we will:

- Discuss the formal mathematical definition of the adjoint as applied to meteorology
- Give a brief description of the components of MAMS
- Describe the synoptic situation of 4-6 April 1982 using National Meteorological Center\(^1\) (NMC) and European for Medium Range Weather Forecasting (ECMWF) analyses
- Explain the experimental methodology
- Discuss the results of the adjoint experiments

---

\(^1\) Currently known as the National Center for Environmental Prediction (NCEP).
Mathematical Background of the Adjoint as Applied to Numerical Weather Prediction Models

A numerical weather prediction model is represented on a discrete, finite grid. Observed meteorological variables and topography are interpolated to each gridpoint to generate the basic state conditions of the model, or initialization at time zero. These basic state variables can be represented as three-dimensional vectors:

\[ x(0) = (u(0), v(0), q(0), T(0), p_s(0)) \]
\[ x_b = (u_b, v_b, q_b, T_b, p_{sb}) \]

Where:
- \( u \) = zonal wind component (m/s)
- \( v \) = meridional wind component (m/s)
- \( T \) = temperature (K)
- \( q \) = water vapor content (g/kg)
- \( p_s \) = surface pressure (mb)

Zero denotes initial conditions, and subscript \( b \) specifies the boundary conditions. The boundary is updated every twelve hours at the new analysis time and is therefore not time dependent in the model integration. From these basic state variables, additional vector quantities can be derived, those pertinent to this study are derived from the components of the wind:

Vertical vorticity (\( \zeta \)), or the rate of "spin" of air on a horizontal plane, defined as:

\[ \zeta = \frac{dv}{dx} - \frac{du}{dy} \]  \hspace{1cm} (1)

Convergence\(^2\) (\(-D\)), or the "coming together" of air, defined as:

\[ -D = \left(-\frac{du}{dx} + \frac{dv}{dy}\right) \]  \hspace{1cm} (2)

where \( dx \) and \( dy \) are the east-west and north-south gridspacing, respectively, in the numerical model.

---

\(^2\) Convergence is also negative divergence.
We define a forecast aspect (FA) as a difference in a specified meteorological variable between the model forecast \( F(\mathbf{x}(\text{model})) \) and the analysis \( F(\mathbf{x}(\text{obs})) \) at verification time. The forecast aspect can be represented as function \( J \), where \( J \) is a squared forecast error:

\[
\bar{J} = \frac{1}{2} \{ F(\mathbf{x}(\text{MODEL})) - F(\mathbf{x}(\text{OBS})) \}^2
\]  

(3)

Now we can take the partial derivative of the forecast aspect with respect to initial and boundary conditions (\( \mathbf{x}(0) \) and \( \mathbf{x}_b \)). To eliminate redundancy, we will carry through this and the following steps with the initial conditions only. Since \( F(\mathbf{x}(\text{obs})) \) is constant with respect to initial conditions:

\[
\frac{\partial \bar{J}}{\partial \mathbf{x}(0)} = \{ F(\mathbf{x}(\text{MODEL})) - F(\mathbf{x}(\text{OBS})) \} \frac{\partial F(\mathbf{x}(\text{MODEL}))}{\partial \mathbf{x}(0)}
\]  

(4)

The adjoint is the last term in the equation. Since \( \mathbf{x}(\text{model}) \) is related to the initial conditions through the model equations and parameterizations (M):

\[
\frac{\partial F(\mathbf{x}(\text{MODEL}))}{\partial \mathbf{x}(0)} = \frac{\partial F\{M(\mathbf{x}(0))\}}{\partial \mathbf{x}(0)}
\]  

(5)

By the chain rule, we write the term on the right hand side of (5) as:

\[
\frac{\partial F\{M(\mathbf{x}(0))\}}{\partial \mathbf{x}(0)} = \frac{\partial F}{\partial \mathbf{x}(M)} \frac{\partial \{M(\mathbf{x}(0))\}}{\partial \mathbf{x}(0)}
\]  

(6)

MAMS generates the last term on the right hand side of (6). The first term on the right-hand side is equal to 1 if our forecast aspect is defined as the total forecast-to-analysis difference in the basic state variables. For wind-derived variables of divergence and vertical vorticity (referred to henceforth as just vorticity), it is defined as the inverse of the gridspacing multiplied by a map factor that adjusts for curvature of the earth. To approximate the change in the forecast aspect (\( \Delta J \)), or total model sensitivity, we can use a Taylor's series approximation:

\[
\Delta \bar{J} \approx \frac{\partial \bar{J}}{\partial \mathbf{x}(0)} \Delta \mathbf{x}
\]  

(7)
In (7), $\Delta x$ is a perturbation in the initial conditions of the model. Substituting from (4) and (6) into (7) yields the final expression that approximates model sensitivity:

$$\Delta \hat{J} \equiv \{F(\hat{x}(\text{MODEL})) - F(\hat{x}(\text{OBS}))\} \frac{\partial F}{\partial \hat{x}(M)} \frac{\partial M(\hat{x}(0))}{\partial \hat{x}(0)} \Delta \hat{x}$$

(8)

For a more in-depth mathematical description of adjoint modeling, we refer the reader to Vukicevic and Errico (1991) and the NCAR Technical note on MAMS (Errico, et. al., 1994).

The Specific Models Used in This Study

The nonlinear forecast model (NLM) incorporated into MAMS is a modification of the Pennsylvania State University (PSU)/NCAR Mesoscale Model (MM4) developed by Anthes and Kuo (1987). This is a regional, primitive-equation, sigma-coordinate, mesoscale model (Vukicevic and Raeder, 1995). The adjoint portion of the model generates the forecast gradient with respect to input ($\partial J / \partial x(0)$) parameters, working backward in time from forecast verification time. Errico, Raeder, and Vukicevic (1994) modified the MM4 to incorporate it into MAMS, but, in principle, the adjoint portion of MAMS should be adaptable to any regional forecast model. We discuss some of the details of the MM4 as they pertain to the definition of $\Delta J$ in the results section of this paper. However, for more detailed information about MAMS and the MM4, we refer the reader to the NCAR Technical Notes listed in the reference section.

---

3 Specifically, our model uses ten sigma levels. The horizontal resolution is 120 km.
The Synoptic Case:
4-6 April 1982

Data Sources for Synoptic Analysis

When one is analyzing the synoptic situation, it is important to consider the standard products the operational meteorologist uses to construct a forecast and how the information is interpreted. The products considered are analysis and computer model output. Examples in both categories include:

Observational information:
- Upper-level observations, specifically the 300-, 500-, and 850-mb levels
- Surface observations
- Isentropic analysis
- Atmospheric cross sections

Model information:
- Forecast precipitation
- Divergence at 700-mb
- 1,000-500 mb thicknesses or 1,000-850 mb thicknesses
- Sea-level pressure
- 500-mb height and vorticity

Unfortunately, the time constraints in the project did not allow us to retrieve all the data that would have been available to the forecaster on that particular date. However, from the various sources available at NCAR, there is quite enough information to diagnose the factors that are contributing to the development and motion of this cyclone. We obtained our synoptic data from two sources. The first was 132 hours of ECMWF analysis data updated every 12 hours already stored on the mainframe computer at NCAR. The ECMWF analysis provided:
- Sea-level pressure (Appendix A)
- 500 mb heights (Appendix B)
- Vorticity at each sigma level\(^4\) (Appendix C)

---

\(^4\) Appendix C contains only the vorticity for \(\sigma = 0.65\).
From the NCAR library, we obtained the NMC archived data for:

- Radar plots
- Surface analyses (Appendix D)
- 850 mb analyses (Appendix E)

These sources are updated every hour, every 3 h, and every 12 h, respectively.

Choosing the Appropriate Forecast Window

Of all 132 hours of analysis data, a practical question arises: at which analysis hour should we start the 48-h forecast? We used three criteria to choose the forecast initial time. First, the forecast begins when the cyclone has already formed; that is, there is a cut-off low. Second, the forecast window for the 48 h forecast is such that it captures the cyclone through all stages of its life cycle: cut-off low, baroclinic leaf, mature cyclone, and occlusion. Third, we wanted to capture the cyclone primarily over land only.

The t=36 h time of the 132 hours of ECMWF analyses accounts for all of these facts. At this time, we see a cut-off low forming in the lee of the Rocky Mountains in the Oklahoma Panhandle. Climatologically, this has proved to be a favored area for cyclogenesis because of its topography (Smith, 1986). A simplified explanation for this is the conservation of potential vorticity. Consider an idealized case of geostrophic flow over a mountain range. Ertel's potential vorticity equation can be written in the form:

\[-g(\zeta + f) \frac{d\Theta}{dp} = P_v\]  \hspace{1cm} (9)

\(\zeta + f\) = Absolute vertical vorticity
\(\Theta\) = Potential temperature (isentrope)
\(p\) = pressure
\(g\) = gravitational force
\(P_v\) = Potential vorticity

\(P_v\) is conserved absent diabatic heating or friction (Holton, 1992).

On the windward slope there is surface divergence as flow constricts and the distance between isentropes decreases. In contrast, on the lee slope, there is surface convergence as the
flow expands, and the distance between isentropes increases. Therefore, westerly, extratropical flow will produce negative vorticity, or anticyclonic flow on the west side of a mountain range, and positive vorticity, or cyclonic flow, on the east side (McIlveen, 1992). The positive vorticity and surface convergence are key factors in the birth of the Oklahoma Panhandle low. Lee cyclogenesis is an observed phenomenon in several other areas in the world. In a recent paper, Vukicevic and Raeder (1995) used the adjoint method to determine the sensitivity of lee cyclogenesis to the Alps in Europe. We incorporated much of the same experimental design from that work into the present study.

**Factors in Cyclone Intensification and Movement**

The quasi-geostrophic omega equation, where omega (ω) is vertical velocity in pressure coordinates, yields five terms that contribute to cyclone intensification. Interpreted for practical forecasting application they are:

- Positive vorticity advection (PVA)
- Warm air (or thickness) advection
- Diabatic heating
- Minimized friction
- Downslope flow

Looking at the 500-mb height analyses and vorticity analyses for sigma = 0.65, there are two fundamental questions we need to ask. First, where are the areas of greatest positive vorticity advection, and, second, how are vorticity maxima amplifying the longwave pattern? The initial time shows a 500-mb height pattern that is relatively "uninteresting" weatherwise. There are two upper-air lows, one just west of British Columbia and the other over Ontario. However, over the majority of the United States there is zonal flow. This pattern persists until t = 60 h of the analysis, when a weak trough begins to form in the northern Great Plains.

From t = 36 h to t = 48 h there are several important changes in the vorticity pattern that give clues why the trough is forming. The vorticity maximum that is over California moves east over the Rockies and increases in magnitude. It also begins to "phase," or join with another vorticity maximum along the Texas Gulf Coast. The strong positive vorticity advection from the Rocky Mountain vorticity maximum causes the trough to continue to amplify. At t = 60 h, a jet

---

5 We chose sigma = 0.65 for vorticity analysis because that is about as close as to the 500 mb level vorticity analysis as the ECMWF data can approximate.
streak is evident over the Oklahoma and Texas panhandles by the vorticity maximum, vorticity minimum couplet. The presence of a jet streak is a significant clue that a storm is brewing in the southern Great Plains. The indirect transverse circulation in the left exit region of the jet maximum is causing upward motion and atmospheric destabilization in central Oklahoma and Arkansas. In the latter portion of the forecast, the phasing of the vorticity maxima is complete by \( t = 72 \) h, and PVA continues to increase over the upper Ohio Valley and into the Mid-Atlantic states. The strengthening of the original vorticity maximum from the Rockies as it moves eastward continues to amplify the longwave pattern.

A crude but quick method to estimate the degree of temperature (or thickness) advection is the "box method." This informal forecasting rule states that the advection occurs where height and thickness patterns cross to form "boxes" on the analysis or model forecast map. The maximum advection occurs where the smallest boxes are located, implying strong winds and a tight temperature gradient in that particular region. Using this method, from the NMC 850-mb analysis from 12Z 4 April to 12Z 6 April, the swath of greatest warm advection follows just south of the track of the 850-mb low, from Arkansas, up the Ohio Valley, and to the mid-Atlantic Coast by verification time (\( t = 84 \) h).

The last three items in the quasi-geostrophic omega equation are not very significant in terms of the cyclone under investigation. Diabatic heating effects are minimized because the forecast window does not include the time period when the center of the cyclone is over the warm open waters of the western Atlantic. Also, because the cyclone is over land of similar topography—the southern Great Plains and Ohio Valley—the effects of friction on the system are relatively constant. In the forecast window the only times when downslope flow, or the lee cyclogenesis phenomenon, is a factor in intensification are at the initial time and at the forecast verification time, when the cyclone is influenced by the Rockies and Appalachians, respectively.

Combining all these factors, the track of the surface low, as indicated by the analysis, is from the Oklahoma Panhandle, up the Ohio Valley, and off Virginia Beach, Virginia, by \( t = 84 \) h. In that period, the sea-level pressure drops from 1004 to 992 mb.

*What is Happening at the Surface*

Although the low develops in the Texas and Oklahoma panhandles by 4 April, the significant weather associated with the cyclone does not begin until 5 April. Looking at the surface analysis for 6Z, 4 April, dewpoints are still low (30s and 40s °F) over Oklahoma and Texas, although there is increasing cloudiness in southern Texas. The surface high over Mississippi and Alabama has not yet moved far enough eastward to develop a moist southerly
flow for significant precipitation. Cold, northwesterly flow around an old cyclone in Ontario is adventing cold, dry air from Canada into the New England, Mid-Atlantic, and southeastern states.

By 18Z, 4 April, a baroclinic leaf is forming in the southern plains over Texas and Oklahoma, indicated by the cloudiness over the region and radar echoes in southeastern Texas. The air is also moistening up rapidly as the atmosphere becomes destabilized with the movement of the surface high in the southeastern states toward the East Coast. For example, the dewpoint in Oklahoma City increases from 40 °F to 50 °F in a six-hour period (18Z, 4 April to 0Z, 5 April). Severe thunderstorms with hail break out over Arkansas and Louisiana during the early evening hours of 4 April. These continue throughout the night and move eastward into the southeastern states by the morning of 5 April. Snow develops during the same periods as warm, moist air overrides the cold surface high pressure dome in the northern plains. 850 mb temperatures are well below 0 °F, supportable for snow.

By 6Z, 5 April, there is a well-defined, intensifying closed circulation over western Oklahoma. By 15Z, occlusion has already occurred as the low moves into southern Missouri. On the northwest side of the low, there is a snowstorm in the upper Midwest, with the heaviest snows in northwest Missouri through Iowa and Wisconsin. Light rains continue over the southeast, while the areas of severe weather move northward into eastern Missouri and Illinois. The Canadian high that brought the cold, dry air into the northern Plains is ridging eastward into the Mid-Atlantic states and Southeast. 850-mb temperatures in the northeastern states are well below 0 °F by 0Z, 5 April. By 21Z, the occluded low, now in southern Indiana, has a secondary trough associated with it extending through the Ohio Valley toward Maryland. The severe weather earlier in the day moves into the Ohio and Tennessee valleys.

All of the ingredients come together for a snowstorm in the Mid-Atlantic states by 6 April: cold, dry air at the surface and warm, moist air overrunning above. The snowstorm that was in the Midwest increases in coverage and intensity as it moves into Ohio, Michigan, and Pennsylvania. More severe weather breaks out in the southeast over Mississippi and Alabama. By 12Z, 6 April, the end of our forecast window, the low in Pennsylvania starts to weaken as a new, even more powerful, surface low forms near Virginia Beach, Virginia. This is indicated by rapid surface pressure drops of over 5 mb/h throughout the Chesapeake Bay, Delmarva Peninsula region. At this point, two of the "insignificant" factors of the quasi-geostrophic omega equation, downslope floe due to the Appalachians and diabatic heating from the Atlantic, cause the greatest upward forcing at the coast.
How Does the NLM Forecast Do?

The results from the 48-h nonlinear forecast model (NLM) show distinct differences between the model forecast and the analysis. Although the track of the surface low is similar, through the Ohio Valley to the Mid-Atlantic Coast, the strength is not. The NLM forecasts a low surface pressure of 979 mb centered in southern Virginia, slightly west of where the actual surface low is observed. The question for investigation is what factors are causing the NLM to forecast a more powerful cyclone than is synoptically observed?

Experiment Method and Procedures

We ran MAMS on a Cray Y-MP8 Supercomputer ("Shavano") at NCAR. The steps involved in data gathering and processing were:

Generating Model Forecast and Perturbations for Initial and Lateral Boundary Conditions

As stated previously, ECMWF analyses for the 132 hour synoptic period were already stored in computer memory and available for use in this project. These analyses, updated at 12 h intervals, are more than adequate to cover the lifetime of the particular cyclone under investigation. Using the NLM within MAMS, we produced two 48-h short range forecasts. We started the first at hour 0 of the analysis, and the second at hour 36. We integrate the adjoint model for the forecast with 36 h as the initial time, when the low pressure system has just moved off the Rockies and into the Great Plains (see synoptics). The first forecast is necessary because we need perturbation values for meteorological variables to calculate the sensitivity. These perturbations are the difference between the first forecast and analysis at t = 36 h. We also wanted to generate boundary perturbations by running a larger-domain version of the NLM, but we were not able to complete this run due the time available to complete this project.

Running the Adjoint Model

From the NLM and analyses, we calculated and plotted a forecast difference at verification time (t = 48 h in the forecast) using a MAMS processor program. With the adjoint model, we calculated the sensitivity gradients of the whole domain and of a smaller domain localized around the area of influence of the cyclone. This "box" about the cyclone is about 500 miles on either
side of the center of low pressure or about half a Rossby wavelength in an amplified upper-level pattern. For our first two adjoint experiments, we defined the forecast aspect \( (J_1) \) as the difference of the entire forecast to the analysis for the whole- \( (J_{1a}) \) and box-domain \( (J_{1b}) \) cases.

For the next set of adjoint experiments, we defined our forecast aspects as the forecast-to-analysis difference in vorticity \( (J_2) \) and convergence \( (J_3) \) in the box domain. In these second set of experiments, we halved the number of sigma levels to capture only the sensitivity of the forecast in the lower troposphere. If we look at it from the point of view of the forecaster, this region of the atmosphere is more important than the upper levels. The 500-mb height-vorticity pattern is typically used to determine the sign of the vorticity advection because 500-mb is near the level of non-divergence. As mentioned, the determination of positive or negative vorticity advection (PVA or NVA) is a factor in cyclone intensification. In their study of alpine lee cyclogenesis, Vukicevic and Raeder (1995) only analyzed the sensitivity to vorticity in the lower troposphere because the vortex of low pressure was most clearly defined there. Divergence is shown at a level of 700 mb on the NMC standard analysis maps because that is in the lower to mid-troposphere, at the level where water condenses to form precipitating clouds. Therefore, determination of upward vertical motion is most critical in this region of the atmosphere.

**Plotting the gradients of the forecast aspects**

The adjoint model provides us with the derivative of a forecast difference \( (\Delta J) \) with respect to an initial or boundary condition basic state variable at each hour of the analysis. Thus, it works backward, integrating from verification time back to initialization with a basic state update. To compare the relative magnitudes of these sensitivity derivatives with the adjoint model, we wrote a subroutine to normalize, or weight, the derivatives by the inverse standard deviations of the NLM-to-forecast difference. We did this for all three adjoint experiments by modifying the adjoint model within MAMS. These gradients are displayed on a map of the model domain, so we can quickly determine:

- The location the forecast is most sensitive at verification time
- Temporal and spatial patterns of sensitivities
- The relative magnitudes of sensitivity
- The differences in sensitivity between the whole domain and box case for forecast aspect \( J_1 \).
Calculating Total Sensitivity and the Sensitivity Gradient

There are two ways we looked at forecast sensitivity. In the first method, we calculated the total sensitivity ($\Delta J$) by multiplying the derivatives of the forecast aspect with respect to basic state conditions by the perturbation values ($\Delta x$) already determined. Using this approach we can look at the contributions of different basic state variables to total sensitivity. The expression for total sensitivity, a scalar value, is:

$$\Delta J_{TOT} = \frac{\partial J}{\partial p_s(0)} \Delta p + \frac{\partial J}{\partial u(0)} \Delta u + \ldots \quad (10)$$

...for all basic state variables

By plotting these total sensitivity components against each other on a histogram plot, we determined which basic state variable our forecast aspect was most sensitive to at the initial time. Note that this portion of the experiment worked for initial conditions only.

In the second method, by squaring the derivatives from the third step and multiplying by the weighting factor, we determined a gradient norm of sensitivity for each forecast aspect. The expression for the total gradient norm is:

$$\left( \frac{d\bar{J}}{d\bar{x}(0)} W \right)^2 = \left( \frac{\partial J}{\partial p_s(0)} w_{ps} \right)^2 + \left( \frac{\partial J}{\partial u(0)} w_{u} \right)^2 + \ldots \quad (11)$$

...for all basic state variables

By looking at the components of this gradient norm, we could view how the sensitivity gradients change in time for each basic state variable. This portion of the experiment successfully worked for both initial and boundary conditions. We added the subroutines to calculate the total sensitivity and gradient norm values within a MAMS processor program.
Experiment Results

Format of Results and Caveats in the Data

Before presenting and discussing the results from our adjoint experiments, we need to explain some of the background and caveats behind the conventions in our data: the adjoint time intervals, the gradients as displayed on maps, the weighting scheme, and the factors included in the measure of total sensitivity.

Since the adjoint model works backward in time, its output is presented in negative time intervals, starting from verification. In our experiment, for example, verification time is denoted as \( t = 0 \) h and initialization as \( t = -48 \) h. These times are equivalent to \( t = 84 \) h and \( t = 36 \) h, respectively, in the ECMWF analysis.

We plotted the gradients of each of our three forecast aspects on geographical maps of the North American model domain. We show the forecast verification box-domain about the cyclone on all gradient field plots. These gradient fields approximate the geographic locations to which the designated forecast aspect is most sensitive. Positive gradients show as solid lines, negative as dashed. The interpretation of these gradient fields is a bit tricky. Recall that we multiply the gradient by the perturbation to determine total sensitivity. The change in the forecast aspect is opposite the gradient if multiplied by a perturbation of the opposite sign, and the same as the gradient if multiplied by a perturbation of the same sign (Vukicevic and Raeder, 1995). To illustrate, we take the gradient of vorticity (\( \Delta J_2 \)) with respect to water vapor content (\( q \)) in the lower levels (\( \sigma = 0.85 \)) for \( t = -48 \) h (Figure 1). We see a large negative gradient over the western Gulf of Mexico. If the forecast has a stronger value of vorticity, in order to decrease vorticity we must lower water vapor content in the Gulf. The boundary gradients are interpreted in the same way, with the boundary being a five point “sponge” region at the edge of the model domain where change is advected in.

The first caveat in our experiment is the determination of the weighting factors (\( w \)). The best physical interpretation of this weighting factor is the trust we place in our measurements of the basic state variables. In this respect, the "perfect weight" would be an accurate representation of data assimilation, or analysis, error. In light of the fact that analysis errors for the 1982 ECMWF analyses are unavailable, we had to devise the weighting scheme of our own, previously described, to yield a "first guess" approximation of this error. We chose as a weight the inverse standard deviation of the forecast-to-analysis difference at verification time. We altered this weighting scheme for the box case, called the reduced-weight box, for the surface pressure and water vapor content because the weights for these variables using the strict standard deviation approach did not make sense. For the water vapor weight, values in the upper atmosphere were
Figure 1: Sensitivity gradient pattern at low levels for FA vorticity ($J_2$) with respect to initial water vapor content ($q$) at $t = -48$ h.

highly inflated, greater than $10^3$, because of the invariability of humidity there. Since upper-air humidity has little impact on meteorological processes in the troposphere anyway, we arbitrarily assigned the upper sigma levels weights of 0. We calculated the standard deviation of surface pressure, defined only two-dimensionally, as 0.5 mb. This measure seemed too small to us, or, in other words, gave more "trust" to the surface pressure measurement. So we divided the calculated surface pressure weight by a factor of 4.

While this fudging in weighting for surface pressure and water vapor content may seem unscientifically arbitrary, we can justify it. First, the weights were derived from a first-order estimate of analysis error. Second, as in the case of water vapor content in the upper levels, the weights make meteorological sense.
Finally, the second caveat is the measure of total sensitivity. It is not just the sensitivity of the NLM to initial and boundary conditions. Total sensitivity includes sensitivity to model error as well, which we cannot quantitatively measure in this experiment. Model error consists of two approximations common to climate and weather models: discretization and parameterization. For the modified version of the MM4, this gridspacing is 120 km. Features and processes on smaller scales than this, either topographic or atmospheric, cannot be resolved by the primitive equations of the model. Fore the unresolvable physical processes, the model requires parameterizations, or estimative schemes. The NLM has four principal parameterizations (from Anthes and Kuo, 1987, and Errico, et.al., 1994):

1. Diffusion of horizontally propagating waves.
2. Net radiative flux
   - Incoming: Function of shortwave transmissivity, solar constant, zenith angle, albedo, and longwave emission from clouds
   - Outgoing: Function of surface emissivity, ground temperature, precipitable water, atmospheric temperature, and absorption and scattering effects of clouds.
3. Sensible and latent heat flux in the planetary boundary layer (PBL).

*Interpreting the Gradient Fields*

The ten experiments we ran generated an enormous amount of data. For each experiment, there are maps of gradient fields for each of the five basic state variables at high, middle, and low levels of the atmosphere\(^6\) (except surface pressure because it is a two-dimensional field). To include the more than 500 gradient field plots from all our experiments in this paper would be neither helpful nor instructive. To condense these results for interpretation, we did several things. First, we chose to look at only the times \(t = -48\) h and \(t = -12\) h in the adjoint model output, the initialization time and period immediately before forecast verification time. Second, rather than consider each experiment individually, we decided to group them together as two ensembles for all forecast aspects: initial condition experiments and boundary condition experiments. The broad agreement in gradient fields within our ensembles allowed us to do this and make generalizations about significant regions to which the forecast is most sensitive. The extraordinary similarity in

\(^6\) \(\sigma = 0.35, 0.65, \text{ and } 0.85\), respectively.
the gradient fields between the whole- and box-domain cases also leads us to the conclusion that the East Coast cyclone is dominating the forecast.

**The Initial Conditions Ensemble**

The most critical region at $t = -48$ h, demonstrated by the gradient plot of water vapor content in the lower levels\(^7\) (Figure 1), is in the western Gulf of Mexico and southeastern Texas. This is not surprising, considering nearly all great East Coast storms, such as March of 1993 or January of 1996, are born in the warm waters of either the Gulf of Mexico or Gulf Stream waters of the Atlantic. The adjoint model suggests several ingredients to generate a more powerful cyclone. Most importantly, humidity is higher just south and west of where the low forms in the Oklahoma Panhandle. Secondly, southerly flow is stronger at the surface to advect the moisture. Finally, greater baroclinicity exists between the Mexican Plateau and the Gulf of Mexico to contribute toward atmospheric instability. These surface factors point to a more vigorous shortwave pattern in the southwestern states, shown by the undulating pattern in the gradient of temperature (Figure 2), amplifying an upper level trough over western Texas. The surface high over the southeast would have to be stronger to increase the southerly flow.

\[ \frac{\partial J_1}{\partial T} \text{ for } t = -48 \text{ h, } \sigma = 0.35 \]

**Figure 2:** Sensitivity gradient pattern at high levels for FA vorticity ($J_2$) with respect to initial temperature at $t = -48$ h.

\(^7\) The gradient plots for temperature, sea-level pressure, and meridional wind also showed the highest sensitivity in this region.
A key synoptic feature affecting model sensitivity before verification time, at \( t = -12 \) h is the ridging surface high pressure system from Canada. This high advects cooler, drier air from the north down the east side of the Appalachians. The NMC surface analysis for 15Z, 5 April, reveals dewpoints in the 20s and 30s °F, over most of the Carolinas (see Appendix D). Sample gradient plots for meridional wind, humidity, and temperature for the three experiments in the lower atmosphere show an area of high sensitivity concentrated in the Carolinas and Georgia (Figure 3). The adjoint results suggest that the surface high inhibits storm development. The northerly flow in the southeastern states deprives the cyclone of the diabatic heating source of the Gulf Stream as it moves into the upper Ohio Valley. Second, the high pressure produces negative vorticity and cold air advection. The stronger storm generated by the NLM is coupled with a more amplified, negatively tilted (southeast to northwest orientation) trough in the longwave pattern. The magnitude of the gradients for both high and low levels of the atmosphere is about the same at this time for each of the three experiments.

**Figure 3:** Sensitivity gradient pattern at low levels for FA convergence \( (J_2) \) with respect to initial water vapor content at \( t = -12 \) h.
Boundary Conditions Ensemble

Unlike initial conditions, we cannot extract any useful information from the boundary gradients for $t = -12$ h. In so short a time from verification, the boundaries have very little effect upon the final solution. What we see instead are nondescript patterns of gradients over all edges of the domain. The gradients are smaller, by several orders of magnitude, compared to gradients at $t = -48$ h. Essentially, the gradient patterns at $t = -12$ h are due to the effect of gravity waves that move at higher speeds than synoptic or mesoscale features. A gravity wave is defined as a wave in the atmosphere in which gravity is the restoring force (McIlvien, 1992). A simple illustration of gravity waves is when air crosses a mountain in a stable atmosphere. The air rises to cross the mountain, then sinks when it encounters warmer air above, rises when it encounters colder air below, and so on.

The forecast is most sensitive to boundary conditions at the initial time, when we see distinct gradient patterns in specific areas. This fact is not surprising, given that the cyclone at this time is closest to the inflow boundaries. For all forecast aspects, there are two areas of interest in the gradient plots. In the upper levels there is a high sensitivity in the eastern Pacific due to the inflow of the jet stream there (Figure 4). In the lower levels, highest sensitivity is in the Bay of Campeche (Figure 5). That fits into our earlier assessment that conditions over the western Gulf of Mexico at the initial time are affecting the forecast at verification. There is probably a low-level jet stream advecting warm air and humidity through that boundary.

Total Sensitivity and Gradient Norm Results

We emphasize that total sensitivity and gradient norm results are the most important aspect of this study. This is the first time MAMS has been modified to generate these results, and, hopefully, may set a precedent for future sensitivity studies.

The results of the total sensitivity calculations (for initial conditions only) are shown in Figure 6. Looking first at the last three sets of data in the histogram, for the first two forecast aspects ($J_{1a}$ and $J_{1b}$), we see some expected and unexpected results. Recall that to get a more accurate representation of the analysis error, we reduced the weights for surface pressure and humidity for the reduced-weight box. Therefore, we should expect that total sensitivity should drop in that case. For humidity it does, but for pressure it does not. In fact, for pressure it is nearly the same result as in the cases with the
Figure 4: Sensitivity gradient pattern at high levels for FA convergence ($J_2$) with respect to boundary zonal wind at $t = -48$ h.

Figure 5: Sensitivity gradient pattern at low levels for FA convergence ($J_3$) with respect to boundary water vapor content ($q$) at $t = -48$ h.
nonreduced weights. We are at a loss to explain why pressure maintains the same sensitivity. Total sensitivity results for vorticity ($J_2$) and convergence ($J_3$) are more explainable. Since these are derived quantities from wind, they are most sensitive to that parameter. The sensitivity to humidity, almost equal to that of wind, is due to the effects of convective activity (i.e. thunderstorms) on wind patterns.

![Total Sensitivity for Forecast Aspects (Initial Conditions Only)](image)

**Figure 6:** Total Sensitivity calculation results (from equation 10).

Plots of the gradient norm with time illustrate how the sensitivity gradient changes in time. The graph of gradient norms with time is similar for all cases. The reduced-weight box domain results with respect to initial conditions is an illustration (Figure 7). A note on this graph: the lines appear nearly horizontal because of the skewed logarithmic scale. We can gain more qualitative information by graphing the change in the gradient norm for pressure and humidity, the two basic state variables with gradient norms of the highest magnitude (Figures 8 & 9). The sensitivity to pressure drops rapid after the initial time to a near constant value; sensitivity to humidity increases in the beginning periods of our forecast, when the cyclone is advecting moisture from the Gulf of Mexico, then drops off as it occludes. For the boundary conditions (Figure 10), the result is the same, and not surprising, for forecast aspects: gradients decrease steadily from the initial time. The boundaries, in short, have more effect on the forecast the longer the forecast runs.
Figure 7: Gradient norm results for FA reduced-weight box, initial conditions (from equation 11).

Figure 8: Change in the humidity gradient for FA reduced-weight box, initial conditions.
Figure 9: Change in the pressure gradient for FA reduced-weight box, initial conditions.

Figure 10: Gradient norm results for FA box, boundary conditions (from equation 11).
Finally, our last result is the comparison of the gradient norm magnitudes of the initial to boundary conditions (Figure 11). To obtain this result, we time-averaged the gradient norms and took a ratio of initial to boundary conditions. Like the total sensitivity plots, there are some results for which we can and some for which we cannot find reasonable explanations. The first conclusion drawn immediately from the plot is that initial conditions dominate almost exclusively for sensitivity to winds and temperature. However, they dominate by varying orders of magnitude for the basic state variables. The sensitivity to humidity and pressure, while still dominated by initial conditions by one to two orders of magnitude, are more influenced by the boundaries (for all forecast aspects). We offer a possible explanation. From our initial condition sensitivity results, we know that pressure is most sensitive to the initial conditions at \( t = -48 \) h. These initial condition sensitivities are concentrated near an inflow boundary, the Bay of Campeche in the southern Gulf of Mexico. The gradients most influenced by the boundaries are pressure for the reduced-weight box forecast aspect and humidity for the convergence forecast aspect. The pressure result is extremely curious, and the only educated guess as to why we get such a result is that the rate of pressure decrease in the reduced weight box with respect to initial conditions far exceeds the rate with respect to boundary conditions. We can offer no reasonable explanation to suggest why convergence is more sensitive to humidity at the boundaries. These last results are perhaps the most revealing in this study. Whether or not a forecast aspect will always be more sensitive to pressure and humidity at the model boundaries is a question that merits further research.
Figure 11: Comparison of initial to boundary condition gradient norms.
Conclusions

For our specific synoptic case study we conclude that:

1. Sensitivity gradients are concentrated in specific locations and patterns for defined forecast aspects. This gives clues to the synoptic prerequisites for cyclogenesis. The stronger storm forecast by the NLM is primarily a result of the enhanced diabatic heating due to convection in the Gulf of Mexico and over the Gulf Stream in the Atlantic.

2. Plotting the sensitivity gradients in time allows us to see cyclone development in time. For example, as the cyclone matures, the sensitivity to humidity and temperature decreases.

3. The effect of boundary conditions on a short-term forecast in the box-verification domain is smallest for winds and temperature. Initial conditions still dominate for humidity and pressure, but not by as much. There are two boundaries that affect the forecast in a significant way. The first is the upper-level boundary in the Eastern Pacific because of the jet stream. The second is the lower-level boundary in the Bay of Campeche. The Bay of Campeche boundary plays a role in moisture advection in the early parts of the forecast, proven by order of magnitude comparison.

4. Divergence and vorticity are most sensitive to winds and humidity. This is not surprising since they are derived from the wind and convective storms influence the wind patterns.

General Conclusions:

1. We cannot say the results for this particular cyclone will be the same for all cases of East Coast cyclogenesis. This procedure needs to be redone on a series of similar cases to collect statistical data to determine if any generalizations can be made for cyclones that form in the lee of the Rocky Mountains. In particular, would the same anomalous results we could not explain appear in a series of cases, or are these values just due to random error or natural variation?
2. There needs to be a better way to estimate analysis error to create appropriate weighting factors in our calculations. Possible alternatives would be to obtain an estimate from the National Center for Environmental Prediction (NCEP) which releases synoptic data to forecasters.

Questions for Further research:

1. If the forecast verification region were moved closer to an inflow boundary, how would that affect sensitivity to the boundary with time?

2. An adjoint model has already been developed for the ETA forecast model, which meteorologists use to make forecasts. A procedure such as the one we have done here on a series of cases would be a superb application of this mathematical technique to aid the weather forecaster and improve model forecasts for public distribution.
References


Acknowledgments

Technical Support

• Dr. Tomislava Vukicevic, NCAR Climate and Global Dynamics Division (CGD), SOARS Scientific Mentor

• Kevin Raeder, CGD

• CGD Systems Staff, particularly Colleen O’Toole and Allen Walker

• Marie Boyko, University of Colorado, Department of Kinesiology, SOARS Technical Writing Instructor

• Brian Bevirt, Scientific Computing Division, SOARS Technical Writing Mentor

Community Support

• Steven Sadler, UCAR Director of Health and Environmental Safety, SOARS Community Mentor

• Dr. Thomas Windham and Sangeeta Mishra, SOARS Coordinators

• SOARS Protégés, in particular Carl Etsitty and Quindi Franco

• Carole and Thomas Reed of Loveland, Colorado

AND THANKS TO ALL THE EMPLOYEES
AT
NCAR MESA LABORATORY
APPENDIX A:

ECMWF Sea-Level Pressure Analyses
Over the Forecast Window
Scaled in millibars (mb)

$t = 36 \text{ h} \quad \text{CUT OFF LOW Initialization}$

$t = 48 \text{ h} \quad \text{BAROCLINIC LEAF}$

$t = 72 \text{ h} \quad \text{OCCLUSION}$

$t = 84 \text{ h} \quad \text{OCCLUSION AND REGENERATION}$

$t = 60 \text{ h} \quad \text{MATURE CYCLONE}$
APPENDIX B:

ECMWF 500-mb Analyses Analyses
Over the Forecast Window
APPENDIX C:

ECMWF Vorticity Analyses for $\sigma = 0.65$
Over the Forecast Window
APPENDIX D:

Sample NMC Surface Analyses
Over Forecast Window
APPENDIX E:

Sample NMC 850-mb Analyses
Over the Forecast Window