Coronal Solar Magnetism Observatory
Technical Note #7, Rev #0

Scattered Light from Internal Reflection in a Coronagraph Objective Lens

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Summary: That the internally reflected light in the objective lens of a coronagraph is a source of scattered light has been recognized since the pioneering work of Lyot[1]. His solution was to block this unwanted light with an occulting spot in the image of the objective lens; this occulter is now known as the "Lyot spot". This note intends to quantify the geometry and magnitude of this internally reflected light.

Lyot Spot Location

Assume a plano-convex thin lens of index n, surrounded by material with index = 1. The small angle approximation will be assumed ($\sin\theta \approx \theta$ and $\tan\theta \approx \theta$). Light is incident at a small angle to the optical axis of the objective lens and strikes the first surface at an angle θ with respect to the surface normal. The geometry of the refracted and internally reflected rays is shown in Figure 1. Upon internal reflection from the back surface of the lens, the front surface of the lens acts as a concave mirror which forms a ghost image some distance behind the lens. The distance of the real image behind the lens, f_i, is proportional to the reciprocal of the tangent of the angle of the refracted ray exiting the lens, ϕ, while the distance of the ghost image behind the lens, f_g, is proportional to the reciprocal of the tangent of the angle of the internally reflected ray exiting the lens, ϕ'.

$$f_i \propto \frac{1}{\tan\phi} \ , \ \text{and} \ \ f_g \propto \frac{1}{\tan\phi'} \ , \ \text{then} \ \frac{f_g}{f_i} = \frac{\phi}{\phi'}$$

$$\text{from Snell's law:} \ \frac{\theta}{\theta'} = n\,, \ \ \frac{\theta-\theta'}{\phi} = \frac{1}{n}, \ \text{and} \ \ \frac{3\theta-\theta'}{\phi'} = \frac{1}{n}$$

$$\text{which results in:} \ \boxed{\frac{f_g}{f_i} = \frac{n-1}{3n-1}}$$

For a typical value of the refractive index, n=1.5, $\frac{f_g}{f_i} = \frac{1}{7}$. Then the ghost image will appear at a distance behind the lens of approximately one seventh of the focal length of the lens. Blocking the Lyot spot with an occulter in the image of the objective will not be optimal since the spot will be slightly out of focus there.

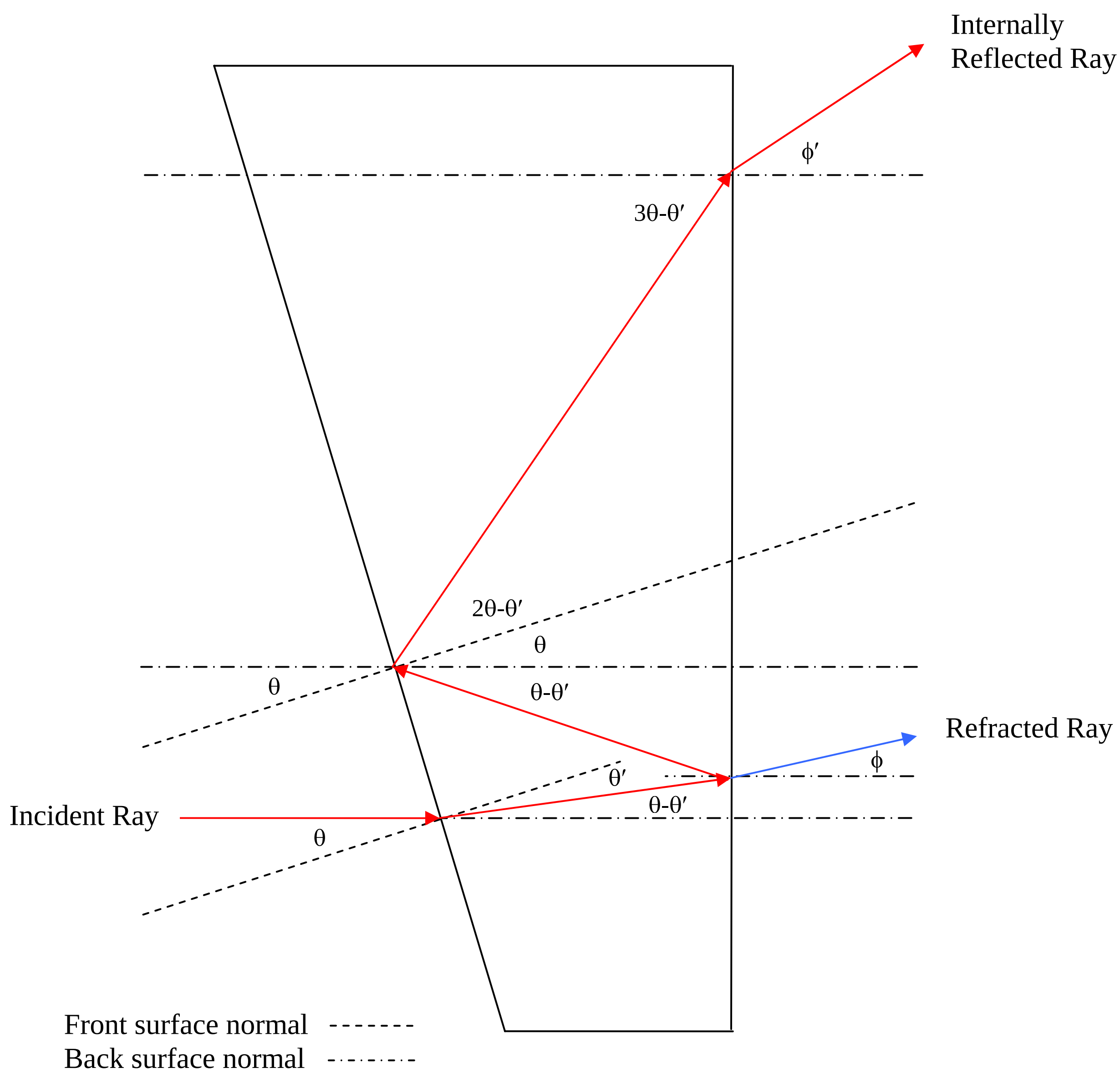


Figure 1. Diagram showing the ray paths for the refracted and internally reflected rays in an off axis section of a lens.

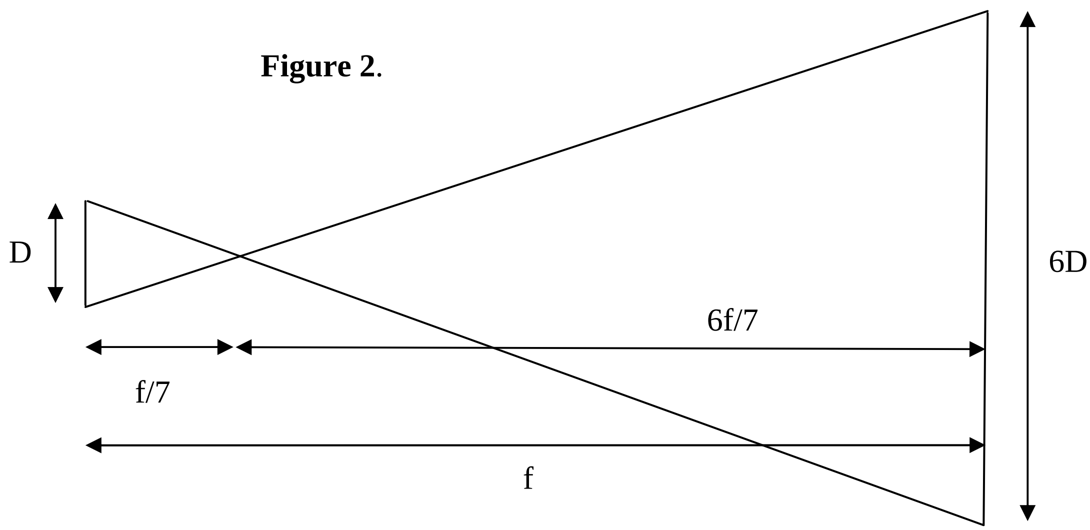
Lyot Spot Brightness

B = solar surface brightness integrated over some wavelength range (phots cm^{-2} $ster^{-1}$ s^{-1})
A = surface area of objective lens (cm^2)
Ω = solid angle subtended by Sun (ster)
r = objective lens reflectivity per surface
f = objective lens focal length (cm)

Solar light passing through the lens forms a solar image with intensity $B\,\Omega A\,(1\text{-}r)^2$ (phots s^{-1}) with an area in the focal plane of Ωf^2 (cm^2), while light internally reflected in the lens has intensity $B\,\Omega A\,(1\text{-}r)^2\,r^2$. Then, the light in the Lyot spot is r^2 less than the solar light, which is of order 10^{-3}; this is large in comparison to the intensity of the solar corona. However, the light in the Lyot spot is diverging quickly since it is focused close to the lens.

Assuming that the Lyot spot is formed at a distance of f/7 from the back of the lens which has a diameter D, then the projected area of the Lyot spot in the focal plane will be 36 A as is shown in Figure 2.

Figure 2.



The energy density of the solar image and the Lyot spot in the focal plane are:

$$E_{sun} = \frac{B\Omega A(1-r)^2}{\Omega f^2}, \quad \text{and} \quad E_{spot} = \frac{B\Omega A(1-r)^2 r^2}{36A} \quad (\text{phots cm}^{-2}\ \text{s}^{-1})$$

then, the ratio of energy density in the spot to the Sun is: $\boxed{\dfrac{E_{spot}}{E_{sun}} = \dfrac{\Omega r^2 \mathrm{f\#}^2}{9\pi}}$

where f# is the focal ratio of the objective.

For a reflectivity of r=0.05 and solid angle $\Omega=6.8\cdot10^{-5}$ ster, $\frac{E_{spot}}{E_{sun}} = 6\cdot10^{-9}\,\mathrm{f}\,\#^2$, which agrees with the result quoted in Newkirk and Bohlin[2].

The scattered light due to internal reflection in the objective lens will be less than 10^{-6} for typical lens reflectivity and focal ratios faster than about f/13. Instruments which will observe the corona through the atmosphere with internally occulted coronagraphs, can probably neglect this source of scattered light.

References

1. Lyot, B., 1932, *l'Astronomie*, **46**, 272. The English translation of this article can be found in Lyot, B, 1933, *Journal of the Royal Astronomical Society of Canada*, **27**, 265.

2. Newkirk, G. Jr, and Bohlin, D., 1963, *App Op*t, **Vol. 2**, No. 2, 131.