An Integer-based Method for Polygon Intersections in the Plane

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July 28, 2010
What I will talk about.
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- Where in atmospheric science is polygon intersection used?
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- O’Rourke’s Chasing Algorithm
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- O’Rourke’s Chasing Algorithm
- Testing
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- Where in atmospheric science is polygon intersection used?
- O’Rourke’s Chasing Algorithm
- Testing
- Further Study
Atmospheric Sciences
Atmospheric Sciences

Regridding: Need to map between two different grids for coupled climate models.\(^a\)

\(^a\)CORE: A Software Component for Conservative Remapping
Semi-Lagrangian transport\(^a\):

\(^a\) [Peter H. Lauritzen, 2010]
Semi-Lagrangian transport\(^a\):

- Grid moves with fluid

\(^a\)[Peter H. Lauritzen, 2010]
Semi-Lagrangian transport$^a$: 
- Grid moves with fluid
- Computing intersections allows one to regrid between Lagrangian and Eulerian grids.

$^a$[Peter H. Lauritzen, 2010]
Robustness Issues

- Double precision and $\epsilon$'s
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Robustness Issues

- Few methods have an easy testing method.
Basic Steps

1. Read in the grid's vertices and convert to rational values.
2. Break polygons up into edges and compare.
3. Determine if edges intersect.
4. If they don't intersect, determine if edge vertices belong to the intersection.
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- If they don’t intersect, determine if edge vertices belong to intersection
Initial vertices are approximated by rational values to within a given $\epsilon$.\(^1\)

\(^1\)[Jerome Spanier, 1987]
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Double to Rational

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\(^{1}[\text{Jerome Spanier, 1987}]\)
Initial vertices are approximated by rational values to within a given $\epsilon$.  

$^{1}$[Jerome Spanier, 1987]
Chasing Algorithm

Uses a Cross Product to define 4 variables for two given edges. 5 different “advancing rules” depending on the values of those variables.
Chasing Algorithm

\[ A \times B = 0 \]
Chasing Algorithm

\[ A \times B > 0 \]
Chasing Algorithm

\[ A \times B < 0 \]
Chasing Algorithm

\[ A \times B > 0, \text{ Vertex-Edge intersection. Advance polygon with vertex.} \]
Chasing Algorithm

$A \times B = 0$, Edge-Edge intersection, advance both polygons.
Chasing Algorithm

\[ A \times B > 0 \]
Chasing Algorithm

\[ A \times B < 0 \]
Chasing Algorithm

\[ A \times B < 0, \text{ proper intersection, advance "outside" polygon.} \]
Chasing Algorithm

\[ A \times B < 0 \]
$A \times B < 0$, compared all edges at least once.
Parameterizing Edges

To find intersection of edges, parameterize into lines...
Parameterizing Edges

\[ A = b - a, \quad B = d - c \]
Parameterizing Edges

\[ f_1(t) = a + tA, \quad f_2(s) = c + sB \]
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If \( 0 \leq t \leq 1 \), then intersection is on edge,
Parameterizing Edges

\[ f_1(t) = a + tA, \quad f_2(s) = c + sB \]

- If \( 0 \leq t \leq 1 \), then intersection is on edge,
- Else intersection is off edge and we don’t care.
Parameterizing Edges

Solving for \( t \) and \( s \) can involve 17 multiplications
Parameterizing Edges

Expressing everything as a rational number reduces multiplications to $5 - 9$.
Multi-Precision Integers

- Even 5 multiplications can create overflow: $9999^5$ needs 20 digits!
Multi-Precision Integers

- Even 5 multiplications can create overflow: $9999^5$ needs 20 digits!
- Creating a multi-precision integer data type with simple arithmetic operations solves this problem!
Area Test

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Area Test
Area Test
Area Test
Area Test

An Integer-based Method for Polygon Intersections in the Plane
Voronoi Grids

An Integer-based Method for Polygon Intersections in the Plane
Visual/Debugging Test
Future Work

2 [Vatti, 1992] and [Jon L. Bentley, 1979]

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Future Work

- Change algorithm to allow for non-convex polygons \(^2\)

\(^2\) [Vatti, 1992] and [Jon L. Bentley, 1979]
Future Work

- Change algorithm to allow for non-convex polygons \(^2\)
- Modify algorithm to work on a sphere, not just a plane

\(^2\)[Vatti, 1992] and [Jon L. Bentley, 1979]
References I


