Applications of Statistics to Modeling the Earth's Climate System

Notes from the Colloquium organized by the National Center for Atmospheric Research Statistics Project, Boulder, Colorado, 6–19 July, 1994. The NCAR Statistics Project is sponsored by the Division of Mathematical Sciences of the National Science Foundation.

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APPLICATION OF STATISTICS TO MODELING THE EARTH’S CLIMATE SYSTEM
NCAR Colloquium — 6 to 19 July 1994

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PREFACE

NCAR’s Statistics Project held a Colloquium “Applications of Statistics to Modeling the Earth’s Climate System” from 6 July through 19 July 1994. The Statistics Project is sponsored by the Division of Mathematical Sciences of the National Science Foundation (DMS-9312686). Its goal is to encourage the use of modern statistical techniques in the geophysical sciences. NCAR participants in the program are Principal Investigators Richard Katz and Roland Madden and Co-Investigators Linda Mearns, Kevin Trenberth, Joseph Tribbia, and David Williamson. Besides encouraging interactions between scientists and statisticians at NCAR, an important mission of the project is to facilitate an exchange of ideas and to interest new researchers in problems involving statistics and modeling and analysis of the climate system.

Twenty-seven graduate students and 5 recent Ph.D. recipients who expressed an interest in applying statistics to the geophysical sciences received financial support to attend the Colloquium. There were tutorials on climate, objective analysis, scaling, and spatial ARMA (autoregressive moving average) processes. In addition, there were more specialized lectures on a wide variety of statistical problems in atmospheric and ocean sciences. There were 19 distinguished visiting scientists speaking at the colloquium, along with nine NCAR scientists.

The participating students were assigned to take notes of the colloquium lectures. These notes, which are contained in this volume, were edited to various degrees by the lecturers and the colloquium coordinators. They are not intended to cover the lectures comprehensively, but it is hoped that they and the listed references will provide good starting points for study of the topics.


We would like to take advantage of the opportunity afforded by the publication of these notes to thank the students and lecturers who made the colloquium successful and Dennis Shea for his reading and constructive comments on the entire document.

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This workshop is an experiment in cross-disciplinary communication between atmospheric and ocean scientists, whom I will refer to simply as "scientists" (or "geophysicists"), and statistical scientists to whom I will refer to as "statisticians". Coming together as we have here presents a rare opportunity for stretching our minds to explore concepts that are new to us and to expand our scientific language base, whether scientists or statisticians.

This is a challenge, with the objective of creating common ground, from which we can pursue collaborative research to understand and to predict evolving states of Earth’s fluid envelope. For studying processes within the fluid envelope of a sphere rotating around a fixed axis while traveling around its heat source, statisticians must translate their conceptual framework to the natural coordinates and interrelated variables of this vast geophysical system. Generally this will mean abandoning the familiar X-Y-Z coordinate system for the earth-oriented coordinates of latitude, longitude, and pressure, and simple scalar or vector variables for arrays of physically related elements. On the other hand, there may still be geophysicists who hold an interpretation of "statistics" as a closed set of formulas and computer algorithms for extracting simple data summary values. This interpretation needs to be let go, in order to see the achievements and potential of statistics for representation of uncertainty and construction of inference techniques in the full space and time dimensions of dynamic systems.

Both the statisticians and the geophysicists who approach this collaboration run very sophisticated businesses which incorporate mathematical rigor and observation-based validation. The differences lie primarily in focus. Statisticians have a habit of divesting variables of all but their most elemental properties and concentrating on the stochastic behavior of their non-deterministic components. Geophysicists are familiar with and are able to model much of the complexity of Earth’s fluid systems: energy inputs, exchanges and losses, and the induces circulation processes, as these influence changes in states of the atmosphere and ocean. Working together, there is a great deal that can be achieved—in refining the algorithms that take in new data as they become available, coalescing the data with geophysical representations of state-space operators, and outputting new information.
In order to participate fully in the collaborative process, statisticians must be familiar with the nature of the data base in relation to the scales and dynamic complexities of the systems studied, as well as to scientific inference objectives. Variables which describe states of the ocean and atmosphere are measured in many different ways: from aircraft and upper air balloons, by satellite-borne instrumentation, from land surface-observing stations, by moored and drifting buoys, from ship sea-surface observations and ocean soundings, etc.

Figure 1 Cartoon depicting some major observing systems used by the atmospheric and oceanographic communities.

(see Figure 1). Due to the variety of observation techniques, statistical properties of errors-of-representativeness and of observation- and transmission-equipment errors, are highly inhomogeneous. For example, sea-surface temperatures (SSTs) are measured by buoys and ships, as well as being inferred from satellite images (Thiébaux, 1980). Although they provide measurements of the same variable, the data from these different sources have dissimilar statistical characteristics. In addition to the differences in observing equipment, observation times, locations and intervals between them vary widely; and this
adds disparity to statistical properties of state-of-the-atmosphere and ocean reports. As another illustration, consider that time-synchronized upper-air soundings are made twice a day from fixed land-based stations, while aircraft reports come from established check points at whatever time aircraft cross them, and polar-orbiting satellites have approximately a 100-minute orbital time and an 8-day progression (to a repeat of orbital track). All of these observing system characteristics must be taken into account in the use of the global data base.

Two major challenges for statisticians who elect to work in the research arena of global change, are the following:

1) Analysis of 3-D observation-minus-predicted increment fields that evolve through time;  
2) Inference with data archives in which model output, reports from the observing network, and their difference fields are correlated over time and space, and are generated by nonstationary processes and inhomogeneous data sources.

The Kalman filter provides a convenient framework with which to describe the first challenge (Thiébaux, 1991). Let

\[ \hat{X}_{s+\Delta} = \Phi_s \cdot \hat{X}_s + K_{s+\Delta} \left( Z_{s+\Delta} - \Phi_s \cdot \hat{X}_s \right) \]

where \( \hat{X}_s \) is the predicted state of the (space/time evolutionary) process at time \( s \), \( \Phi_s \) is the (estimated) system operator, \( K_s \) is called the transfer function and

\[ Z_{s+\Delta} = X_{s+\Delta} + V_{s+\Delta} \]

is the observation of the system, with \( V_{s+\Delta} \) random noise. The true underlying system evolves as

\[ X_{s+\Delta} = \Phi_s^* \cdot X_s + W_{s+\Delta} \]

with \( W_{s+\Delta} \) independent random fluctuations and \( \Phi_s^* \) the true system operator. Many different methods fit within this general framework, as described in Thiébaux (1991).

The second challenge makes it impossible for the statistician to use the “classical” tools for inference, which typically require independent and identically distributed data. Instead, a useful approach can be found in bootstrapping. In this context, bootstrapping refers to a growing class of “estimation and testing procedures that base their construction on information in the data set itself, without involving external distribution assumptions” (Thiébaux, 1995). These procedures make extensive use of computers to approximate the distributions and properties of the distributions for statistics of interest, by using resampling and simulation techniques.
References and Related Publications


There is a distinction between weather and climate:

- Weather involves short-term highly variable atmospheric phenomena.
- Climate is determined by the statistics of weather variations and by slower interactions between the atmosphere, ocean, cryosphere, land surface, and other "climatic components" that contribute to the dynamics of the Earth's environment.

Two of the main influences that determine weather and the climate are the radiation budget and the dynamics of the atmosphere and ocean. Because the Earth receives far more solar energy in the tropics than it does at the poles, a main role of the atmosphere and the ocean is to transport this excess energy and distribute it more equally over the globe. The circulation that accomplishes the transport is controlled by Newton's second law of motion; namely that the acceleration of a parcel is equal to the net force acting on the parcel. The most important forces are the pressure force and the gravitational force. Newton's second law holds for motion in a fixed coordinate system. Because the earth is rotating there is an important apparent force, the Coriolis force. It tends to turn parcels to the right (left) of their motion in the Northern (Southern) Hemisphere.

Some important aspects of weather and climate:

- Since land heats faster than the oceans, air above land will tend to absorb heat and expand faster than air above water. This is one source of a Pressure Gradient. At upper levels, the expanded air will be at higher pressure and move towards neighboring cooler ocean areas which are at lower pressure. Pressure forces like this and the Coriolis force are the two dominant driving forces in the Earth's atmosphere.
• Land surface irregularities can work to confound our attempts at forecasting. Near the Earth's surface, features such as mountains, etc. can interfere with the atmosphere's smooth transfer of energy.

• Sometimes temperature gradients can cause instabilities in flow. Small random perturbations that have certain spatial scales, determined by the temperature gradient and vertical wind shear, can grow at the expense of the mean flow. This process is known as Baroclinic Instability. It is very important in forming and maintaining the low and high pressure cells that make up the weather.

There are certain structures that are recurrent or essentially permanent. This is evidence that the climate does contain some predictable elements. For example, the ocean experiences periodic events known as El Niño and La Niña, which are quasi-periodic pools of warm and cold water (respectively) in the eastern, equatorial Pacific Ocean, and which have been shown to affect weather and climate far from this apparent point of origin.

So what are the primary problems with climate modeling that statisticians can help resolve? An example is the fact that there are considerable problems with the data. The amount of data collected is huge:

• The time record for each point is relatively short.

• Methods for data collection and reporting have changed frequently, and many of these changes have gone undocumented.

• Certain parts of the globe have more observations than do others. Proper analysis of the data must take this into account.

• Many of the classical assumption of "cookbook" statistics may be invalid when applied to the climate; perhaps it is nonstationary, for instance. Problems with spatial correlation give us trouble counting the "sample size," etc.

• Statisticians bring a different perspective to the table than do physicists, due to their different training.

If a physicist is uncertain about the nature of a particular phenomenon, he or she will observe that phenomenon and attempt to postulate deterministic "laws of nature"
which it obeys. When a statistician is uncertain about a given phenomenon, he or she will
observe that phenomenon and attempt to make statements about the uncertainty, so that
a statistical model that includes those uncertainties can be built. Although both of these
views are easily defensible, they represent quite different ways of attacking a problem,
and if meaningful cooperation between atmospheric scientists and statisticians is to be
achieved, then some consensus must be reached on when each approach is to be applied.

Related Publications

511 pp.
Trenberth, K.E., 1992: *Climate System Modeling*, Cambridge University Press, New York,
788 pp.
The atmospheric circulation is governed by certain dynamic equations. Some very basic ones are:

a) Ideal gas Law,

\[ p = \rho RT \]  \hspace{1cm} (1)

where \( p \) is the pressure, \( \rho \) is the density. \( R \) the gas constant for dry air and \( T \) is the temperature.

b) Hydrostatic relation,

\[
\frac{\partial p}{\partial z} = -\rho g \partial z \\
= -\rho \partial \Phi
\]  \hspace{1cm} (2)

where 
\[ \Phi \equiv \text{geopotential} \]
\[ z \equiv \text{geopotential height} \]

The hydrostatic relation describes how pressure falls off with height when the vertically directed pressure force is balanced by gravity, a suitable approximation for large-scale flow.

c) Geostrophic relation,

\[ V_g = \frac{1}{f} k \times \nabla_p \Phi \]  \hspace{1cm} (3)

where \( V_g \) is the vector geostrophic wind, a good approximation to the real wind away from the surface, \( f \) is the Coriolis parameter, \( k \) is a unit vector positive upward, and \( \nabla_p \) is the gradient operator on a constant pressure surface. The geostrophic wind is proportional to the gradient in \( \Phi \) and has high \( \Phi \) to the right (left) in the Northern (Southern) Hemisphere. It represents a flow balanced by pressure and Coriolis forces. The Coriolis parameter is given by \( f = 2\Omega \sin \phi \) where \( \Omega \) is the angular velocity of the Earth and \( \phi \) is the latitude.
d) Thermal wind relation

From the geostrophic and hydrostatic relations and the Ideal Gas Law one can derive the thermal wind relation

\[ \frac{dV_g}{d\ln p} = -\frac{R}{f} \mathbf{k} \times \nabla_p T \]  \hspace{1cm} (4)

which relates the vertical geostrophic wind shear, or change, to the horizontal temperature gradient. Since temperature generally decreases toward the poles the thermal wind relation demands that winds become more westerly (directed toward the east) with height. In the Northern (Southern) Hemisphere this increase in westerly winds with height occurs with warmer temperatures to the right (left). This relation is governed by (4) and changes at the equator because \( f \), the Coriolis parameter, changes sign at the equator. Equations (2), (3) and (4) explain many of the large-scale features of the general circulation.

An important phenomenon that affects interannual variability is the so-called ENSO. This is short for El Niño-Southern Oscillation. The Southern Oscillation is the atmospheric part of ENSO. It is basically an oscillation of pressure and air mass from the eastern Pacific to the western Pacific-Indian Ocean and back again. It is also accompanied by large-scale atmospheric wave patterns that affect mid-latitudes far from the Pacific. The El Niño is the ocean part of ENSO and is manifest by warm pools of water along the equator far east from their usual locations in the western Pacific. La Niña is the name of the opposite or cold water phase of the oscillation. Because of its long time scale (3-6 years), understanding ENSO offers some hope for long-range prediction.

Figure 1 is the zonally averaged (averaged around a latitude circle) temperature. In the troposphere (below about 300 hPa near the poles and 100 hPa near the equator: 1 mb = 1 hPa), the temperature gradient is directed equatorward. Figure 2 shows the zonally averaged west to east winds (negative values means winds are directed toward the west). One can see the jet streams near 30N and 50S. Comparing Figures 1 and 2 we note that, consistent with (4), the westerlies increase to the level where the temperature gradient is directed equatorward and then decrease above where the temperature gradient is directed poleward.
Figure 1. Zonal average temperature for January from 1987-1989 in °Kelvin (from Trenberth, 1992).

Figure 2. Zonal average of the east-west or u-wind for January from 1987-1989 in m/s. Positive values mean the wind is blowing from west to east ("westerlies"), (from Trenberth, 1992).
Figure 3. Contours are for the average sea-level pressure for January from 1979-1989 in hPa (or millibars). Arrows represent wind stress at the surface over the oceans for the same 11 Januaries. No data is indicated over land. The arrows point in the direction that the surface wind is blowing and their lengths are proportional to the wind speed. (Adapted from Hurrell et al., 1993, courtesy of J. Hurrell.)

Figure 3 is presented to illustrate the geostrophic relation at the surface. The contours (lines of constant pressure-isobars) are the mean January sea-level pressure and the vectors are the surface wind stress. The vectors point in the direction that the wind is blowing and their lengths are proportional to the wind strength. It is clear that the winds are at least qualitatively similar to geostrophic winds defined by (3). In the Northern Hemisphere they blow with higher pressure (or geopotential height) on their right and they can be seen to be stronger in regions of stronger pressure gradients. Because the Coriolis parameter $f$ is a function of latitude, this last is strictly true only when comparing winds and pressure gradients from the same latitude.
A major discrepancy in the balance of (3) is the importance of the frictional force at the surface. It causes the winds to blow not quite perpendicular to the pressure gradient as suggested by the cross product in (3), and it introduces a slight divergence away from or out of high pressure areas and a convergence into low pressure areas. This phenomenon contributes to the fact that low pressure areas tend to have more converging air at the surface resulting in upward motion, clouds, and rain than high pressure areas.

References and Related Publications


Objective Analysis/Combining Information

Part 1: Background

Dr. Grace Wahba

Hulin Wu and Lixin Zeng, Rapporteurs

This talk introduced some basic concepts of smoothing techniques related to variational methods for objective data analysis.

Subjective analysis: Analyze data based on the analyzer’s personal experience and subjective opinions, usually by hand.

Objective analysis: Use a computer algorithm for the above (see Daley, 1991).

In this talk, smoothing methods on one to four dimensions are introduced.

• One Dimension — usually time dimension

Let

\[ y_i = f(x_i) + e_i, \ i = 1, 2, \ldots n \]

where \( y_i \) is observation, \( x_i \) is state variable, \( \{e_i\} \) are “white noise”, \( f \) is a “smooth” function.

We want to find \( f \) on a function space to minimize:

\[
\frac{1}{n} \sum_{i=1}^{n} (y_i - f(x_i))^2 + \lambda \int_{a}^{b} (f^{(m)}(x))^2 \, dx .
\]

The minimizer, \( f_\lambda \), is well known to be a natural polynomial spline of degree 2m-1. The “smoothing” parameter \( \lambda \) may be chosen by the “leave-one-out” technique.

• Two Dimensions — usually space dimensions

\[ y_i = f(x_1(i), x_2(i)) + e_i, \ x = (x_1, x_2) \]

Assumptions are similar to the above.

We want to find \( f \) to minimize:

\[
\frac{1}{n} \sum_{i=1}^{n} [y_i - f(x(i))]^2 + \lambda J_m(f)
\]
where

$$J_m(f) = \sum_{\nu=0}^{m} \binom{m}{\nu} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{\partial^m f}{\partial x_1^{m-\nu} \partial x_2^\nu} \right) dx_1 dx_2$$

$$m = 2, 3, \ldots$$

The smoothing parameters, $\lambda$ and $m$, can be chosen by the generalized cross validation (GCV) technique. The "ordinary" CV is defined as

$$V_0(\lambda, m) = \sum [f_{\lambda,m}(x(k)) - y_k]^2$$

The GCV is

$$V(\lambda, m) = \frac{\sum [f_{\lambda,m}(x(k)) - y_k]^2}{\text{trace} (I - A(\lambda, m))^2}$$

where $A(\lambda, m)$ is the influence matrix.

The GCV is theoretically the same as or better than the ordinary cross validation method and much easier to compute (see Wahba and Wendelberger, 1980).

- Three and Four Dimensions — three space and/or time dimensions, usually (but not always) need a dynamical model.

Our goal is to find $f$ to minimize

$$\frac{1}{n} \sum_i [y_i - f(p(i))]^2 + \lambda \int_{\text{sphere}} (\Delta^{m/2} f) dp$$

Generally, for a strictly positive definite function $R(s, t)$, find $f \in H_R$ to minimize

$$\frac{1}{n} \sum_{i=1}^{n} [y_i - f(t(i))]^2 + \lambda \| f \|^2_R, \quad t \in \tau$$

where $\| f \|^2_R$ is the penalty associated with $R$.

$$f_\lambda(t) = \sum_{i=1}^{n} C_i R(t, t(i))$$

with $C = (R_n + \lambda I)^{-1} y$ is the solution.

Finally, the connection to Bayesian estimation was briefly discussed.

References and Related Publications


Objective Analysis/Combining Information

Part 2: Recent Results

Dr. Grace Wahba

Lixin Zeng and Hulin Wu, Rapporteurs

Introduction

After illustrating the basic principles of objective analysis in part 1, its application to medium-range numerical weather prediction (NWP) models is addressed in this talk. The goal of objective analysis here is to combine model forecasts, observations and other information in a three and/or four dimensional global scale. Some recently devised techniques, such as general cross validation (GCV), are described below. However, the choice of the topics here is idiosyncratic; thus, it is not the intention of this talk to provide a balanced overview of objective analysis. The reader is referred to Daley (1991) for a more comprehensive coverage.

Objective Analysis for NWP Models

Because the uncertainties are inherent in NWP models as well as in observations, they need to be combined to give an optimal description of the atmosphere. The algorithm and procedure of combining the information is called objective analysis. A brief summary of the state variables used in the NWP models describing the state of the atmosphere is now given. The basic state variables include wind components \((u, v)\), temperature \((T)\), sea level pressure \((p_s)\), relative humidity \((q)\), etc. For numerical and other reasons, they may be written in alternative forms. For example, the wind components \((u, v)\) can be replaced by stream function \((\psi)\) and velocity potential \((\Phi)\). In the spectral models, each of the variables are expanded with regard to a set of orthogonal functions, such as Fourier or spherical harmonics. As a result, the value of the variables on grid points are replaced by the expansion coefficients of orthogonal functions. Whatever type of state variables are selected, they define a model state vector \(x\), with dimension equal to the total number of the state variables. The merit of the NWP model is to obtain a future \(x\) based on the information available now and in the past.
There are also various types of observations to be used in objective analysis, including
the state variables themselves and other parameters which demand a sometimes complex
transfer function to be comparable with the state variables. The observational errors
usually come from the instruments that collect the data and from the imperfect transfer
functions.

Let
\[ x'_f = \text{forecast of state vector at time } t \text{ (obtained by the NWP model)} \]
\[ = \xi'_f + x_{\text{true}}, \text{ and} \]
\[ y_t = \text{observations at time } t \]
\[ = K(x_{\text{true}}) + \xi_{\text{obs}}, \]
where \( K \) is the transfer function mentioned the last paragraph and \( \xi_{\text{obs}} \) and \( \xi'_f \) are the
observation and forecast errors, respectively. The goal of objective analysis, or data assim-
ilation, is to combine \( x'_f \) and \( y_t \) to form an analyzed state vector at time \( t, x_a^t \). Then \( x_a^t \)
serves as the input to the NWP model to obtain the \( x_{f+\delta t}^t \) for the next time \( t + \delta t \). Weather
forecasts are based on this type of cycle.

The analyzed state vector \( x_a^t \) is usually obtained by minimizing the following cost
function \( J \) with respect to \( x \). (The time index is omitted.)

\[ J = (y - K(x))' S^{-1} (y - K(x)) + (x - x_f)' \Sigma^{-1} (x - x_f) + C(x). \]

The two terms on the right-hand side measure the “closeness to observations” and the
“closeness to forecast” of the state vector \( x \). The third term describes possible constraints
on \( x \), such as physical laws. However, since this talk is not intended to discuss the physical
constraints, this term will be omitted. ( )' denotes matrix transpose.

If the forecast and observation errors are normally distributed random variables with
zero mean and covariance matrices \( S \) and \( \Sigma \), respectively, then \( x_a \), which minimizes \( J \), is
the Bayes or Gandin estimate for \( x \) (Gandin, 1965 and Lorenc, 1986). The approach of
minimizing \( J \) can be found in Parrish and Derber (1992) given \( S \) and \( \Sigma \).

**Tuning Problem**

Two important components of the above procedure are the covariance matrices \( S \) and
\( \Sigma \), which are theoretically unknown. However, the result \( x \) is sensitive to the choices of
them. Traditionally, the information on \( \Sigma \) is collected by long-term comparison of forecast
and observations and parametric covariances are fit to the comparisons (Hollingsworth and
Lonnberg, 1986; Lonnberg and Hollingsworth, 1986). \( S \) represents the instrument error covariance and may be well known. However, the estimation of \( S \) can be harder when newer observations such as satellite measurement of radiance are introduced. There may be poorly known quantities in \( K \), adding more uncertainties in \( S \). In this section, the application of the general cross validation (GCV), unbiased risk (UBR) and maximum likelihood (ML) methods to the estimation of \( S \) and \( \Sigma \) is introduced.

Let \( \delta = x - x^f \) and \( y^* = y - Kx \), where \( K \) is assumed to be a linear operator on \( x \). Then \( J \) can be written as

\[
J = \left( y^* - K\delta \right)' S^{-1} \left( y^* - K\delta \right)
\]

Assume \( S = \delta^2 S \), where \( S \) is a known matrix.

Let

\[
\begin{align*}
\tilde{z} &\equiv \tilde{S}^{-1} y^* \\
\tilde{K} &\equiv \tilde{S}^{-1} K \\
\tilde{\Sigma} &\equiv \frac{1}{\sigma^2} \Sigma \\
\tilde{J} &\equiv \|\tilde{z} - \tilde{K}\delta\|^2 + \delta' \tilde{\Sigma}^{-1} \delta
\end{align*}
\]

Then the \( \delta \) that minimizes \( \tilde{J} \) will also minimize \( J \).

Further assume

\[
\tilde{\Sigma} = \tilde{\Sigma} (\lambda, \theta) = \sum_{\alpha} \lambda_{\alpha} J_{\alpha}(\theta)
\]

where \( \lambda_{\alpha} \)'s are smoothing parameters and \( J_{\alpha}(\theta) \) are penalty functionals. The minimizer \( \hat{\delta} \) is given by

\[
\hat{\delta} = \hat{\delta} (\lambda, \theta) = (\tilde{K}'\tilde{K} + \tilde{\Sigma}^{-1})^{-1} \tilde{K}'\tilde{z}.
\]

The next step is to find the parameters \( \sigma^2, \lambda, \) and \( \theta \) that ensure \( \hat{\delta} \) an optimal estimate of \( \delta \). The details of this method are described in Wahba et al. (1994), and only an outline is provided here. Three approaches are applied to reach this goal; namely, GCV, UBR, and ML methods. Each of the methods calculates the minimizers of a special function, as listed below.

\[
GCV: V(\lambda, \theta) = \frac{\| [I - A(\lambda, \theta)] \tilde{z} \|^2}{\text{tr} (I - A(\lambda, \theta))^2}
\]

where \( A(\lambda, \theta) \equiv \tilde{K} [\tilde{K}'\tilde{K} + \tilde{\Sigma}^{-1}_{(\lambda, \theta)}]^{-1} \tilde{K}' \), and \( \text{tr}(\cdot) \) denotes the trace of a matrix.

\[
UBR: U(\lambda, \theta) = \| [I - A(\lambda, \theta)] \tilde{z} \|^2 + 2\sigma^2 \text{tr}(A)
\]
Matrix $A$ usually has large dimensions in NWP models, and evaluating its trace involves matrix inversions. Thus, it may be numerically difficult to perform the above minimizations. Fortunately, the randomized trace estimation method (Girard, 1987 and Hutchinson, 1989) can be used to estimate $tr(A)$, based on the following facts. Let $\xi$ be a pseudo-Gaussian vector

$$\xi \sim N (0, \sigma^2 I)$$

where $\sigma^2$ is a preassigned number. It can be proven that

$$E \left( \frac{1}{\sigma^2 \xi^t A \xi} \right) = tr(A)$$

As a result, the left-hand side of the above equation may be applied as an unbiased estimate of $tr(A)$. This approximation makes the implementation of GCV, UBR, or ML methods to NWP models feasible.

Summary

Objective analysis is crucial to NWP because it provides an optimal state of the atmosphere as the initial condition for the NWP models. One major problem is that some important parameters in the objective analysis algorithms are not known exactly, such as observation and forecast covariance matrices. There are methods that can provide good estimates of them but are not used in NWP models since they demand too much computer power (e.g., Kalman filter). The optimal interpolation method used in many operational NWP models (e.g., at the National Meteorological Center) uses prescribed covariance matrices, which may not be optimal. The objective analysis method described in Section 3 provides the possibility of tuning the covariance matrices and the feasibility of operational implementation.

References


**Related Publications**


Overview of Statistical Problems in Climate

Dr. Paul Julian

Karen S. Kelly and Katrin Müller, Rapporteurs

The objective of Dr. Julian’s discussion was to review some selective applications in the interface between statistics and meteorology. In applying statistics to meteorology, an essential open-ended question is the characterization of the atmosphere as chaotic and/or stochastic. The processes involved are highly non-linear, but they are at least in part deterministic because short-range weather predictions are quite successful. Statistics is applied to three types of meteorological problems: i) numerical weather prediction, ii) climate models, and iii) heuristic investigation and data analysis.

Numerical weather prediction is a classical initial value problem in physics. Here, the initial state plays a critical role. To derive an initial state, an assimilation model is required to combine measurements from many different sources, locate them on equidistant grid points, and make them physically consistent. Given an initial state, a dynamic physical model is implemented to make weather predictions. For short-range predictions these models are quite accurate, yet they remain dependent upon the initial state.

Climate models, in contrast, make long-range time integrations and are less dependent on the initial state. These models are not used to make forecasts, but rather to investigate the interaction among the components of the atmosphere.

Heuristic investigations can be applied to both model output and observed data. The observed data is either raw data or an integrated data set obtained from an assimilation model. An integrated data set offers some advantages over raw data: i) it is physically consistent, ii) it contains estimates of variables for which no observations are available, and iii) errors in the observations have been mitigated. There is, of course, a tradeoff involved in that the integrated data is dependent upon the assimilation model. Because these models are always being improved, resulting data are inhomogeneous. A way to resolve this problem is to select one model and run the historical data set through it to produce an historically integrated data set that is homogeneous (with respect to the model). This, of course, does not solve the problem that the measurements themselves are not homogeneous.
One type of investigation involves analyzing a time series. Spectral analysis has been used predominantly in three ways: (i) to identify wave modes, (ii) to search for "cycles" or narrow frequency bands, and (iii) to model ARMA-type series for forecasting. The extension to cross-spectral analysis allows two or more time series to be jointly examined.

To investigate the space and space-time dimension, several methods may be employed. The simplest statistical tool consists of computing empirical orthogonal functions (EOFs). This involves calculation of eigenvectors and eigenvalues of a covariance or correlation matrix (Karhunen-Loeve basis functions). These basis functions are orthogonal over space (or time, depending on how they were computed). Each eigenvalue can be ranked according to the proportion of total space-time variability it explains. When translating over both space and time, the covariance or correlation matrix becomes complex.

The empirical method of EOFs has more recently been supplemented by a more physical one called Principle Oscillation Patterns (POPs). This involves first estimating a covariance matrix by fitting a linear first order Markov process to the data. However, the covariance matrix is no longer symmetric and eigenvectors no longer orthogonal.

### Related Publications


Atmospheric Science and Statistical Science:
Some Historical Connections

Dr. Allan H. Murphy

Igor Perisic and Todd Ringler, Rapporteurs

This talk described some historical connections between atmospheric science and statistical science, focusing on the 19th and early 20th centuries. Important individual contributions from famous researchers during the period were identified, as were developments in statistical science, and applications of statistical concepts and methods in atmospheric science.

Some contributions of several noteworthy 19th century individuals were discussed. Pierre Simon Laplace (1749–1827) analyzed the effects of the moon on tides in the atmosphere using the method of weighted least squares (Laplace, 1823). This work demonstrated the surprising depth of Laplace’s understanding of statistical concepts and their application to a delicate scientific problem (Stigler, 1975a). Adolphe Quetelet (1796–1874) was perhaps best known for his statistical/sociological studies of the “average man”, but he also worked as a meteorologist at the Royal Observatory in Brussels. He fit both symmetric and asymmetric distributions to meteorological data and developed simple statistical models to demonstrate that day-to-day weather was persistent (Stigler, 1975b). Francis Galton (1822-1911) made important contributions to the development of regression analysis and discovered the correlation coefficient. He, too, worked as a meteorologist during the period 1855-1865, and published a book (Galton, 1863) containing daily weather maps with ingenious (for the time) representations of various weather conditions and events. Galton also gave the name “anticyclone” to areas of high pressure.

Among a variety of activities and events at the interface between meteorology and statistics in the 19th century, it was interesting to note that statistical meteorology was identified as a scientific discipline by J.B. Lamarck in 1802. Moreover, although meteorologists/climatologists not surprisingly focused their attention in these early days primarily
on the problem of describing average weather conditions, considerations of variability in such conditions were not entirely neglected (e.g., Buys Ballot, 1850).

An interesting event in the 1880s related to the application of statistical methods in meteorology was the so-called "Finley affair". J.P. Finley, a sergeant in the U.S. Army, produced tornado/no tornado forecasts around this time, and he described some results of his work in a paper published in 1884 (Finley, 1884). He used 2x2 contingency tables to summarize his forecasting performance and reported an accuracy (percent correct) of 96.6%. It was soon pointed out that if he had adopted a strategy of never forecasting tornadoes he would have achieved a level of accuracy (according to this measure) of 98.2%! Finley's paper stimulated considerable interest among mathematicians, statisticians, and philosophers as well as meteorologists (e.g., Doolittle, 1885; Gilbert, 1884; Peirce, 1884), and several new measures of forecasting performance for this 2x2 situation were proposed. The first applications of statistical concepts and methods to the problem of forecast verification in meteorology appear to date from this event.

The uncertainties inherent in weather forecasting have also served as a focal point for interactions between meteorologists and statisticians. For example, it was interesting to note that probabilities and odds were used in connection with weather forecasts at least as early as the late 18th century, and it was evidently common practice to label descriptions of future weather conditions with the heading "probabilities" in the latter half of the 19th century. Pioneering work by W.E. Cooke, A. Ångström, C. Hallenbeck, and others in the first quarter of the 20th century (see Liljas and Murphy, 1994), followed up by methodological and experimental studies conducted by a small cadre of meteorologists and statisticians in the period 1940-1965, laid the groundwork for the current relatively widespread use of probability forecasts in meteorology.

In the first half of the 20th century several well-known scientists worked in both statistics and meteorology. For example, Lewis Fry Richardson (1881–1953) is well known by meteorologists for his pioneering efforts to develop methods of numerical weather prediction (Richardson, 1922). However, he later used statistical methods and models to study fatal quarrels (i.e., wars) (Richardson, 1950). On the other hand, Harold Jeffreys (1891–1989) is a familiar name in statistics for his book on probability theory (Jeffreys, 1939), but he
published several papers on meteorological problems (e.g., the relation between wind and
the distribution of pressure) in the period 1915–1925. P.C. Mahalanobis (1893–1972) was
a statistician well-known for his contributions to multivariate analysis (e.g., Mahalanobis
distance); he also wrote several papers on meteorological problems, including a memoir
ettitled “Correlation of Upper Air Variables.” Other names that could be mentioned
from this period include A.N. Kolmogorov, who made important contributions to both
probability theory and the theory of atmospheric turbulence, and G.T. Walker, who is
known by meteorologists for his studies of tropical circulations (and for giving the name
to the Southern Oscillation) and by statisticians for the Yule-Walker equations (relating
the parameters in autoregressive models to autocorrelations).

Although the connections between atmospheric and statistical science appear to have
weakened to some extent in the period 1925-1950, they were strongly re-established in
the late 1940s and early 1950s, due initially to the application of statistical methods in
weather forecasting and weather modification. Probabilistic models and statistical methods
were taken up by many subdisciplines of the fields of meteorology and climatology in the
period 1950–1975. The last 20 years have witnessed a rapid increase in the range, scope,
and depth of applications of statistical concepts and methods, particularly in areas such
as four-dimensional data assimilation, weather/climate prediction, detection of climate
change, and chaos-related research. This colloquium—and the climate/statistics project
at NCAR—are important steps in building a permanent relationship between these two
sciences.

[Note: Three very useful secondary references with which to begin a study of the historical
connections between atmospheric science and statistical science are the books by T.M.
Porter (1884) and S.M. Stigler (1986), and the review paper by O.B. Shenyin (1985).]

References and Related Publications
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What is Climate?

Dr. Edward Lorenz

Karla Nobrega and Kevin Walsh, Rapporteurs

The old saying that "climate is what you expect; weather is what you get" may provide a colloquial definition of climate. More properly, it might be restated as, "climate is what you ought to expect when you are not in a position to make a skillful weather forecast". There are fundamental processes which limit our ability to make skillful predictions. Weather is an intrinsically chaotic phenomenon; that is, future weather conditions are sensitive to initial conditions in that small differences in starting values between two weather forecasts lead to large differences after a finite time.

An example of a simple system whose chaotic nature is easy to verify is the difference equation

\[ x_{n+1} = x_n^2 - c , \]  

(1)

where \( x \) is a scalar value, \( n \) is the number of time steps, and \( c \) is a constant. This equation exhibits chaotic behavior for some value of \( c \) between 1.4 and 2.0. Although the atmosphere is a more complex system, it is similarly chaotic. Optimum weather forecasts typically have an error doubling time of about 2 days, which, for temperature or wind prediction, implies that the error after 8 days is comparable in magnitude to the range of the predicted variable.

Thus, when considering what the weather will be like 30 days in advance, our best guess is usually the climatological long-term mean. The concept of a climatology assumes that one has an accepted definition of climate, preferably one that facilitates comparison with theoretical results. One possible definition is that climate is the set of all infinite-term statistical properties of the atmosphere and its surroundings. Much theoretical work has been performed on infinite-term statistics (often displayed as attractors) of various systems. Unfortunately, if one averages over an infinite time period, climate change is excluded by definition. Changes in climate might be included by modifying this definition,
using the statistics that would occur if phenomena not controlled or influenced by the weather, such as solar output, volcanic eruptions, and atmospheric composition, could be held constant with time. This revised definition would permit climate change, but would however not allow the climate to be determined from data records, because external conditions have changed in the past at unknown times. In addition, this definition would imply that all climate changes are externally produced, thus ignoring the possibility of internally generated climate change.

An alternative definition would replace infinite-term statistics by statistics calculated over a long but finite time interval. The specification of this time interval is subject to some debate; too short an interval would not yield a representative climate, while too long an interval might span a real climate change. A more robust definition uses the concept of ensemble statistics. An ensemble is produced by starting with a single state of the atmosphere-ocean-earth system. A large number of states is then constructed by perturbing the initial state many times, and subsequently permitting each new state to evolve for the same fixed amount of time. Climate may then be defined as the set of all statistical properties of the new ensemble. This definition does not permit evaluation of the climate from observations, but is convenient for the calculation of the climatology produced by a model.

The choice of integration time is governed by the predictability limits of phenomena in the earth-atmosphere-ocean system. Ideally the chosen time interval should be longer than the range of predictability of phenomena like El-Niño/Southern Oscillation (ENSO) (see for example Philander, 1990), but not so long that real changes of climate are likely to have intervened. A value of four years may be chosen as a compromise.

The output of a simple model has been examined to assess the suitability of the various definitions of climate. This model possesses none of the physics of weather. It is a very simple mathematical model that exhibits a behaviour known as “almost-intransivity”. An almost-intransitive system is characterized by very infrequent transitions between two or more regimes, while time spent in any one regime is long but not infinite.

The simple almost-intransitive system used here is based upon the game called “Bull in a China Shop”. In this game, several small pins are placed in two or more compartments
of a box, the compartments being separated by a divider containing a small gap. Play begins when a small top is spun in one of the compartments, and points are scored when pins are knocked down by the top. Extra points are scored if the top travels through the gap into the second compartment and knocks over pins there. One can relate this simple game to characteristics of the climate system. Lorenz (1994) states that

...the irregular motions of the top within a compartment will serve as an analogue of the irregular progressions of migratory storms; in due time they will lead to a change of regime, as they lead the top to and through a door. Regime changes may or may not be considered “climatic”, according to their typical frequency of occurrence. Note that in selecting an almost-intransitive system as an analogue, we are making no claim that the climate system itself is almost intransitive. Almost intransitivity ensures the presence of two or more time scales, which the climate system certainly possesses.

The numerical model is developed from the above game, but certain modifications are made. In order to create a chaotic system, the walls of the playing surface are made to bulge inward. A “northern” boundary is defined by an infinite chain of circular arcs

\[ y = b + a - \sqrt{a^2 - (x - 2m)^2}, \]

and a “southern” by

\[ y = -b - a + \sqrt{a^2 - (x - 2m)^2}, \]

where \( 2b \) defines the smallest distance between the boundaries, \( a = \sqrt{4/3} \) and \( m \) assumes every integer value. The spinning top is replaced by a particle traveling at a constant speed. The system effectively has three variables: two position coordinates \( x \) and \( y \), and a velocity consisting of two components \( u \) and \( v \), with \( u^2 + v^2 = 1 \).

The final modification involves changes in the direction of the particle as it crosses the \( x \)-axis. If \( \theta = \tan^{-1}(u/v) \) is the angle between the path and a line perpendicular to the \( x \)-axis, before crossing, and \( \theta' \) is the angle after crossing, the direction is altered using the relation

\[ \tan \theta' = k \tan \theta - l f(x) \]  

(2)

where

\[ f(x) = (x - x^*) \]  

(3)

in model A, and

\[ f(x) = (x - x^*) - c \tan^{-1}((x - x^*)/c))/\tan^{-1}(1) \]  

(4)

with \( 0 \leq k \leq 1 \) and \( l > 0 \), in model B, where \( k \) is a damping constant, \( l \) is the restoring constant, and \( x^* \) is the desired climatological mean value of \( x \). Models A and B are
both supposed to represent the climate system, while model B better captures internally generated climate changes.

Both model A and model B produced oscillating time series with two or more time scales, reflecting those found in the climate system. The frequency distribution of $x$ for model A is mound shaped, with symmetric bench-shaped tails. The attractor of A, having a larger number of points in the central region, reflects this empirical density function. Model B's frequency distribution of $x$ is bimodal. The dispersion of points within model B's attractor also illustrates this bimodality, though it is uncertain whether the bimodality would remain with other parameter values or with a longer simulation of the model. Distributions generated using ensembles of forecasts remain similar to those generated using infinite-term statistics for both model A and model B, except that the distribution of the ensemble statistics is sensitive to the initial conditions. These simulations illustrate that a simple nonlinear model can generate some observed characteristics of the climate system, and can reveal some of the strong and weak points of various definitions of climate.

References and Related Publications

Scale is an important consideration in the study of climatic processes and is integral to the mission statement of the NCAR Climate and Global Dynamics Division as expressed by director Warren Washington: “to understand climate at all scales, leading to long term weather and climate predictions”.

In particular, precipitation is a process that crosses a number of scale boundaries ranging from the microscale to the synoptic (continental) scale.

The subject of this and the following two lectures is the conceptual and mathematical development of random cascade theory as it applies to both turbulence and the distribution of rainfall over an area. Aspects of this material are covered in detail in a variety of references, but particularly in Gupta and Waymire (1993).

This first lecture is chiefly concerned with the historical development of rainfall modeling.

Approaches to the Modeling of Physical Processes

Some simple examples show us that partial differential equations are not always appropriate for modeling the workings of the physical world:

1) Newton’s law, which accurately describes the path of a thrown object (i.e., a relatively heavy one for which the effect of the atmosphere can be ignored).

2) The binomial distribution, which accurately describes the probability behavior of balls ‘walking’ randomly through a Galton Board—a stochastic process (Bhattacharya and Waymire, 1991).

Note that the path of the thrown object is not sensitive to initial conditions, but the path taken by the ball through the Galton board is.
Rainfall, modeled stochastically, has a considerably more complicated probability density function (pdf) behavior than the examples above.

**Early Attempts at Rainfall Modeling**

Many of these attempts are reviewed by Katz (1985) and Waymire and Gupta (1981).

The aim of rainfall modeling in the 1950’s and 60’s was to fit parameters for various time-series models to rainfall data on various timescales (hourly, daily, monthly), as in, for example, the Neymann-Scott Poisson Cluster Model, which treats the initial rain event as a parent followed by sister showers. This model has also been used to describe earthquakes and their aftershocks.

Although second-order moment characteristics (i.e., variability) of the rainfall were modeled reasonably well in these attempts, it was impossible to determine WHY any given model worked. More physically-based models were therefore required.

**Early Physically-Based Models**

Le Cam (1961) provided the first of these, a stochastic description (a cluster-point process model) of precipitation with a hierarchic rainfield structure in which:

1) Clustering takes place in both space and time.
2) Smaller areas are “children” of larger areas.
3) The hierarchical spatial structure is described by four nested levels, with rainfall intensity increasing as the size of the structure decreases:
   - Synoptic ($> 10^4 \text{ km}^2$)
   - Large Mesoscale Areas ($10^3 - 10^4 \text{ km}^2$)
   - Small Mesoscale Areas ($10^2 - 10^3 \text{ km}^2$)
   - Cell ($< 50 \text{ km}^2$)

Empirical evidence from radar and raingauge analysis supports this structure (Austin and Houze, 1972), and suggests roughly a doubling of rainfall intensity from one scale to the next. ‘Cells’, however, may support up to a tenfold increase in rainfall intensity within supercells.
Self-Similarity

The existence of this 'doubling rule' across order-of-magnitude changes in spatial scale suggests that the appropriate stochastic model for rainfall may be 'statistical self-similarity'.

Self-similarity means, basically, that the process or object in question appears to have the same structure regardless of the scale at which it is analysed or viewed. In the context of rainfall, self-similarity means that the spatial structure of rainfall intensities will look the same statistically regardless of the size of the area being observed. Another way of expressing this (Gupta and Waymire, 1993) is that the “observed organization of fluctuations at distinct spatial scales is an artifact of the scales of measurement and are nonexistent in the natural processes.”

The mathematical test for self-similarity involves determining the moments of rainfall intensity for a series of \( i.e., \) square observation areas of varying side-length. The log of the moments of rainfall intensity is plotted against the log of length-of-side. If the resulting relationships are linear for each order moment and if the slopes of these relationships also vary linearly with the order of moment, then the process is considered to be self-similar (obeying 'power law scaling' where the exponent of the power law is the slope of the linear relationship for each moment's slope).

Examples of self-similar processes in other contexts include:
1) Brownian motion
2) Lévy stable processes
3) Fractional Brownian motion.

Is Rainfall Self-Similar?

Lovejoy (1981; 1982; 1983), Lovejoy and Mandlebrot (1985), Waymire (1985), Schertzer and Lovejoy (1987), Gupta and Waymire (1990), and others have investigated the application of self-similarity theory to empirical rainfall data and found a number of difficulties.

Most recently, Gupta and Waymire (1993) have investigated the GATE radar/raingauge data (Hudlow and Patterson, 1979), which was collected in the tropical Atlantic ocean by 5 ships covering an area of 400 \( km^2 \) at a resolution of 4 \( km^2 \).
They have found log-log linearity for each moment, but it departs from simple scaling behavior in that the relationship between these slopes and the moment order is nonlinear.

**What other possible stochastic model for the rainfall process matches this departure from self-similarity?**

Gupta and Waymire (1993) suggest that cascade geometry, a multiplicative process, seems to work for both turbulence and rainfall. Cascade models were also emphasized by Schertzer and Lovejoy (1987) based on analogies drawn to turbulence.

**Some Questions and Comments**

1) A comment was made that exploratory analysis of the GATE data has not shown it to depart significantly from linearity.

2) One questioner asked (with reference to fractal analysis of topography and the influence of underlying geological structure), if the displayed plot could represent two self-similar relationships with different slopes over different ranges of scale, with a breakpoint due to some characteristic of the rainfall process.

3) There was some discussion as to the likelihood that rainfall should display scaling behavior over such a large range of scales, given the differences between, for example, convective and frontal precipitation.

Taylor hypothesis is the *frozen field* hypothesis that \( R(x, t) = X(x + ut) \) where \( u \) is the large scale average velocity of the initial radar field \( X(x) \).

**References**


Related Publications


With reference to some of the session’s other lectures on global climate modeling, it is worth noting that the discussion of random cascade theory belongs as part of the debate about the quantification of subgrid-scale processes.

This second lecture covers the physical structure and mathematical foundations of random cascade theory [Gupta and Waymire, 1993; Waymire and Williams, 1994; and She and Waymire, 1994].

Redistribution of Mass in a Random Cascade Model

The random cascade as applied to rainfall is a multiplicative process that models the redistribution of mass over progressively finer scales. It is analogous to the redistribution of energy in turbulence into smaller and smaller eddies.

By ‘multiplicative’ we mean that the results at a given level of the cascade are determined by multiplication from the results at the previous level. This is in contrast to self-similarity, which is commonly a property of additive processes.

This is how a random cascade model works for rainfall: The mass of rainfall at the initial level has unit one and is distributed uniformly throughout a unit area. The mass is redistributed to the next level by dividing the initial area into subregions determined by a specified branching number. The mass making it to the next level is determined by a non-negative random weighting factor with expectation one. The values of the random variable are independent from each other on the same level, as well as between levels. The process iterates again with the same branching number throughout each level, so that if $b = 4$, the second level has 4 subregions, and the third level has 16. The probability that a subregion in any level will receive any mass from the level above is constant throughout the process. When $W$ is Bernoulli zero/nonzero distributed, it can be shown that this probability must be greater than .25 for any rain to reach the surface. Because of the
iteration procedure that defines the process, a subregion at each level may be considered to be the initial level. The iteration process may continue an infinite number of times to the surface, leading to a limit. Statistical inference for branching processes is given a thorough treatment in Peter Guttorp's recent book of the same name (Guttorp, 1991).

In the Bernoulli zero/nonzero case, another critical value of concern in the limit is that for which there is a non-zero probability of obtaining a coherent structure. Coherence is measured by having a continuous 'rain band' from one edge of the original area to the opposite one. There is a non-zero probability of coherent structures, if the probability of rain reaching the next level is at least equal to 0.71.

**Intermittency**

Intermittency is the term that describes the distribution of wet/dry squares in a partitioned region. We are interested in the probability that a square is wet. In the case of independently occurring wet squares the log probability of a wet square is log linear with the coefficient determined by the dimension. For instance, in two dimensions, the log (lambda) coefficient is 2, and in three dimensions, it is 3. In the multiplicative cascade case, the log (probability) of a wet cell is also exactly log linear, with the log (lambda) coefficient determined by the probability that the weighting value is non-zero.

**References**


**Related Publications**


In this lecture, Dr. Waymire develops the idea of the cumulant generating function of $W$. The rainfall mass that reaches the surface may be entirely determined by the modified cumulant generating function,

$$-(h - 1) + \log \{\text{base } (b)\} EW^h,$$

of the weighting random variable $W$. References on the modified cumulant generating function include Mandelbrot (1974) and Kahane and Peyriere (1976).

He derives the modified cumulant generating function when the distribution of $W$ is as follows: $W = 1/p$ with probability $p$ and $W = 0$ with probability $1 - p$. For this model, the modified cumulant generating function takes the form:

$$[\log \{\text{base } (b)\} (bp)] [h - 1].$$

Then, for various other distributions of $W$, including the binomial, beta, lognormal, and uniform, he gives the forms of the modified cumulant generating function. A note of interest is that when $W$ is distributed uniformly, the total mass of rainfall follows a gamma distribution.

Dr. Waymire then goes into the calculation of tail probabilities for the expected value of the mean number of subregions in a level receiving any mass from the previous level. A tail probability is the probability that the mean exceeds a specified value. Holley and Waymire (1992) provide a complete discussion on the topic.

The concluding discussion showed that the log-Poisson distribution is the proper correction to Kolmogorov's log-normal hypothesis for turbulence. This is explained in She and Waymire (1994).
References


Related Publications


This talk combines ideas of statistical mechanics with conventional numerical ocean modeling. The key ideas from probability revolve around the concept of maximum entropy. An example that studies velocities and eddy flow is presented, based on the North Atlantic. Finally, a summary and discussion of unresolved issues is given.

Suppose we collect all the dependent variables that we want to use to describe the oceans and collect all their values in a vector \( Y \). These variables could be temperatures, velocities or chemical concentrations. One issue that arises is: what are the real dependent variables? For example, we measure a temperature at a specific point. Does that temperature reflect things at that exact point, or is it an expected temperature over an entire block? We will come back to this issue.

Given \( Y \), the evolution of the ocean is the trajectory \( Y(t) \) in the phase space, which may obey

\[
\frac{d}{dt} Y = G(Y) .
\]

This is difficult to solve for several reasons: \( G \) is often nonlinear; we never know \( Y(t = 0) \) precisely, nor the boundary conditions.

So, perhaps our focus should be on the equations of motion of the dependent variables. If we are discussing expected values as before, then we are interested in obtaining the equations of evolution of the moments.

To attempt to do this, let \( p(Y) \, dY \) be the probability that the actual state of the ocean lies within volume \( dY \) of \( Y \). what we want to know we can express as an expectation,

\[
E(Y) = \int Y p(Y) \, dY .
\]
The problem is: Given some $p(Y; t = 0)$, how can we find $p(Y; t = \text{later})$? Or, in keeping with the previous notation, what is
\[ \frac{d}{dt}p(Y), \]
and how can we solve it? This is even harder to do than the previous problem.

To motivate a possible solution to the problems, a "toy" arctic model was presented that was completely isolated from wind, forcing and was not connected to the Atlantic. It had small scale random eddies at $t = 0$. The problem was the typical parameterizations of eddy effects (viscosity, drag) tend to drag models towards rest, while the model described was starting nearly at rest. But, upon running the model, it did not remain at rest, indicating we should change our approach to things. This is discussed below.

In statistical geofluid mechanics, we can examine two different situations. The first is equilibrium statistical mechanics, where we have an isolated system that is unforced. The second is disequilibrium statistical mechanics, which is sometimes marginally tractable if, for example, the situation is nearly statistically homogeneous.

When dealing with the ocean, we have a body of water that is forced, dissipative and very inhomogeneous. So, to solve an ocean problem, we will attempt a hybrid solution.

A key idea in the solution is entropy, defined as $S = -\int p(Y) \log p(Y)$, where $Y$ is our set of dependent variables. Entropy applies near or far from the maximum entropy equilibrium. We can write
\[ \frac{\partial S}{\partial y_i} = \frac{F_i}{\lambda}, \]
where $F_i$ is the "conjugate force" to $y_i$.

In practice, what we will do is attempt to move towards the state of maximum entropy. If the model variable $y_i$ differs from $y_i^*$, where $y^*$ is evaluated at the state of maximum entropy $S_{\text{max}}$, then we should obtain a force acting on $y_i$ given by $\lambda (y_i^* - y_i)$. Two questions are what is $\lambda$, and why do we choose to be linear in $y_i^* - y_i$? For small $|y_i^* - y_i|$, linearity in $y_i^* - y_i$ is strictly defensible. In reality, we are probably far from maximum entropy, so choosing a linear form is based on simplicity.

A practical oceanographic exercise in global ocean climatology was then used to illustrate this entropy-tendency approach. The approach involved making a guess of the maximum entropy state $Y^*$, and revised the eddy parameterization to point towards $Y^*$,
rather than the state-of-rest. Recall from the Arctic model discussed before that state-of-
rest is the traditional approach.

To implement this parameterization, it was first noted that $Y^*$ is depth independent
because of the grid coarseness. So, if we let $\psi$ denote the streamfunction, we can recover
$\psi^*$ (its maximum entropy value) reasonably well if we know the depth of the fluid $h$ we
can express the streamfunction as

$$\left(\frac{\alpha}{\beta} - \nabla^2\right) \psi^* \approx h,$$

where $\nabla$ is the horizontal Laplacian, $h = \int \frac{\delta H}{H_0}$ is a potential vorticity due to variations of
$H$, a variable depth bottom about a reference depth $H_0$, $L^2 = \frac{\beta}{\alpha}$ is a length scale, and $\alpha$
and $\beta$ are Lagrange multiples.

If we then find that the grid is coarser then $L$, we can set $\psi^* = L^2h$. At the full depth
transport, $\psi^* = -fL^2H$, which leads us towards the higher entropy at $u^*$. The choice of $L$
is obviously important. In this particular example (a global ocean model), $L = 9 + 6 \cos \alpha \phi$,
and is measured in kilometres.

Our next step is to replace $Am \nabla^2 u$ with $Am \nabla^2 (u - u^*)$. We should note that although
$u^*$ is depth-independent, $u - u^*$ is depth-dependent. At completion, we compare our
solution using this method (the topographic stress case) with the control case, which uses
$L = 0$. Two unresolved questions are why do we operate under $\nabla^2$, and should we be
scaling $(u - u^*)$ by a different factor? A point to be made is that this work is in the
introductory stages, so much work is still left to be done.

A variety of plots were presented comparing the control and topographic stress cases. The first plot showed how the transport streamfunction was recovered differently in the
two cases, with a notable difference in the Gulf Stream.

A series of plots of the Atlantic Ocean at different depths was presented. At the shallow
depths, surface dominates over the entropy tendency. Nevertheless, the new method sometimes gave flows going opposite to traditional models. At increasing depth, the entropy
tendency dominates, recovering tighter boundary currents. This was noticeable along the Gulf Stream.
Zonal velocities were examined at 60°W. Here the stress case, by recognizing the eddy tendencies, forced the Gulf Stream further offshore than in the control case.

These ideas can also be extended to coupled models, such as those including surface velocity and heat flux.

In conclusion, a very different approach is stressed in this talk, based on the concepts of moments of distributions and entropy. The key is that processes like eddy viscosity should not be centered about a state of mean rest. Rather, they should tend toward higher entropy mean flow. It should be noted that the higher entropy states described are only approximate. A key parameter to choose is $L^2$, as we discussed previously. There are a number of ways to parameterize the tendency toward higher entropy — Laplacian and biharmonic are two examples. It is not known at this point which is best for a particular problem. One final point to bring up is how to extend these concepts to smaller scales, and possibly stratification.

**Related Publications**


Statistics of Coherent Structure in Turbulence

Dr. James McWilliams

Jianping Mao and Greg Walker, Rapporteurs

Turbulence is ubiquitous in fluid media, including the Earth’s atmosphere and oceans, due to flow instabilities. The importance of turbulence in the climate system is as a transport agent on all scales that acts to maintain the partially mixed equilibrium distribution of physical and chemical properties in the atmosphere and oceans.

‘Coherent structures’ or ‘coherent vortices’ in turbulence are characterized by: the circumstances of their occurrence; their localized spatial structure; the statistical distribution of their properties (abundance, size, amplitude, shape) and its time evolution; the nonlinear dynamics associated with individual vortices in isolation and their interactions among a few neighboring vortices; and the aggregate dynamic behavior of structured turbulence.

This talk addressed two parts: the phenomenon and the statistics of the coherent structure in turbulence.

Phenomenon

There are many examples of the organized behaviors of turbulence or coherent vortex motions, at both large and small scales, in the atmosphere and oceans.

A well-known lab example is the Karman vortex street for flow passing an obstacle, such as an island, for atmospheric and oceanic flow. The turbulence is caused by the horizontal shear of the flow vorticity. Another type of lab example is the buoyant plumes which are forced in a fluid heated from below.

We can see other geophysical examples. When vertical shear of horizontal velocity occurs in a stably stratified fluid on scales large enough to be influenced by planetary rotation, it can be baroclinically instable. Products of this instability include familiar synoptic-scale cyclones of the extra-tropical atmosphere and oceanic vortices seen in the marginal ice zones.
Another potent means of producing coherent vortices is by the intensification of the ambient vorticity by suction-induced inflow. This is the cause of the intense winds that develop in hurricanes and tropical cyclones, as well as in tornadoes on a somewhat smaller scale, and in dust devils and water spouts on even smaller scales. Also, photographs of sunglint patterns from space show many spiral eddy patterns on the ocean surface, which are probably the result of a surface organized by boundary-layer inflow in small atmospheric cyclonic vortices.

Atmospheric blocking situations and the polar vortex in the stratosphere are other examples of coherent vortices in the atmosphere.

**Coherent Structures**

Three fundamental issues come up in the statistical study of coherent structures (CS):

1) How does one measure the intermittency in a turbulent fluid (*i.e.*, the number of CS)?

2) How does one detect, recognize and quantitatively measure CS?

3) How do we measure transport behavior in a turbulent flow with CS (*i.e.*, the probability density function of \( x(t) - x(0) \), where \( x(t) \) denotes the trajectory of a material parcel)?

One measure of intermittency is the spatial kurtosis, \( K \), of the potential vorticity or some other flow property, but there are more refined measures than \( K \); namely, the single point PDF and moment structure functions. For the moment structure functions, three types of fits are consistent with data: multifractal size scaling, shell models, and analytical statistical models.

Several difficulties face the detection of CS: (1) the definition of CS varies with the regime, (2) indeterminacy in size, amplitude and location of CS, (3) the deformability of CS. Nevertheless, there are three analysis techniques we can consider: (1) wavelet analysis, which improves the signal/noise ratio, (2) empirical orthogonal functions which handle deformations, but are vulnerable to indeterminacies, (3) a census which uses a specification of ideal shape to test for a degree of local conformity; a high enough degree indicates a vortex is a CS and its properties can then be measured.
The problem of measuring the PDF of \( x(t) - x(0) \), i.e., the issue of Lagrangian transport distributions, is directly relevant to climate models. The conventional measure is dispersion, but there are probably better measures.

Related Publications

Wavelets


EOFs


Identification of Flow Structures


Climate Modeling

*Dr. Philip Rasch*

*Barbara Kess and Todd Ringler, Rapporteurs*

This talk provided an overview on modeling the climate system. It described the fundamental driving radiative forces, and followed with a description of climate models, which range from very simple box, 1-D, 2-D through the more elaborate 3-D general circulation models. Simulations with these models were then briefly compared to observations.

The climate system can be thought of as being driven by solar radiation. As radiant energy enters the atmosphere it is absorbed, reflected and re-emitted by the atmosphere, ocean, land and ice. Some of the re-emitted radiation is reabsorbed in the atmosphere, creating a greenhouse effect. The constituents that cause the greenhouse effect are mostly water vapor, clouds, and CO$_2$.

The absorbed radiation minus the re-emitted radiation is positive in the tropics and negative in the extratropics. The role of the climate system is (in a sense) to transport heat from the tropics to the poles.

The climate system is modeled through application of the physical laws and driving forces that are thought to apply to this system. There are many components in the climate system and it must be decided which components are the most important and how to model them accurately. The components must be simple enough so the computation on a computer is feasible. The vast number of scales of importance mean it is impossible to explicitly represent every process thought to be important in the climate system. There is a hierarchy of models used to understand the climate system, ranging from simple to complex. The simplest models are termed box models in which energy in equals energy out. The first extension from the box model is the 1-D model, in which energy balance is applied not only in time but also in latitude. Two-dimensional models include latitude and height as independent variables.

The most sophisticated climate models use the three dimensions of latitude, longitude and height. The governing equations are integrated in time until a statistically relevant...
climatology is obtained. Many of the components of the climate system which are not solved explicitly in the models enter the model as prescribed boundary conditions (i.e., ocean, ice, biosphere). Some phenomena occur on large scale and are resolved explicitly by the model while other phenomena occur on smaller scales and must be parameterized. In most GCMs (General Circulation Models) the horizontal representation is spectral using a spherical harmonic basis. The coarsest horizontal resolution typically used is approximately equivalent to a 4.5° latitude by 7.5° longitude grid. Current workhorse integrations use 2.8° × 2.8° grid. Higher resolutions are being explored. The vertical coordinate is often a sigma or hybrid system. The advantage of these coordinates is that they follow the Earth's surface. Model coordinates do not intersect the ground.

One of the more sophisticated recent GCMs is NCAR's Community Climate Model, version 2 (CCM2). It is possible to couple the CCM2 to an ocean model and a biosphere model to form a more complex climate systems model.

The last portion of the talk compared model results to observations. GCM results obtained from CCM0, CCM1 and CCM2 were all compared. All of the models have deficiencies, but CCM2 seemed to compare best to the observations.

Related Publications


A sample of climate statistics from this model can be found in Climate statistics from the NCAR Community Climate Model (CCM2), by J. J. Hack, B.A. Boville, J.T. Kiehl, P.J. Rasch, and D.L. Williamson, Journal of Geophysical Research, in press.


A good description of the fundamentals of climate system modeling can be found in Climate System Modeling. (K. Trenberth, ed.) Cambridge University Press, 788 pp.
Motivation for and Approaches to Downscaling of GCM's

The scale of interest may be local compared with the GCM scale for some applications. For example, hydrologists may be concerned with the within grid cell probability distribution of precipitation, particularly in relation to catchment areas. This problem has implications in estimating probabilities of flooding, saturation and desertification.

Precipitation contours may be obtained from 5° x 5° GCM output, but precipitation is a rough field which is not yet modelled well with GCM's. Subgrid scale precipitation may be described in terms of a stochastic model depending on smooth atmospheric variables. GCM output of the smooth variables can be used as input for the stochastic model. This is the downscaling approach discussed by Dr. Guttorp.

Other approaches to downscaling include subgrid parametrization (where properties such as topography are used in the grid square), and scaling (as discussed by Dr. Waymire in his talks).

Stochastic Precipitation Models

Dr. Guttorp overviewed 3 types of rainfall models: namely, Markov chain models, point process models, and hidden Markov models. Hughes and Guttorp (1994a,b) use the last of these approaches, with some extensions.

Markov Chain Models

Markov chain models date back at least to Quetelet in 1852. A first order Markov chain model was used by Gabriel and Neumann (1962) to model the presence or absence of rainfall. Their first order Markov assumption was that
Pr (rain today | history of presence or absence of rainfall up to and including yesterday) 
= Pr (rain today | presence or absence of rainfall yesterday).

This model performed poorly for persistence of wet (or dry) states, and for representing spatial structure. Precipitation is the only atmospheric variable included in the model.

**Point Process Models**

If one assumes that precipitation data are continuous in time, one may consider the start of each rainfall event when a storm passes over a station, as a point in time. A mark representing the volume of rainfall may be associated with each point, giving a marked point process. These models have several drawbacks, including that precipitation measurements are often discontinuous in time. Difficulties also exist in considering spatial structure using these models.

**Hidden Markov Model**

The hidden Markov model is discussed in Baum (1972), and was introduced into the hydrological literature by Zucchini and Guttorp (1991). Hughes and Guttorp (1994 a,b) use hidden Markov models based on unobserved weather states to include atmospheric information in the model. Weather states are defined as theoretical constructs which summarize atmospheric conditions, and are modeled as unobserved Markov chains. These models will be discussed further in the next section.

**Including Atmospheric Variables**

Using clustering algorithms to define atmospheric regimes have traditionally not worked well for precipitation studies, even with the inclusion of precipitation data in the clustering (Hay et al. 1991; Hughes et al. 1993).

Non-homogeneous hidden Markov models are another approach for including atmospheric variables (Hughes and Guttorp, 1994 a,b). Atmospheric variables are used in determining weather states. The following notation is used:

For time $t$ in an observation window $\{1, \ldots, T\}$, and $n$ sites, let

- $S_t$ = weather state at time $t$
- $X_t$ = atmospheric measurements or summaries at time $t$
- $R_t$ = a vector (length $n$) of precipitation at time $t$
- $X^T_t$ = atmospheric history from time 1 through $T$
- $S^T_t$ = history of states from time 1 through $t$
- $R^{t-1}_t$ = precipitation history from time 1 through $t - 1$
Hughes’ model (Hughes and Guttorp, 1994 a,b) makes the following two simplifying assumptions

\[
\begin{align*}
\Pr (S_t | St^{t-1}, X_t^T) &= \Pr (S_t | St_{t-1}, X_t) \\
\Pr (R_t | St^T, R^{t-1}_t, X_t^T) &= \Pr (R_t | S_t).
\end{align*}
\]

**Parameterization**

The conditional state transition probabilities \(\Pr (S_t | S_{t-1}, X_t)\) are parameterized using logistic regression, \(i.e.,\)

\[
\Pr (S_t | S_{t-1}, X_t) \propto \exp (a_{S_{t-1}, S_t} + b_{S_{t-1}, S_t} X_t).
\]

If one considers the presence or absence of rainfall, one can parameterize the spatial precipitation structure using the autologistic model (see references in Cressie, 1991). This gives

\[
\Pr (R = r | S_t = s) \propto \exp \left( \sum_{i=1}^{n} \alpha_i r_i + \beta_s \sum_{j<i} \frac{r_i r_j}{d_{ij}} \right),
\]

where \(r = [r_1, \ldots, r_n]^T\) with

\[
r_i = \begin{cases} 1, & \text{if rain at site } i \\ 0, & \text{otherwise} \end{cases}
\]

and where

\(\alpha_i\) controls the model probability of rain at site \(i\), conditional on weather state \(s\),

\(\beta_s\) is a dependence parameter controlling the probability of rain at both stations \(i\) and \(j\), a distance \(d_{ij}\) apart, conditional on weather state \(s\).

Parameter estimation is discussed in detail in Hughes and Guttorp (1994a,b). Rainfall volumes could be used instead of the presence or absence of rainfall.

**Example**

Hughes and Guttorp (1994b), investigated daily precipitation data from 24 stations in Washington state for the months January through March of the years 1965 through 1985. Five weather states are derived based on EOF’s of the sea-level pressure field and the 500 mb geopotential height field.

Plotting the sea-level pressure contours, the 500 mb geopotential height contours, and the site-specific mean precipitation separately for each weather state, showed that the derived weather states may be meteorologically interpretable in terms of weather systems. For example, precipitation patterns for weather state 3 capture the rain shadow east of
Puget Sound, while weather state 4 captures a low precipitation state characterized by a high mean sea-level pressure over British Columbia.

Plotting between-site correlations versus distance separately for each of the five states, indicates some spatial correlation for 4 of the 5 states. This highlights the need for a spatial component, such as the autologistic component in Hughes' model. Further details, including descriptions of the weather states, their derivation, and possible interpretations, are given in Hughes and Guttorp, 1994b.

Future Work

These models might be extended to the continental scale, although further validation of the approach is still needed on all scales. Possible continental extension approaches include splicing together independent regional models; using K-variate weather states for K regions; using a dependence model where the weather state in particular region depends on the previous day’s weather in surrounding regions; and using regional and continental weather states.

References


Estimating Global Mean Temperature
From Instrument Data

Dr. Richard Gunst

Richard Levine and David Montroy, Rapporteurs

The problems associated with estimating a spatial mean from a set of observations may be shown through the example of temperature anomalies over a given region. The temperature anomaly $a_t(s)$ at location $s$ and time $t$ is given by

$$a_t(s) = T_t(s) - \tau(s)$$

where $T_t$ is the temperature and $\tau$ is the mean temperature at $s$. The temperature anomaly is used as a measure of temperature to help eliminate local effects such as altitude, "urban-heat island" warming, and varying measurement techniques.

Temperature anomalies are measured at irregularly spaced locations. These data, like all spatial data, are a realization of a statistical process driven by physical forces and measurement techniques. Because a process can be associated with the data, there is interest in predicting the particular variable either as a value at a specific point or as a mean over a specified region $R$. The spatial mean temperature anomaly is given by:

$$z_t = \frac{\int_D a_t(s) \, ds}{\int_D ds}.$$  \hspace{1cm} (2)

Due to the discrete nature of observations, it cannot be calculated directly, but may instead be estimated by:

$$\hat{z}_t = \frac{\sum w_ia_t(s_i)}{\sum w_i}.$$  \hspace{1cm} (3)

where $s_i$ is the $i^{th}$ spatial location at which an observation is taken and $w_i$ is a weighting factor applied at that location. Selection of the weighting factor is crucial in computing the prediction $\hat{z}_t$. Ideally, $w_i$ is chosen so that $\hat{z}_t$ is an unbiased predictor having minimum variance.

When spatial data are gathered, two classical textbook assumptions are sometimes implicitly utilized. First, it is assumed that the observations are indeed a representative sample from a fixed spatial population with constant mean $\mu$ and constant variance.
Additionally, the observations are assumed to be statistically independent. Neither assumption holds in practice, however. The population is evolving as the number of observing stations and the density thereof changes. Also, due to variations in distributions of stations over land and sea, it is nearly impossible to obtain a representative sample of observations from all regions of the globe. The second assumption does not hold, either, as observations are correlated to varying degrees due to the physical properties of the system.

One approach for predicting the spatial mean is optimal averaging, whereby the weights in (3) are chosen in order to minimize the variance of the difference between the estimated and actual mean. If anomalies are uncorrelated, the solution is given by:

\[ w_i = \frac{C(s_i)^{-1}}{\sum C(s_j)^{-1}} \]  

where \( C(s_i) \) is the variance of the temperature anomaly at \( s_i \). (It is assumed all weights must sum to one.) If the variance of the anomalies is a constant, \( w_i = n^{-1} \) (where \( n \) is the total number of locations) and an ordinary average is applied. If the variance is a function of distance, the average is distance-weighted. Certain deficiencies are associated with the usual application of distance weighting, however. The spatial correlation structure ordinarily is not used in selecting the distance weighting, the weights are not optimal unless data variances are increasing functions of distance, and no criteria exist for determining the uncertainty of the predictor.

An alternative to the methods previously discussed for predicting spatial mean temperature anomalies considers the spatial correlation structure in the data. As stated by Cressie (1991), we can separate the stochastic and deterministic components that describe the changes in temperature anomalies over space at a particular instant in time \( t \), say

\[ a_t(s) = \mu(s) + e_{ss}(s) + e_{ms}(s) + e_{me}(s) \]  

Here, \( s \) denotes spatial location of the station where observations are taken; \( \mu(s) \) represents the large-scale variability in the data such as topography and persistent spatial trends; \( e_{ss}(s) \) consists of small scale continuous variation, such as regional and local changes; \( e_{ms}(s) \) models microscale discontinuous variation, such as station inhomogeneities; and \( e_{me}(s) \) allows for measurement error. Unfortunately, even if we can combine the stochastic components into one general error term, \( e(s) \), which models the spatial correlation between observations at different stations, the model is overparameterized. Here, to fit the model, we need to either assume the spatial covariances are known or simplify the correlation.
structure, thus diminishing the number of unknown parameters. Such a reduction can be accomplished by presuming a functional form for the correlation between observations over distance (see Cressie (1991) and Cherry's talk in this report, in particular with respect to variograms). Then, using the estimates obtained for the spatial correlations, we can use standard statistical techniques to estimate the trend components, \( \mu(s) \), of model (5). For example, if we assume \( \mu(s) \) is a linear function of location or other nonstochastic spatial variates so that

\[
\mu(s_i) = x(s_i)^T \beta ,
\]

where \( i \) is the station number and \( T \) denotes the transpose, then generalized least squares methodology can be implemented to estimate the vector parameters \( \beta \). The vector \( x(s_i) \) in this representation contains location (e.g., latitude and longitude) and other nonstochastic spatial variates corresponding to location \( s_i \).

Critical to the optimal prediction of spatial means using either optimal averaging or kriging methods is the estimation of spatial correlations. When data are collected over both space and time, there is some ambiguity about how to correctly estimate spatial correlations. Models more general than (5) can be proposed in which both spatial and temporal effects are included. For example, model (5) could represent the spatial component of the model for a fixed point in time and additional temporal components could be added; e.g., linear trend or autoregressive terms. Difficulties occur in separating the effects of spatial and temporal changes on the changes in anomalies. Moreover, both the temporal and the spatial correlation structures would have to be modeled in order to overcome the overparameterization problem.

These difficulties notwithstanding, there has been much work done on modeling spatial correlation structures. A common and popular method is to assume that the covariance between two anomalies \( a(s_i) \) and \( a(s_j) \) can be estimated as

\[
\text{cov}(a(s_i), a(s_j)) = \hat{\sigma}_i \hat{\sigma}_j \hat{g}(r_{ij})
\]

where \( \hat{\sigma}_i \) is the estimated standard deviation of anomalies at location \( s_i \) and \( \hat{g} \) is a curve fitted to the correlations \( r_{ij} \) calculated from anomaly time series between pairs of stations. Such methods must be used with care since the correlations \( r_{ij} \) are affected not only by the spatial autocorrelation but also by temporal autocorrelation and nonstationarity. It is easily demonstrated that temporal autocorrelation or nonstationarity can dominate the
value of $r_{ij}$, regardless of the strength of spatial correlations. Thus, this method should only be used when the meteorological variable under study is known to lack strong temporal structure. Depending on the regions and the time span, temperature anomalies might or might not possess temporal structure.

Fortunately, there is an effective alternative to correlation function modeling that does not require the use of time series correlations. Structure function (variogram) modeling is based on the calculation of half the variance of the difference of two anomalies a fixed difference apart:

$$\gamma(s_i, s_j) = \text{var} \{ a(s_i), a(s_j) \} / 2.$$  

This quantity is estimated for groups of stations that are binned according to their distance and direction from one another. Since the anomalies used in the calculation of the estimate are all from the same time point or period, the temporal structure, if any, in the anomalies does not affect the estimates. From the estimated structure function, spatial correlations can be estimated. The references (e.g., Cressie, 1991; Gandin, 1963; Matheron, 1963) detail this method of estimating spatial correlations.

References and Related Publications


GCM Modeling Experiments

Dr. Joseph J. Tribbia
Katrin Müller and Barbara Kess, Rapporteurs

The talk discussed three types of GCM modeling experiments. These experiments corresponded to equilibrium climate sensitivity, climate variations and climate forecasting. The experiments presented were not meant to be a comprehensive treatment of GCM modeling.

Equilibrium Climate Sensitivity (Signal to Noise)

The following experiment is described in detail in Blackmon et al. (1983). It attempts to answer the question of whether midlatitude correlations with the El Niño Southern Oscillation (ENSO) can be explained by atmospheric response to anomalous sea surface temperature (SST). For this experiment a control integration was performed using climatological SST's as a lower boundary condition in a climate model. The results from the control run were compared with another model integration using Tropical Pacific SST anomalies associated with a warm ENSO event. An additional model experiment was performed with the same anomalies doubled.

Changes in the 500 hPa height field associated with the twice as warm El Niño event were compared to observed anomalies. The patterns showed some similarity, but were not identical. A two-sided t-test revealed that the response of the model with ENSO-like SST was significantly different from that of the control.

Climate Variations

Structure of Low Frequency (10-90 Days) Variability

A traditional way of looking at low-frequency variability is by estimating empirical orthogonal functions (EOF's) because the leading EOF's are typically of large spatial scale and they vary relatively slowly with time. The four leading EOF's of wintertime monthly mean 500 hPa height fields were shown. It turns out that the forecast skill of
models improves if only the leading EOF’s have to be forecast (Branstator et al. 1993). In connection with EOF’s it should be mentioned that they do not necessarily identify different patterns when the amount of variability explained by them is almost equal (North et al. 1982).

**Existence of Multimodality and Regimes**

A paper by Sutera (1986) deals with the possible bimodality of the atmosphere. The data used were 16 Northern Hemispheric winters of 500 hPa height fields. The index chosen for the investigation was wavenumber 3 amplitude. The probability density distribution (number of occurrence of a specific amplitude) in the 42°S-60°S zone was derived by using maximum penalized likelihood (MPL) technique (Good and Gaskins, 1980). The curve shows two relative maxima. The statistical significance of these maxima estimated by Monte Carlo techniques exceeds 95%, but does not take into account the serial correlation of the data.

Distribution fitting MPL was also applied to model output. The NCAR CCM0 also showed bimodality whereas the probability density distribution for the NCAR CCM1 only had one single peak.

The extension of this technique to more than one index was done by Trevisan (1994). The probability density distribution of EOF1 and EOF2 of monthly mean 500 mb height shows clusters which are related to climate anomalies, but the significance of this approach is questionable, due to sampling considerations.

**Low-Frequency/High-Frequency Relations**

Can high-frequency eddies (highs and lows on a weather map) influence the low-frequency structure of the atmosphere? Cross-correlations between high-frequency variance and 500 hPa streamfunctions confirm this influence. Model experiments with the NCAR CCM2 also show this effect (work in progress, Gerardo Decanio and Joseph Tribbia).

**Forecasting Experiments (Non-Equilibrium Statistical Mechanics)**

An experiment of prediction beyond the deterministic limit was performed. The integrations of two slightly different initial states show great differences after 11 days of model integration (Tribbia and Baumhefner, 1988).
References


Comparison of Model Output with Observations

Dr. James W. Hurrell

Richard Levine, Rapporteur

The climate system consists of the atmosphere, ocean, cryosphere, land and the biosphere. Each component interacts with one another in complex ways on various space and time scales. As a result, the climate system evolves continuously with complicated feedback loops existing between the interacting components. Climate models attempt to study and predict future states of this system. These models rely on basic physical laws and mathematical equations to formulate the physical, chemical, and biological processes governing the climate system, and they represent the only available means to consider simultaneously the wide range of interacting processes. However, computing power constrains even the most sophisticated model in terms of the amount of detail that can be resolved. For example, the horizontal resolution of a typical climate model may be 4.5° latitude by 7.5° longitude with 10 vertical levels, which only allows for interpretation of large-scale climatic features. Subgrid-scale phenomena are represented collectively by parameterization rather than individually. Thus, even our “primitive equation” models contain empirical statistical elements, and uncertainty about the parameterization of feedback mechanisms is one reason reliable, verified forecasting of key climate variables is difficult to obtain.

These difficulties in modeling the climate system necessitate the use of validation techniques to ensure the appropriateness of the models. One method for validating a climate model is to check its ability to simulate the present day climate. This approach demands extensive global data sets of high quality, a criterion that is difficult to meet for such fields as moisture, clouds and precipitation over the oceans. In addition, these data need to be of high quality, long-time length, and proper scale (i.e., compatible with model output in space and time resolution), and many different fields need to be examined in an attempt to gain a fairly complete view of a particular simulation. Existing historical land and marine records often contain errors and do not provide global coverage. Satellite measurements are
an important, relatively new source of information and often provide global coverage, but short data records and precision and calibration errors introduce uncertainties. An alternative is the global analyses produced using four-dimensional data assimilation at the major operational forecast centers, such as National Meteorological Center (NMC) and European Centre for Medium Range Weather Forecasts (ECMWF). Data assimilation procedures attempt to reduce errors in observations by combining the observations with short-term numerical forecasts using optimum-interpolation methods. While inhomogeneities may be introduced by operational changes to the numerical model used in the data assimilation procedure, such global analyses are useful for validating the output from sophisticated 3-D climate models.

In addition to considerations of accuracy and precision of existing data sets utilized in model validation, a major issue is the significance of discrepancies between model output and observed data. How serious can a model’s errors be in order for the model to still be useful? Mitchell et al. (1987) pointed out that some discrepancies between simulated and observed climates may be allowable, provided the patterns are sufficiently alike that relevant physical mechanisms can be identified. For example, most climate models indicate that, with increased carbon dioxide and warmer surface temperatures, precipitation will increase in areas where it was already large. Thus, even if regions of heavy precipitation are located differently in the models, the implications for real climate change would be the same. It is clear, nonetheless, that the most reliable conclusions concerning climate change will be made from models with the most realistic simulations.

Examples of recent model validation efforts include the work of Boer et al. (1991, 1992), Gates et al. (1990, 1992), and Gates (1992). Hurrell, Hack, and Baumhefner (1993) and Hurrell (1994) compare the simulated climates of several NCAR atmospheric general circulation models (AGCMs) to data obtained from ECMWF. Though these comparisons provide much insight into the ability of these AGCMs to accurately represent atmospheric processes, research still needs to be done in the area of model validation. In particular, most validation efforts focus on comparison of means and variances of model variables. These analyses do not examine the models’ ability to account for feedback loops or interactions between variables nor phenomena such as cyclones or monsoons. Appropriate statistical methods may allow us to more fully compare the model output with observations.
Additionally, reliable observations of climatological variables and processes are necessary. If the data used for comparison are incomplete or biased, it is difficult to determine whether discrepancies between the model output and observations are due to the model or observational errors. Furthermore, model validation procedures can be extremely computer intensive and in some cases unmanageable. Statistical techniques and efficient numerical algorithms may reduce some of the difficulties of comparing model output and observations and, hence, lead to more manageable and complete model validation procedures.

References


Some Aspects of Climate Intercomparison

Francis W. Zwiers
Jianping Mao and Kevin Walsh, Rapporteurs

Statistical comparisons of means are frequently conducted in climatology to intercompare observed and/or simulated climates among themselves or against fixed reference values. These comparisons are conducted by employing a paradigm in which (a) a statistical model is imposed upon the samples of climate data; (b) a null hypothesis $H_0$ which is to be tested is specified; (c) an alternate hypothesis $H_a$ which guides the interpretation of the test statistic is also specified; and (d) a test statistic is computed to determine how unusual the observed difference of means is in the context of the model and the null hypothesis.

This talk described some details of statistical methods which are commonly used for climate intercomparison, and showed that some of them are based upon ad hoc assumptions. The first part of this summary reviews scalar techniques for evaluating the difference of means. The second part discusses the evaluation of field significance.

Scalar Significance Tests of the Difference of Means

Scalar tests are conducted at grid points, and can be combined with a re-sampling procedure to aid interpretation of results. The classic test used is the well-known student’s $t$ test, which assumes that the observations are statistically independent and Gaussian distributed. When observations are serially correlated, the standard error estimator employed in the $t$-statistics underestimates the true variability of the difference of means with the result that magnitude of the $t$-statistic is inappropriately inflated and the significance level of the test is greater than specified by the user. In addition, in some climate studies, the data are not Gaussian distributed. A usual approach to the problem of serial correlation is to calculate the time or space interval between effectively independent data. A sample of dependent data of size $N$ is then equivalent, to some extent, to a sample of independent data of size $N_e$, where $N_e$ is calculated from $N$ and appropriate values of the autocorrelation function.
A relatively clear and simple-minded solution to this problem is to form subsamples of independent observations and then use the usual $t$ test with this new, smaller sample size. Unfortunately, this solution is achieved by throwing away some of the information contained in the data. An alternative, heuristically-derived approach is to scale the ordinary $t$-statistic by a factor which depends upon an estimate of $N_e$ and then to compare the resulting statistic against critical values from a $t$-distribution with degrees of freedom which depend upon the estimated $N_e$ (e.g., Thiebaux and Zwiers, 1984). This "heuristic" test usually does not work well, because (1) the heuristic $t$ does not have a student's $t$ distribution; (2) the equivalent sample size $N_e$ is hard to estimate well; and (3) simulations show the test has the wrong size in that it may be conservative or liberal depending upon various assumptions.

A more formal approach to the problem of testing the means of a time series is to base inference on the likelihood ratio (LR) test. The main idea of this test is that the likelihood of the observations is maximized under two scenarios, one in which the null hypothesis is true ($H_0$) and the other in which it is false ($H_a$). The ratio of these two quantities is calculated. Asymptotic theory provides a large sample reference distribution for the natural logarithm of the likelihood ratio, and also demonstrates that the LR test is asymptotically optimal. In this test, improvements are also possible by means of small sample corrections.

The problems mentioned above in discussion of the heuristic test could also be ameliorated by the use of an empirical test procedure, the table lookup test. This test is based on the following ideas: (1) the use of the ordinary difference of means statistic which is not adjusted for serial correlation; (2) determination of critical values which are conditional low-variance indicators of serial correlation instead of high variance indicators such as the estimated equivalent sample size; (3) the serial correlation indicator which is chosen is the lag one serial correlation coefficient.

**Field Significance Tests**

Scalar tests evaluate significance at a grid point, whereas field significance tests (as their name indicates) evaluate the statistical significance of the entire simulated or observed field. One could use the multivariate Hotelling $T^2$ test for this purpose, which is a multivariate analogue of student's $t$ test. Unfortunately, this test lacks power for the number of realizations and grid points typical of climate studies. Moreover, in many
climate problems the dimensionality of the observed field is greater than the sample size with the consequence that Hotelling's $T^2$ cannot be computed because the sample variance/covariance matrix is of less than full rank. One solution to this problem requires the use of simplified covariance structure models with a small number of free parameters. Another solution is to reduce the dimensionality of the fields. This may be accomplished by projecting the fields onto EOFs or other basis functions. It is important however that the basis functions are derived from a data set independent of those being compared.

A more usual approach is to evaluate the local significance of each grid point and then to determine the field significance based on a statistical interpretation of the number of points at which the local test is rejected or accepted. If there is no spatial correlation, this is easily performed by assuming that the number of reject decisions is binomially distributed, or, for a larger but still reasonable number of grid points, that the number of reject decisions $L$ is approximately Gaussian distributed with a known standard error. Nevertheless, in the real world the data is spatially correlated, which means that $L$ is more variable than the binomial distribution predicts.

An example of a method which corrects for spatial correlation is provided in Livezey and Chen (1983), in which they discuss the interpretation of a map of correlations between a scalar index (the Southern Oscillation Index, SOI; see for example, Rasmusson and Carpenter, 1982) and a series of two dimensional fields of winter mean 700 mb geopotential. It is assumed that serial autocorrelation is not present in either the SOI or the geopotential fields; this is a reasonable assumption, given that each field is separated by a year in time. If one first assumes that the grid points are independent (i.e., have no spatial correlation), the significance levels depends (as above) on the standard error of a Gaussian distribution. If the points are not independent, a different approach needs to be taken, one based upon a parametric re-sampling technique. This method works when one component of the pair being tested is a scalar or low-dimensional vector, and the other is a field which is not simulated by the re-sampling technique, and hence which preserves its spatial structure.

For "unpaired" fields (i.e., where unlike the previous example there is no a priori hypothesized connection between any two pairs of the compared data sets) one can use a permutation procedure or bootstrap technique to generate distributions from data. In both cases, it is important that the spatial covariance structure of the data is maintained. It is also necessary that there is no serial correlation in the data. An example is found in Zwiers and Boer (1987), in which a climate model simulation with a seasonal cycle of solar
forcing was compared to a simulation performed in so-called “perpetual” mode, in which the solar forcing is set to some fixed value, typically a seasonal mean.

It is concluded that multivariate tests are usually not useful for climate problems because of their lack of power, while re-sampling procedures are sensitive to serial correlation. Ensembles of pointwise tests may be interpreted globally, but this assessment depends upon the power of the local tests and the spatial structure of the fields so compared. The methodology of the evaluation of field significance is thus still in its formative stages.

References and Related Publications


Climate Signal and Weather Noise

Dr. Cecil E. Leith, Jr.

Jianjian Gong and Jean Opsomer, Rapporteurs

Introduction

In this lecture, the differences between weather noise and climate signal are discussed. A simple model based on the fluctuation dissipation equation is proposed to capture and separate both effects. An important potential shortcoming of current modeling efforts, known as the "infrared climate problem", is briefly addressed.

Weather Noise

Weather is the prototypical chaotic system with essentially no predictability beyond a week, while climate is the expected or averaged weather, i.e., consists of all of the probabilistic properties of weather fluctuations.

Ideally one would like to consider an average over an ensemble of the weather of a myriad of earths. In practice, we are limited to finite time average statistics for our single earth in order to estimate climate probabilities. We thus face a sampling problem.

Weather anomalies can be considered as a stationary time series, $x(t), t \geq 0$, with time-lagged correlation $R(\tau) = X(\tau)/X(0)$ where $\langle x(t) \rangle = 0$, and $X(\tau) = \langle x(t), x(t + \tau) \rangle$. We can consider an effective time between independent samples as

$$S = \int R(\tau) d\tau$$

and thus the effective number of independent observations is

$$N = T/S$$

where $T$ is the total time interval of observations.

Since $S$ is of the order of a week, it is to be expected that there will be sizable fluctuations in seasonal average of the observations from year to year based solely on sampling fluctuations, even with no change in the underlying climate probabilities.

This is weather noise, and it tends to obscure the possible real changes in climate probabilities.
Climate Signal

A key problem in climate system modeling is the detection of a change in the underlying climate probabilities in response to changing external influences, especially those induced by human activities.

For the climate system, consider the symbolic evolution equation for the state vector $x$

$$\frac{dx}{dt} = Q(x) + f(t)$$

where $Q(x)$ represents complicated nonlinear internal dynamical and physical processes and $f$ represents some external forcing influences. The existence of a stationary base climate is assumed, and without loss of generality we assume for the base climate that $\langle x(t) \rangle = 0$.

The simplest question for the climate system is its sensitivity as given by the infinitesimal response of $\langle x \rangle$ to an infinitesimal change in $f$, i.e., the linear probabilistic response of the mean climate.

We look then for the Green’s matrix function $G(\tau)$ such that

$$\delta(x)(t) = \int_{-\infty}^{t} G(t-s) \delta f(s) ds .$$

Note that an impulsive $\delta f(0)$ introduced at time $\tau = 0$ will induce a jump in $\langle x \rangle = 0$ to $\langle x \rangle(0+) \langle x \rangle(0+)$ which relaxes back to the base state $\langle x \rangle = 0$ by the relation

$$\langle x \rangle(\tau) = G(\tau) \langle x \rangle(0+) .$$

The state vector $x$ may have of the order of a million components. The corresponding $G(\tau)$ provides the linear response of any one component to a perturbation in any other. The determination of all of the elements of $G$ is therefore an overwhelming task by the usual method of making long integrations with numerical models.

In practice, instead of perturbing each component separately, a collective perturbation is introduced, such as of sea surface temperature over a domain or of perturbed heating induced by a change in CO$_2$ concentration in the global atmosphere.

Fluctuation Dissipation Relation

An alternative approach to estimate $G(\tau)$ is to use the fluctuation dissipation relation, which states that

$$G(\tau) = R(\tau) \text{ for } \tau \geq 0$$
where $R(\tau)$ is the time-lagged correlation matrix for the system. This relation is theoretically only valid for systems in thermodynamical equilibrium, however, so that it is unclear whether this approach is better than direct model estimation of $G(\tau)$.

In any case for model estimation of $G(\tau)$, it is important for the model generated $R(\tau)$ to be realistic, otherwise it is unlikely that the $G(\tau)$ of the model would agree with that of the real atmosphere.

**Stochastic Climate Model**

A stochastic climate model that incorporates the fluctuation dissipation relation can be devised and would provide a crude estimate of climate sensitivity. This model would be of the Langevin type, with random white forcing and specified damping that mimic all first and second moments of the real atmosphere. This model could then be refined and expanded to improve prediction ability for mean flow and for spatial and temporal dependencies.

**Infrared Climate**

Unfortunately, there is evidence that such a model would be unsatisfactory to capture some of the low-frequency phenomena observed in the atmosphere. This is referred to as the *infrared climate problem* and appears to be caused by non-linear interactions of the chaotic internal weather frequencies that potentially induce a “piling up” of extra variance at the low frequencies.

This infrared climate problem further obscures the predictability of climate change induced by slowly changing external influences, anthropogenic or not.

**Related Publications**


Summary

Dr. Jones described several models for continuous time processes with both regularly and unevenly spaced observations. He then extended these model for processes on a circle and processes in space. He indicated how to carry out computations for estimating the parameters of these models with either a single realization or several independent realizations of the process.

In his second talk, Dr. Jones continued by describing time series analysis on a 2-D sphere, using concepts of spherical harmonics and axial symmetry. Also discussed were the modeling of spatial random fields using spatial ARMA models. These models are illustrated by three data sets.

Time Series Analysis

Let $x(t)$ be a time series in continuous time. The mean function is

$$ E \{ x(t) \} = \mu(t) $$

The variance function is

$$ E \{ [x(t) - \mu(t)]^2 \} = V(t) $$

The mean function is typically handled by regression. For a periodic function with period $P$, this might be a Fourier series truncated after some harmonic.

$$ \mu(t) = \beta_0 + \beta_1 \cos(2\pi t/P) + \beta_2 \sin(2\pi t/P) $$

$$ + \beta_3 \cos(4\pi t/P) + \beta_4 \sin(4\pi t/P) $$

The mean value function could be considered to be climatology, the function to which statistical predictions converge as the prediction interval increases.
The estimation of the regression is complicated by variance heterogeneity and serial correlation. The variance may also be periodic

\[ V(t) = \gamma_0 + \gamma_1 \cos(2\pi t/P) + \gamma_2 \sin(2\pi t/P) + \cdots \]

In this expression, certain values of the \( \gamma \)'s may give negative variances, so a better periodic model for a variance function is

\[ V(t) = \exp \{ \gamma_0 + \gamma_1 \cos(2\pi t/P) + \gamma_2 \sin(2\pi t/P) + \cdots \} \]

This is sometimes called a log linear model, and can also be written

\[ V(t) = \sigma^2 \exp \{ \gamma_1 \cos(2\pi t/P) + \gamma_2 \sin(2\pi t/P) + \cdots \} \]

where

\[ \sigma^2 = \exp(\gamma_0) \]

The correlation function of the time series at two different times is

\[ \rho(s, t) = \frac{E \{ [x(s) - \mu(s)] [x(t) - \mu(t)] \}}{\sqrt{V(s)V(t)}} \]

A stationary process has

\[ \mu(t) = \mu \quad \text{constant} \]
\[ V(t) = V \quad \text{constant} \]

\( \rho(s - t) \) depends only on the time difference; however, a stationary error process can also be defined where the mean value function is not constant, but the variance function is constant and the correlation function depends only on the time difference. In this case, the covariance function is

\[ C(s - t) = E \{ [x(s) - \mu(s)] [x(t) - \mu(t)] \} \]

Periodic structure, Gladyshev (1961), Pagano and others.

**Frequency domain analysis**

If there are equally spaced observations with spacing \( \delta t \), the spectral density can be estimated up to frequency \( 1/2\delta t \), and higher frequencies are aliased and added to spectral density at lower frequencies. The estimation can be carried out using the fast Fourier transform (FFT) with smoothing in the frequency domain to reduce the variance of the estimate, or via estimation of the covariance function, applying a weight function (lag window) and Fourier transforming.
**Parametric models**

**Autoregressive models**

\[ y(t_j) - \mu(t_j) = \phi_1 [y(t_{j-1}) - \mu(t_{j-1})] \]
\[ + \phi_2 [y(t_{j-2}) - \mu(t_{j-2})] \]
\[ + \cdots + \phi_p [y(t_{j-p}) - \mu(t_{j-p})] + \epsilon(t_j) \]
\[ \epsilon(t_j) \sim \mathcal{N}(0, \sigma^2) \]

Standard estimation methods are least squares, maximum likelihood, maximum entropy and Yule-Walker equations.

Spectral density

\[ s(f) = \frac{\sigma^2}{\left| 1 - \sum_{k=1}^{p} \phi_k e^{2\pi ikft} \right|^2} \]

**Autoregressive, Moving Average (ARMA) Models**

An Autoregressive, Moving Average (ARMA) model, or order \((p, q)\) with the mean function subtracted is

\[ y(t_j) - \sum_{k=1}^{p} \phi_k y(t_{j-k}) = \epsilon(t_j) + \sum_{k=1}^{q} \theta_k \epsilon(t_{j-k}) \]

and has spectral density

\[ s(f) = \frac{\sigma^2}{\left| 1 - \sum_{k=1}^{p} \phi_k e^{2\pi ikft} \right|^2} \left| 1 - \sum_{k=1}^{q} \theta_k e^{2\pi ikft} \right|^2 \]

This is referred to as a rational spectrum. ARMA models are often used in business applications for the purpose of forecasting.

**Unequally spaced observations**

If observations are unequally spaced, frequency domain methods tend to break down. The orthogonality of the sines and cosines used in the Fourier transform is lost. Suppose there are \(n\) unequally spaced observations

\[ y(t_1), y(t_2), \ldots, y(t_n) \]

Assume that the observations are Gaussian, have a periodic mean function, a periodic variance function, and a first order autoregressive error structure (AR(1)). With unequally
spaced observations, it is necessary to assume that the underlying process is a continuous
time process. A continuous time AR (1) error structure has an exponentially decaying
correlation, as does a discrete time AR (1) structure. The correlation matrix of AR (1)
errors for unequally spaced observations is of the form

$$C_{ij} = \begin{cases} 1 & \text{if } i = j \\ \exp \{-\alpha |t_i - t_j|\} & \text{if } i \neq j \end{cases}$$

**Random Process on a Circle**

Let $s$ be a point on the unit circle, $0 \leq s \leq 1$, and $y(s)$ be a random function on the
circle. Expand the random function in a Fourier series

$$y(s) = \xi_0 + \xi_1 \cos (2\pi s) + \xi_2 \sin (2\pi s) + \xi_3 \cos (4\pi s) + \xi_4 \sin (4\pi s) + \xi_5 \cos (6\pi s) + \xi_6 \sin (6\pi s) + \cdots$$

where the $\xi$'s are random variables with

$$E \{\xi_i\} = \beta_i$$

The mean function of the random function is

$$\mu(s) = \beta_0 + \beta_1 \cos (2\pi s) + \beta_2 \sin (2\pi s) + \cdots$$

If the spatial covariance structure is stationary, i.e., depends only on the spatial
distance between two points, the covariance matrix of the $\xi$'s will be diagonal, and the
diagonal elements will be the discrete wave number spectrum of the spatial process $g_\nu^2$ and
the covariance function is

$$C(s_1, s_2) = \sum_{\nu=0}^{\infty} g_\nu^2 \cos [2\pi \nu(s_1 - s_2)] .$$

where

$$\text{var} (\xi_0) = g_0^2$$
$$\text{var} (\xi_1) = \text{var} (\xi_2) = g_1^2$$
$$\text{var} (\xi_3) = \text{var} (\xi_4) = g_2^2$$
$$\vdots$$

So

$$\text{var} (y(s)) = \sum_{\nu=0}^{\infty} g_\nu^2$$
Without spatial stationarity, the covariance matrix of the $\xi$'s will not be diagonal. If we diagonalize this matrix (principal components), the linear combinations of the sines and cosines formed using the eigenvectors are *empirical orthogonal functions*.

**Unequally Spaced Observations in Space**

Suppose we have discrete observations at unequally spaced spatial locations

$$y(s_i), \quad i = 1, n$$

We would like to interpolate to a regular grid. Assume a stationary covariance structure truncated at some wave number, $p$.

The elements of the error covariance matrix $V$ are

$$V_{ij} = \delta_{ij} \sigma_o^2 + \sum_{\nu=2}^{p} g_\nu^2 \cos[2\pi\nu(s_i - s_j)]$$

where $\sigma_o^2$ is the observational error variance (nugget effect). The error covariance matrix has $p$ unknown parameters, $\sigma_o, g_2, \ldots, g_p$.

Suppose we have independent realizations of the random field observed at unequally spaced locations, not necessarily the same for each realization

$$y_k(s_{ik}), \quad i = 1, n_k, \quad k = 1, m.$$  

Now we can separate the mean function from the covariance structure at the same harmonic (Jones, 1993)

$$y_k(s_{ik}) = \xi_0 + \xi_1 \cos(2\pi s_{ik}) + \xi_2 \sin(2\pi s_{ik}) + \epsilon(s_{ik})$$

$$\text{cov} \{\epsilon(s_{ik}), \epsilon(s_{jk})\} = \sum_{\nu=0}^{p} g_\nu^2 \cos[2\pi\nu(s_{ik} - s_{jk})]$$

Consider a space-time process. The $\beta$'s depend on time

$$y(s, t) = \xi_0(t) + \xi_1(t) \cos(2\pi s) + \xi_2(t) \sin(2\pi s) + \cdots$$

The mean value function is, as before,

$$\mu(s) = \beta_0 + \beta_1 \cos(2\pi s) + \beta_2 \sin(2\pi s) + \cdots$$

This is an expansion of the field using orthogonal functions. The $\xi(t)$'s are a multivariate time series. Assume it is a multivariate AR (1) process. Using a finite number of sines and cosines in the expansion, the vector of $\xi(t)$'s will be the state vector. Unequally spaced point observations can be written as linear combinations of the state elements giving a state space representation. The Kalman filter can be used to calculate the likelihood, and nonlinear optimization used to obtain M.L. estimates of the parameters.
Random Fields on a Sphere (2D)

Longitude $0 \leq \theta \leq 2\pi$

Polar colatitude $0 \leq \phi \leq \pi$

In the east-west direction, we have $\sin(\nu \theta)$ and $\cos(\nu \theta)$. In the north-south direction we have Legendre polynomials and associated Legendre functions (Press et al., 1988).

Spherical Harmonics

$$Y_{n}^{\nu}(\theta, \phi) = \frac{\cos(\nu \theta)}{\sin(\nu \theta)} \left\{ P_{n}^{\nu}(\cos \theta) \right\}$$

If the entire random field is observed, it can be expanded in a series of spherical harmonics. Let $P$ be a point on the sphere,

$$y(P) = \sum_{n=0}^{\infty} \sum_{\nu=-n}^{n} \xi_{\nu n} Y_{n}^{\nu}(P)$$

The condition for isotropy (the covariance depends only on the spherical distance between the two points given by Obukhov in 1947 (Yaglom, 1961) is

$$\text{cov}(\xi_{\nu n}, \xi_{\mu m}) = \delta_{\nu \mu} \delta_{mn} g_{n}^{2}$$

The covariance between two points an angle $\gamma$ apart is

$$C(\gamma) = \sum_{n=0}^{\infty} g_{n}^{2} P_{n}(\cos \gamma),$$

where $P_{n}(\cos \gamma)$ is a Legendre polynomial of degree $n$. Suppose we would like to estimate the isotropic covariance function from unequally spaced observations of a single realization. There is a confounding between the mean field and the covariance structure. Some spherical harmonics can be used to model the mean, and others to model the covariance structure. For given values of the $g$'s, we can calculate the covariance matrix, $\mathbf{V}$, of the observed data. Use nonlinear optimization to estimate the $g$'s.

Axial Symmetry

The correlation is the same between any two points that can be rotated into each other (Jones, 1963).

$$C(P, Q) = \sum_{\nu=-\infty}^{\infty} \sum_{m=\nu}^{\infty} \sum_{n=\nu}^{\infty} f_{mn\nu} e^{i\nu(\theta_{P} - \theta_{Q})} P_{m}(\cos \phi_{P}) P_{n}^{\nu}(\cos \phi_{Q})$$
Continuous Spatial ARMA Models

Spatial Autoregressive Moving Average (ARMA) models are obtained formally as the solution to a certain class of stochastic linear partial differential equations. Whittle (1954) first considered one such model arising from the solution to a 2-dimensional Laplace equation. He showed that the covariance between two points a distance \( r \) apart is:

\[
\gamma(r) = \frac{p^2_r}{2\sqrt{\theta}} K_1 \left( r\sqrt{\phi} \right)
\]

where \( \phi \) is positive, \( K_1 \) a modified Bessel function of the second kind, order 1 and \( \epsilon(x, y) \) two-dimensional "white noise" with variance \( \sigma^2 \). This happens to be an isotropic process with a rational spectral density. In our analysis we considered models (Vecchia, 1985) which are more general versions of Whittle's. The \( \theta \)'s and \( \phi \)'s are either real or come in complex conjugate pairs which represent a generalization from Vecchia's approach, but increase the computational difficulty. The covariance between two points separated by a distance \( r \) is now

\[
\gamma(r) = -\sigma^2 \sum_{j=1}^{p} \theta_j K_0 \left( r\sqrt{\phi_j} \right)
\]

where

\[
w_j = \frac{D^2(-\phi_j)}{\prod_{l=1}^{p} (\phi_j - \phi_l)^2}
\]

These ARMA models are further used to model random fields (see Jones and Vecchia, 1993a, for details). The procedure involved is as follows:

If one has a spatial random process of the form

\[
z(x, y) = \sum_{j=0}^{p-1} \beta_j f_j(x, y) + \zeta(x, y)
\]

where \( (x, y) \in \mathbb{R}^2 \), \( \zeta(x, y) \) is a homogeneous Gaussian random field with mean zero and covariance function \( \sigma^2 R(u, v) \) and \( \beta_j \) the unknown regression coefficients. \( \zeta(x, y) \) is then modeled by the aforementioned ARMA models. The \( n \) observed data points can be written in matrix form,

\[
Z = X\beta + \xi + \eta
\]

where \( Z \) is a column vector of observations, \( X \) the design matrix for trend regression, \( \xi \) the column vector of the actual values of the random field assumed independent of the
nugget effect \( \eta \). The estimates \( \hat{\beta} \) and \( R\hat{\sigma}^2 \) are found using maximum likelihood techniques based on Cholesky factorization of the covariance matrix augmented by \( X \) and \( Z \). As an alternative, one could use the method described below to estimate the parameters.

**Nearest Neighborhood Methods**

These are used when the entire covariance matrix does not fit in storage. The approximate likelihood is calculated by considering the '\( m \)' nearest neighbors of a particular point. An algorithm is constructed that produces an approximate orthogonal transformation where each row of the design matrix is orthogonal to those of the '\( m \)' nearest neighbors.

**Anistropic Processes**

One could introduce anisotrophy by stretching and rotating the axes (Vecchia, 1985). This involves scaling and rotation of the axes which do not introduce too many additional parameters into the non-linear optimization.

**Prediction on the Field**

The methods described above can be used to obtain minimum mean square error unbiased predictions of the true field. The prediction of the field at a location consists of the estimated trend surface plus a linear combination of the deviations from the trend surface at the observed locations. The prediction error is the difference of prediction and the true trend surface plus the level of the random field. (See Jones and Vecchia, 1993a, b, for details.)

**Examples**

The first example considered is 193 observations of the top elevation of a coal bed in Wyoming. Various models including anisotropic and isotropic ones were fitted to the data set. The best model is chosen using likelihood ratio test and the Akaike's Information Criterion (AIC). The model chosen was isotropic and had a significant quadratic trend surface with six parameters. It is close to a AR (2) process. It performed favorably against standard Kriging models.
The second example considered is the one modeling snowmass data (Jones and Vecchia, 1993b). Forty years of snowpack data were recorded at up to 104 locations in the Colorado Rocky Mountains. The spatial covariance structure is assumed to be that of an anisotropic continuous spatial process as extreme anisotrophy was exhibited by the data. The structure is estimated using maximum likelihood. Also, two different methods were used to estimate the yearly effect which indicated no significant trend in the mean yearly snowfall. Prediction of snowfall at unobservable locations was made possible. Quadratic trend surface and elevation were significant in all models considered. An ARMA (1, 0) performed well among those models that did not consider individual yearly mean levels, while an ARMA (2, 0) did well when yearly means were incorporated into the models. The graphs indicated that the strong anisotropy was due to the fact that the prevailing winds blow from west to east.

The last example considered was Aquifer data consisting of 96 points. The best fit was an isotropic AR (2) model and the field has a quadratic trend surface.

References

Related Publications


The concept of "trend" is offered by Brillinger (1994) as an example of a vague concept in statistics of the type that Mosteller and Tukey (1977) refer to: "Effective data analysis requires us to understand vague concepts ... that may be made definite in many ways".

The notion of trend finds importance in the study of repetitions and variations in space and time which is relevant to statistical studies of climate for example. The proper development of a statistical model involves several steps including defining variables, data collection, exploratory data analysis, model construction, criticism, and perhaps multiple reconstruction, and probabilistic inference. Because all probabilistic inferences depend critically on the probability model assumed, all inferences are limited in their validity by that model.

While the modeler should seek to minimize the influence of a priori biases that enter into model construction, this is never entirely possible. In fact this compromise between subjective and objective elements belongs to a broad set of "complementarities" which enter into model construction including formal vs. informal approach, problem centered vs. data centered analysis, and exploratory vs. confirmatory data analysis. These complementarities also reflect the distinction between Bayesian and non-Bayesian viewpoints.

Models used for statistical inference represent a formal expression of evidence, a complementarity of deterministic and probabilistic components. Inference (e.g., prediction) arises from combining "independent" evidence and hence is always relative to evidence introduced. Observational studies often seek to attribute cause through statistical inference based on tests that are based on probability models. However, "causal thinking" may play a fundamental role in motivating a particular probability model. Studies investigating links between smoking and health problems during the 1950s–1960s, and studies seeking to link observed trends in global temperature and increasing concentrations of greenhouse gases represent such "causal thinking".

The IPCC time series of estimated annual global average temperature from 1861 to 1988 is frequently analyzed for inferences regarding global warming. Conventional studies
which seek to describe the observed warming evident in the time series in terms of a non-stationary trend are influenced by \textit{a priori} assumptions regarding the causal influence of increasing greenhouse gases on global climate. The observed behavior may however be equally well described in terms of stationary, purely stochastic models with long-range dependence (Beran, 1992; Bloomfield and Nychka, 1992; Künsch, 1986; Smith, 1993). This possibility is investigated in more detail.

The IPCC record is analyzed in the context of fractional Gaussian noise as a model for long-range dependence. Specifically, one assumes a spectral density function, which depends on frequency as,

\[ S(f) \sim f^{-d} \]

A fit of such a dependence to the periodogram of the IPCC time series is suggestive of such a scaling with \( d \approx 2.8 \). A variety of realizations of such a process with \( d = 0.8, 1.8, 2.8 \) are generated with variable-amplitude additive white noise for comparison with the IPCC record, and some similarities are observed. In particular, trends of similar magnitude to that observed over 128 year period are generated. Moreover, the phase-structure of the periodogram is shown to exhibit a similar “banding” for both the synthetic and IPCC time series. When piecewise linear trends are fit to the time series, the residuals exhibit a phase structure roughly consistent with white noise.

Long-range dependence is thus offered as an alternative model to explain observed global warming that does not invoke assumptions of a causal relationship with greenhouse gas production.

\textbf{References}


Motivation

An important statistical issue related to global climate change is the detection of trends in observed meteorological time series, particularly temperature. Detecting trend is important both for deciding if the climate is indeed changing and as a means for validating GCMs. Unfortunately, the detection of trends in observed data is complicated by numerous factors, including variation in measured trends by location and for different time periods. A classic example is the steady rise in average global temperature from 1910 to 1940, followed by steady temperature or near cooling until 1975, followed by yet another distinct rise. Effects such as these suggest that the observed fluctuations in temperature may be part of long-term natural variability, and thus may not be indicative of anthropogenic climate change.

In this talk, these issues are approached from the perspective of time series with long-range dependence (LRD), i.e., a spectral density which is proportional to $\omega^{1-2H}$ as $\omega \downarrow 0$, where $\omega$ is frequency and the parameter $H$ (the Hurst coefficient, after the 1950's study of the River Nile by the English physicist H.E. Hurst) is between 0.5 and 1. Much of the material covered in this talk can be found in Smith (1993).

Trend in Climate Series

Consider the following linear trend model

$$y_n = \alpha + \beta x_n + \zeta_n, \quad 1 \leq n \leq N,$$

where $y_n$ is the observed temperature value in month $n$, $x_n$ is the month number expressed as a change in temperature per century, and $\{\zeta_n\}$ is a stationary time series such that $E\{\zeta_n\} = 0$ and $E\{\zeta_n \zeta_{n+k}\} = \gamma_k$.

If $\{\zeta_n\}$ are uncorrelated ($\gamma_k = 0$ for $k \neq 0$), then $\hat{\beta}$ is estimated by the ordinary least squares (OLS) estimator with the familiar OLS variance. Since the $\{\zeta_n\}$ are correlated in
practice, the OLS estimators are na"ive and the estimated variance of $\hat{\beta}$ underestimates the true variance.

If $\{\zeta_n\}$ are correlated, $\hat{\beta}$ can be shown to have variance

$$\text{var}(\hat{\beta}) = \left\{ \frac{1}{\sum_{n=1}^{N} (x_n - \bar{x})^2} \right\}^2 \sum_{m=1}^{N} \sum_{n=1}^{N} (x_m - \bar{x})(x_n - \bar{x}) \gamma_{m-n}.$$ 

Equivalently, let $f(\omega)$ be the spectral density of $\{\zeta_n\}$. Then the relation between the autocovariances $\{\gamma_k\}$ and the spectral density is

$$f(\omega) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \gamma_n e^{-i\omega n},$$

where

$$\gamma_n = \int_{-\pi}^{\pi} e^{i\omega n} f(\omega) d\omega.$$ 

In this case,

$$\text{var}(\hat{\beta}) = \left\{ \frac{1}{\sum_{n=1}^{N} (x_n - \bar{x})^2} \right\}^2 \int_{-\pi}^{\pi} f(\omega) \left| \beta_N(\omega) \right|^2 d\omega,$$

where

$$\beta_N(\omega) = \sum_{n=1}^{N} (x_n - \bar{x}) e^{i\omega n}.$$ 

The spectral density at a particular frequency $\omega$ may be estimated by means of the periodogram,

$$I_n(\omega) = \frac{2}{N} \left| \sum_{n=1}^{N} y_n e^{i\omega n} \right|^2 = \frac{2}{N} \left[ \sum_{n=1}^{N} y_n^2 + 2 \sum_{k=1}^{N-1} \sum_{m=1}^{N-k} y_m y_{m+k} \cos(k\omega) \right],$$

whose mean converges as $N \to \infty$ to

$$2\gamma_0 + 4 \sum_{k=1}^{\infty} \gamma_k \cos(k\omega) = 4\pi f(\omega).$$

There are several possible approaches to model $\{\zeta_n\}$:

- Assume a simple model, e.g., AR (1) (Bloomfield, 1992). This model does not adequately model low-frequency variation.

- Fit a higher-order model, e.g., AR (p), ARMA (p,q). These models do a better job of modeling low-frequency variation, but at the cost of more parameters.

- Use a spectral version of the formula with various forms of $f(\omega)$ (Bloomfield and Nychka, 1992). $f(\omega)$ should satisfy $f(\omega) \to \infty$ as $\omega \to 0$ (i.e., LRD).
Models for Long-Range Dependence

We can define the long-range dependence by either of the relations:

\[ \gamma_k \sim a k^{2H-2} , \ k \to \infty \]

or

\[ f (\omega) \sim b \omega^{1-2H} , \ \omega \downarrow 0 , \]

where \( \frac{1}{2} < H < 1 \). After some algebraic manipulation, we can express the relation between the positive constants \( a \) and \( b \) as

\[ b = \frac{a}{\pi} \Gamma (2H - 1) \sin (\pi - \pi H) . \]

The above definition of long-range dependence relates to the historical R/S analysis pioneered by Hurst in the 1950’s. This method subsequently led to more mathematically mature ideas such as fractional Brownian motion and fractional differencing.

Now, suppose our model is

\[ y_n = \alpha + \beta x_n + \zeta_n , \ 1 \leq n \leq N \]

with

\[ f (\omega) \sim b \omega^{1-2H} , \ \omega \downarrow 0 \left( \frac{1}{2} < H < 1 \right) . \]

Then, for OLS \( \hat{\beta} \), the variance can be found as

\[ \text{var}(\hat{\beta}) \sim \frac{36a(1-H)}{H(1+H)(2H-1)} N^{2H-4} = \frac{36b\pi (1-H) N^{2H-4}}{H(1+H)\Gamma (2H) \sin (\pi - \pi H)} \]

Yajima (1988) considered the efficiency of the least squares estimate as compared with the best linear unbiased estimate (BLUE). Efficiencies were obtained of 1, .991, .969, .942, .915 and .889 for \( H = .5 \) to 1 in increments of .1. These seem to confirm that the efficiency is near 1, so that there is no need to do the extra work required to compute the BLUE.

According to well-known theory, \( I_n (\omega_j) \) for \( j = 1, 2, \ldots \) are approximately independent exponential random variables with mean \( 4\pi f (\omega_j) \). A regression of \( \log I_n (\omega_j) \) on \( \log \omega_j \) for \( 1 \leq j \leq n \), where \( n \leq N \), will give an estimate of \( H \) from the fact that

\[ \log f (\omega) = \log b + (1-2H) \log \omega . \]

Application to Temperature Data

This method was applied to three data sets with the following results.

(1) Central England Temperature Data 1907-1973 (Manley, 1974)

\( H = .65 \), \( \hat{\beta} = .275 \), s.e. (\( \hat{\beta} \)) = .2
(2) USA Temperature Data 1895-1987 (Historical Climate Network Data)
\[ \hat{H} = .57, \hat{\beta} = .6, s.e. (\hat{\beta}) = .3 \]

(3) Global Temperature Data 1854-1989 (IPCC, 1990)
\[ \hat{H} = .9, \hat{\beta} = .4, s.e. (\hat{\beta}) = .1 \]

Note that for cases (1) and (2) there is some evidence of LRD compared to a null-hypothesis of \( H = .5 \) (no LRD). However, this evidence is not particularly strong in light of the standard error and data quality problems. Case (3) shows significant \( H \), implying LRD in the global average IPCC data.

Possible Extensions

One possible extension of this work would entail the simultaneous estimation of both the trend parameter and \( H \). Other work could extend LRD to spatial-temporal models. As a reference, Haslett and Raftery (1989) worked with spatial-temporal models of wind-speed within the fractional differencing framework.

References and Related Publications

The Art of Forecast Skill Evaluation

Dr. Robert Livezey

Gábor Kis-Kovács, Rapporteur

The subject of the talk was the problem of estimating the skill of different climate forecast systems. It is important to emphasize that the skill and not the value of the prediction is considered. “Skillful” models have the ability (in contrast with e.g., fixed climatology forecasts) to discriminate one forecast situation from another. Generally skill is a necessary condition for value, but the latter also depends on at least the user’s cost/benefit situation. Verification of probability forecasts is also not considered here (cf. Daan, 1985).

Elements of Objective Verification

The basic elements and considerations for objective verification can be summarized as follows:

Quantification

Forecasts and observations must be precisely defined, so that there is no ambiguity whatsoever whether a particular forecast is right or wrong, and, for the latter, what its error is.

Authentication

True forecast schemes must not use any future information. In the case of hindcasts, which are predictions based on information preceding and/or following the beginning of the target forecast period (this can be the case in laboratory experiments), uninflated estimates of skill can be determined only if one assumes the climate is reasonably stationary and the tested prediction sample is independent of the model development sample.

Probability Distributions

It is not possible to make objective assessments of forecast skill without some information about the joint probability distributions of forecasts and observations. Two examples were given. The distribution of the monthly mean precipitation is at many stations in the U.S. negatively skewed. That is, the median is less than the mean. The skewness
may cause, say, 60% below the mean and 40% above. Verification assuming a 50-50 split will lead to incorrect measures of skill. A similar situation arises for locations with anthropogenic urban heat-island trend. An accuracy of about 90% can be achieved only by forecasting above normal all the time, but this is meaningless information about the skill of the forecast.

**Comparison of Forecasts**

Two types of comparisons can be made: a) to reference or control forecasts and b) to other forecasts.

a) The following four control forecasts are the most common: 1. random forecast, 2. climatology (the forecast is the mean), 3. persistence (the forecast is the current departure from the mean), and 4. damped persistence (e.g., AR (1) model). The measure used in the forecast evaluation excludes some references from the comparison process as a matter of course (e.g., by using MSE as a measure damped persistence is the correct choice). The skill of the control should be hard to beat.

b) In the case of two (or more) forecasts the comparisons must be homogeneous. That is, the forecasts have to be made for the same place, the same time and the same variables. Another important point is that neither forecast should be arbitrarily excluded from using data that is available to the other. For example, a monthly forecast which is completed several days before the target period begins cannot be fairly compared to a persistence control forecast that uses the data from the whole month.

**Concluding Remarks**

In all cases, the usefulness of forecasts needs to be questioned, regardless of whether or not they can be shown to have real skill.

**Some Measures**

Two types of forecasts are treated: continuous and categorical forecasts (forecast of a range of values of the predictand) and examples for scores defined for each type are given.

**Categorical Forecasts — The Heidke Score**

This measure is based on the so-called contingency table that contains all the possible forecast/observation outcomes sorted in columns and rows and divided by the number
of forecasts. (A similar table was used in the famous Finley-Affair; see also notes for Murphy.) The Heidke score is defined as \( S = (H - E)/(T - E) \), where \( H \) is the number of correct forecasts, \( E \) is the number of correct random forecasts, \( T \) is the total number of observations. The main problems with this score in that form are that a) with non-equiprobable classes the score is inequitable, i.e., advantage can be gained by forecasting one class all of the time, and b) cannot distinguish between two forecast systems with the same correct forecasts but different numbers of 1-, 2-, etc., class misses. The Heidke score can be modified to eliminate these drawbacks, and alternatives to it are recommended (cf. Barnston (1992) for two such alternatives).

**Continuous Forecasts — Pattern Verification and the Murphy-Epstein Decomposition**

See Livezey (1994) or Barnston (1992) for more basic considerations for verification of continuous forecasts. Here the more complicated case of a forecast of a field of variables is considered. The MSE between the field of the forecast \( f \) and the field of observations \( x \) is defined as follows.

\[
MSE(f, x) = \langle(f_i - x_i)^2\rangle
\]

where \( i \) denotes the gridpoints and the angle bracket a spatially weighed mean. Let \( ' \) denote climate anomalies (i.e., the deviation from the climatological mean) and introduce the following quantities.

\[
\begin{align*}
    s_f^2 &= \langle f'^2 \rangle - \langle f' \rangle^2, \\
    s_x^2 &= \langle x'^2 \rangle - \langle x' \rangle^2, \\
    s_{fx'} &= \langle(f' - \langle f' \rangle)(x' - \langle x' \rangle)\rangle, \\
    \rho_{fx'} &= \frac{s_{fx'}}{s_f s_x}.
\end{align*}
\]

The first two can be regarded as spatial standard deviations, the third as spatial covariance and the last line as the anomaly correlation. Murphy and Epstein (1989) linked these quantities to a score \( \beta \), as

\[
\beta = \frac{A^2 - B^2 - C^2 + D^2}{1 + D^2}
\]

where
\[ \beta = \frac{MSE(c, x) - MSE(f, x)}{MSE(c, x)} \]  
Percent reduction of MSE from climate reference forecasts

\[ A^2 = \rho_{f'x'}^2 \]  
Skill in forecasting phase of anomaly

\[ B^2 = [\rho_{f'x'} - s_{f'}/s_{x'}]^2 \]  
Conditional bias, contribution of amplitude error to MSE

\[ C^2 = [((f') - (x'))/s_x]^2 \]  
Unconditional bias

\[ D^2 = [(x')/s_x]^2 \]  
Normalization factor

As an example the M-E scores were shown for model simulations of 10 years of seasonal mean 700 mb heights. The scores were much worse on average than random forecasts as a result of large model bias errors \( (B^2 \text{ and } C^2)^* \). After post-processing the forecasts by redefining \( f' \) as

\[ f_i' = (f_i - \bar{f}_i) \]

using the model climatology \( \bar{f}_i \), which in general reduces \( C^2 \), the unconditional bias was on the average still large. This is because there was a clear trend in \( (\langle f' \rangle - \langle x' \rangle) \). The large conditional bias error was caused by a tendency for \( s_{f'} > s_{x'} \). The latter problem was eliminated by forming ensemble average forecasts. The example illustrates the diagnostic power of the M-E decomposition.

For a final topic, the use of cross-validation for the evaluation of hindcasts, the student is referred to Livezey (1994) and Michaelson (1987).

References and Related Publications


* It should be mentioned that smoothing or an inappropriate climatology can inflate \( \beta \).


Fingerprint methods are useful in determining whether some model-predicted climate change signal is evident in observed records of climate change, and is uniquely attributable to changes in greenhouse gases. To implement fingerprint methods, statistical techniques are required to reduce the dimensionality of large space-time data sets, to identify climate variables with high signal-to-noise ratios, and to separate greenhouse-gas (GHG) signals from natural variability noise.

An example of the use of statistical methods in detection studies is the recent work by Santer et al. (1994). This study makes use of data from time-dependent greenhouse warming experiments recently performed with the Hamburg coupled atmosphere-ocean model (Cubasch et al. 1992). The atmospheric and oceanic components were the ECHAM-1 AGCM and the LSG OGCM, respectively. Four numerical experiments were performed. Scenario A ("Business as Usual") assumed unrestricted emissions of equivalent CO$_2$, while Scenario D ("Draconian Measures") assumed severe emissions restrictions after the year 2000. An experiment with instantaneous doubling of CO$_2$ and a control run with fixed CO$_2$ were also carried out.

The four runs, each of length 100 years, yielded different global-mean surface temperature changes. Scenario A showed a slow initial increase in temperature, with a more rapid, linear increase after about 40 years. The CO$_2$-doubling experiment had a characteristic two-time scale response. The temperature increase in Scenario D was much smaller than in Scenario A, while the control run cooled by roughly 0.4°C in 100 years.

Empirical Orthogonal Functions (EOFs) were used to determine the amount of space-time variability explained by different model-simulated climate variables. Pattern correlations between the spatial EOFs of near-surface temperature and sea-level pressure (SLP) illustrated that SLP had a similar EOF 1 pattern in both Scenario A and the control run,
making it a poor candidate for inclusion as a component of a multivariate GHG fingerprint. In contrast, the dominant modes of surface temperature variability were dissimilar in Scenario A and the control, which is a favorable situation for detecting a GHG signal.

To study the time evolution of GHG signals and natural variability noise, the Santer et al. study used cumulative explained spatial variance, principal component time series, and projections in two-dimensional EOF-space. Cumulative explained spatial variance is a measure of how the contribution of different EOFs to the total signal variance changes with time. In the case of surface temperature for Scenario A, this showed that a single dominant signal pattern is established after roughly 50 years. Principal Component (PC) time series were used to compare the evolution of the different signals and the control run noise on a common basis. The PC analysis showed that for surface temperature, the dominant EOF 1 mode of Scenario A is relatively uncontaminated by natural variability noise, while Scenario A and the control run have some (small) common component of variability in the EOF 2 direction. Projections in two-dimensional EOF space were used to examine the joint evolution of signals and noise in time and space. Such projections indicated that the Scenario A surface temperature signal separates from the internally-generated noise in the control run in roughly 20 years.

Previous investigations have had a lack of success in detecting a GHG surface temperature signal in observed records of temperature change (Santer et al. 1993). One possible explanation for failure to detect a GHG signal is that anthropogenic sulfate aerosols may have masked the expected signal.

Recent experiments by Taylor and Penner (1994) may help to resolve this issue. Taylor and Penner used an AGCM coupled to a tropospheric chemistry model to perform four separate integrations: present-day sulfate aerosols and pre-industrial CO₂, no sulfate aerosols and present-day CO₂, present-day sulfate aerosols and CO₂, and a control with no sulfate aerosols and pre-industrial CO₂. Various pattern correlations were then computed to determine whether the model-predicted equilibrium signals were increasingly evident in observed temperature data. The results of this study lend support to the hypothesis that sulfate aerosols may have masked a greenhouse warming signal.
References and Related Publications


Complex Quality Control of Meteorological Data

Dr. Lev Gandin

Peihua Qiu and Gary Sneddon, Rapporteurs

Many meteorological centers worldwide perform numerical weather analysis and prediction operationally using powerful computers. The analysis-forecast cycle consists of following major steps: data reception and collection from the Global Telecommunication System (GTS), preprocessing and decoding of received reports, quality control of this information, objective analysis of meteorological fields and, finally, numerical forecasting by means of dynamical models of the atmosphere. Ultra-short range, usually 6 hour, forecasts are also applied as so-called first-guess fields in addition to reported observation data, forming what is called a data assimilation system. The first guess information is also used in the course of the quality control.

The quality control step is a set of automatically performed procedures used in order to detect and correct the so-called rough errors in reported meteorological data. The rough errors in meteorological reports occur comparatively rarely. Each rough error has its definite cause, that may happen in the course of measurement, or processing, or communicating the data. It is the task of the quality control (QC) to automatically detect each rough error and, if possible, to automatically correct erroneous data. If unable to correct an error, the QC algorithm should either preclude the assimilation of suspected datum or to make it assimilated with a smaller weight.

The random errors in meteorological data are, unlike the rough errors, inherent in all data. They are usually more or less independent of each other at different points and times, forming what is called random noise. These errors can not be corrected, but the overall noise level is usually known and is taken into account in data assimilation as well as in the quality control. Errors of the last category, called systematic ones, persist, to some extent, in time or/and in space, and this improves the possibility to correct even small such errors. As to the rough errors, it is impossible even to detect a rough error unless it substantially exceeds the noise level. That is why the errors dealt with by QC have been given the name 'rough errors'.
The necessity to have an automatic QC incorporated into the operational analysis-forecast cycle was recognized at the beginning of the numerical weather prediction era, and the first of such algorithms were proposed more than 40 years ago. They used a so-called sequential or hierarchical approach; a sequence of various checks, beginning with the least sensitive ones, was applied to the same set of data, leading to successive detection of questionable information and to flagging it in order to be not included into the data assimilation set.

The sequential QC does not investigate the cause of each detected rough error and is therefore unable to correct any errors. This aim is achieved by a different approach called the complex quality control (CQC). Unlike the traditional, sequential QC, the CQC does not make any changes in a report until it performs all checks of this report. The CQC algorithm thus consists of two major parts: the checks and the Decision Making Algorithm (DMA), the latter being an advanced code with complicated logic. Results of all checks are expressed not by flags, but quantitatively, by so-called residuals. The DMA then analyses all residuals. If none of them is large (by absolute value), as is the case in most reports, then the DMA concludes that there is no reason to suspect any error in this report. If at least one residual is large, then the DMA analyzes the pattern of various residuals, trying to locate an error (or errors), to explain its origin and, if possible, to correct the erroneous datum (or data). Corrections made by the DMA are directed towards the restoration of correct values which were distorted by errors made while processing the data or originated along communication channels.

The design of a new QC system, called the complex quality control of rawinsonde height and temperature (CQCHT), began in 1990. It contains a symbiosis of quasi-functional checks (like the hydrostatic one) and statistical checks (e.g., horizontal and vertical interpolation checks for deviations of height and temperature from the forecast first guess). After extensive testing and improvements, the CQCHT became operational at NMC in 1991.

Related Publications


Economically Optimal Strategies for
Users of Meteorological Information

Dr. Lev Gandin

Karen S. Kelly and Gábor Kis-Kovács, Rapporteurs

Users of Meteorologic Information

The problem of determining economically optimal strategies for the users of meteorological information is one that deserves more attention. It is the users of forecasts (those who make decisions based on the forecast information) that ultimately determine their value. Often forecasts are either accepted at face value (believed as perfect), or not trusted at all (ignored). Neither of these strategies will guarantee that economically optimal decisions are made.

In just turning to these users, one can see that economically optimal strategies are rarely pursued in practice. A typical answer to the question “How important for your business are weather forecasts and other data provided by meteorologists?” is “Very important”. In contrast to this response, when those same users are asked “How often do you make or change your decisions on the basis of the meteorological information?”, the answer is often “Almost never”. The implication is that the forecasts are of little value to the users. How can this situation be improved? What can be done to enable users to derive and apply economically optimal strategies based on meteorological information?

Statistical Decision Making

The first step is to properly formulate the problem – how exactly can users define their economically optimal strategies on the basis of meteorological information? The general framework for this problem is expressed in terms of statistical decision making. The essential components are as follows:

1. The user may be an individual, a company, or any entity that makes decisions based on meteorological information. The user must determine what meteorological parameters should influence the decisions.
2. **Climatological information** about the influencing parameter(s) must be available and described in terms of probability distribution laws.

3. **Statistical information about the forecast performance** must be available and presented in terms of joint probability distribution laws. In other words, there must be data on joint occurrence of actual and predicted values of the influencing meteorological parameter(s).

4. The user has to have a set of available actions/decisions to react on meteorological information. This set may be either discrete (consisting of a finite number of available actions) or continuous. The user's objective is to select, in each case, the economically optimal action. The user's rule of doing so is called his or her economically optimal strategy (EOS). In order to be able to define his EOS, a user has to know his or her loss (or gain) function and to specify the optimization principle to be applied in decision making.

5. The **loss function** describes the economic consequences of user's actions in terms of the mean (in probabilistic sense) losses as a function of the values of influencing parameter(s) assumed by the action and of those actually happened (in other words, of what was believed would occur and what actually did occur). Although the main information on a loss function comes from empirical data on user's economic losses, the design and application of simple theoretical models for the loss functions proves rather useful as well. Usually, the general form of a loss function is implied by theoretical consideration, while the coefficients are found from empirical data.

6. The most often used optimization principle is that of minimizing the user's expected (mean) loss, (usually called the Bayesian approach) thus achieving optimal results in a long run. Some other principles, like the minimization of the probability of big losses (sometimes referred to as the minimization of risk) have been also applied. It is up to the user to select the optimization principle on the basis of general economical considerations.

Although the first investigations on the economically optimal use of meteorological information have been made several decades ago, the attention paid nowadays to this problem by the meteorological community and the level of its understanding needs substantial improvement. The biggest stumbling block is connected with the users' loss functions, the existing information about them is far from sufficient, and more investigations are strongly needed.
An Example

It is useful to consider a simple, "two by two" example of how one actually select an economically optimal strategy. A dichotomic situation $x$, i.e., that with only two alternatives $x = 1$ and $x = 2$ (like rain and no rain) is assumed in this example, and the user's decision $y$ and the forecast $z$ are dichotomic as well. Knowing the forecast, $z = 1$ or $z = 2$ (or ignoring it), the user has to make his or her decision, $y = 1$ or $y = 2$, and his or her loss will depend on which decision $y$ he or she made and which alternative $x$ happened in reality. If the correlation between forecasts and reality is positive and if the user's strategy is definite, i.e., each decision $y$ is entirely determined by the forecast $z$, then he or she can select among three strategies defining $y$ as a function of $x$:

- $S_1: y(1) = y(2) = 1$
- $S_2: y(1) = y(2) = 2$
- $S_3: y(1) = 1; y(2) = 2$

The strategy $S_3$ is that of believe in all forecasts, while $S_1$ and $S_2$ both ignore the forecast always believing in one of the two possible alternatives.

If the joint probability of actual state $x$ and forecast $z$ is denoted $p(x, z)$ and the (mean) user's loss with actual state $x$ and user's decision $y$ is $l(x, y)$ then the equations for mean losses $L_1 - L_3$ with the strategies $S_1 - S_3$ are

\[
\begin{align*}
L_1 &= p l(1,1) + q l(2,1); \\
L_2 &= p l(1,2) + q l(2,2); \\
L_3 &= p(1,1) l(1,1) + p(1,2) l(1,2) + p(2,1) l(2,1) + p(2,2) l(2,2)
\end{align*}
\]

where

\[
p = p(1,1) + p(1,2) \quad \text{and} \quad q = p(2,1) + p(2,2) = 1 - p
\]

are marginal probabilities of $x = 1$ and of $x = 2$.

The economically optimal strategy is the one with the smallest mean loss. One can then define the relative gain due to the optimal strategy by comparing the minimal mean loss when a forecast is available to that when there is no forecast (the optimal strategy under this latter scenario is called the climatologically optimal strategy).
Related Publications


Some Comments on Turbulence and Statistics or Loose Ends Others Have Not Touched Upon

Jackson Herring

Jianjian Gong and Igor Perisic, Rapporteurs

This talk will try to fill the gaps other speakers left on the subject of turbulence (see McWilliams, Guttorp, Waymire). We start with definitions of turbulence given by various luminaries:

What is Turbulence?

Turbulence is a three-dimensional time dependent motion in which vortex stretching causes velocity fluctuations to spread to all wave lengths between a minimum determined by viscosity and a maximum determined by the boundary conditions of the flow. It is the usual state of fluid motion except at low Reynolds numbers. P. Bradshaw, An Introduction to Turbulence and Its Measurement, Pergamon Press, p. 17, 1972.

The only short but satisfactory answer to the question “what is turbulence” is that it is the general-solution of the Navier-Stokes equations. P. Bradshaw, The Understanding and Prediction of Turbulent Flow, Aeronaut. J. p. 406, 1972.


... turbulence is a phenomenon of instability... nothing less than a thorough understanding of the statistical stratification of the system of all their solutions would seem to be adequate to elucidate the phenomenon of turbulence... Turbulence proper is tied to three dimensionality. J. von Neumann, Recent theories of turbulence—A report to the office of Naval Research, in collected work 6, pp. 439, 441, 448, 1949.

Finally, we should not altogether neglect the possibility that there is no such thing as ‘turbulence’. That is to say, it is not meaningful to talk of the properties of a turbulent flow independently of the physical situation in which it arises. In searching for a theory of

The distinguishing feature of turbulent flow is that its velocity field appears to be random and varies unpredictably. The flow does, however, satisfy a set of differential equations, the Navier Stokes equations, which are not random. This contrast is the source of much of what is interesting in turbulence theory. A. J. Chorin, *Lectures on Turbulence Theory*, Publish or Perish, Berkeley, p.I, 1975.

We have therefore defined turbulence as random fluctuations of the thermodynamic characteristics of vortex flows, thereby distinguishing it at the onset from any kind whatever of random irrotational i.e., potential flows. A.S. Monin, *On the Nature of Turbulence*, *Sov. Phys. Uspekhi*, p.430, 1978.

We note in these definitions some contradictions (as between von Neumann and Heisenberg), and a stress on three dimensionality with random fluctuations in vorticity.

**What are Fluctuations?**

A simple definition would be that fluctuations are variations around an “average.” In general mathematical form $u = \bar{u} + u'$, where the overbar and prime represent the average and variation from the average, respectively. It is important here to make a suitable definition of an average. For example of the flow consists of a mean cellular circulation together with random fluctuations about the cellular circulation, it is vital for the average to specify the proper spacing of the cells as well as their shapes.

**The Traditional Statistics Point of View**

Average properties of the flow, together with averages of its moments (Reynolds stresses, etc.) are the quantities to be computed. Only certain statistical features of the fluctuation field, $u'$ (such as variances) are computed. The hope is that the full distribution function of the flow may be complex, but the first few moments may obey (approximately) relatively simple equations. Effects of structures on averages are inferred from an approximation of the full distribution function.
Some Dogmas

The centerpiece of the turbulent energy distribution is eddy viscosity. It is a parameterization of the extraction of energy from large scales to small scales (see cascade theory). A molecular analogy would imply a viscosity term: $\nu_{eddyl} \Delta$. The dimensions of $\nu$ is $uL$, and the molecular viscosity takes $u$ to be the thermal molecular speed, and $L$ the mean free path. For turbulent flows, $u$ would be some measure of the flows speed, and $L$ a length scale.

Holloway's Axiom

In his talk, Holloway stressed certain ideas drawn from statistical mechanics. We shall characterize them here as an axiom, but not in any serious mathematical way. Briefly, flows seek the symmetry of the equations of motion. For example, a turbulent patch spreads out and becomes homogeneous, reflecting the fact that the equations of motion are indifferent to uniform translation. Similarly, if there is no viscosity, the flow spreads uniformly to all scales, since the equations are invariant to scale transformations. This tendency applies only to ensemble averages and not to the details of the flow. From this axiom we may infer that large scales (where viscous effects are small) resemble more inviscid equipartitioning than small scales. If the viscosity is 0 and Euler equations are projected onto a finite wave number band, the equilibrium distribution function of the independent degrees of freedom of velocity amplitudes ($x_n \sim u(k)$) is of a Gaussian form: $P_0(x_1, x_2, \ldots) = \exp(-\Sigma c_n x_n^2)$. The energy of a specific wave number is then $E_n = \int dx x_n^2 P = Ak^2$.

How to apply this to real flows? Holloway has given examples in his lecture, and we give another here. Consider the large scales of a real flow. Then, we know that the small scales affect the large by way of an approximate eddy viscosity. We can then guess that as the wave number goes to 0 the rate of change in energy is $\partial_t E(k, t) = Ak^4 - \nu_{eddyl} k^2 E(k, t)$. From this, we conclude that as the flow evolves, $E(k) \rightarrow k^p$, $p \leq 4$, and that $E_{total} \sim t^{-n}$, $1 \leq n \leq 1.37$, (Lesieur and Schertzer, 1978).

Comments on Perturbation Theory for Distribution Functions

Let $x_n$ represent the velocity field, in some collective sense. The basic problem is to determine the probability density for $(x_1, x_2, \ldots)$, given the Navier-Stokes equations
in the form: \((\partial_t + \nu_n) x_n = \Sigma_{m,l} C_{nml} x_n x_m + F_n(t)\). Denoting this probability density as \(P\) we find its equation of motion to be, \((\partial_t + \mathcal{L}_o) P = \Sigma_{nml} \partial_n x_m x_l P \equiv VP\), where \(\mathcal{L}_o = -\Sigma_n \partial_n \nu_n X_n\). Its (steady state) solution can be constructed formally via a Neumann series is \(P = P_0 + \sum \mathcal{L}_o^i V P_0 + \cdots\). But this diverges, for interesting values of \(C_{ijk}\). One can interpret \(\mathcal{L}_o\) as a representation of viscous processes, and \(V\) as interactions among modes (strain, pressure effects and advection). With this view in mind \(|V|/|\mathcal{L}_o| \sim\) Reynolds number. Hence, the above Neumann series is actually one in powers of Reynolds number, not a good idea, if the Reynolds number is large. The situation is improved if we base the iteration on rewriting the equation as \([\mathcal{L} + \{V + (\mathcal{L}_o - \mathcal{L})\}] P = 0\), and adjust \(\mathcal{L}\) so that its basic state is the actual distribution, \(P\), instead of \(P_0\). The higher order terms of such a series are (nominally) each of the same order of magnitude, so there is a hope that the series could be asymptotic, for large Reynolds number. Such a procedure is a primitive renormalized perturbation theory (RPT), but its structure is much like the more sophisticated theories, for example the direct interaction approximation of Kraichnan (1959).

Kraichnan (1962) already had some strong suspicions that even renormalized perturbations series such as that presented might also diverge. Nevertheless, 2nd order renormalized theory is still used. Some of the reasons are that the equations are recognizable as a generalized Fokker-Planck type, their Lagrangian generalizations yield good comparisons with many high Reynolds number experiments and some stochastic models have the same variances which are simple to interpret.

**Stochastic Models**

We now discuss some stochastic models of \(x_n\), which have variance equations closely similar to the renormalized perturbation theory discussed above. There are two types of stochastic models. The random coupling model (RCM) and the Langevin models. Langevin models are “structureless” (controlled by Gaussian processes) while the RCM is nearly so, but is able to estimate certain 4th multivariate cumulants. The Langevin stochastic models have the form, \((\partial_t + \nu_n + \eta_n) X_n = \sigma(t) \sum C_{nml} X_m X_e \theta_{nmle}\), where \(\sigma\) is a white noise function, \(\eta\) an eddy viscosity, and \(\theta\) relaxation factors which are determined by a comparison with RPS. We note that the presence of \(\eta\) allows the variance of \(X_n\) to satisfy energy conservation.
Comments on Structures and Their Effects in 3-D Turbulence

Suppose that the inertial range form of $E(k) \sim k^{-n}$, where $n$ is less than 3. Let's then look at the energy transfer function: $T(k) = (\partial_t + 2\nu k^2) E(k)$. As the viscosity approaches 0, and at long times, we have: $E \equiv 2\nu \int_0^\infty k^4 E(k) dk \approx \int_0^\infty k^2 T(k) > 0$. For two-dimensional flow, $\int k^2 T(n) = 0$. The speaker speculates that a randomly-oriented isolated vortex tube is not capable of satisfying $\epsilon = \int k^2 T(k) dh$. What is needed is an additional strain field, which stretches out vorticity to small scales. Notice, in this connection, that for strictly two-dimensional flow, $\int_0^\infty k^2 T(k) = 0$.

If in 3-D turbulence vortex tubes are Burgers vortices, then their cross-sectional dimension would be the dissipation scale. Hence, the structures may be close to the dissipation scale, and there is a chance that Holloway's axiom is applicable. It would imply that large scale $\sim$ Gaussian and small scale are intermittent (strongly non-Gaussian).

In 3-D isotropic turbulence, the probability distribution $P(u(x))$ is $\sim$ Gaussian, but $P(\text{grad} \times u(x))$ is more nearly exponential.

References and Related Publications


Optimal Spatial Averaging

Rudolf O. Weber

Michael Mann, Rapporteur

Typically, observation errors and small-scale spatial variability complicate the problem of producing estimates of spatially averaged meteorological or climatic fields. Because data are spatially-dependent, error estimation is difficult. A number of methods have been employed to estimate these spatial averages and associated errors.

One typical method for calculating spatial averages of a field \( f \) is the simple area average,

\[
a = \sum_{j=1}^{N} \tilde{g}_i f_i
\]

where the \( \tilde{g}_i \) are area weights for the associated gridpoints and are constrained by the normalization \( \sum_{i=1}^{N} \tilde{g}_i = 1 \). The Thiessen average (Thiessen, 1911)

\[
T = \sum_{i=1}^{N} t_i f_i \quad \text{with} \quad \sum_{j=1}^{N} t_i = 1
\]

attempts to improve on the area-average by fitting polygonal area regions (Voronoi polygons) to the gridpoint sampling, thus taking geometry into account. Presently, we will focus on a third scheme, the optimal average (Kagan, 1979; Vinnikov et al. 1990; Gandin, 1993) which attempts to use both the geometry and spatial covariance structure of the field,

\[
O = \sum_{j=1}^{N} w_j f_j
\]

This is similar to the method of statistical or optimal interpolation (Gandin, 1965; Thiébaux and Pedder, 1987; Daley, 1991). The optimal average will be shown to outperform the other two methods in several examples.

In determining the optimal weights, one assumes \( \langle f_i \rangle = 0 \) for all gridpoints \( i = 1, \ldots, M \) where \( \langle \cdot \rangle \) denotes time-averaging. We seek a solution of minimum variance,

\[
D^2 = \langle (O - \mu)^2 \rangle
\]
where \( \mu \) denotes the true sample spatial mean. Minimizing \( D^2 \) leads to the normal equations,

\[
\sum_{j=1}^{N} w_j C_{jk} = \Omega_k \quad k = 1, \ldots, N
\]  

(5)

where

\[
C_{jk} = \langle [f_j - \langle f_j \rangle] [f_k - \langle f_k \rangle] \rangle
\]  

(6)

is the sample spatial covariance matrix and \( \Omega_k \) is the mutual covariance vector between the observed values at gridpoints and the true spatial mean,

\[
\Omega_k = \langle [\mu - \langle \mu \rangle] [f_k - \langle f_k \rangle] \rangle.
\]  

(7)

It is convenient to assume a covariance structure,

\[
\hat{C}_{jk} = \sigma_j \sigma_k B_{jk}
\]  

(8)

where the spatial standard deviations \( \sigma_i \) are allowed to vary spatially in an arbitrary fashion, but the spatial correlation function is assumed homogeneous and isotropic, \( B_{jk} = B(r) \) over some domain of area \( A \). One then fits a correlation function model \( b(r) \) which is positive definite on \( A \), (Yadrenko, 1983; Yaglom, 1986; Weber and Talkner, 1993) and we have

\[
\Omega_k = \frac{1}{A} \int \int \hat{C}(x, y, x_k, y_k) dxdy
\]  

(9)

Examples of possible definite functions are

\[
b(r) = \exp(-\alpha r) \cos(\omega r) \begin{cases} \mathbb{R}^2 : \alpha \geq \omega \\ \mathbb{R}^3 : \alpha \geq \sqrt{3}\omega \end{cases} \]  

(10a)

and

\[
b(r) = \exp(-2\alpha a \sin(\theta/2)) \cos(\omega r) \quad (\mathbb{S}^2) \]  

(10b)

\[
\alpha \geq \sqrt{3} \omega
\]

Studies (Kim and North, 1991) show that in fact for actual observational and modeled data, the spatial correlation structure is neither very homogeneous nor isotropic. The sampling error in the optimal averaging can also be estimated,

\[
D^2 = m^2 + \sum_{k=1}^{N} \sum_{j=1}^{N} w_j w_k \hat{C}_{jk} - 2 \sum_{j=1}^{N} w_j \hat{\Omega}_j
\]  

(11)

where

\[
m^2 = \frac{1}{A} \int \int \int \int \hat{C}(x, x', y, y') dxdx'dyd'y'
\]  

(12)
If we assume measurement errors $\delta_i$ that obey
\[
\langle \delta_i \rangle = 0 \quad i = 1, \ldots, M
\]
\[
\langle \delta_i \delta_k \rangle = 0 \quad \text{and} \quad \langle \delta_i f_k \rangle = 0
\]
we have to solve the normal equations
\[
\sum_{j=1}^{N} w_j C_{jk} + w_k \langle \delta_k^2 \rangle = \Omega_k
\]
\[
D^2 = \ldots + \sum_{j=1}^{N} w_j^2 \langle \delta_j^2 \rangle .
\]
Ultimately, we can generalize to the case of non-zero temporal mean, $\langle f_j \rangle \neq 0$ in which case both biased and unbiased estimates are possible. For the biased case, one minimizes
\[
D^2 = \langle (O - \langle O \rangle - \mu + \langle \mu \rangle)^2 \rangle
\]
while in the unbiased case one uses the additional constraint,
\[
\langle O \rangle = \langle \mu \rangle .
\]
If $\langle f_j \rangle$ is approximately constant in the area A, this constraint is equivalent to
\[
\sum_{j=1}^{N} w_j = 1 .
\]

We first consider the example of a random field on a $10 \times 10$ $(x, y)$ grid, with the model correlation function,
\[
B(r) \exp \left(- (r/dc)^{1.5} \right), \quad \sigma^2 = \text{constant}
\]
Many (1000) realizations of spatial random fields with prescribed covariance matrix (Johnson, 1987) are generated. In this case, both Thiessen and optimal averages approximate the true spatial mean well, as measured by the mean-square error. The area-average performs relatively poorly. For the anisotropic variance structure, $\sigma^2 = y$, the optimal averaging considerably out-performs the Thiessen average, with the area average again performing relatively poorly. In cases where an incorrect correlation model is used, $\left( B(r) = \exp \left(- (r/dc)^2 \right) \right)$, or an incorrect variance structure is assumed, $\left( \sigma^2 = 5 \right)$, optimal averaging still performs relatively well.

Optimal averaging thus out-performs both area-averaging and Thiessen weighting and seems to be relatively insensitive to the choice of correlation model. It also gives an intrinsic estimate of sampling error.
The optimal spatial average is tested on the globe based on a perpetual January climatological sea-surface temperature GCM simulation, and from historical data (Madden et al. 1993). The GCM results show a covariance structure which might be only crudely approximated as isotropic and homogeneous. The results based on observational data demonstrate that mean-squared error decreases essentially monotonically with the number of gridpoints used in the optimal averaging, suggesting that it is advantageous in general to use all gridpoint data available.

References and Related Publications


Kriging

Dr. Steve Cherry
Wendy Meiring, Rapporteur

Introduction

Dr. Cherry presented an introduction to geostatistics, including kriging. Kriging provides a tool for predicting at unsampled sites given observations from other sampled spatial locations. Prediction requires a measure of spatial association. In geostatistics spatial association is typically modelled by variograms. Variogram estimation is one of the main topics Dr. Cherry discussed. Much of the notation and discussion follows Cressie, 1991.

Techniques similar to kriging have been developed in many fields. One of these techniques is optimal linear interpolation, which is used in the atmospheric sciences.

Geostatistics

Geostatistical data has a continuous spatial domain. In geological applications there might be only one realization of the process, whereas atmospheric applications have several realizations through time. In atmospheric applications, variogram estimation techniques may use the repeated observations through time to better estimate the spatial structure (see list of references). Data are assumed to be a realization of a stochastic process

\[
\{Z(x) : x \in D \subset \mathbb{R}^d\},
\]

for some fixed spatial domain \(D\) within \(d\)-dimensional Euclidean space \(\mathbb{R}^d\).

Variograms and Semivariograms

Definitions

The variogram is defined as the variance of the increments of the process, \(\text{Var}[Z(x_i) - Z(x_j)]\), for spatial points \(x_i\) and \(x_j\). The semivariogram \(\gamma(x_i, x_j)\) is defined as half the variogram

\[
\gamma(x_i, x_j) = \frac{1}{2} \text{Var}[Z(x_i) - Z(x_j)]
\]
Commonly-Made Assumptions

Assumptions such as intrinsic stationarity, second order stationarity, and isotropy are often made. These properties are defined below.

An **intrinsically stationary process** is one with a constant mean, and a semivariogram which is only a function of the spatial difference between $x_i$ and $x_j$, *i.e.*, $E(x_i) = \mu$ and $\gamma(x_i, x_j) = \gamma(x_i - x_j)$ for some constant $\mu$ and all points $x_i, x_j$ in $D$.

If the process has a constant mean, and a spatial covariance function which only depends on the spatial lag between points, the process is **second order stationary**. These properties may be written as $E(x_i) = \mu$ and $cov(Z(x_i), Z(x_j)) = C(x_i - x_j)$, for some constant $\mu$ and all points $x_i$ in $D$.

If the process is second order stationary, it may be shown that

$$\gamma(h) = C(0) - C(h) = C(0) \left[ 1 - \rho(h) \right],$$

where $C(0)$ is the variance of $Z(x)$, $C(h)$ is the covariance at spatial lag $h$, and $\rho(h)$ is the spatial correlation at spatial lag $h$. Second order stationarity thus implies that the spatial covariance function depends only the spatial lag between points, thus second order stationarity implies intrinsic stationarity. The converse is not true however. A Wiener process offers one counterexample, since its variogram only depends on the spatial lag between points, while its covariance function depends on the locations themselves.

If the semivariogram is only a function of the Euclidean distance between points, the process is said to be **isotropic**. In many applications isotropy assumptions will be violated, in which case the process is called anisotropic. In atmospheric data, factors such as topography and wind contribute to anisotropy. Sampson and Guttorp (1992) and Monestiez and Switzer (1991) discuss two approaches to dealing with a non-stationary anisotropic covariance structure.

Estimating Semivariograms

Intrinsic stationarity implies $\gamma(x_i, x_j) = \frac{1}{2} E \left[ (Z(x_i) - Z(x_j))^2 \right]$. For regularly spaced data a natural estimator is Matheron’s classical method of moments estimator, given by

$$\hat{\gamma}(h) = \frac{1}{2 |N(h)|} \sum_{N(h)} (Z(x_i) - Z(x_j))^2$$

where $N(h) = \{(x_i, x_j) : x_i - x_j = h\}$. A simple redefinition of $N(h)$ accommodates irregularly spaced data (see Cressie, 1991).
Variograms must satisfy several theoretical properties, including conditional negative definiteness (see Cressie for details). The estimate given above is not conditionally negative definite, so a valid semivariogram must be fit to this sample estimate of the semivariogram to obtain such a property.

**Valid Semivariogram Functions**

Valid semivariograms include the spherical, exponential and power models, but there are many others, both parametric and non-parametric. (See Cressie 1991, for a list of some of these and the properties that they must satisfy.) Spherical, exponential and power models are examples of semivariogram functions which assume that the spatial correlation never increases as the distance between sites increases. There are other semivariogram functions which accommodate periodic structure in the spatial correlation.

**Methods of Fitting**

Many fitting techniques have been used. These include maximum likelihood, restricted maximum likelihood, weighted and unweighted nonlinear least squares. Non-parametric methods have been developed by researchers such as Sampson and Guttorp (1992), and Hall et al. (1993). Possibly the most common method currently used by practitioners to fit semivariograms is “by eye”.

**Kriging**

Many types of kriging have been developed to address different problems. Cressie (1991) overviews many of these types. Only two types, namely ordinary kriging and universal kriging were overviewed in this talk. In both cases we consider data \( \{Z(x_1), \ldots, Z(x_n)\} \) at sites \( x_1, \ldots, x_n \). We wish to predict \( Z(x_0) \) for some site \( x_0 \) in \( D \).

**Ordinary Kriging**

The ordinary kriging model is

\[
Z(x) = \mu + \epsilon(x),
\]

where \( \mu \) is assumed constant for the whole field, and where \( \text{E} [\epsilon(x)] = 0 \) and \( \gamma(h) \) is the semivariogram.
Ordinary kriging predicts $Z(x_0)$ as a linear combination $\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)$ of the other values, with weights $\{\lambda_1, \ldots, \lambda_n\}$. The weights minimize $E\left[ (Z(x_0) - \hat{Z}(x_0))^2 \right]$, subject to the constraint $\sum_{i=1}^{n} \lambda_i = 1$. This constraint is needed to insure unbiasedness of the predictor $\hat{Z}(x_0)$. A system of equations is solved for $\{\lambda_1, \ldots, \lambda_n\}$. One of the main perceived advantages of kriging as a spatial prediction procedure is that an estimate for the mean squared prediction error is obtained. This estimate of variability may be misleading, however. Kriging assumes a known semivariogram, and the estimate of prediction error takes no account of uncertainty in the semivariogram estimate.

Ordinary kriging was discussed in terms of an illustrative example of Iowa precipitation. More detailed examples are given in the referenced papers.

**Universal Kriging**

Ordinary kriging assumed a constant mean. One possible approach to dealing with nonstationarity in the mean, is universal kriging. This models the quantity of interest as

$$Z(x) = \sum_{j=1}^{p} f_j(x) \beta_j + \epsilon(x).$$

$Z(x)$ is again estimated as a linear combination

$$Z(x) = \sum_{i=1}^{n} \lambda_i Z(x_i).$$

Equations can be solved for $\{\lambda_1, \ldots, \lambda_n\}$, subject to an unbiasedness constraint. A major problem arises, however, in that $\{\beta_1, \ldots, \beta_p\}$ is needed to estimate the semivariogram, but the semivariogram is also needed to estimate $\{\beta_1, \ldots, \beta_p\}$. Many semivariogram estimators are now biased.

**Future Work**

Dr. Cherry highlighted some areas for further research. These include robust semivariogram estimation, and non-parametric semivariogram estimation.

**References and Related Publications**


