Error Sensitivity of Fixed- and Variable-Path Lyman-Alpha Hygrometers

Arden Buck
A Lyman-alpha hygrometer development project, carried out by the author at the Research Systems Facility of NCAR, is now nearing completion. The present study, an outgrowth of that work, may have general usefulness to other users of Lyman-alpha hygrometers. In addition, the equations developed here can be applied to other absorption devices operating anywhere in the electromagnetic spectrum.
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INTRODUCTION

The Lyman-alpha absorption hygrometer, operated in a variable path mode, has the potential of offering both high spatial/temporal resolution and high absolute accuracy in a relatively simple instrument. Operating at the 121.6 nm Lyman-alpha line in the far UV, the hygrometer uses a hydrogen glow discharge lamp as a source and a nitric oxide ion chamber as a detector. Windows are made of magnesium fluoride, and together with the detector form a 115- to 135-nm bracket for the Lyman-alpha radiation. Typical path lengths range from 0.05 to 10 cm.

There is no hysteresis effect, except for possible window sorption and desorption lags, and no inherent loss in resolution at very low humidities, since path length can be increased to maintain a constant sensitivity. In practice, operation is limited by the $O_2$ and $O_3$ correction terms.

Major problems in operating the instrument are spectral purity of the source and window degradation in certain field environments.

The optimum solution to these problems will be a function of the particular application and of the desired size and cost of the instrument.

In this note, the operation of absorption hygrometers in general will be examined, along with some schemes for allowing self-calibration. The effects of various parameter measurement errors on humidity accuracy will be calculated for both ideal and practical instruments, and error limits to achieve some arbitrary level of performance will be listed.

BASIC EQUATIONS

Beer's law, on which all spectroscopic determination must be based, can be expressed as:

$$I = I_0 e^{-k_o x/o}$$  \hspace{1cm} (1)
where $I_o$ = transmitted intensity
$I$ = received intensity over distance $x$
$k_1$ = absorption coefficient at STP
$\rho_1$ = actual density of absorbing medium
$\rho_o$ = density of absorbing medium at STP

Plane wave propagation through a uniform medium is assumed. For convenience, let $K_1$ include both $k_1$ and $\rho_o$ terms, and let $I_o$ and $I$ be replaced by their sensed voltage equivalents. Then (1) becomes

$$V = V_o e^{-k_1 \rho_1 x}$$

or

$$\ln V = \ln V_o - k_1 \rho_1 x.$$ 

Here

$$V_o = I_o \gamma DT^2 E$$

where $\gamma$ = detector conversion efficiency
$D$ = detector current per unit of input intensity
$T$ = window transmission
$E$ = electronic processing factor

In the real world, one deals with source/detector combinations which allow more than one spectral line and thus more than one absorption coefficient. In addition, several absorbing materials may be present in the sensing volume, each having its own array of absorption coefficients. For the more general case, (2) becomes

$$V = V_o (\lambda_1) e^{\rho_1 x} - K_1 (\rho_1) \rho_1 x - K_2 (\rho_2) \rho_2 x - K_3 (\rho_3) \rho_3 x - \ldots$$

$$+ V_o (\lambda_2) e^{\rho_1 x} - K_1 (\rho_1) \rho_1 x - K_2 (\rho_2) \rho_2 x - K_3 (\rho_3) \rho_3 x - \ldots$$

$$+ V_o (\lambda_3) e^{\rho_1 x} \ldots + \ldots$$

$$= \sum_{i=1}^{m} V_o (\lambda_i) e^{\rho_i x} - \sum_{j=1}^{n} K_j (\rho_j) \rho_j x$$

(3)
where $V_o (\lambda_i), K_j (\lambda_i)$, and $\rho_j$ represent values for the $i$th spectral line and the $j$th absorbing gas. Let $\rho_1$ represent water vapor density. For a hygrometer, where one is looking for $\rho_1$ as a function of $x$ and $V$, an expression such as (3) is virtually unmanageable. In practice, it is sufficient to use an empirically determined function

$$\rho_1 x = f_V(V'')$$

(4)

where $V''$ is $\ln V$ corrected for the effects of $V_o$, nonplanar propagation, and the other absorbers. Although the effects of the $\rho_j x$ terms are not mutually independent except for the case $m = 1$, their interaction is weak for a sufficiently narrow range of concentrations. A detailed analysis of this interaction is given later.

MODE OF OPERATION

Consider the simplified equation (2). If $V_o$ is known, (2) suffices to determine $\rho_1$ from measured voltage $V$. If $V_o$ is unknown, an additional equation is needed to eliminate it. $V_o$ can change by changes in any of the factors which determine system gain: source output $I_o$, detector performance $D$, window transmission $T$, or electronic performance $E$. A second equation may be generated by (a) a second (known) value for $K_1$, (b) a second value for $x$, or (c) a second value for $\rho_1$ (Substituting known values for $\rho_1$ of course, is simply a calibration.)

In all three cases, the first equation is

$$\ln V_1 = \ln V_o - K_1 \rho_1 x$$

and the second is

$$\ln V_2 = \ln V_o - K'_1 \rho_1 x$$

for (a)

$$\ln V_2 = \ln V_o - K_1 \rho_1 x'$$

for (b)

or

$$\ln V_2 = \ln V_o - K_1 \rho'_1 x$$

for (c)

(5)
depending on which variable is changed. The solutions are

\[
\rho_1 = \frac{\ln(V_1/V_2)}{(K_1' - K_1)x} \quad (6a)
\]

\[
\rho_1 = \frac{\ln(V_1/V_2)}{K_1(x' - x)} \quad (6b)
\]

\[
\rho_1 = \frac{\ln(V_1/V_2) + K_1\rho_1'x}{K_1x} \quad (6c)
\]

respectively.

For the special cases in which \(K_1, x\) or \(\rho_1\) go to zero or are changed by a known multiplicative factor \(m\), the solutions in (6) reduce to \(\ln(V_1/V_2)/K_1x\) and \(\ln(V_1/V_2)/(m - 1)K_1x\), respectively.

The approach applies to the more general expression (4), except that altering the operating spectrum alters the entire function \(f_V\) rather than \(K_1\), and the variables are not mutually independent.

Let us designate four basic operating modes: (I) fixed path, fixed spectral distribution, (II) fixed path, variable spectral distribution (changing \(K_1\) or \(f_V\) commonly called the two-wavelength mode), (III) variable path, fixed spectral distribution, and (IV) variable \(p_1\). Additional systems could be proposed which employ various combinations of these modes.

In a Mode I system, if the effects of other gases and changes in \(V_0\) are negligible or well known, then \(V\) accurately reflects atmospheric water vapor. Otherwise, unknown values for these terms introduce error.

Mode II and III systems appear from Eqs. (6a) and (6c) to be similar, but when design constraints are considered there are important differences. Mode II has been commonly used in the IR. For \(V\) to accurately reflect \(\rho_1\) in a Mode II instrument, we require that (a) the effect of extraneous absorbers be negligible or well known and (b) between-channel errors be negligible. Requirement (a) is adequately satisfied
in the IR; however, (b) is subject to some problems. The spectral differences required to generate the two equations usually involve two sources or two spectral filters placed alternately in the sensing path. Uncorrectable changes in $V_0$ can occur by dirt accumulating unequally on the two sources or filters, variations in the spectral response of the detector (this commonly occurs), dispersive material such as liquid water collecting on the optics, or any other effective gain change occurring unequally for the two channels.

For a Mode III system, requirements (a) and (b) still apply. Corrections must be made for extraneous absorbers ($O_2$ and $O_3$ in the UV). Between-channel error is not a serious problem in a Mode III system, however, since all physical elements of the system are common to both channels. The only extraneous variable between channels is collimation variation in the detector, an easily determined function of source/detector geometry.

In a Mode IV system, one is constrained to changing $\rho_1$ by a multiplicative factor or by referring to a known $\rho_1$ (baselining). The former could be accomplished by a variable pressure scheme, while the latter would involve injecting a known absorber into the sensing volume. If one could contrive to change the density $\rho_1$ without changing the densities of extraneous absorbers, it would be possible to correct for the other absorbers as well as for $V_0$.

The characteristics of the four operating modes are summed up below:

<table>
<thead>
<tr>
<th>Parameter varied</th>
<th>Mode I</th>
<th>Mode II</th>
<th>Mode III</th>
<th>Mode IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrects for $V_0$ changes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Corrects for effect of other absorbers</td>
<td>no</td>
<td>to extent that $K_2'=K_2$</td>
<td>no</td>
<td>to extent that $\rho_2'=\rho_2$</td>
</tr>
<tr>
<td>Control of between-channel errors</td>
<td>n.a.</td>
<td>difficult</td>
<td>easy</td>
<td>not known</td>
</tr>
</tbody>
</table>
In this error analysis we will be comparing Lyman-alpha system Modes I and III for several representative operating conditions and source spectral makeups.

EQUATIONS USED IN ERROR ANALYSIS

The error analysis is done for the general case of Eq. (4) and for the more ideal case of Eq. (2), except that the effect of a second absorber is included. If we assume a polynomial solution for \( f_v \) in (4), we have

\[
\rho_1 x = f_v(V'')
\]

\[
= a_1 + a_2 (V'') + a_3 (V'')^2 + a_4 (V'')^3 + \ldots \quad (7)
\]

where

\[
V'' = \ln V - \ln V_0 - f_{21} - f_c.
\]

The oxygen correction term \( f_{21} \) can be approximated by another polynomial:

\[
f_{21} = \ln_1 + \ln_2 (P_x/T) + \ln_3 (P_x/T)^2 + \ldots \quad (9)
\]

The effects of other extraneous absorbers in the troposphere can be largely neglected. Above the tropopause, ozone becomes significant.

The effects of nonplanar propagation (imperfect collimation) can be represented by the expression

\[
f_c = c_1 + c_2 x + c_3 x^2 + \ldots \quad (10)
\]

In practice, \( f_v, f_{21}, \) and \( f_c \) are determined from experimental data.
Let us now find the solutions for \( f_2 \), \( f_c \), and \( f_v \) for the monochromatic case of Eq. (2). If we include the effects of a second absorber \( \rho_2 \) in Eq. (2), we have

\[
V = V_0 e^{-K_1\rho_1 x} - K_2\rho_2 x
\]

or

\[
\ln V = \ln V_0 - K_1\rho_1 x - K_2\rho_2 x
\]  

(11)

Substituting into (7) gives us:

\[
f_v = V'' = -\frac{V''}{K_1}
\]

\[
V'' = \ln V - \ln V_0 + \rho_2 K_2 x
\]

\[
f_{21} = -\rho_2 K_2 x
\]

\[
f_c = 0
\]  

(12)

It should be noted that even for a monochromatic source, there is a slight pressure dependence in \( K_2 \).

In the variable-path system, the non-ideal case requires that we obtain first an iterative numerical solution for \( V_o \), then substitute into (7) to obtain \( \rho_1 \). An ideal source allows the following analytical solution:

\[
V_1 = V_0 e^{-(K_1\rho_1 + K_2\rho_2)x_1}
\]

\[
V_2 = V_0 e^{-(K_1\rho_1 + K_2\rho_2)x_2}
\]

\[
\rho_1 = \frac{\ln(V_1/V_2) - K_2\rho_2(x_2 - x_1)}{K_1(x_2 - x_1)}
\]  

(13)
Detailed analysis was performed for the effects of variations in the four basic parameters V, x, P, and T. Other causes of error were related to these four.

For Mode I (fixed path) operation, analytical solutions were obtained; analytical solutions for Mode III operation could not be so easily obtained, and numerical solutions were resorted to in most cases.

SOURCE CONFIGURATIONS

Three representative source configurations were analyzed: source 1 is a standard commercial source, source 2 is an experimental source, containing uranium hydriden (UH$_3$), and source 3 is an ideal source emitting radiation only at 1215.6 angstroms. The effects of water vapor and oxygen were assumed not to interact; the effect of their interaction is analyzed separately later.

The slope of the UH$_3$ source is about 5% lower than that of the ideal source; we have as yet no physical explanation for this. The effect is to make some aspects of source 2 performance appear about 5% better than ideal.

The values of the coefficients used in the analysis are given in Table I, p.17, and $f_v$ for the three sources are plotted in Figs. 1-3, pp. 28-30.

OPERATING CONDITIONS

The following four sets of operating conditions were chosen for analysis. They are typical of the locales indicated, and also represent a wide range of meteorological parameter values.

Temperature is in degrees C, $\rho$ is in millibars, $\rho_1$ is in grams per cubic meter, and x is in centimeters.
Path length $x$ was optimized for minimum voltage noise sensitivity (See Appendix 1), but limitations at both extremes were accommodated: insufficient airflow at the short end and loss of signal due to beam spreading at the long end. The sets of path lengths $x_1$ and $x_2$ were chosen somewhat arbitrarily but reflect approximately optimum values for noise insensitivity. An interesting question, left unexplored, is the effect on error sensitivity of changing the relationship of $x_1$, $x_2$ and $\bar{x}$.

**VOLTAGE ERROR SENSITIVITY**

From Eq. (7) we obtain

$$\frac{\partial \rho_1}{\partial V} = \frac{\partial f(V'')}{\partial V''} \cdot \frac{1}{xV}$$

which reduces for the ideal case of Eq.(11) to

$$\frac{\partial \rho_1}{\partial V} = \frac{-1}{K_1 xV}$$
This form is appropriate for the $\rho_1$ error due to an error in voltage reading. For an unrecognized change in gain, the following form is more useful:

$$\frac{\partial \rho_1}{\rho_1} = \frac{\partial \mathbf{v}}{\mathbf{v}} \left[ \frac{f'(v''(\mathbf{v}'))}{\rho_1 x} \right]$$  \hspace{1cm} (16)

$$= \frac{\partial \mathbf{v}}{\mathbf{v}} \left[ \frac{-1}{\rho_1 K_1 x} \right]$$ \hspace{1cm} (17)

No analytical solution was developed for the variable path case.

Voltage error sensitivity is evaluated in Table II, p. 18, for the three source types and four test conditions, defined in the previous sections, for operating Mode I. The second set of values is somewhat arbitrary from source to source, since fractional error per millivolt is inversely proportional to system gain $V_o$, which was picked arbitrarily for the ideal source.

Voltage error sensitivity is evaluated for Mode III operation in Table III, p. 19. The error resulting from voltage bias or offset error is found by adding the equivalent fractional changes in $V_1$ and $V_2$. Note the anomalous sign change for source 2, condition a.

**PATH LENGTH ERROR SENSITIVITY**

Equation (7) can be written

$$\rho_1 = \frac{u(w(x))}{v}$$, \hspace{1cm} where $u = f_\mathbf{v} = \rho x$$

$v = x$

$w = v''$
Then
\[
\frac{\partial \rho_1}{\partial x} = v \frac{\partial u}{\partial w} \frac{\partial w}{\partial x} - u \frac{\partial v}{\partial x} = \frac{f'_{V}}{x} (V'')( - f'_{C} - \frac{P_{T} f'_{21}}{T_{21}}) - \rho_1 \tag{19}
\]

and
\[
\frac{\partial \rho_1}{\rho_1} = \frac{\partial x}{x} \left[ - \frac{f'_{V}(V'')}{\rho_1} \left( \frac{f'_{C} + \frac{P_{T} f'_{21}}{T_{21}}}{\rho_1} \right) - 1 \right] \tag{20}
\]

In the ideal case,
\[
\frac{\partial \rho_1}{\partial x} = - \frac{1}{x} \left[ \frac{\rho_1 K_1 + \rho_2 K_2}{K_1} \right] \tag{21}
\]

\[
\frac{\partial \rho_1}{\rho_1} = \frac{\partial x}{x} \left[ - 1 - \frac{\rho_2 K_2}{\rho_1 K_1} \right] \tag{21}
\]

Table IV, p. 20, gives fixed-path versions of (19) and (20), while Table V, p. 21, evaluates (19) for Mode III for balanced and unbalanced changes in path length. As expected, error drops drastically when change is equal for both path lengths.

PRESSURE ERROR SENSITIVITY

Using a similar approach to the above, solutions for the non-ideal case are:
\[
\frac{\partial \rho_1}{\partial p} = \frac{- f'_{V}(V'') f'_{21}(P x / T)}{T_{21}} \tag{22}
\]
\[
\frac{\partial \rho_1}{\rho_1} = \frac{\partial p}{p} \left[ \frac{- f'_{V} f'_{21} P}{\rho_1 T} \right] \tag{22}
\]
where T is in degrees K.

The ideal case is not expressed in terms of P and T, so that solution is not so meaningful.

Table VI, p. 22, gives evaluations. As can be seen, pressure affects humidity errors only slightly.

**TEMPERATURE ERROR SENSITIVITY**

\[
\frac{\partial \rho_1}{\partial T} = \frac{f'_{V''} f'_{21} P}{T_2}
\]

\[
\frac{\partial \rho_1}{\rho_1} = \frac{\partial T}{T} \left[ \frac{f'(V'') f'_{21} P}{\rho_1 T} \right]
\]

Evaluations are given in Table VII, p. 23. As with pressure, humidity is a very weak function of ambient temperature.

**SENSITIVITY TO ERROR IN CORRECTION TERMS**

For convenience, Table VIII, p. 24, gives sensitivity to errors in V'', i.e., errors in \( f_c \), \( f_{21} \), \( \ln V \), or \( \ln V \) for Mode I, or errors in \( f_c \), \( f_{21} \) or \( \ln V \) for Mode III. The error equation is simply

\[
\frac{\partial \rho_1}{\partial V''} = \frac{\partial \rho_1}{\partial f_c} + \frac{\partial \rho_1}{\partial f_{21}} + \ldots = \frac{f'(V'')}{x}
\]

Errors in Mode III operation cancel out for changes equal to both channels.
SPECTRAL PURITY OF SOURCE

Spectral purity affects system performance in several ways. A spectrally pure source exhibits the following characteristics, as compared to a spectrally impure source:

(a) more linear $f_v(V'')$
(b) more predictable $f'_v(V'')$ and thus more accurate determination of $V_o$ and $\rho_1$ in Mode III
(c) more linear and predictable $f_{21}$
(d) less interaction between $f_{21}$ and $f_v$

While the sensitivity to these effects can be seen in the other error terms, a more important question is: "What errors can we expect under real conditions?" Although it is difficult to evaluate error due to "spectral change" per se, it is possible to evaluate two related effects:

(a) change in effective absorption coefficient, and
(b) interaction between $f(V'')$ and $f_{21}$ (PX/T)

For (a), approximating $f_v(V'')$ at a given point by a short straight-line segment having a slope equal to $f'_v(V'')$, and defining an equivalent absorption coefficient $K'_1 = 1/f'_v(V'')$, we find that fractional change in indicated humidity is equal to fractional change in $K'_1$. This holds for operation in Modes I and III.

For all sources, the Lyman-alpha line dominates when absorption is very small, and effective $K'_1$ is very close to ideal $K_1$ in this region. As $\rho_x$ becomes larger, the Lyman-alpha is absorbed faster than other lines, and the impurities become exponentially more important. Therefore, the variability of effective $K'_1$ with increasing $\rho_1x$ is a good measure of spectral impurity (related to water vapor; the effect of the same lines on $K_2$ is entirely different). It is also a measure of the susceptibility of a given source to spectral instability, since the
instability is due to changing line strengths relative to the Lyman alpha line.

Table IX, p. 25, gives the change of $K_1'$ over two operating ranges. For (b), the effect of $H_2O - O_2$ interaction was evaluated by modeling sources 1 and 2, substituting line strengths found experimentally, and their absorption coefficients for $H_2O$ and $O_2$ into Eq. (3), and generating a two-dimensional matrix of output voltage vs. $H_2O$ and $O_2$ concentrations. Figs. 4-9, pp. 31-36, give the results, and Table X, p. 26, extracts from those results the equivalent errors that result from interaction between the two. One can think of it in terms of a change in $f_{v'}(V')$ as one changes $O_2$ concentration, or a change in $f_{21}$ as one changes $p_x$. They are equivalent. Here we refer to an error in $f_{21}$. In the absence of interaction (ideal source), the differences in the two right-hand columns of Figs. 4-6 and the curves of Figs. 7-9 remain constant with changing pressure (height). The $UH_3$ source curve in Fig. 7 differs appreciably from actual behavior, so there is reason to doubt the numbers from that particular model.

OTHER ERROR EFFECTS

Ozone: $O_3$ absorbs Lyman alpha radiation very strongly, and as one approaches the tropopause, its effect rapidly overpowers the instrument's response. Figure 10, p. 37, gives one projection of the effect of $O_3$ in terms of equivalent $p_x$ error. The actual $O_3$ distribution fluctuates strongly from day to day; average data were used here. We can expect at least 2% error at 10 km without any $O_3$ correction.

Temperature Expansion: Assuming a metal expansion length of 5 cm and a temperature coefficient of 10 ppm/°C, we have a resultant path-length error of $5 \times 10^{-5}$ cm/°C.

Amplifier Linearity: This is equivalent to predictability of $f_{v'}(V')$. Normally, nonlinearities in the electronics will be absorbed into the empirical determination of $f$.

Temperature Control of $UH_3$ Source: The output intensity of the $UH_3$ source changes roughly 7% per degree of $UH_3$ temperature change.
SOME TYPICAL ERROR VALUES

Assume the following uncertainties for the parameters analyzed above:

- \( P: \pm 3 \text{ mb} \)
- \( T: \pm 1^\circ \text{C} \)
- \( X: \pm 0.003 \text{ cm} \)
- \( V \text{ bias}: \pm 3 \text{ mv, condition b} \)
- \( V_o: 20\% \)
- ambient temperature range: 40°C
- temperature control of UH_3 source: \( \pm 0.05^\circ \text{C} \)

Percent errors in \( \rho_1 \) resulting from these uncertainties are evaluated in Table XI, p. 27, for Boulder conditions, sources 1 and 2, Modes I and III. Assuming that correction is made for half the temperature expansion and that path-length error is split between balanced and unbalanced, the total root sum square measurement error can be calculated. Error is maximum (19.6%) for source 1, Mode I; minimum error (0.46%) is found for source 2, Mode III. This is given at the bottom of Table XI, p. 27.

An important error source is not included in these values. The rate of change of the spectral distribution of the sources is not evaluated, since it depends upon the frequency with which one recalibrates. Spectral change rate fluctuates greatly from source to source, but seems to be about an order of magnitude lower for the UH_3 source tested.

CONCLUSIONS

From Table XI, p. 27, the three most important contributors to measurement error are, in order of importance:

- errors in system gain (fixed path only)
- \( f_v, f_{21} \) interaction (fixed path only)
- path length

The variable path system shows its advantage primarily in its insensitivity to \( V_o \) changes.
The $f_v/f_{21}$ dependence is a strong source of error for the fixed-path system using a standard source, but drops drastically for the $\text{UH}_3$ source and/or variable-path operation. If the operating altitude range had been more limited, this error would have been less. At any rate, it seems appropriate to include a correction term for this effect in future tests.

For the variable-path system, path length is the primary contributor to error. Two means for reducing this error suggest themselves: (1) measure path length more carefully, (2) increase path length, which reduces the sensitivity to $x$ error at the expense of increasing sensitivity to $V$ error.

Rather than the fractional or percentage units used in most of this study, error expressed in grams per cubic meter may be more significant from a scientific point of view. In other cases dew-point error is important (see Appendix 2).

In summary, the advantages of operating in Mode III, and using the $\text{UH}_3$ source are clear from this study.
### TABLE I
COEFFICIENTS FOR EQUATIONS 7 - 10

<table>
<thead>
<tr>
<th></th>
<th>Standard Source</th>
<th>UH$_3$ Source</th>
<th>Ideal Source</th>
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</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>7.027</td>
<td>4.1387</td>
<td>4.15</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-4.540</td>
<td>-1.9401</td>
<td>-2.0747 ($= -1/K_1$)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>1.047</td>
<td>-0.0135</td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>-0.154</td>
<td>-0.0242</td>
<td></td>
</tr>
<tr>
<td>$a_5$</td>
<td>0.000</td>
<td>0.0123</td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.1123</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.3165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_3$</td>
<td>-0.0532</td>
<td>$f_{21} = -\rho_2 K_2 x$</td>
<td>$f_{21} = -\rho_2 K_2 x$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.00554</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_5$</td>
<td>0.00021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.5856</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_2$</td>
<td>-0.6157</td>
<td>same</td>
<td>same</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.0304</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE II

VOLTAGE ERROR SENSITIVITY OF HUMIDITY, MODE I

(FIXED PATH)

<table>
<thead>
<tr>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
</tr>
<tr>
<td>Standard</td>
<td>-0.54</td>
<td>-0.92</td>
<td>-1.21</td>
<td>-1.80</td>
</tr>
<tr>
<td>UH$_3$</td>
<td>-0.37</td>
<td>-0.66</td>
<td>-1.00</td>
<td>-1.58</td>
</tr>
<tr>
<td>Ideal</td>
<td>-0.14</td>
<td>-0.69</td>
<td>-1.04</td>
<td>-1.66</td>
</tr>
</tbody>
</table>

**Fractional change per millivolt**

<table>
<thead>
<tr>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
</tr>
<tr>
<td>Standard</td>
<td>-0.0028</td>
<td>-0.0004</td>
<td>-0.0011</td>
<td>-0.0056</td>
</tr>
<tr>
<td>UH$_3$</td>
<td>-0.1320</td>
<td>-0.0003</td>
<td>-0.0006</td>
<td>-0.0024</td>
</tr>
<tr>
<td>Ideal</td>
<td>-0.0217</td>
<td>-0.0003</td>
<td>-0.0007</td>
<td>-0.0027</td>
</tr>
</tbody>
</table>
### TABLE III

**VOLTAGE ERROR SENSITIVITY, MODE III**

*(VARIABLE PATH)*

Fractional change per millivolt, $\Delta V$, only

<table>
<thead>
<tr>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropics Boulder 4 km 7 km</td>
<td>Tropics</td>
<td>Boulder 4 km</td>
<td>7 km</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>0.006</td>
<td>0.0005</td>
<td>0.0015</td>
<td>0.001</td>
</tr>
<tr>
<td>UH$_3$</td>
<td>-0.0167</td>
<td>0.0002</td>
<td>0.0007</td>
<td>0.0037</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.0025</td>
<td>0.0003</td>
<td>0.0008</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Fractional change per millivolt, $\Delta V_2$ only

<table>
<thead>
<tr>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropics Boulder 4 km 7 km</td>
<td>Tropics</td>
<td>Boulder 4 km</td>
<td>7 km</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>-0.036</td>
<td>-0.0017</td>
<td>-0.0048</td>
<td>-0.0030</td>
</tr>
<tr>
<td>UH$_3$</td>
<td>0.151</td>
<td>-0.0008</td>
<td>-0.0021</td>
<td>-0.0095</td>
</tr>
<tr>
<td>Ideal</td>
<td>-0.406</td>
<td>-0.0009</td>
<td>-0.0023</td>
<td>-0.0106</td>
</tr>
</tbody>
</table>

Equal fractional changes in $V_1$ & $V_2$ (gain change)

<table>
<thead>
<tr>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropics Boulder 4 km 7 km</td>
<td>Tropics</td>
<td>Boulder 4 km</td>
<td>7 km</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>UH$_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ideal</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
**TABLE IV**

PATH-LENGTH ERROR SENSITIVITY, MODE I

(FIXED PATH)

<table>
<thead>
<tr>
<th>Source</th>
<th>a (Tropics)</th>
<th>b (Boulder)</th>
<th>c (4 km)</th>
<th>d (7 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>-1.36</td>
<td>-1.61</td>
<td>-2.73</td>
<td>-5.26</td>
</tr>
<tr>
<td>( \text{UH}_3 )</td>
<td>-1.14</td>
<td>-1.25</td>
<td>-2.07</td>
<td>-3.70</td>
</tr>
<tr>
<td>Ideal</td>
<td>-1.05</td>
<td>-1.26</td>
<td>-2.11</td>
<td>-3.83</td>
</tr>
</tbody>
</table>

**Fractional change per cm**

<table>
<thead>
<tr>
<th>Source</th>
<th>a (Tropics)</th>
<th>b (Boulder)</th>
<th>c (4 km)</th>
<th>d (7 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>-2.26</td>
<td>-2.68</td>
<td>-1.37</td>
<td>-1.04</td>
</tr>
<tr>
<td>( \text{UH}_3 )</td>
<td>-1.90</td>
<td>-2.08</td>
<td>-1.04</td>
<td>-0.76</td>
</tr>
<tr>
<td>Ideal</td>
<td>-1.75</td>
<td>-2.10</td>
<td>-1.06</td>
<td>-0.76</td>
</tr>
</tbody>
</table>
TABLE V
PATH-LENGTH ERROR SENSITIVITY, MODE III
(VARIABLE PATH)

Fractional change per cm, $\Delta x_1$ only

<table>
<thead>
<tr>
<th>Source</th>
<th>a Tropics</th>
<th>b Boulder</th>
<th>c 4 km</th>
<th>d 7 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>18.3</td>
<td>7.55</td>
<td>3.95</td>
<td>3.24</td>
</tr>
<tr>
<td>$\text{UH}_3$</td>
<td>-14.4</td>
<td>3.00</td>
<td>2.14</td>
<td>2.16</td>
</tr>
<tr>
<td>Ideal</td>
<td>2.64</td>
<td>3.18</td>
<td>2.20</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Fractional change per cm, $\Delta x_1 = \Delta x_2$

<table>
<thead>
<tr>
<th>Source</th>
<th>a Tropics</th>
<th>b Boulder</th>
<th>c 4 km</th>
<th>d 7 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>6.61</td>
<td>1.67</td>
<td>0.47</td>
<td>0.68</td>
</tr>
<tr>
<td>$\text{UH}_3$</td>
<td>-7.96</td>
<td>-0.42</td>
<td>0.12</td>
<td>0.48</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.01</td>
<td>0.03</td>
<td>0.12</td>
<td>0.52</td>
</tr>
</tbody>
</table>
### TABLE VI
PRESSURE ERROR SENSITIVITY, g/m$^3$ per mb

<table>
<thead>
<tr>
<th>Mode I</th>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
</tr>
<tr>
<td>Standard</td>
<td></td>
<td>-0.0044</td>
<td>-0.0017</td>
<td>-0.0009</td>
<td>-0.00091</td>
</tr>
<tr>
<td>UH$_3$</td>
<td></td>
<td>-0.0004</td>
<td>-0.00014</td>
<td>-0.00014</td>
<td>-0.00015</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>-0.00014</td>
<td>-0.00014</td>
<td>-0.00014</td>
<td>-0.00015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode III</th>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
</tr>
<tr>
<td>Standard</td>
<td></td>
<td>-0.0067</td>
<td>-0.0010</td>
<td>-0.0008</td>
<td>-0.0007</td>
</tr>
<tr>
<td>UH$_3$</td>
<td></td>
<td>0.0014</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>-0.0002</td>
</tr>
</tbody>
</table>
### TABLE VII

TEMPERATURE ERROR SENSITIVITY, \( g/m^3 \) per °C

#### Mode I

<table>
<thead>
<tr>
<th>Source</th>
<th>a (Tropics)</th>
<th>b (Boulder)</th>
<th>c (4 km)</th>
<th>d (7 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>0.0147</td>
<td>0.0050</td>
<td>0.0020</td>
<td>0.0015</td>
</tr>
<tr>
<td>( UH_3 )</td>
<td>0.0013</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.0005</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

#### Mode III

<table>
<thead>
<tr>
<th>Source</th>
<th>a (Tropics)</th>
<th>b (Boulder)</th>
<th>c (4 km)</th>
<th>d (7 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>0.0219</td>
<td>0.0027</td>
<td>0.0023</td>
<td>0.0014</td>
</tr>
<tr>
<td>( UH_3 )</td>
<td>-0.0039</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.0005</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
TABLE VIII

FRACTIONAL CHANGE IN $\rho$ PER UNIT CHANGE IN:

$f_{21}^*$, $f_c^*$, $V''$, $\ln V$, or $\ln V_o$

<table>
<thead>
<tr>
<th>Mode I</th>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>-0.54</td>
<td>-0.92</td>
<td>-1.21</td>
<td>-1.80</td>
<td></td>
</tr>
<tr>
<td>UH$_3$</td>
<td>-0.37</td>
<td>-0.66</td>
<td>-1.00</td>
<td>-1.58</td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td>-0.14</td>
<td>-0.69</td>
<td>-1.04</td>
<td>-0.83</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode III, unbalanced (arbitrary sign)</th>
<th>Source</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tropics</td>
<td>Boulder</td>
<td>4 km</td>
<td>7 km</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>3.31</td>
<td>2.28</td>
<td>3.18</td>
<td>5.12</td>
<td></td>
</tr>
<tr>
<td>UH$_3$</td>
<td>-1.74</td>
<td>0.94</td>
<td>1.98</td>
<td>4.00</td>
<td></td>
</tr>
<tr>
<td>Ideal</td>
<td>0.21</td>
<td>1.04</td>
<td>2.06</td>
<td>4.20</td>
<td></td>
</tr>
</tbody>
</table>
TABLE IX

VARIABILITY OF EFFECTIVE NET ABSORPTION COEFFICIENT

\[ (K'_1 = f'_v(y'')^{-1}) \]

% change over range of \( \rho x \) values

<table>
<thead>
<tr>
<th>Source</th>
<th>( \rho x = 1.25 ) to 3.0</th>
<th>( \rho x = 1.25 ) to 10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>23.</td>
<td>165.</td>
</tr>
<tr>
<td>( \text{UH}_3 )</td>
<td>0.5</td>
<td>63.</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.</td>
<td>0.</td>
</tr>
</tbody>
</table>
### TABLE X

EFFECT OF INTERDEPENDENCE OF $f(V'')$ and $f_{21}(P_x/T)$

Units: equivalent $f_{21}$ error

<table>
<thead>
<tr>
<th>Source</th>
<th>Altitude range</th>
<th>0 - 3.5 km</th>
<th>0 - 6 km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho x$ range: 1 - 5 g/m$^3$·cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td></td>
<td>0.073</td>
<td>0.121</td>
</tr>
<tr>
<td>$UH_3$</td>
<td></td>
<td>0.010</td>
<td>0.016</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>0.010</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>$\rho x$ range: 1 - 10 g/m$^3$·cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td></td>
<td>0.238</td>
<td>0.380</td>
</tr>
<tr>
<td>$UH_3$</td>
<td></td>
<td>0.017</td>
<td>0.026</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>0.010</td>
<td>0.016</td>
</tr>
<tr>
<td>Parameter</td>
<td>Assumed Error</td>
<td>Fixed-Path Instrument</td>
<td>Variable-Path Instrument</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---------------</td>
<td>-----------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Std. Source</td>
<td>UH$_3$ Source</td>
</tr>
<tr>
<td>Pressure</td>
<td>± 3 mb</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Temperature</td>
<td>± 1°C</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Path Length</td>
<td>± 0.003 cm</td>
<td>0.81</td>
<td>0.64</td>
</tr>
<tr>
<td>Voltage Offset</td>
<td>± 3 mv</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>System Gain (window transmission, source output, etc.)</td>
<td>20%</td>
<td>18.4</td>
<td>18.4</td>
</tr>
<tr>
<td>Temperature Expansion</td>
<td>2 x 10$^{-3}$ cm for 40°C range</td>
<td>0.54</td>
<td>0.43</td>
</tr>
<tr>
<td>Temperature UH$_3$</td>
<td>± 0.05°C</td>
<td>0</td>
<td>0.23</td>
</tr>
<tr>
<td>Interaction of $f_v$, $f_{21}$</td>
<td>range: 0 - 3.5 km, 1 - 5 g/m$^3$.cm</td>
<td>6.7</td>
<td>0.66</td>
</tr>
<tr>
<td>Total Root Sum Square Error</td>
<td>see text, p. 15 for assumptions</td>
<td>19.6</td>
<td>18.4</td>
</tr>
</tbody>
</table>
Fig. 1. $f_v(v'')$ vs $v''$ for Standard Source.
Fig. 2. $f_v(V'')$ vs $V''$ for $\text{UH}_3$ Source.
Fig. 3. $f_{\rho_X}(V'')$ vs $V''$ for Ideal Source.
COMMERCIAL SOURCE 3

\[ K(X) = K_2(X) \]

\[
\begin{array}{cccc}
\text{NO.} & V_p(\lambda) & K(\lambda) & K_2(\lambda) \\
1 & 10.00 & 387.00 & .34 \\
2 & .19 & 500.00 & 6.00 \\
3 & .23 & 110.00 & 10.00 \\
4 & .40 & 70.00 & 10.00 \\
5 & .25 & 240.00 & 9.00 \\
6 & .38 & 320.00 & 10.00 \\
7 & .12 & 280.00 & 9.00 \\
8 & .20 & 200.00 & .90 \\
9 & .25 & 150.00 & 8.00 \\
10 & .30 & 170.00 & 20.00 \\
11 & .28 & 180.00 & 15.00 \\
12 & .20 & 200.00 & 10.00 \\
13 & .25 & 200.00 & 10.00 \\
14 & .15 & 200.00 & 9.00 \\
15 & .15 & 200.00 & 8.50 \\
16 & .20 & 200.00 & 8.00 \\
17 & .20 & 200.00 & 2.50 \\
18 & .18 & 200.00 & 2.00 \\
19 & .22 & 200.00 & 4.00 \\
20 & .20 & 200.00 & 4.00 \\
21 & .30 & 200.00 & 8.00 \\
22 & .20 & 200.00 & 9.00 \\
23 & .12 & 190.00 & 10.50 \\
24 & .20 & 180.00 & 10.50 \\
\end{array}
\]

\[ z = 6 \text{ Km} \]

\[ z = 3.5 \text{ Km} \]

\[ z = 0 \text{ Km} \]

\[ 300-200 \]

\[ 300-150 \]

\[ 250 \]

\[ 225 \]

\[ 250 \]

\[ 275 \]

\[ 300 \]

\[ 300 \]

\[ 200 \]

\[ 150 \]

\[ 125 \]

\[ 100 \]

\[ 75 \]

\[ 50 \]

\[ 25 \]

\[ 10 \]

\[ 5 \]

\[ 1 \]

\[ 30 \]

\[ 29 \]

\[ 28 \]

\[ 27 \]

\[ 26 \]

\[ 25 \]

\[ 24 \]

\[ 23 \]

\[ 22 \]

\[ 21 \]

\[ 20 \]

\[ 19 \]

\[ 18 \]

\[ 17 \]

\[ 16 \]

\[ 15 \]

\[ 14 \]

\[ 13 \]

\[ 12 \]

\[ 11 \]

\[ 10 \]

\[ 9 \]

\[ 8 \]

\[ 7 \]

\[ 6 \]

\[ 5 \]

\[ 4 \]

\[ 3 \]

\[ 2 \]

\[ 1 \]

\[ \text{N=24} \]

\[ \text{LOF} \]

\[ \text{N=24} \]

\[ \text{Spectral} \]

\[ \text{lines} \]

\[ \text{normalized} \]

\[ \text{V}_p(\text{total}) \]

\[ \text{assume: } x = 1.0 \]

\[ \text{Fig. 4. Model of Commercial Source.} \]
Fig. 5. Model of Experimental Source.
### Ideal Lyman-Alpha Source with Ideal Atmosphere

<table>
<thead>
<tr>
<th>NO.</th>
<th>$v_0(X)$</th>
<th>$K_1(X)$</th>
<th>$K_2(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.000</td>
<td>387.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>$K_2(X)$</th>
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**Fig. 6. Model of Ideal Source.**
Fig. 7. Standard Source Modeled at Two Altitudes.
Fig. 8. UH₃ Source Modeled at Two Altitudes.
Fig. 9. Ideal Source Modeled at Two Altitudes.
Fig. 10. $\rho_1 \rho_{21} \rho_{31}$ vs altitude, in equivalent units.
Among the possible criteria for selecting a particular value for \( x \) is that of minimizing voltage noise sensitivity (maximizing voltage response).

Taking the simplest case, a fixed path ideal system, we find the point at which

\[
0 = \frac{\partial}{\partial x} \left( \frac{\partial \rho}{\partial \alpha} \right)
\]

\[
= \frac{\partial}{\partial x} \left[ \frac{\partial \left\{ \ln(V_o/V) - \rho_2 K_2 x \right\}}{\partial \alpha} \right]
\]

\[
= \frac{\partial}{\partial x} \left[ \frac{\exp \left( K_1 \rho_1 x + K_2 \rho_2 x \right)}{K_1 V_o x} \right]
\]

\[
= \frac{\exp \left( K_1 \rho_1 x + K_2 \rho_2 x \right)}{K_1 V_o x} \left[ K_1 \rho_1 + K_2 \rho_2 - \frac{1}{x} \right]
\]

which gives

\[
x_o = \frac{1}{K_1 \rho_1 + K_2 \rho_2} \tag{2}
\]

An approximation for the non-ideal case is:

\[
x_o = \frac{-f'(V'' \rho)}{\rho_1} \tag{3}
\]

For the variable-path case, not only do we want maximum voltage sensitivity at each path length, but we also want to define \( V_o \) as well as possible. The first requirement alone would lead us to \( x_1 = x_2 = x_o \).
and the second would lead us to $x_1 = x_0$, $x_2 = 0$. We need an optimal compromise.

Taking the ideal simplification again, and letting $a = K_1 \rho_1 + K_2 \rho_2$, we have

$$v_1 = v_0 e^{-ax_1}$$

$$v_2 = v_0 e^{-ax_2}$$

and

$$v_1 = \frac{v_1}{v_2} = e^{-a(x_2 - x_1)}$$ \hspace{1cm} (4)

We hedge here by separating what should be a simultaneous solution into two parts. First, solving for the optimum separation gives us (without putting down the mathematical steps):

$$x_2 - x_1 = \frac{2}{a}$$ \hspace{1cm} (5)

Secondly, solving for optimum average gives:

$$\frac{x_1 + x_2}{2} = \frac{1}{a} \left[ 1 + \frac{e^{0.5} - e^{-0.5}}{2(e^{0.5} + e^{-0.5})} \right]$$ \hspace{1cm} (6)

$$= \frac{1.23}{a}$$ \hspace{1cm} (7)

The optimum values derived from (5) and (7) are:

$$x_1 = \frac{0.73}{K_1 \rho_1 + K_2 \rho_2}$$

$$x_2 = \frac{1.73}{K_1 \rho_1 + K_2 \rho_2}$$ \hspace{1cm} (8)
In practice, there are other considerations, such as:

(a) path length error sensitivity
(b) dynamic range of data logging equipment compared to range of humidity to be measured
(c) constriction of air flow at small \( x \)
(d) reduction of signal at large \( x \)

For a non-ideal system having nonlinear behavior, the above analysis fails. Optimum values for the more general case have not been worked out.
APPENDIX 2 - HUMIDITY CONVERSIONS

CONVERSIONS TO ABSOLUTE HUMIDITY

To convert to \( \rho \) (in g/m\(^3\)) from:

**mixing ratio** \( r \):

\[
\rho = r \cdot d \quad \text{(} r \text{ in g/g)}
\]
\[
= r \cdot d \times 10^{-3} \quad \text{(} r \text{ in g/kg)}
\]
\[
= r \cdot d \times 10^{-6} \quad \text{(} r \text{ in ppm)}
\]

\( d \) = density of dry air in g/m\(^3\).

**precipitable cm:**

\[
\rho = \text{mm/km} = \mu/m
\]
\[
= 10 \times \text{cm/km} = 10^6 \times \text{cm/m}
\]
\[
= 10^6 \times \text{m/m}
\]

**vapor pressure** \( e \):

\[
\rho = \frac{217 e}{T + 273.16} \quad \text{e in mb, } T \text{ in } ^\circ C
\]

**dew point** \( T_d \):

\[
\rho = \frac{1322.3}{T + 273} \exp \left[ \frac{22.51 T_d}{273 + T_d} \right] \quad \text{over ice}
\]
\[
= \frac{1322.3}{T + 273} \exp \left[ \frac{17.27 T_d}{237.3 + T_d} \right] \quad \text{over water}
\]

\( T, T_d \) in \( ^\circ C \).
CONVERSION FROM ABSOLUTE HUMIDITY

To convert from $\rho$ (in g/m$^3$) to:

**mixing ratio** $r$:

\[ r = \frac{\rho}{d} \quad \text{g/g} \]
\[ r = \rho \times 10^3/d \quad \text{g/kg} \]
\[ r = \rho \times 10^6/d \quad \text{ppm} \]

$d = $ dry air density in g/m$^3$

**precipitable cm**:

\[ \text{cm/km} = 0.1\rho \]
\[ \text{mm/km} = \mu/m = \rho \]
\[ \text{cm/m} = 10^{-6}\rho \]

**vapor pressure** $e$:

\[ e = \frac{\rho(T + 273.16)}{216.5} \text{mb} \]

$T$ in °C

**dew point** $T_d$:

\[ T_d = \frac{273 \ln[(T + 273)\rho/1322.3]}{22.51 - \ln[(T + 273)\rho/1322.3]} \quad \text{over ice} \]
\[ = \frac{237.3 \ln[(T + 273)\rho/1322.3]}{17.27 - \ln[(T + 273)\rho/1322.3]} \quad \text{over water} \]

$T, T_d$ in °C.
APPENDIX 2
Graph la
Absolute Humidity vs.
Dew Point (Over Ice)
APPENDIX 2
Graph 1b
Absolute Humidity
vs.
Dew Point
(Over Water)
Appendix 2

\[ \frac{\partial \rho_1}{\partial T_d} \] for Conditions a - d *

<table>
<thead>
<tr>
<th>( \rho_1 )</th>
<th>( \frac{\partial \rho_1}{\partial T_d} ) g/m³ per °K</th>
<th>%/°C</th>
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<td>6.2</td>
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*As described on page 9.