Description of NCAR Community Climate Model (CCM1)

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1. INTRODUCTION

This report presents the details of the governing equations, physical parameterizations, and numerical algorithms defining the version of the NCAR Community Climate Model designated CCM1. The presentation is designed to follow the flow of the model code, but it does not provide details of the code itself. Such details are given in separate reports described later in this section. It is NCAR's intention that this model provide NCAR and the university research community with a general circulation model with strong emphasis on documentation and reliability. The CCM1 version has evolved over the last few years from the earlier CCM0.

a. Brief History

The first version of the NCAR Community Climate Model (designated CCM0A) was based on the Australian spectral model developed by W. Bourke, B. McAvaney, K. Puri, and R. Thurling (Bourke et al., 1977; McAvaney et al., 1978) and was described in Washington (1982). The model was adapted to the NCAR computers by K. Puri (Australian Numerical Meteorological Research Centre—ANMRC) during an extended visit and subsequently modified by E. Pitcher (University of Miami) and R. Malone (Los Alamos/Department of Energy) to adopt more efficient Fourier transform routines and the improved radiation/cloudiness parameterizations of Ramanathan and Dickinson. The radiation and cloud models were subsequently revised to their CCM0A versions by Ramanathan. Results of January and July simulations produced by CCM0A are presented by Pitcher et al. (1983). The response of the model to refinements in the radiative processes is described by Ramanathan et al. (1983).

An important broadening of the concept of the NCAR community model occurred in late 1981 with NCAR's decision to utilize the same basic code for global forecast studies (both medium- and long-range) and for climate simulation. Economy and increased efficiency could then be achieved by documenting and maintaining only one set of codes. Changes from one application to the other could be relatively straightforward in a model with modular design. The use of one basic
model for both forecasting and climate studies has potential scientific value since a major part of long-range (one- to two-week) forecast errors is due to the drift toward a model climate which differs from that of the atmosphere. Thus, improvements in the climate aspects of the model should lead to improvements in forecasts. Similarly, many physical parameterizations are deterministic rather than statistical in the sense that they are based on the details of the current model state rather than on some past statistical properties. Thus, some of their properties can be studied, improved, and verified by examining them in a forecast mode.

Because of the extension of the role of the CCM to include forecast studies as well as climate studies, and because of the expected widespread use for both purposes by university as well as NCAR scientists, a versatile, modular, and well-documented code is essential. The version designated CCM0B was developed to meet these requirements. This code originated with an adiabatic, inviscid version of the spectral model developed at the European Centre for Medium Range Weather Forecasts (ECMWF) by A. P. M. Baede, M. Jarraud, and U. Cubasch (Baede et al., 1979). Physical parameterizations and numerical approximations matching those of CCM0A were added to this model. The physical parameterizations included the radiation and cloud routines developed at NCAR (Ramanathan et al., 1983) and convective adjustment, stable condensation, vertical diffusion, surface fluxes, and surface-energy–balance prescription developed at the Geophysical Fluid Dynamics Laboratory (GFDL) (Smagorinsky, 1963; Manabe et al., 1965; Smagorinsky et al., 1965; Holloway and Manabe, 1971). The vertical and temporal finite differences matched those of the Australian spectral model (Bourke et al., 1977). This combination was designated the NCAR CCM0B. It was described in a series of technical notes which included a User's Guide (Sato et al., 1983) that provided details of the code logic, flow, and style and illustrated how to modify and run that model, and a note that described each subroutine in the model library (Williamson et al., 1983). That note was designed to be used in conjunction with the code itself. The details of the algorithms were given in Williamson (1983), and circulation statistics from long January and July simulations were presented in Williamson and Williamson 2.
The latter report provided prospective users with a brief, concise summary of the climate produced by the model. In addition, it demonstrated that the climate produced by the CCM0B version matched the original CCM0A version to within the natural variability.

The advantages of the community model concept in which many scientists use the same basic model for a variety of scientific studies were demonstrated in a workshop held at NCAR in July 1985 (Anthes, 1986). Many diverse experiments were seen to relate to each other when performed with the same model, and much constructive dialogue took place between experts in several disciplines. The experiences of many individuals have led to the improvements incorporated in the model to bring it to CCM1.

b. Summary of Differences between CCM1 and CCM0

As mentioned earlier, the CCM1 version described in this report has evolved from the earlier CCM0B. Substantial changes have been incorporated in the radiation scheme, in both the physical parameterizations and the actual structure of the code. The solar albedo parameterization of Briegleb et al. (1986), which includes a solar zenith–angle dependence of albedo on various surface types, has been included in CCM1. The absorption due to solar radiation by H$_2$O and O$_2$ has been improved. The method of Kratz and Cess (1985) has been adopted to account for direct–beam absorption by H$_2$O. For O$_2$ absorption, the method of Kiehl and Yamanouchi (1985) has been used. For longwave radiation, the H$_2$O–absorptivity scheme in CCM0 has been completely replaced with the new nonisothermal emissivity scheme of Ramanathan and Downey (1986). The CO$_2$–absorptance model has also been replaced with the model of Kiehl and Briegleb (1987). Absorption by ozone has been replaced with the model of Ramanathan and Dickinson (1979). The emissivity of stratiform clouds is now determined by their liquid–water content. Finally, a new finite–difference scheme has been implemented in the longwave part of the radiation model. The radiation code itself has been restructured to make it more modular. Thus, many of the physical parameterizations are now self–contained.

Based on the comparative study of Williamson (1987), the vertical finite–
difference approximations have been modified to conserve energy without adversely affecting the model simulations. Frictional heating has been included such that the momentum diffusion produces a corresponding heating term in the thermodynamic equation. These last two improvements result in the energy in the model being conserved to the order of one W m\(^{-2}\) and moisture to one-hundredth W m\(^{-2}\) energy equivalent over 90-day periods.

The horizontal diffusion has been converted to a \(\nabla^4\) form except in the top few levels of the model where the \(\nabla^2\) form is retained. This diffusion in the upper levels is an important component of the climate balance (Boville, 1984). In addition, there is a partial correction toward evaluating the \(\nabla^4\) form on pressure surfaces rather than sigma surfaces. This correction is done with local values in grid-point space rather than with global averages in spectral space. It tends to decrease spurious diffusion associated with steep mountains.

A second moisture-adjustment procedure has been included in CCM1 which ensures that moisture is nonnegative. The additional adjustment provides for a global horizontal borrowing (Royer, 1986) in a conserving manner.

The vertical diffusion has been converted to a nonlinear form evaluated in grid-point space from the earlier linear form evaluated in spectral space. The eddy-mixing coefficient now depends on local shear and stability. The diffusion is applied through the atmosphere rather than only below 500 mb as done in CCM0B. A dry convective adjustment is not applied in the troposphere. As a column becomes unstable, the vertical diffusion coefficient becomes large enough that the diffusion stabilizes the column. The vertical diffusion transfers momentum as well as heat and moisture vertically. Formally, the dry convective adjustment has also been modified to completely mix moisture whenever dry adjustment takes place. This term is of little consequence, however, as the adjustment is now applied only in the stratosphere where the moisture amounts are very small and adjustments are likely to occur only for numerical reasons, if at all.

Condensation occurs at 100 percent relative humidity rather than at the 80 percent of CCM0B. Evaporation is decreased over deserts and grasslands from the
CCM0B rates. The drag coefficient has been made a function of stability following Deardorff (1972). The equation of state has been converted from that for a dry atmosphere to that for a moist atmosphere so that virtual temperature is used where appropriate and the variation with moisture of the specific heat at constant pressure is accounted for.

CCM1 can be used in a perpetual January or July simulation mode or in a seasonal mode in which the specified surface conditions vary with time. In addition, an optional interactive surface hydrology (Budyko, 1956) is included that follows the formulation presented by Manabe (1969).

The code itself has also undergone substantial revision. As before, a Users' Guide (Bath et al., 1987a) is available that provides details of the code logic, flow, and style and explains how to modify and run CCM1. A technical note (Bath et al., 1987b) is also available that describes each subroutine and common block in the model library; it is designed to be used in conjunction with the code itself. These two reports and the present report, by themselves, are designed to document CCM1. No reference to the earlier reports documenting CCM0B should be required. A document that includes all information on the radiation and cloud routines is also available (Kiehl et al., 1987).
2. CONTINUOUS GOVERNING EQUATIONS

The continuous equations are similar to those used by Bourke (1974) and Hoskins and Simmons (1975) and adopt the σ-vertical coordinate proposed by Phillips (1957).

a. Momentum Equations

The zonal and meridional components of the momentum equations may be written in σ coordinates as

\[
\frac{\partial u}{\partial t} = \eta v - \frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} (\Phi + E) - \frac{RT_v}{a \cos \phi} \frac{\partial}{\partial \lambda} \ln p_s - \sigma \frac{\partial u}{\partial \sigma} + F_{uv} + F_{uH}, \tag{2.a.1}
\]

\[
\frac{\partial v}{\partial t} = -\eta u - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\Phi + E) - \frac{RT_v}{a} \frac{\partial}{\partial \phi} \ln p_s - \sigma \frac{\partial v}{\partial \sigma} + F_{ov} + F_{vH}, \tag{2.a.2}
\]

where the absolute vorticity \( \eta \), relative vorticity \( \zeta \), divergence \( \delta \), and kinetic energy \( E \) are given by

\[
\eta = \zeta + f, \tag{2.a.3}
\]

\[
\zeta = \frac{1}{a \cos \phi} \left[ \frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi} (u \cos \phi) \right], \tag{2.a.4}
\]

\[
\delta = \frac{1}{a \cos \phi} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos \phi) \right], \tag{2.a.5}
\]

\[
E = \frac{1}{2} (u^2 + v^2). \tag{2.a.6}
\]

Here \( f \) is the Coriolis parameter \( (2\Omega \sin \phi) \), \( t \) is time, \( \phi \) latitude, \( \lambda \) longitude, \( \sigma \) vertical coordinate \( (p/p_s) \), \( p_s \) surface pressure, \( \dot{\sigma} \) vertical velocity in \( \sigma \) coordinates, \( u \) zonal wind component, \( v \) meridional wind component, \( \Phi \) geopotential, \( a \) mean radius of the earth, \( R \) gas constant for dry air, and \( T_v \) virtual temperature. The virtual temperature is given by

\[
T_v = \left[ 1 + \left( \frac{R_v}{R} - 1 \right) q \right] T, \tag{2.a.7}
\]
where $T$ is temperature, $q$ specific humidity, and $R_v$ gas constant for water vapor. The vertical friction terms $F_{uV}, F_{oV}$, which include surface fluxes and vertical diffusion, and the horizontal diffusion terms $F_{uH}, F_{oH}$ will be described shortly.

b. Vorticity and Divergence Equations

The momentum equations of the form (2.a.1) and (2.a.2) are not directly used by the model but rather, following Bourke (1974), their vorticity and divergence counterparts are obtained from (2.a.1) and (2.a.2) with the relations (2.a.3), (2.a.4), and (2.a.5). In addition, the variable

$$\mu = \sin \phi$$  \hspace{1cm} (2.b.1)

is used for the meridional independent variable rather than latitude $\phi$, and

$$U = u \cos \phi$$  \hspace{1cm} (2.b.2)

$$V = v \cos \phi$$  \hspace{1cm} (2.b.3)

are used when velocity components are needed. These forms are more convenient for the spectral representation. The equations for absolute vorticity and divergence can be written

$$\frac{\partial \eta}{\partial t} = \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (N_v + \cos \phi F_{uV}) - \frac{1}{a} \frac{\partial}{\partial \mu} (N_u + \cos \phi F_{uV}) + F_{\eta H}, \hspace{1cm} (2.b.4)$$

$$\frac{\partial \delta}{\partial t} = \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (N_u + \cos \phi F_{oV}) + \frac{1}{a} \frac{\partial}{\partial \mu} (N_v + \cos \phi F_{oV}) + F_{\delta H}$$

$$- \nabla^2 (E + \Phi + RT_0 \ell n p_s). \hspace{1cm} (2.b.5)$$

The spherical horizontal Laplacian operator is denoted $\nabla^2$.

$$\nabla^2 = \frac{1}{a^2(1 - \mu^2)} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial}{\partial \mu}\right]. \hspace{1cm} (2.b.6)$$

The virtual temperature has been divided into two parts, one of which $T_o$ is a function of $\sigma$ only, in order to facilitate the incorporation of the semi-implicit time-integration scheme,

$$T_v'(\lambda, \mu, \sigma, t) = T_v(\lambda, \mu, \sigma, t) - T_o(\sigma). \hspace{1cm} (2.b.7)$$
The mean temperature $T_o$ for the linearization associated with the semi-implicit scheme is specified a priori, usually to be $300^\circ$K (Simmons et al., 1978). The nonlinear dynamical terms are

$$N_u = \eta V - RT_v' \frac{1}{a} \frac{\partial}{\partial \lambda} \ln p_\sigma - \sigma \frac{\partial U}{\partial \sigma}, \quad (2.8)$$

$$N_v = -\eta U - RT_v' \frac{(1-\mu^2)}{a} \frac{\partial}{\partial \mu} \ln p_\sigma - \sigma \frac{\partial V}{\partial \sigma}. \quad (2.9)$$

The horizontal diffusion terms $F_{\eta_H}$ and $F_{\delta_H}$ are formulated directly using $\eta$ and $\delta$ rather than $u$ and $v$. They are converted to the equivalent $F_{uH}$ and $F_{vH}$ forms for use in the frictional heating term in the thermodynamic equation.

c. Thermodynamic and Mixing-Ratio Equations

The thermodynamic equation is for the perturbation temperature $T'$ calculated about the same mean $T_o$ as used earlier for the virtual temperature.

$$\frac{\partial T'}{\partial t} = -\frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \lambda} (UT') - \frac{1}{a} \frac{\partial}{\partial \mu} (VT') + T' \delta - \sigma \frac{\partial T}{\partial \sigma} + \frac{RT_v \omega}{C_p^*}$$

$$+ Q_S + Q_{tw} + F_{TV} + F_{TH} - \frac{1}{C_p^*} [u(F_{uV} + F_{uH}) + v(F_{vV} + F_{vH})], \quad (2.1)$$

$$T' (\lambda, \mu, \sigma, t) = T(\lambda, \mu, \sigma, t) - T_o(\sigma), \quad (2.2)$$

$$C_p^* = \left[ 1 + \left( \frac{C_{p_s}}{C_p} - 1 \right) \right] C_p, \quad (2.3)$$

where $C_p$ is the specific heat of dry air at constant pressure and $C_{p_s}$ is the specific heat of water vapor at constant pressure. The vertical diffusion $F_{TV}$, which includes the sensible heat flux from the surface and the horizontal diffusion term $F_{TH}$, will be defined shortly. The pressure vertical velocity $\omega$ and the sigma coordinate vertical velocity $\sigma = \frac{d\sigma}{dt}$ are given in the next section [(2.d.7) and (2.d.5)]. The solar atmospheric heating rate $Q_S$ is given by (2.g.34) to (2.g.36) and the longwave atmospheric heating rate $Q_{tw}$ by (2.g.60).
The moisture forecast equation for the specific humidity $q$ is

$$\frac{\partial q}{\partial t} = - \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda}(Uq) - \frac{1}{a} \frac{\partial}{\partial \mu}(Vq) + q\delta - \delta \frac{\partial q}{\partial \sigma} + S + F_{qv} + F_{qH}. \quad (2.c.4)$$

We shall discuss later the mathematical expressions for the source/sink term $S$ for water vapor and the horizontal and vertical water--vapor diffusion terms $F_{qH}, F_{qV}$.

d. Vertical Velocities and Surface Pressure

The continuity equation in the $\sigma$--system is

$$\frac{\partial \ln p_s}{\partial t} = -\mathbf{V} \cdot \nabla \ln p_s - \delta - \frac{\partial \sigma}{\partial \sigma}, \quad (2.d.1)$$

where $\mathbf{V}$ is the horizontal velocity vector with components $(u, v)$ and

$$\mathbf{V} \cdot \nabla \ln p_s = \frac{U}{a(1 - \mu^2)} \frac{\partial \ln p_s}{\partial \lambda} + \frac{V}{a} \frac{\partial \ln p_s}{\partial \mu}. \quad (2.d.2)$$

The continuity equation (2.d.1) is not used directly but, when integrated in the vertical, gives equations for the surface--pressure tendency and for $\dot{\sigma}$. Integrating (2.d.1) from $\sigma = 0$ to $\sigma = 1$, with the boundary conditions

$$\dot{\sigma} = 0 \quad \text{at} \quad \sigma = 0 \quad \text{and} \quad 1, \quad (2.d.3)$$

gives the prognostic equation for surface pressure,

$$\frac{\partial \ln p_s}{\partial t} = - \int_0^1 (\delta + \mathbf{V} \cdot \nabla \ln p_s) d\sigma. \quad (2.d.4)$$

The diagnostic equation for the sigma vertical velocity $\dot{\sigma}$ is derived by integrating the continuity equation (2.d.1) vertically from the top of the atmosphere ($\sigma = 0$) to $\sigma$ and substituting the surface--pressure--tendency equation (2.d.4),

$$\dot{\sigma} = \sigma \int_0^1 (\delta + \mathbf{V} \cdot \nabla \ln p_s) d\sigma - \int_0^\sigma (\delta + \mathbf{V} \cdot \nabla \ln p_s) d\sigma. \quad (2.d.5)$$

The pressure vertical velocity is obtained from (2.d.5) and (2.d.4) using

$$\frac{\omega}{\rho} = \frac{\dot{\sigma}}{\sigma} + \frac{d \ln p_s}{dt}, \quad (2.d.6)$$
The hydrostatic equation is
\[ \frac{\partial \Phi}{\partial \ln \sigma} = -RT_v, \] (2.e.1)
or in integral form,
\[ \Phi = \Phi_s - \int_{\sigma=1}^{\sigma} RT_v d\ln \sigma, \] (2.e.2)
where \( \Phi_s \) is the geopotential at the earth's surface.

The equation of state is that for a moist atmosphere,
\[ p = \rho RT_v, \] (2.e.3)
where \( \rho \) is the density and \( T_v \) was given earlier (2.a.7).

f. Vertical Diffusion

The form of the vertical diffusion terms of momentum, sensible heat, and moisture follows those described by Smagorinsky et al. (1965) and Manabe et al. (1965):

\[ F_{uv} = -\frac{1}{\rho} \frac{\partial \tau_\lambda}{\partial z} = \frac{g}{p_s} \frac{\partial \tau_\lambda}{\partial \sigma}, \] (2.f.1)

\[ F_{uv} = -\frac{1}{\rho} \frac{\partial \tau_\mu}{\partial z} = \frac{g}{p_s} \frac{\partial \tau_\mu}{\partial \sigma}, \] (2.f.2)

\[ F_{Tv} = -\frac{\sigma^\kappa}{C_p^* \rho} \frac{\partial H}{\partial z} = \frac{\sigma^\kappa}{C_p^* p_s} \frac{g}{\partial \sigma}, \] (2.f.3)

\[ F_{qv} = -\frac{1}{\rho} \frac{\partial R}{\partial z} = \frac{g}{p_s} \frac{\partial R}{\partial \sigma}, \] (2.f.4)

where \( C_p^* \) is given by (2.c.3). The sigma factor \( \sigma^\kappa = (p/p_s)^\kappa \) is included in (2.f.3) to account approximately for formulating the vertical diffusion of heat in terms of
potential temperature rather than temperature. The potential temperature is given by

\[ \theta = T \left( \frac{p_*}{p} \right)^\kappa, \]  

(2.f.5)

where \( p_* \) is a reference pressure. Formally in (2.f.3), \( p_* \) is taken to be \( p_* \), but its actual value is irrelevant since it cancels out of the vertical diffusion problem, as seen in section 3 where the discrete equations are given. Above the surface layer, the upward fluxes of momentum, sensible heat, and moisture due to turbulent motions are given by

\[ r_\lambda = -\rho K \frac{\partial u}{\partial z} = \frac{g p^2}{p_*} K \frac{\partial u}{\partial \sigma}, \]  

(2.f.6)

\[ r_\mu = -\rho K \frac{\partial v}{\partial z} = \frac{g p^2}{p_*} K \frac{\partial v}{\partial \sigma}, \]  

(2.f.7)

\[ H = -C_p^* \rho K \frac{\partial \theta}{\partial z} = C_p^* \frac{g p^2}{p_*} K \frac{\partial}{\partial \sigma} (T/\sigma^\kappa), \]  

(2.f.8)

\[ R = -\rho K \frac{\partial q}{\partial z} = \frac{g p^2}{p_*} \frac{\partial q}{\partial \sigma}, \]  

(2.f.9)

where \( K \) depends on the local stability as follows. The Richardson number \( R_I \) is given by

\[ R_I = \frac{g}{\theta} \frac{\partial \theta}{\partial z} / \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right] \]

\[ = -R^* \sigma^\kappa \frac{\partial}{\partial \sigma} \left( T/\sigma^\kappa \right) / \left[ \left( \frac{\partial u}{\partial \sigma} \right)^2 + \left( \frac{\partial v}{\partial \sigma} \right)^2 \right], \]  

(2.f.10)

and the modified gas constant \( R^* \) is

\[ R^* = R \left[ 1.0 + \left( \frac{R_v}{R} - 1 \right) q \right], \]  

(2.f.11)

\( R \) being the gas constant for dry air and \( R_v \) that for water vapor. The coefficient \( K \) for the stable case is

\[ K = K_n (1.0 - 5R_I), \quad R_I > 0, \]  

(2.f.12)
and for the unstable case

\[ K = K_n \sqrt{1.0 - 18.0R_I}, \quad R_I < 0, \]  

with a minimum value limited by

\[ K \geq 0.1. \]  

The neutral \( K_n \) is given by

\[ K_n = \ell^2 \sqrt{\frac{\partial^2 u^2}{\partial z^2} + \frac{\partial v^2}{\partial z^2}} = \ell^2 \frac{g \sigma}{RT_v} \sqrt{\frac{\partial u^2}{\partial \sigma^2} + \frac{\partial v^2}{\partial \sigma^2}}, \]  

with

\[ \ell = 30.0 \text{ m.} \]  

In the preceding, the derivatives with respect to \( z \) are converted to ones with respect to \( \sigma \) with the relation

\[ \frac{1}{\rho} \frac{\partial}{\partial z} = -\frac{g}{p_s} \frac{\partial}{\partial \sigma}. \]  

At the top of the model, the fluxes are assumed to be zero.

\[ \tau_\lambda = \tau_\mu = H = R = 0. \]  

At the earth’s surface, the fluxes are given in terms of the bulk aerodynamic parameterization, following Deardorff (1972). The formulation assumes knowledge of the height \( h \) of the planetary boundary layer, but this height can be approximately specified in a model without a diurnal cycle. Hence,

\[ \tau_\lambda = -\rho h C_D |V| u_h, \]  

\[ \tau_\mu = -\rho h C_D |V| v_h, \]  

\[ H = C_p^* \rho h C_H |V| (T_s - T_h/\sigma_h^{\xi}), \]  

\[ R = D_W \rho h C_H |V| (g_s(T_s) - q_h), \]
where the subscript $h$ denotes the first atmospheric grid level above the earth's surface, $T_s$ is the temperature at the earth's surface, $q_s(T_s)$ is the saturation specific humidity at temperature $T_s$, and $C_p$ is from (2.c.3). The $\sigma_h^\kappa$ factor in (2.f.21) arises from using potential temperature rather than temperature for the vertical diffusion.

The velocity magnitude $|V|$ is given by

$$|V|^2 = |V_h|^2 + |V_c|^2,$$  \hspace{1cm} (2.f.23)

with a minimum value

$$|V| > 1.0 \text{ m s}^{-1}.$$  \hspace{1cm} (2.f.24)

$$|V_c| = \begin{cases} 2(T_s - T_h/\sigma_h^\kappa)^{1/2} & T_s > T_h/\sigma_h^\kappa, \\ 0 & T_s \leq T_h/\sigma_h^\kappa, \end{cases}$$  \hspace{1cm} (2.f.25)

and

$$|V_h|^2 = u_h^2 + v_h^2.$$  \hspace{1cm} (2.f.26)

The drag coefficients depend on the bulk Richardson number,

$$R_{IB} = \frac{gh}{T_h/\sigma_h^\kappa} \frac{(T_h/\sigma_h^\kappa - T_s)}{|V_h|^2},$$  \hspace{1cm} (2.f.27)

where $h$ is the height of the planetary boundary layer, assumed given by

$$h = \frac{RT_{vh} \Delta \sigma_h}{g \sigma_h},$$  \hspace{1cm} (2.f.28)

with a minimum value,

$$h \geq 500.0 \text{ m}.$$  \hspace{1cm} (2.f.29)

The Richardson number itself also has an upper bound,

$$R_{IB} \leq .5R_{IC},$$  \hspace{1cm} (2.f.30)

with the critical Richardson number,

$$R_{IC} = 3.05.$$  \hspace{1cm} (2.f.31)
$\Delta \sigma_h$ is the thickness of the lowest model layer. For the standard 12-layer model, the minimum value of 500.0 m applies.

The neutral drag coefficients are given by

$$C_{UN} = \left[ k^{-1} \ln \left( \frac{0.025h}{z_o} \right) + 8.4 \right]^{-1},$$  \hspace{1cm} (2.f.32)

$$C_{\theta n} = \left[ k^{-1} \ln \left( \frac{0.025h}{z_o} \right) + 7.3 \right]^{-1},$$  \hspace{1cm} (2.f.33)

where the roughness length,

$$z_o = \begin{cases} 
0.25 \text{ m} & \text{over land, sea ice, and snow cover,} \\
0.001 \text{ m} & \text{over water,} 
\end{cases}$$  \hspace{1cm} (2.f.34)

and

$$k = 0.4.$$  \hspace{1cm} (2.f.35)

For the stable case ($R_{IB} \geq 0$),

$$C_U = C_{UN} \left( 1 - \frac{R_{IB}}{R_{IC}} \right),$$  \hspace{1cm} (2.f.36)

$$C_{\theta} = C_{\theta n} \left( 1 - \frac{R_{IB}}{R_{IC}} \right),$$  \hspace{1cm} (2.f.37)

while for the unstable ($R_{IB} < 0$),

$$C_U = \left[ C_{UN}^{-1} - 25.0 \exp(0.26\zeta - 0.03\zeta^2) \right]^{-1},$$  \hspace{1cm} (2.f.38)

$$C_{\theta} = \left[ C_{\theta n}^{-1} + C_U^{-1} - C_{UN}^{-1} \right]^{-1},$$  \hspace{1cm} (2.f.39)

$$\zeta = \log_{10}(-R_{IB}) - 3.5,$$  \hspace{1cm} (2.f.40)

and finally,

$$C_D = C_U^2,$$  \hspace{1cm} (2.f.41)

$$C_H = C_U C_{\theta}.$$  \hspace{1cm} (2.f.42)
The parameter $D_w$ in (2.f.22) is a wetness factor that sets the evaporation from the surface to be a specified fraction of the evaporation from a saturated surface. This parameter is set to be 1.0 over snow, sea ice, and ocean, 0.1 over grassland and scrub, 0.01 over desert, and 0.25 over all other land types.

**g. Physical Parameterization of Radiation**

**Solar Radiation.** The radiative flux in the solar spectral region is divided into two regions. Radiation between 0–0.9 μm is denoted as ultraviolet and visible (UVV), while radiation between 0.9–4.0 μm is denoted as near-infrared (NIR). The net downward flux for these two intervals is $F_{UVV}^S$ and $F_{NIR}^S$, respectively. Each of these fluxes is, in turn, separated into three components—clear sky, nonoverlapped clouds, and overlapped clouds. These are denoted $F_{UVVO}^S$, $F_{UVV1}^S$, and $F_{UVVM}^S$ for the ultraviolet and visible wavelength region and $F_{NIR0}^S$, $F_{NIR1}^S$, and $F_{NIRM}^S$ for the near-infrared region.

**Solar Insolation.** The solar insolation at the top of the model atmosphere is determined by

$$S_I = S_o \cos \zeta f_d \epsilon,$$

where $S_o$ is the solar constant, $\zeta$ is the solar zenith angle, $f_d$ is the fractional amount of daylight, and $\epsilon$ is the eccentricity factor which is dependent on the time of year. $f_d$ and $\zeta$ depend on latitude. These quantities are calculated from information on the calendar day. The value of $S_o$ is 1370 W m$^{-2}$. No diurnal cycle is included in the standard version of the model.

**Cloud Cover.** Fractional cloud cover is calculated assuming random overlap of cloud layers. The clear-sky fraction for the total atmospheric column is given by

$$f_{\text{clear}} = \prod_{i=1}^{N} (1 - A_i),$$

where $A_i$ is the fractional cloud cover in layer $i$ and $N$ is the total number of atmospheric layers. The fractional cloud cover is calculated based on the convective parameterization and is described in sections 2l and 4s.

The level structure employed for radiation calculations is shown in Figure 1. Note that the level indices for the radiation calculations are the inverse to those
Figure 1. Level structure for radiation calculations.
used by the rest of CCM1 (see Figure 2). Fluxes are calculated on half-sigma levels, while heating rates are calculated on full-sigma levels. Clouds are formed between half-sigma levels, and thus centered about full-sigma levels.

The two other cloudy-sky conditions that are needed for the solar calculations are the nonoverlapped and overlapped cases. The nonoverlapped (single layered) case is defined as that fraction of the atmosphere having a cloud in a given layer $k$, but clear sky above and below the cloud; the fraction of sky $f_{1L}(k)$ with this condition is

$$f_{1L}(k) = A_k \prod_{i \neq k}^N (1 - A_i).$$  \hspace{1cm} (2.g.3)

The overlapped (multiply layered) cloud case is defined as that fraction of the atmosphere having a cloud in layer $k$ overlain by cloud in one or more higher levels and clear sky below the cloud,

$$f_{ML}(k) = A_k \prod_{i=k+1}^N (1 - A_i) \left\{ 1 - \prod_{i=1}^{k-1} (1 - A_i) \right\}$$  

$$= A_k \prod_{i=k+1}^N (1 - A_i) \left\{ 1 - \frac{\prod_{i=1}^N (1 - A_i)}{\prod_{i=k-2}^N (1 - A_i)} \right\}$$ \hspace{1cm} (2.g.4)

$$f_{ML}(k) = A_k \prod_{i=k+1}^N (1 - A_i) \left\{ 1 - \frac{f_{\text{clear}}}{\prod_{i=k-2}^N (1 - A_i)} \right\}.$$

**Absorption Due to Gases.** Fractional absorption in the ultraviolet and visible spectral regions due to ozone defined as $\xi_{O_3}(p)$ is obtained from the expressions of Lacis and Hansen (1974). Absorption in the near-infrared spectral region by water vapor $\xi_{H_2O}(p)$ is calculated from the parameterization of Kratz and Cess (1985) for the direct beam, and from the formulation of Lacis and Hansen (1974) for the reflected beam. The use of these two different formulations is for computational efficiency. Absorption in the near-infrared region due to carbon dioxide $\xi_{CO_2}(p)$ is calculated from the expression of Sasamori et al. (1972). This expression agrees
very well with the newer absorptance formulation of Kiehl et al. (1985) below 10 mb. However, above 10 mb, the Sasamori et al. expression underestimated near-infrared CO₂ heating. Absorption due to oxygen ξₐ₂ is based on the parameterization of Kiehl and Yamanouchi (1985).

**Clear Sky.** The approach of modeling clear-sky flux at the surface follows Lacis and Hansen (1974) and accounts for direct-beam solar radiation reaching the surface and reflection of diffuse radiation between the surface and molecular (Rayleigh) scatter from the atmosphere.

For the ultraviolet and visible spectral region, this is given by

\[
F_{UVV0}^S = S_I f_{clear} \left( 0.647 - ξ_{o_2}(p_s) - \alpha_{R}^{dr} \right) \left[ (1.0 - \alpha_{UVV}^{df}) + \frac{\alpha_{UVV}^{df} \alpha_{R}^{df} (1.0 - \alpha_{UVV}^{df})}{1.0 - \alpha_{UVV}^{df} \alpha_{R}^{df}} \right],
\]

(2.6.5)

where \( S_I \) is the solar insolation, \( f_{clear} \) is the fractional amount of clear sky, \( ξ_{o_2}(p_s) \) is the absorption due to ozone from the top of the atmosphere to the surface \( (p_s) \), and \( \alpha_{R}^{dr} \) is the direct-beam Rayleigh albedo,

\[
\alpha_{R}^{dr} = \frac{0.28}{1.0 + 6.43 \cos \zeta},
\]

(2.6.6)

where \( \zeta \) is the zenith angle. The factor 0.647 accounts for the fractional amount of energy of the solar spectrum that lies between 0 and 0.9 \( \mu \)m. The term \( (1.0 - \alpha_{UVV}^{df}) \) accounts for the amount of direct-beam ultraviolet and visible radiation absorbed at the surface; the second term accounts for the amount of diffuse radiation absorbed at the surface due to multiple reflections between the surface and Rayleigh scatterers in the atmosphere; \( \alpha_{UVV}^{df} \) is the direct-beam surface albedo for the ultraviolet and visible spectral region. Downward Rayleigh scattered radiation is included in the direct beam. This factor depends upon the following surface types: ocean, land, sea ice, and snow. \( \alpha_{UVV}^{df} \) is the diffuse-beam surface albedo, and \( \alpha_{R}^{df} \) is the diffuse-beam Rayleigh albedo,

\[
\alpha_{R}^{df} = 0.0685.
\]

(2.6.7)
The near-infrared clear-sky net surface flux is

\[ F_{NIR0}^S = S_I f_{clear} \left( 0.353 - \xi_{H_2O}(p_s) - \xi_{CO_2}(p_s) - \xi_{O_2}(p_s) \right) (1 - \alpha_{NIR}^{dr}). \quad (2.9.8) \]

The 0.353 factor accounts for the fractional amount of solar radiation between 0.9 and 4 \( \mu \)m. The terms \( \xi_{H_2O} \), \( \xi_{CO_2} \), and \( \xi_{O_2} \) are the absorptivities for \( H_2O \), \( CO_2 \), and \( O_2 \), respectively. The absorption is over the total atmospheric column. (N.B. Although \( O_2 \) absorbs in the visible, the \( \xi_{O_2} \) term is included in the near-infrared region for simplicity, since accounting for multiple scattering with \( O_2 \) absorption would complicate the ultraviolet and visible calculations.) The term \( \alpha_{NIR}^{dr} \) is the near-infrared direct-beam surface albedo.

**Nonoverlapped Fraction of Clouds.** The contribution to the surface flux from the ultraviolet and visible spectral regions where nonoverlapped clouds are present is

\[ F_{UV1}^S = S_I (0.647 - \xi_{O_3}(p_s)) \left( 1.0 - \alpha_{UVV}^{df} \right) \sum_k \frac{(1.0 - \alpha_c^{dr}(k))}{(1.0 - \alpha_c^{df}(k)\alpha_{UVV}^{df})} f_{1L}(k), \]

where the summation is over all layers containing clouds, while \( f_{1L}(k) \) is the fraction in any of these layers that contains nonoverlapped cloud; \( \alpha_c^{dr}(k) \) is the direct-beam cloud albedo,

\[ \alpha_c^{dr}(k) = \frac{\alpha_{cl}(k)}{\alpha_{cl}(k) + \cos \zeta}, \quad (2.9.10) \]

where \( \alpha_{cl}(k) \) is a parameter dependent on cloud level. Presently, \( \alpha_{cl}(k) \) can assume the values 0.6, 0.3, and 0.15 for levels 1–4, levels 5 and 6, and levels 7–13, respectively. The term \( \alpha_c^{df}(k) \) is the diffuse-beam cloud albedo,

\[ \alpha_c^{df}(k) = \frac{\alpha_{cl}(k)}{\alpha_{cl}(k) + 0.5}. \quad (2.9.11) \]

The term in Eq. (2.9.9) after the summation sign accounts for attenuation by the clouds multiple scattering between the surface (\( \alpha_{UVV}^{df} \)) and the cloud layer.

For the near-infrared spectral region, water–vapor absorption by the cloud is accounted for by enhancing the path length; for this region, the near-infrared
surface flux for nonoverlapped clouds is

\[
F_{\text{NIR}}^S = S_I (1.0 - \alpha_{\text{NIR}}^{df}) \sum_k \frac{(1.0 - \alpha^{dr}_c(k))}{1.0 - \alpha^{df}_c(k) \alpha_{\text{NIR}}^{df}} (0.353 - \xi_{\text{CO}_2}(p_s)) \\
- \xi_{\text{O}_2}(p_s) - \xi_{\text{H}_2\text{O}}(p_k, \bar{u})) f_{1L}(k). 
\]

(2.g.12)

\(\alpha_{\text{NIR}}^{df}\) is the diffuse-beam near-infrared surface albedo. The term \(\xi_{\text{H}_2\text{O}}(p_k, \bar{u})\) is the water-vapor absorption for a path from the top of the atmosphere to the cloud top plus an enhanced path through the cloud, plus the water-vapor path from the cloud base to the surface. The path factor through the cloud with top at level \(k\) is given by

\[
\bar{u} = \frac{1}{0.8} (\bar{\tau} + 10.0 \times 1.8) (U_{\text{H}_2\text{O}}(p_{k+1}) - U_{\text{H}_2\text{O}}(p_k)), 
\]

(2.g.13)

where \(U_{\text{H}_2\text{O}}\) is the water-vapor column amount,

\[
U_{\text{H}_2\text{O}} = \int_k^{\infty} \left( \frac{p}{p_o} \right) \rho_{\text{H}_2\text{O}} dz = \int_0^k \mu_{\text{H}_2\text{O}} \frac{dp^2}{2g}, 
\]

(2.g.14)

where \(\mu_{\text{H}_2\text{O}}\) is the specific humidity, and

\[
\bar{\tau} = \frac{1}{\sqrt{\tau^2 + 3.6 \times 10^{-4}}}. 
\]

(2.g.15)

The factor of 10.0 multiplying the diffusivity factor of 1.8 accounts for scattering by liquid droplets. \(\bar{\tau}\) is approximately the reciprocal of the cosine of the zenith angle. (N.B. The 0.8 factor is the cloud relative humidity. In CCM1, the cloud relative humidity is 1.0. This discrepancy is not viewed as important, but should be changed in future versions of the model.)

Overlapped Fraction of Clouds. For overlapped clouds, the surface flux evaluation is complicated by the multiple reflections not only from clouds to surface, but between the various cloud layers. The surface flux for the ultraviolet and visible region in the presence of multiply overlapped clouds is

\[
F_{\text{UVVM}}^S = S_I (0.647 - \xi_{\text{O}_2}(p_s)) (1.0 - \alpha_{\text{UVV}}^{df}) \sum_k \frac{(1.0 - \alpha(k))}{1.0 - \alpha_{\text{UVV}}^{df} \alpha_{\text{clmax}}^{df}(k)} f_{ML}(k), 
\]

(2.g.16)
where the "effective" cloud albedo of the multiply overlapped system $\bar{\alpha}$ is

$$
\bar{\alpha}(k) = \alpha^d(k) + \alpha^d_{clmax}(k)(1.0 - \alpha^d(k))(1.0 - \alpha^d(k)) - \Delta U(p_k)
$$

(2.g.17)

$\alpha^d_{clmax}(k)$ is the maximum cloud albedo of all cloud layers to direct radiation for all multiply overlapped layers, and $\alpha^d_{clmax}(k)$ is the analogous quantity for diffuse radiation. The factor $1.0 - \Delta U(p_k)$ accounts for absorption from cloud top to the surface by ozone, carbon dioxide, oxygen, and an enhanced water–vapor path,

$$
\Delta U(p_k) = \xi_{O_2}(p_s) - \xi_{O_2}(p_k) - \xi_{H_2O}(p_s, \bar{\alpha}) - \xi_{H_2O}(p_k) + \xi_{CO_2}(p_s)
$$

(2.g.18)

where $\bar{\alpha}$ is the water–vapor path from the cloud top to the surface and accounts for the enhanced path through the cloud due to scattering [see Equation (2.9.13)]. (N.B. This path length accounts for absorption due to all gases irrespective of the wavelengths in which they absorb.)

The near-infrared surface–flux contribution for multiply overlapped clouds is

$$
F_{NIRM}^S = S_I \sum_k (0.353 - \xi_{H_2O}(p_k, \bar{\alpha}) - \xi_{CO_2}(p_s) - \xi_{O_2}(p_s))
$$

(2.g.19)

$$
\frac{(1.0 - \bar{\alpha}(k))(1 - \alpha^d_{NIR})}{1.0 - \alpha^d_{clmax}(k)\alpha^d_{NIR}} f_{ML}(k).
$$

Solar Surface Flux. The total ultraviolet and visible surface flux is

$$
F_{UVV}^S = F_{UVV0}^S + F_{UVV1}^S + F_{UVVM}^S,
$$

(2.g.20)

while that for the near–infrared spectral region is

$$
F_{NIR}^S = F_{NIR0}^S + F_{NIR1}^S + F_{NIRM}^S.
$$

(2.g.21)

The total net downward solar flux is

$$
F^S = F_{UVV}^S + F_{NIR}^S.
$$

(2.g.22)
Solar Atmospheric Heating. Solar heating within the atmosphere is separated into three regions—above clouds, within clouds, and below clouds. The “within cloud” case is further divided into the nonoverlapped and multiply overlapped cases.

Heating above cloud tops occurs through three processes—(1) direct-beam absorption of the downward flux, (2) absorption of radiation directly reflected off the cloud tops, and (3) a contribution from solar radiation that is scattered between clouds and the surface and eventually reaches above the clouds. Absorption of the directly reflected beams is accounted for by water vapor only. Absorption of the multiply reflected radiation is accounted for by ozone only.

The flux difference across a layer bounded by the pressure levels $p_k, p_{k+1}$ above clouds is given by

$$
\Delta F_{av}(k) = R_{ref}^{dr} [\xi H_2 O(p_{k+1}) - \xi H_2 O(p_k)] + f_{clear} [F^\uparrow(p_{k+1}) - F^\uparrow(p_k)]
$$

$$+ \left( R_{ref}^{dr} + R_{ref M}^{df} + R_{ref 1}^{df} \right) [\xi O_3(p_{k+1}) - \xi O_3(p_k)],
$$

(2.23)

where $F^\uparrow(p_k)$ is the downward clear-sky flux and $R_{ref}^{dr}$ is the fraction of incident solar radiation that is reflected off nonoverlapping and overlapping clouds,

$$R_{ref}^{dr} = S_L [\alpha_{cl}^{dr}(p_{kc}) f_{1L}(p_{kc}) + \alpha_{clmax}^{dr}(p_{kc}) f_{ML}(p_{kc})].
$$

(2.24)

$R_{ref M}^{df}$ is the fractional amount of radiation multiply reflected by overlapped clouds which escapes above the highest cloud top,

$$R_{ref M}^{df} = S_I f_{ML}(p_{kc}) \frac{(1.0 - \alpha_{clmax}^{df}(p_{kc}))(1.0 - \alpha_{clmax}^{df}(p_{kc})) \alpha_{UVV}^{df}}{1.0 - \alpha_{UVV}^{df} \alpha_{clmax}^{df}(p_{kc})}.
$$

(2.25)

$R_{ref 1}^{df}$ is the fractional amount of radiation multiply reflected by nonoverlapped clouds

$$R_{ref 1}^{df} = S_I f_{1L}(p_{kc}) \frac{(1.0 - \alpha_{cl}^{df}(p_{kc}))(1.0 - \alpha_{c}^{df}(p_{kc})) \alpha_{UVV}^{df}}{1.0 - \alpha_{UVV}^{df} \alpha_{c}^{df}(p_{kc})}.
$$

(2.26)
The flux difference between layers located within nonoverlapped clouds is

\[
\Delta F_{cld}(p_{kc}) = F^S(p_{kc}) \left\{ \xi_{H_2O}(p_{kc}, \overline{u}) - \xi_{H_2O}(p_{kc}) + \xi_{CO_2}(p_{kc-1}) - \xi_{CO_2}(p_{kc}) \\
+ \xi_{O_3}(p_{kc}) - \xi_{O_3}(p_{kc-1}) + \xi_{O_8}(p_{kc-1}) - \xi_{O_8}(p_{kc}) \right\} \\
+ F^S(p_{kc}) \alpha_{NIR}^d \frac{\left( \xi_{H_2O}(\overline{u}) - \xi_{H_2O}(\overline{u}) \right) \left( 1 - \alpha_c^{df}(p_{kc}) \right)}{1 - \alpha_c^{df}(p_{kc}) \alpha_{NIR}^{df}},
\]

where \( \overline{u} \) is the water-vapor path through the cloud with top at \( p_{kc} \),

\[
\overline{u} = (u(p_{kc-1}) - u(p_{kc})) \left\{ \frac{1}{0.8} \left( \zeta + 10.0 \times 1.8 \right) \right\},
\]

where the factor in brackets accounts for enhanced path due to scattering. The term \( \overline{u} \) is the water-vapor path through the cloud and down to the surface and back to the cloud base,

\[
\overline{u} = u + 2.0 \times 1.8 \times (u(p_{s}) - u(p_{kc-1})).
\]

\( u \) is the water-vapor path through the cloud, down to the surface and back to the cloud top at \( p_{kc} \),

\[
u = u + 1.8(u(p_{kc-1}) - u(p_{kc})) \frac{10.0}{0.8},
\]

where once again the last term accounts for beam enhancement due to scattering. \( F^S(p_{kc}) \) is the absorbed solar flux incident at the top of the clouds,

\[
F^S(p_{kc}) = S_l(1.0 - \alpha_c^{df}(p_{kc})) f_{1L}(p_{kc}).
\]

The heating below the clouds is calculated from the total absorption of solar radiation in the atmosphere from the cloud base to the surface. The total columnar absorption is then distributed within each layer below the clouds by weighting by the mass fraction within each layer.

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The absorbed flux for the entire column between the cloud base at \( p_{kc-1} \) and the surface \( p_s \) is given by

\[
\Delta F_{btw}(p_{kc}) = S_f f_{ML}(p_{kc})(1 - \bar{\alpha}(p_{kc})) \left\{ \Delta U(p_{kc}) + \frac{.353 - [\xi_{H_2O}(\bar{v}) + \xi_{CO_2}(p_s) + \xi_{O_2}(p_s)]}{1 - \alpha_{NIR}^{df} \alpha_{clmax}(p_{kc})} \right\},
\]

(2.g.32)

where \( \bar{\alpha}(p_{kc}) \) is defined in Equation (2.g.17), and \( \Delta U(p_{kc}) \) is defined in Equation (2.g.18). This total absorbed flux is then distributed within each layer below the cloud base by mass weighting \( \Delta F_{btw}(p_{kc}) \) as follows:

\[
\Delta F_{btw}(p_{kc}) = \Delta F_{btw}(p_{kc}) \frac{p_{k-1} - p_k}{p_s - p_{kc-1}}.
\]

(2.g.33)

The solar heating is then obtained from

\[
Q_{S}^{abv}(k) = \frac{g}{C_p} \frac{\Delta F_{abv}(k)}{\Delta p_k},
\]

(2.g.34)

above clouds, where \( g \) is the acceleration due to gravity and \( C_p \) is the specific heat of air. Solar heating within a nonoverlapped cloud is

\[
Q_{S}^{cld}(k_c) = \frac{g}{C_p} \frac{\Delta F_{cld}(k_c)}{\Delta p_{k_c}},
\]

(2.g.35)

while solar heating below clouds is given by

\[
Q_{S}^{blw}(k) = \frac{g}{C_p} \frac{\Delta F_{blw}(k)}{\Delta p_k}.
\]

(2.g.36)

**Longwave Radiation.** Fluxes are calculated at each model level in both up and down directions. The approach to solving the transfer equations is to employ absorptivities and emissivities. Thus, the clear–sky fluxes at a half–level \( k \) are

\[
F_{citr}^\perp(p_k) = B(0)\epsilon(0, p_k) + \int_0^{p_k} \alpha(p', p_k) \frac{dB}{dp'}(p') dp',
\]

(2.g.37)
and

\[ F_{cl}^{T}(p_k) = \sigma_B T_S^4 - \int_{p_k}^{p_t} \alpha(p',p_k) \frac{dB}{dp'}(p') dp', \quad (2.9.38) \]

where \( B(p) = \sigma_B T^4(p) \) is just Stefan–Boltzmann’s law, and the absorptivity is defined as

\[ \alpha(p,p') = \frac{1}{\frac{dB}{dT}(p')} \int A_\nu(p',p) \frac{dB_\nu}{dT}(p') d\nu, \quad (2.9.39) \]

and the emissivity is

\[ \epsilon(0,p) = \frac{1}{B(0)} \int A_\nu(0,p) B_\nu(0) d\nu, \quad (2.9.40) \]

where \( A_\nu \) is the absorptivity due to a given gas, \( B_\nu(p') \) is Planck’s function, and \( \nu \) is the wavenumber in \( \text{cm}^{-1} \). An isothermal layer is assumed to exist at the model top \( p = 0 \). For \( \text{CO}_2 \) and \( \text{O}_3 \), the band–absorptance technique is used to evaluate \( \alpha \) and \( \epsilon \). This method uses the fact that gas absorption is limited to a finite spectral width. The Planck functions are evaluated at the center of the bands, and integration over \( \nu \) is carried out for \( A_\nu \). Thus,

\[ \alpha_{\text{CO}_2}(p,p') = \frac{1}{4\sigma T^3(p')} \frac{dB_{\text{CO}_2}}{dT}(p') A_{\text{CO}_2}(p',p), \quad (2.9.41) \]

\( B_{\text{CO}_2} \) is evaluated for \( \nu = 667 \text{ cm}^{-1} \), where \( A_{\text{CO}_2}(p',p) \) is the broad–band absorptance from Kiehl and Briegleb (1987). Similarly,

\[ \epsilon_{\text{CO}_2}(0,p) = \frac{1}{\sigma T^4(0)} B_{\text{CO}_2}(0) A_{\text{CO}_2}(0,p). \quad (2.9.42) \]

For ozone,

\[ \alpha_{\text{O}_3}(p,p') = \frac{1}{4\sigma T^3(p')} \frac{dB_{\text{O}_3}}{dT}(p') A_{\text{O}_3}(p',p), \quad (2.9.43) \]

and

\[ \epsilon_{\text{O}_3}(0,p) = \frac{1}{\sigma T^4(0)} B_{\text{O}_3}(0) A_{\text{O}_3}(0,p), \quad (2.9.44) \]

where \( A_{\text{O}_3} \) is the ozone broad–band absorptance from Ramanathan and Dickinson (1979). Water vapor cannot employ the broad–band absorptance method since \( \text{H}_2\text{O} \) absorption extends throughout the entire longwave region. The method of
Ramanathan and Downey (1986) is used for water-vapor absorptivities and emissivities. The overlap treatment between water vapor and other gases is described in Ramanathan and Downey (1986). Thus, the total absorptivity is given by

$$\alpha(p, p') = \alpha_{CO_2}(p, p') + \alpha_{O_2}(p, p') + \alpha_{H_2O}(p, p'),$$  \hfill (2.g.45)

and the total emissivity is

$$\epsilon(0, p) = \epsilon_{CO_2}(0, p) + \epsilon_{O_2}(0, p) + \epsilon_{H_2O}(0, p).$$  \hfill (2.g.46)

Clear-sky fluxes are thus obtained by integrating Equations (2.g.37) and (2.g.38). Details of the integration of these equations are given in section 4b.

**Surface Fluxes.** The downward longwave clear-sky flux at the surface is, thus,

$$F_{clr}(p_s) = B(0)\epsilon(0, p_s) + \int_0^{p_s} \alpha(p', p_s) \frac{dB}{dp'}(p') dp',$$  \hfill (2.g.47)

while the upward flux at the surface is just

$$F^\uparrow(p_s) = \sigma_B T_S^4.$$  \hfill (2.g.48)

The downward cloudy-sky flux at the surface is

$$F^\downarrow(p_s) = F_{clr}(p_s) f_{clear} + \sigma T^4(p_{clb2}) \epsilon(p_{clb2}) A(p_{clb2})$$

$$+ \sum_{k=3}^{K_{MAX}} \left\{ \sigma T^4(p_{clbk}) + \int_{p_{clbk}}^{p_s} \alpha(p_s, p') \frac{dB(p')}{dp'} dp' \right\} \epsilon(p_{clbk}) f_{cld}(k),$$  \hfill (2.g.49)

where $p_{clbk}$ is the pressure level of the cloud base at $k$ (see Figure 1), $\epsilon(p_{clbk})$ is the cloud emissivity obtained from

$$\epsilon(p_{clbk}) = \begin{cases} \frac{1.0 - \exp(-r_{cl}q_c(k))}{1.0 - \exp(-2.5)} & r_{cl}q_c(k) \leq 2.5, \\ 1.0 & r_{cl}q_c(k) > 2.5, \end{cases}$$  \hfill (2.g.50)

where $r_{cl} = 1000$ and $q_c(k)$ is the cloud liquid-water content per time step in gm cm$^{-2}$ s$^{-1}$. This formulation for $\epsilon$ is from Ramanathan et al. (1983) and is
based on the measurements of Griffith et al. (1980). \( f_{cld}(k) \) is the probability of a cloud existing in layer \( k \) and clear sky below this layer,

\[
f_{cld}(k) = A_k \prod_{i=2}^{k-1} (1 - A_i), \quad (2.g.51)
\]
or

\[
f_{cld}(k) = A_k \frac{\prod_{i=2}^{KMAX} (1 - A_i)}{\prod_{i=k}^{KMAX} (1 - A_i)}. \quad (2.g.52)
\]

The net longwave flux at the surface is

\[
F^N(p_e) = F^\uparrow(p_e) + F^\downarrow(p_e). \quad (2.g.53)
\]

Fluxes within the atmosphere are evaluated at each model half-layer for longwave heating-rate calculations on full \( \sigma \)-levels. The upward flux between the surface and the lowest cloud layer is equal to the clear-sky upward flux,

\[
F_{blu}^\uparrow(p_k) = F_{clr}^\uparrow(p_k). \quad (2.g.54)
\]

Within the layers that contain clouds, the upward flux is

\[
F_{cld}^\uparrow(p_k) = F_{clr}^\uparrow(p_k) f_{clear}(k) + \sum_{\ell=k_{low}}^{k} \left\{ \sigma T^4(p_{c舅舅}) - \int_{p_{c舅舅}}^{p_k} \alpha(p', p_{c舅舅}) \frac{dB}{dp'} (p') dp' \right\} \times \epsilon(p_\ell) f_{cld}(\ell) \quad p_{k_{low}} \leq p_k \leq p_{k+1}, \quad (2.g.55)
\]

where \( f_{cld}(\ell) \) is the probability that a cloud is in layer \( \ell \) and clear sky exists above layer \( \ell \).

\[
f_{cld}(\ell) = A_\ell \prod_{i=\ell+1}^{k} (1 - A_i) = A_\ell \frac{\prod_{i=\ell+1}^{KMAX} (1 - A_i)}{\prod_{i=k+1}^{KMAX} (1 - A_i)}. \quad (2.g.56)
\]

\( k_{low} \) is the lowest level of cloud, \( k_{HI} \) is the highest layer of cloud, and \( p_{c舅舅} \) is the cloud-top level (see Figure 1). The upward flux above the clouds is obtained from
a similar expression. The downward flux above the cloudy region is equal to the clear-sky flux,
\[ F_{abu}^{\downarrow}(p_k) = F_{clr}^{\downarrow}(p_k). \quad (2.g.57) \]

Within the clouds and below the clouds, the downward flux is
\[
F_{cld,blw}^{\downarrow}(p_k) = F_{clr}^{\downarrow}(p_k) f_{cld}(k) + \sum_{t=k}^{k_{HI}} \left\{ \sigma T^4(p_{cld}) + \int_{p_{cld}}^{p_k} \alpha(p', p) \frac{dB}{dp'}(p')dp' \right\} \\
\times \epsilon(p_t) F_{cld}(\ell), \quad p_s \leq p_k \leq p_{HI}, \quad (2.g.58)
\]
where \( p_{cld} \) is the cloud-base level, \( f_{cld}(\ell) \) is the probability of a cloud existing in layer \( \ell \) and clear sky below this level,
\[
f_{cld}(\ell) = A_\ell \prod_{i=k}^{K_{MAX}} (1 - A_i) = A_\ell \frac{\prod_{i=k}^{K_{MAX}} (1 - A_i)}{\prod_{i=\ell}^{K_{MAX}} (1 - A_i)}. \quad (2.g.59)
\]

The longwave atmospheric heating rate is obtained from
\[
Q_{lw}(p_k) = \frac{g \left[ F^\uparrow(p_{k+1}) - F^\downarrow(p_{k+1}) - F^\downarrow(p_k) + F^\uparrow(p_k) \right]}{C_p(p_{k+1} - p_k)}. \quad (2.g.60)
\]

h. Surface-Temperature Calculation

Over open ocean, the surface temperature is specified as climatological values. Over land, snow, and sea ice, the surface temperature is computed from an instantaneous surface-energy balance (Holloway and Manabe, 1971),
\[
\sigma_B T_s^4 - F^S - F^\downarrow(p_s) + H + LR + Q_{ICE} = 0, \quad (2.h.1)
\]
where \( \sigma_B \) is the Stefan-Boltzmann constant, \( F^S \) is the total net downward solar flux from (2.g.22), \( F^\downarrow(p_s) \) downward longwave flux from (2.g.49), \( H \) vertical flux of sensible heat given by (2.f.21), \( R \) evaporation given by (2.f.22), and \( L \) latent heat of condensation of water. The term \( Q_{ice} \) represents the heat conduction into and out of pack ice and is defined as
\[
Q_{ice} = \begin{cases} 
  kI^{-1}(T_s - T_{sw}) & \text{over pack ice,} \\
  0 & \text{over all other surfaces,}
\end{cases} \quad (2.h.2)
\]
where \( k \) is the thermal conductivity of ice taken to be 2.092 W m\(^{-2}\) °K\(^{-1}\), \( I \) is the ice thickness chosen as 2 m, and \( T_{sw} = 271.2^\circ\text{K} \) is the temperature of sea water below the pack ice. The implicit equation for \( T_8 \) (2.h.1) is solved by the Newton–Raphson iterative procedure.

1. Horizontal Diffusion

The horizontal diffusion is a linear \( \nabla^2 \) form on sigma surfaces in the top few levels of the model and linear \( \nabla^4 \) form with a partial correction for temperature and specific humidity to pressure surfaces elsewhere. The \( \nabla^2 \) form has improved the CCM0 model simulation in the upper few levels of the model (Boville, 1984) and, therefore, is retained in CCM1. At these levels, the diffusion is given by

\[
F_{\eta H} = K_2 [\nabla^2 \eta + 2(\eta/a^2)] ,
\]

\[
F_{\delta H} = K_2 [\nabla^2 \delta + 2(\delta/a^2)] ,
\]

\[
F_{TH} = K_2 \nabla^2 T ,
\]

\[
F_{qH} = K_2 \nabla^2 q .
\]

Since these terms are linear, they are easily calculated in spectral space. The undifferentiated correction term is added to the vorticity and divergence diffusion operators to prevent damping of uniform \( (n = 1) \) rotations (Orszag, 1974; Bourke et al., 1977).

The \( \nabla^4 \) form is applied at all other levels and, in particular, throughout the troposphere.

\[
F_{\eta H} = -K_4 [\nabla^4 \eta - (2\eta/a^2)^2] ,
\]

\[
F_{\delta H} = -K_4 [\nabla^4 \delta - (2\delta/a^2)^2] ,
\]

\[
F_{TH} = -K_4 \left[ \nabla^4 T - \sigma \frac{\partial T}{\partial \sigma} \nabla^4 \ell n p_s \right] ,
\]
\[ F_{qH} = -K_4 \left[ \nabla^4 q - \sigma \frac{\partial q}{\partial \sigma} \nabla^4 \ell n p_s \right]. \quad (2.i.8) \]

The second term in \( F_{TH} \) and \( F_{qH} \) consists of the leading term in the transformation of the \( \nabla^4 \) operator on pressure surfaces to that on \( \sigma \) surfaces. It is included to offset partially a spurious diffusion of \( T \) and \( q \) over mountains. As with the \( \nabla^2 \) form, the \( \nabla^4 \) operator can be conveniently calculated in spectral space. The correction term \( \sigma \frac{\partial T}{\partial \sigma} (\nabla^4 \ell n p_s) \) is then completed after transformation of \( T \) or \( q \) and \( \nabla^4 \ell n p_s \) back to grid-point space. As with the \( \nabla^2 \) form, an undifferentiated term is added to the vorticity and divergence diffusion operators to prevent damping of uniform rotations. For the R15–version of the model, the diffusion coefficients \( K_2 \) and \( K_4 \) are taken to be

\[ K_2 = 2.5 \times 10^5 \text{ m}^2 \text{ s}^{-1}, \quad (2.i.9) \]

\[ K_4 = 2.0 \times 10^{16} \text{ m}^4 \text{ s}^{-1}. \quad (2.i.10) \]

**j. Convective Adjustment**

After the new temperatures and mixing ratios are computed from (2.c.1) and (2.c.4), the convective and nonconvective adjustment scheme of Manabe et al. (1965) is applied to create mutually adjusted \( T \) and \( q \) fields. If the predicted atmosphere is not saturated and the lapse rate exceeds the dry adiabatic lapse rate, the temperatures are reset to give a dry adiabatic lapse rate. The moisture field is also assumed to be mixed by this process and is reset to the average value. This dry convective adjustment is only applied in the stratosphere of the model. The stability–dependent vertical diffusion is allowed to do the vertical mixing in the troposphere. Thus, in practice, momentum is mixed as well as heat and moisture. If the atmosphere is supersaturated and the lapse rate exceeds the moist adiabatic lapse rate, the moisture and temperature fields are simultaneously modified so that the atmosphere is just saturated and satisfies the moist adiabatic lapse rate. If the atmosphere is stable but supersaturated, the moisture field is adjusted to be saturated and the temperature field is simultaneously adjusted to reflect the heating
due to the release of latent heat. When the moisture field is changed to eliminate supersaturation, the change is assumed to go into precipitation. The details of these processes are presented later when the numerical algorithms are described.

k. Moisture Source/Sink and Heating Terms

The source/sink term $S$ in the moisture equation (2.c.4) essentially represents the condensation processes discussed in the previous section. The vertical diffusion $F_{qv}$ includes the source from the earth's surface. Similarly, the temperature change due to the convective adjustment represents a heating process implicitly included in $Q$, as does the release of latent heat corresponding to the change in moisture from supersaturation to saturation. $Q$ also includes the heating or cooling due to radiation. In the following, the radiative heating is explicitly included in $Q$ in the tendency equations. The vertical diffusion $F_{TV}$ includes the source from the earth's surface. The other sources mentioned enter after the tendency equation has been solved through the stable and unstable adjustment parameterizations.

l. Cloud Parameterization

The model forms clouds that interact with the radiation parameterization described in section 2g. The algorithm is essentially that used in CCM0 and described in Ramanathan et al. (1983). Convective clouds are formed when one or more layers undergo moist convective adjustment. The model assumes that the clouds in each layer are randomly overlapped with a maximum cloud cover of 30 percent in the convective column and cloud emissivity of 1.

Nonconvective clouds are formed wherever stable condensation occurs. The fractional cloud cover of these nonconvective clouds is assumed to be 95 percent and their emissivity is a function of liquid water content. No clouds of any type are formed in the very thin surface layer of the model nor in the top two layers of the standard 12-layer version.

m. Optional Interactive Surface Hydrology

An interactive surface hydrology prescription, following Manabe (1969) and Budyko (1956), is included in the model as an option. If this option is chosen, the wetness factor $D_W$ for the latent heat flux at the surface over land (2.f.22) is
determined from a calculated soil moisture rather than specified and held fixed as described following (2.f.42). Over ocean and over snow cover, the wetness factor $D_W$ is set to 1.0 as before, i.e., evaporation is assumed to be from a saturated surface.

Over land, the soil moisture is forecast by adding rainfall and snowmelt to the existing soil moisture and subtracting the evaporation and runoff. The precipitation from the convective adjustment is assumed to be rain if either the surface temperature or the temperature in either of the first two atmospheric layers above the surface is greater than or equal to 273.16 K. Otherwise, the precipitation is assumed to be snow.

The water equivalent depth of snow ($S$) is predicted by

$$\frac{dS}{dt} = S_f - R - M_e,$$  \hspace{1cm} (2.m.1)

where $S_f$ is the rate of snowfall, $M_e$ is the rate of snowmelt, and $R$ is the evaporation (sublimation) rate determined from (2.f.22). The rate of snowmelt is calculated from a heat-balance relation at the snow-covered surface and is given by

$$M_e = \begin{cases} 
E_x/L & \text{if } E_x > 0, \\
0 & \text{if } E_x \leq 0,
\end{cases}$$  \hspace{1cm} (2.m.2)

where the excess heat energy available to melt snow $E_x$ represents the energy available after the surface temperature reaches 273.16 K. It is obtained from the surface-energy balance (2.h.1)

$$E_x = - \left[ \sigma_B T_*^4 - F^S - F^L(p_s) + H + LR + Q_{ICE} \right]_{T_*=273.16},$$  \hspace{1cm} (2.m.3)

in which the surface temperature $T_*$ is set to the melting temperature of ice. The liquid water-holding capacity of snow is assumed to be zero, so all the snowmelt $M_e$ goes into the soil moisture. Soil moisture begins to evaporate as described next after all the snow has sublimated or melted.

The rate of change of the soil moisture ($W$) is determined from

$$\frac{\partial W}{\partial t} = P_r + M_e - R,$$  \hspace{1cm} (2.m.4)
where the total rainfall rate is \( P_r \), the snowmelt rate \( M_e \) is from (2.m.2), and \( R \) is the evaporation rate determined from (2.f.22).

When the soil moisture reaches the soil field capacity \( (W_{FC}) \), the maximum amount of water that can be stored in the soil, the excess above the field capacity is assumed to form runoff \( (r_f) \). If \( W > W_{FC} \), the runoff over a given time step is given by \( r_f = W - W_{FC} \) and \( W \) is then reset to \( W_{FC} \). The field capacity is taken to be \( W_{FC} = 0.15 \) m. This runoff is assumed to flow directly into rivers and the sea and does not affect any other points.

The wetness factor \( D_{W} \) in the evaporation (2.f.22) is then determined from the soil moisture,

\[
D_{W} = \begin{cases} 
1 & \text{if } W \geq W_C \\
W/W_c & \text{if } W < W_C,
\end{cases} \tag{2.m.5}
\]

where \( W_C \) is a critical fraction of the soil field capacity at which the evaporation rate becomes that from a saturated surface,

\[
W_C = 0.75 \ W_{FC}. \tag{2.m.6}
\]
3. NUMERICAL ALGORITHMS

The vertical and temporal aspects of the model are represented by finite-difference approximations, while the horizontal aspects are treated by the spectral-transform method. Thus, at certain points in the code the prognostic variables $\eta, \delta, T, q,$ and $\ell np_s$ are known in terms of coefficients of truncated series of spherical harmonic functions, while at other points they are given by grid-point values on a corresponding Gaussian grid. In general, physical parameterizations and nonlinear operations are carried out in grid-point space. Horizontal derivatives and linear operations are performed in spectral space. Externally, the model appears to the user to be a grid-point model as far as data required and produced by it. Similarly, since all nonlinear parameterizations are developed and carried out in grid-point space, the model also appears as a grid-point model for the incorporation of physical parameterizations, and the user need not be too concerned with the spectral aspects. The algebra involved in the spectral transformations has been presented in several publications (Daley et al., 1976; Bourke et al., 1977; Machenhauer, 1979). In this report, we present only the details relevant to the model code; for more details and general philosophy, the reader is referred to these earlier papers.

We describe the numerical algorithms of the model sequentially starting with an overview of the temporal aspects, followed by the details of the vertical finite differences, and finally by the complete flow of the model, including details of the time differences coupled with the vertical discrete operations and the horizontal spectral representation and operations.

a. Overview of Time Differencing

The time differences are basically centered with the terms responsible for fast-moving gravity waves treated semi-implicitly (Hoskins and Simmons, 1975). The vertical and horizontal diffusion terms are treated implicitly using a time-splitting procedure, vertical before the advection processes and horizontal after. After the time step is completed, the temperatures and mixing ratios are convectively adjusted and finally a time filter is applied to the prognostic variables.
\$lnp_s, T, \eta, \delta, \text{ and } q\$. The time filter was originally designed by Robert (1966) and later studied by Asselin (1972).

The time filter provides filtered values of the prognostic variables at time \(n\) after the values at time \(n + 1\) are computed. For an arbitrary variable \(\psi\), it has the form,

\[
\bar{\psi}^n = \psi^n + \alpha(\psi^{n-1} - 2\psi^n + \psi^{n+1}),
\]

where \(n\) is the time index and \(\alpha\) a small coefficient typically 0.060. Such a linear filter may be applied in either grid or spectral space. Because the model is designed to have data in spectral space at only one time level and in grid space at two time levels, the filter is applied in grid space and then it must be applied in two steps, as all three time levels are never available simultaneously,

\[
\check{\psi}^n = \psi^n + \alpha(\check{\psi}^{n-1} - 2\psi^n),
\]

\[
\bar{\psi}^n = \check{\psi}^n + \alpha(\psi^{n+1}).
\]

Note that \(\check{\psi}^n\) is never used by the model except in the second half of the time filter (3.a.3). Each step involves only two time levels. The actual point of application of the two parts in the flow of the code will be described shortly.

We now describe the flow of the model with respect to the temporal approximations. In general, the prognostic equations for a generic variable \(\psi\) have the form,

\[
\frac{\partial \psi}{\partial t} = F_{\psi_G}(\psi) + \Gamma(\psi) + F_{\psi_S}(\psi).
\]

The \(F_{\psi_G}\) and \(F_{\psi_S}\) terms are time split. The first term \(F_{\psi_G}\) includes the vertical diffusion and surface fluxes (nonlinear operations done in grid-point space), while the last \(F_{\psi_S}\) includes the linear horizontal diffusion calculated in spectral space. The operator \(\Gamma\) includes all the remaining terms.

The starting point for the forecast consists of grid variables \(\psi^n\) and time-smoothed grid variables \(\bar{\psi}^{n-1}\). The half-time-smoothed \(\check{\psi}^n\) have also been computed and are stored in a special buffer to be made available later in the sequence.
In general, the model is designed to provide for time splitting as follows:

\[ \psi^{n+1} = \overline{\psi}^{n-1} + 2\Delta t \ F_{\psi G}^{n} (\psi^{n+1}) , \]  
\[ \psi^{**n+1} = \psi^{n+1} + 2\Delta t \ \Gamma \left( \overline{\psi}^{n-1} , \psi^{n} , \psi^{*n+1} , \psi^{**n+1} \right) , \]  
\[ \psi^{***n+1} = \psi^{**n+1} + 2\Delta t \ F_{\psi S} \left( \psi^{***n+1} \right) . \]  

The first step (3.a.5) represents the calculation of surface fluxes and the implicit nonlinear vertical diffusion. The superscript \( n \) on the operator \( F_{\psi G} \) implies that the variables in the coefficients of the nonlinear diffusion are taken at time level \( n \) and can vary in grid space while the prognostic variable \( \psi \) being diffused is \( \psi^{n+1} \).

The second step (3.a.6) includes all terms except \( F_{\psi G} \) and \( F_{\psi S} \). The operator \( \Gamma \) uses variables \( \overline{\psi}^{n-1} \), \( \psi^{n+1} \), and \( \psi^{n} \) for the explicit part of the computation and \( \psi^{**n+1} \) for the semi-implicit part. This will be more obvious below when the details of the semi-implicit approximations are described. It is during this second step that variables are transformed from grid space to spectral space so that \( \psi^{***n+1} \) is in the form of spectral coefficients. The integrals involved in the transformation are described below. The solution of the semi-implicit operator is completed in spectral space. The third step (3.a.7) is the linear horizontal diffusion and is performed in spectral space. The operator \( F_{\psi S} \) includes only \( \psi^{***n+1} \), and any coefficients involved are independent of \( \lambda \) and \( \mu \).

After completion of (3.a.5) to (3.a.7), \( \psi^{***n+1} \) is available in spectral space. This is the only time level available in spectral space. The corresponding grid–point values are now computed and the horizontal diffusion of \( T \) and \( q \) completed. These newly completed values are denoted \( \hat{\psi}^{n+1} \). The time index is decremented so that \( \hat{\psi}^{n+1} \) becomes \( \hat{\psi}^{n} \), and the half-smoothed \( \hat{\psi}^{n} \) is brought in from its special buffer and its time index decremented so that it becomes \( \hat{\psi}^{n-1} \).

\[ \hat{\psi}^{n+1} \rightarrow \hat{\psi}^{n} , \]  
\[ \hat{\psi}^{n} \rightarrow \hat{\psi}^{n-1} . \]  

(3.a.8)
Following this index shift, the dry mass of the atmosphere is corrected to a specified value, negative moisture values are eliminated in a conserving manner, and the convective adjustment is performed on these new values,

\[ \psi^n = \text{Convectively adjusted} \left( \tilde{\psi}^n \right) , \]  

(3.a.9)

and the second step of the time filter (3.a.3) is completed for time level \( n - 1 \),

\[ \tilde{\psi}^{n-1} = \tilde{\psi}^{n-1} + \alpha(\psi^n) . \]  

(3.a.10)

Grid-point variables \( \psi^n \) and \( \tilde{\psi}^{n-1} \) are now available for output to history files if desired for post-processing later. Only \( \tilde{\psi}^{n-1} \) are actually written to the history tape. Finally, the first half of the time filter (3.a.2) is applied to the time level \( n \),

\[ \tilde{\psi}^n = \psi^n + \alpha(\tilde{\psi}^{n-1} - 2\psi^n) , \]  

(3.a.11)

and the result stored in a special buffer to be brought in after the next time step, as described preceding (3.a.8). Note that \( \tilde{\psi}^n \) is never used by the model except in the second step of the filter (3.a.10).

At this point, a complete cycle has been performed and the logic proceeds back to the starting point described immediately before (3.a.5).

b. Vertical Finite Differences

The vertical discrete grid and the distribution of variables on this grid are shown in Figure 2. All prognostic variables and geopotential are carried at full-index \( \sigma \)-levels while the vertical velocity \( \dot{\sigma} \) is carried at half-index \( \sigma \)-levels. The vertical finite differences are a combination of those adopted by the ANMRC model (Bourke, 1974; Bourke et al., 1977) and thus by CCM0 and those used in the first ECMWF model (Burbridge and Haseler, 1977). The particular combination was chosen to include the most desirable properties of each (Williamson, 1987). The full-index \( \sigma \)-values of the grid levels are specified as input to the model. The half-index levels are given as the average of the adjacent full-index levels with the end values set to 0 and 1, as indicated in Figure 2.
<table>
<thead>
<tr>
<th>Vertical Index</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>$\sigma = 0$</td>
</tr>
<tr>
<td>$1$</td>
<td>$\Delta \sigma_1, \sigma_1$</td>
</tr>
<tr>
<td>$1 1/2$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$2$</td>
<td>$\Delta \sigma_2, \sigma_2$</td>
</tr>
<tr>
<td>$2 1/2$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$3$</td>
<td>$\Delta \sigma_3, \sigma_3$</td>
</tr>
<tr>
<td>$k - 1/2$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$k$</td>
<td>$\Delta \sigma_k, \sigma_k$</td>
</tr>
<tr>
<td>$k + 1/2$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$K - 1$</td>
<td>$\Delta \sigma_{K-1}, \sigma_{K-1}$</td>
</tr>
<tr>
<td>$K - 1/2$</td>
<td>$\phi$</td>
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<tr>
<td>$K$</td>
<td>$\Delta \sigma_K, \sigma_K$</td>
</tr>
<tr>
<td>$K + 1/2$</td>
<td>$\sigma = 1$</td>
</tr>
</tbody>
</table>

Figure 2. Vertical grid and variable placement.
Vertical Advection. Vertical advection of momentum, temperature, and moisture at the level \(k\) is approximated by

\[
\left( \frac{\partial \psi}{\partial \sigma} \right)_k = \alpha_{k+} \dot{\sigma}_{k+1/2} \left( \frac{\partial \psi}{\partial \sigma} \right)_{k+1/2} + \alpha_{k-} \dot{\sigma}_{k-1/2} \left( \frac{\partial \psi}{\partial \sigma} \right)_{k-1/2}, \quad (3.b.1)
\]

\[1 \leq k \leq K\]

where

\[
\left( \frac{\partial \psi}{\partial \sigma} \right)_{k+1/2} = \frac{\psi_{k+1} - \psi_k}{\sigma_{k+1} - \sigma_k} \quad \text{for} \quad 1 \leq k \leq K - 1, \quad (3.b.2)
\]

and

\[
\alpha_{k+} = \frac{\sigma_{k+1} - \sigma_k}{2\Delta \sigma_k} \quad 1 \leq k \leq K, \quad (3.b.3)
\]

\[
\alpha_{k-} = \frac{\sigma_k - \sigma_{k-1}}{2\Delta \sigma_k} \quad 1 \leq k \leq K, \quad (3.b.4)
\]

where

\[
\Delta \sigma_k = \sigma_{k+1/2} - \sigma_{k-1/2}. \quad (3.b.5)
\]

Note that

\[
\dot{\sigma}_{1/2} = \dot{\sigma}_{K+1/2} = 0, \quad (3.b.6)
\]

according to the boundary conditions (2.d.3), so \(\alpha_{K+}\) and \(\alpha_{1-}\) are never needed.

Vertical Integrals. The vertical integrals in the \(\dot{\sigma}\)-equation (2.d.5) and surface-pressure-tendency equation (2.d.1) are given by

\[
\dot{\sigma}_{k+1/2} = \sigma_{k+1/2} \sum_{j=1}^{K} \left( \delta_j + \vec{V}_j \cdot \nabla \ell \ln p_s \right) \Delta \sigma_j \quad (3.b.7)
\]

\[- \sum_{j=1}^{k} \left( \delta_j + \vec{V}_j \cdot \nabla \ell \ln p_s \right) \Delta \sigma_j, \quad 1 \leq k \leq K - 1,
\]

and

\[
\frac{\partial \ell \ln p_s}{\partial t} = - \sum_{j=1}^{K} \left( \delta_j + \vec{V}_j \cdot \nabla \ell \ln p_s \right) \Delta \sigma_j, \quad (3.b.8)
\]

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where
\[ \Delta \sigma_k = \sigma_{k+1/2} - \sigma_{k-1/2}, \quad 1 \leq k \leq K. \] (3.b.9)

The horizontal derivatives of \( \ln p_s \) are available on the Gaussian grid when needed for these computations. As will be discussed later, these horizontal derivatives are obtained in spectral space, then transformed to grid-point space along with the other variables.

**Hydrostatic Equation.** Integration of the hydrostatic equation (2.e.2) to full-index levels gives
\[ \Phi_k = \Phi_s - R \int_{\sigma_1}^{\sigma_k} T_v d\ln \sigma. \] (3.b.10)

This can be approximated, in general, by
\[ \Phi_k = \Phi_s + R \sum_{j=1}^{K} B_{kj} T_{vj}. \] (3.b.11)

The matrix \( B \) is triangular (\( B_{kj} = 0 \) for \( j < k \)). \( T_v \) is assumed to vary linearly with \( \ln \sigma \) between full-index points, with extrapolation to the ground assuming an isothermal first-half layer. This relationship gives for the first level \( k = K \),
\[ \Phi_K = \Phi_s + RT_{vK} (-\ln \sigma_K), \] (3.b.12)

and for \( k \leq K - 1 \),
\[ \Phi_k = \Phi_{k+1} + R \frac{\ln (\sigma_{k+1}/\sigma_k)}{2} (T_{v_{k+1}} + T_{vk}). \] (3.b.13)

The matrix \( B \) is thus given by
\[ B_{K,K} = -\ln \sigma_K, \] (3.b.14)
\[ B_{K,k} = 0 \quad \text{for} \quad k < K - 1, \] (3.b.15)

and for \( k \leq K - 1 \),
\[ B_{k,k} = \frac{\ln (\sigma_{k+1}/\sigma_k)}{2}, \] (3.b.16)
\[ B_{k,k+1} = B_{k+1,k+1} + \frac{\ell n(\sigma_{k+1}/\sigma_k)}{2}, \quad (3.b.17) \]

\[ B_{k,\ell} = B_{k+1,\ell} \quad \text{for} \quad \ell \geq k + 2, \quad (3.b.18) \]

\[ B_{k,\ell} = 0 \quad \text{for} \quad \ell < k. \quad (3.b.19) \]

**Energy Conversion Term.** The vertical integral associated with \( \omega/p \) (2.d.7) in the conversion term in the thermodynamic equation (2.c.1) can be written in a general form as

\[
\left( \frac{\omega}{p} \right)_k = \left[ \mathbf{V}_k \cdot \nabla \ell n \rho_s - \sum_{j=1}^{k} C_{kj} (\delta_j + \mathbf{V}_j \cdot \nabla \ell n \rho_s) \right]. \quad (3.b.20)
\]

The coefficients \( B \) in the hydrostatic integral (3.b.11) and \( C \) in the conversion term integral (3.b.20) are not unrelated. In order to conserve energy \textit{a priori} during conversion from potential to kinetic, they should satisfy

\[ C_{kj} = B_{jk} \frac{\Delta \sigma_j}{\Delta \sigma_k}. \quad (3.b.21) \]

The coefficients in CCM1 are taken to satisfy this relationship and are determined accordingly from the \( B_{jk} \).

c. **Horizontal Spectral Representation**

The horizontal representation of an arbitrary variable \( \psi \) consists of a truncated series of spherical harmonic functions,

\[
\psi(\lambda, \mu) = \sum_{m=-M}^{M} \sum_{n=|m|}^{N(m)} \psi_{n}^{m} P_{n}^{m}(\mu) e^{im\lambda}, \quad (3.c.1)
\]

where \( M \) is the highest Fourier wavenumber included in the east–west representation and \( N(m) \), which can be a function of the Fourier wavenumber \( m \), is the highest degree of the associated Legendre functions included in the north–south representation. The properties of the spherical harmonic functions used in the representation have been reviewed by Machenhauer (1979). The model is coded for a
general pentagonal truncation illustrated in Figure 3 defined by three parameters: $M$ the largest Fourier wavenumber, $K$ the highest degree of the associated Legendre polynomials, and $N$ the highest degree of the Legendre polynomials for $m = 0$. The common truncations are subsets of this pentagonal case:

**Triangular**: $M = N = K$,

**Rhomboidal**: $K = N + M$,  \hspace{1cm} (3.c.2)

**Trapezoidal**: $N = K > M$.

Furthermore, the model may be integrated either hemispherically or globally by making use of the parity of the dependent variables and prognostic equations.

The associated Legendre polynomials used in the model are normalized such that

$$\int_{-1}^{1} [P_n^m(\mu)]^2 d\mu = 1. \hspace{1cm} (3.c.3)$$

With this normalization, the Coriolis parameter $f$ is

$$f = \frac{\Omega}{\sqrt{0.375}} P_1^0, \hspace{1cm} (3.c.4)$$

which is required for the absolute vorticity (2.a.3).
The coefficients of the spectral representation (3.c.1) are given by
\[ \psi_n^m = \int_{-1}^{1} \int_{0}^{2\pi} \psi(\lambda, \mu)e^{-im\lambda}d\lambda P_n^m(\mu)d\mu. \] (3.c.5)

The inner integral represents a Fourier transform,
\[ \psi^m(\mu) = \frac{1}{2\pi} \int_{0}^{2\pi} \psi(\lambda, \mu)e^{-im\lambda}d\lambda, \] (3.c.6)
which is performed by a Fast Fourier Transform (FFT) subroutine. The outer integral is performed via Gaussian quadrature,
\[ \psi_n^m = \sum_{j=1}^{J} \psi^m(\mu_j)P_n^m(\mu_j)w_j, \] (3.c.7)
where \( \mu_j \) denotes the Gaussian grid points in the meridional direction, \( w_j \) the Gaussian weight at point \( \mu_j \), and \( J \) the number of Gaussian grid points from pole to pole. The Gaussian grid points \( (\mu_j) \) are given by the roots of the Legendre polynomial \( P_j(\mu) \), and the corresponding weights are given by
\[ w_j = \frac{2(1 - \mu_j^2)}{[J P_{j-1}(\mu_j)]^2}. \] (3.c.8)

The weights themselves satisfy
\[ \sum_{j=1}^{J} w_j = 2.0. \] (3.c.9)

The Gaussian grid used for the north-south transformation is generally chosen to allow unaliased computations of quadratic terms only. In this case, the number of Gaussian latitudes \( J \) must satisfy
\[ J \geq (2N + K + M + 1)/2 \quad \text{for} \quad M \leq 2(K - N), \] (3.c.10)
\[ J \geq (3K + 1)/2 \quad \text{for} \quad M \geq 2(K - N). \] (3.c.11)

For the common truncations, these become
\[ J \geq (3K + 1)/2 \quad \text{for triangular and trapezoidal}, \] (3.c.12)
\[ J \geq (3N + 2M + 1)/2 \quad \text{for rhomboidal}. \] (3.c.13)
In order to allow exact Fourier transform of quadratic terms, the number of points \( P \) in the east–west direction must satisfy

\[
P \geq 3M + 1. \quad (3.c.14)
\]

The actual values of \( J \) and \( P \) are often not set equal to the lower limit in order to allow use of more efficient transform programs.

Although, in the next section of this model description, we continue to indicate the Gaussian quadrature as a sum from pole to pole, the code actually deals with the symmetric and antisymmetric components of variables and accumulates the sums from equator to pole only. The model requires an even number of Gaussian grid points to easily use the symmetry conditions. This may be slightly inefficient for some spectral resolutions. We define a new index which goes from \(-I\) at the point next to the south pole to \(+I\) at the point next to the north pole and not including 0 (there are no points at the equator or pole in the Gaussian grid), i.e., let \( I = J/2 \) and \( i = j - J/2 \) for \( j \geq J/2 + 1 \) and \( i = j - J/2 - 1 \) for \( j \leq J/2 \); then the summation in (3.c.7) can be rewritten

\[
\psi^m_n = \sum_{i=-I}^{I} \psi^m(\mu_i) P^m_n(\mu_i) w_i. \quad (3.c.15)
\]

The symmetric (even) and antisymmetric (odd) components of \( \psi^m \) are defined by

\[
\psi E_i^m = \frac{1}{2} (\psi_i^m + \psi_{-i}^m), \quad (3.c.16)
\]

\[
\psi O_i^m = \frac{1}{2} (\psi_i^m - \psi_{-i}^m).
\]

Since \( w_i \) is symmetric about the equator, (3.c.15) can be rewritten to give formulas for the coefficients of even and odd spherical harmonics.

\[
\psi^m_n = \begin{cases} 
\sum_{i=1}^{I} \psi E_i^m(\mu_i) P^m_n(\mu_i) 2w_i & \text{for } n - m \text{ even}, \\
\sum_{i=1}^{I} \psi O_i^m(\mu_i) P^m_n(\mu_i) 2w_i & \text{for } n - m \text{ odd}.
\end{cases} \quad (3.c.17)
\]
The model uses the spectral transform method (Machenhauer, 1979) for all nonlinear terms. However, as mentioned in section 3a, the model can be thought of as starting from grid-point values at time $t$ (consistent with the spectral representation) and producing a forecast of the grid-point values at time $t + \Delta t$ (again, consistent with the spectral resolution). The forecast procedure involves computation of the nonlinear terms including physical parameterizations at grid points, transformation via Gaussian quadrature of the nonlinear terms from grid-point space to spectral space, computation of the spectral coefficients of the prognostic variables at time $t + \Delta t$ (with the implied spectral truncation to the model resolution), and transformation back to grid-point space. The details of the equations involved in the various transformations are given in the next section.
4. DETAILED FLOW OF THE MODEL

In the previous sections, we described in general the flow of the model indicating the time-stepping procedures and the points at which various computations are performed. In this section, we present the details of the formulas used by the model including the physical parameterizations, with emphasis on the transformations between grid and spectral space and the semi-implicit algorithm. Reference is made to the vertical finite differences and time filter described in the preceding sections.

a. Initial Grid Values

The starting point for this description consists of grid-point values of the prognostic variables at time \( n \), \((\eta^n, \delta^n, T^n, q^n, \ln p^n)\), and surface geopotential \((\Phi_s)\) which have been obtained from the spectral representation and convectively adjusted. In addition, the wind components \( U^n \) and \( V^n \) are available on the grid, having been computed from the spectral representation of \( \eta^n \) and \( \delta^n \), as are the derivatives of the surface pressure, \( \delta \ln p^n / \partial \lambda \) and \( \delta \ln p^n / \partial \mu \), which have been computed from the spectral representation of \( \ln p^n \). The formulas giving the grid values from the spectral representation are presented later in this description (4.k.1)-(4.k.11). The procedure to obtain these values for the first time step is given in section 5.

In addition, the time-smoothed values of the prognostic variables on the grid at time \( n - 1 \) are also available. These are \( \eta^{n-1}, \delta^{n-1}, T^{n-1}, q^{n-1}, \) and \( \ln p_s^{n-1} \).

b. Radiation

The model first computes the solar and longwave heating/cooling rates \( Q_S^n, Q_{lw}^n \), using the state variables at time \( n - 1 \). These heating rates are saved for later use in the thermodynamic energy equation. In addition to the state variables, the radiation needs the cloud amounts calculated following the convective adjustment described in section 4s. The net downward longwave flux and absorbed solar flux at the surface \( (F_1(p_s), F_S) \) are also computed for use in the surface-energy balance described in section 4c.
Two types of radiation calculations are performed in the model: partial and full. Full radiation calculations are performed every 12 h of a model day. During a full radiation calculation, fluxes and atmospheric heating rates are evaluated. Partial calculations are performed each model time step. During a partial calculation, only the radiative fluxes at the surface are evaluated. These are then used in the evaluation of the surface temperature over non-ocean points.

The equations for calculating the solar flux and heating rates described in section 2g are those explicitly employed in the CCM1 radiation model. The continuous equations for the infrared calculations require a more sophisticated vertical finite-differencing scheme due to the integral term \( \int a \, dB \) in Equations (2.g.37–2.g.38). The reason for the additional care in evaluating this integral arises from the nonlinear behavior of \( a \) across a given model layer. For example, if the flux at half-level \( p_k \) is required, an integral of the form \( \int_{p_k}^{p_k} a(p', p_k) \, dB(p') \) must be evaluated. Now for the nearest layer to level \( p_k \), the following terms will arise:

\[
\int_{p_{k+1}}^{p_k} a(p', p_k) \, dB(p') = \frac{[\alpha(p_{k+1}, p_k) + \alpha(p_k, p_k)]}{2} [B(p_k) - B(p_{k+1})],
\]

employing the trapezoidal rule. The problem arises with the second absorptivity \( \alpha(p_k, p_k) \) since this term is zero. It is also known that \( a \) is nearly exponential in form within a layer. Thus, to accurately account for the variation of \( a(p, p') \) across a layer, many more grid points are required than are available in the CCM1. The nearest layer must, therefore, be subdivided and \( \alpha \) must be evaluated across the subdivided layers. The algorithm that is employed in CCM1 is to use a trapezoidal method for all layers except the nearest layer. For the nearest layer, a subdivision as illustrated in Figure 4 is employed.
Figure 4. Nearest subdivided layer for Equations (4.b.2, 4.b.3).

For the upward flux, the nearest layer contribution to the integral is evaluated from

\[
\int_{\sigma^k}^{\sigma^{k+1}} \alpha dB(\sigma') = \alpha_{22} [B(\sigma_{H}^{k+1}) - B(\sigma^{k})] + \alpha_{21} [B(\sigma^{k}) - B(\sigma_{H}^{k})], \quad (4.b.2)
\]

while for the downward flux, the integral is evaluated according to

\[
\int_{\sigma_{H}^{k}}^{\sigma_{H}^{k+1}} \alpha dB(\sigma') = \alpha_{11} [B(\sigma^{k}) - B(\sigma_{H}^{k})] + \alpha_{12} [B(\sigma_{H}^{k+1}) - B(\sigma^{k})]. \quad (4.b.3)
\]

The \(\alpha_{ij}, i = 1, 2; j = 1, 2\), are absorptivities evaluated for the subdivided paths shown in Figure 4. The path–length dependence for the absorptivities arises from the dependence on the absorptance \(A(p, p')\) [e.g., Eq. (2.g.39)]. Temperatures are known at \(\sigma\)-levels. Temperatures at half-sigma levels are determined through linear interpolation in \(\log \sigma\) between \(\sigma\)-level temperatures. Thus, \(B(\sigma_k) = \sigma_B T_k^4\) can be evaluated at all required levels. The most involved calculation arises from the evaluation of the fraction of layers shown in Figure 4. In general, the absorptance of a layer can require the evaluation of the following path lengths:

\[
\xi(p_k, p_{k+1}) = f(T)\bar{p} \Delta p, \quad (4.b.4)
\]

and,

\[
u(p_k, p_{k+1}) = g(T)\Delta p, \quad (4.b.5)
\]

and,

\[
\beta(p_k, k_{k+1}) = h(T)\bar{p}, \quad (4.b.6)
\]
where \( f, g, \) and \( h \) are functions of temperature due to band parameters (see Kiehl and Ramanathan, 1983), and \( \bar{T} \) is an absorber mass-weighted mean temperature.

These path lengths are, in particular, used extensively in the evaluation of \( A_{O_2} \) (Ramanathan and Dickinson, 1979) and \( A_{CO_2} \) (Kiehl and Briegleb, 1987). But path lengths dependent on both \( p^2 \) (i.e., \( \xi \)) and \( p \) (i.e., \( u \)) are also needed in calculating the water-vapor absorptivity, \( \alpha_{H_2O} \) (Ramanathan and Downey, 1986).

To account for the subdivided layer, a fractional layer amount must be multiplied by \( \xi \) and \( u \), e.g.,

\[
\bar{\xi}_{11} = \xi(\sigma_H^k, \sigma_H^{k+1}) \times UINPL(1,k), \quad (4.\text{b.7})
\]

\[
\bar{u}_{11} = u(\sigma_H^k, \sigma_H^{k+1}) \times WINPL(1,k), \quad (4.\text{b.8})
\]

and

\[
\bar{\beta}_{11} = \beta(\sigma_H^k, \sigma_H^{k+1}) \times PINPL(1,k), \quad (4.\text{b.9})
\]

where \( UINPL, WINPL, \) and \( PINPL \) are factors to account for the fractional subdivided layer amount. These quantities are derived for the case where the mixing ratio is assumed to be constant within a given layer (\( CO_2 \) and \( H_2O \)). For ozone, the mixing ratio is assumed to interpolate linearly in physical thickness; thus, another fractional layer amount \( ZINPL \) is required for evaluating \( A_{O_2}(p, p') \) across subdivided layers.

Consider the subdivided path for \( \alpha_{22} \); the total path length from \( \sigma_H^k \) to \( \sigma_H^{k+1} \) for the \( p^2 \) path length will be

\[
\xi(\sigma_H^k, \sigma_H^{k+1}) \approx \bar{\sigma}_H \left[p_H^k - p_H^{k+1}\right], \quad (4.\text{b.10})
\]

where \( \bar{\sigma}_H \equiv \frac{p_H^k + p_H^{k+1}}{2} \), or in terms of \( \sigma \)-coordinates,

\[
\xi(\sigma_H^k, \sigma_H^{k+1}) \approx \bar{\sigma}_H \left[\sigma_H^k - \sigma_H^{k+1}\right], \quad (4.\text{b.11})
\]

where \( \bar{\sigma}_H \equiv \frac{\sigma_H^k + \sigma_H^{k+1}}{2} \). The total layer path length is, therefore, proportional to

\[
\xi(\sigma_H^k, \sigma_H^{k+1}) \approx \frac{1}{2}(\sigma_H^k - \sigma_H^{k+1^2}). \quad (4.\text{b.12})
\]
Now the path length $\xi$ for $\alpha_{22}$ requires the mean pressure,

$$\bar{p}_{22} \approx \frac{1}{2} \left\{ \frac{\sigma_k + \sigma^{k+1}_H}{2} + \sigma^{k+1}_H \right\}, \quad (4.b.13)$$

and the pressure difference

$$\Delta p_{22} \approx \frac{\sigma_k + \sigma^{k+1}_H}{2} - \sigma^{k+1}_H. \quad (4.b.14)$$

Therefore, the path $\xi_{22}$ is

$$\xi_{22} \approx \bar{p}_{22} \Delta p_{22} = \frac{1}{2} \left\{ \left( \frac{\sigma_k + \sigma^{k+1}_H}{2} \right)^2 - \sigma^{k+1}_H \right\}. \quad (4.b.15)$$

The fractional path length is obtained by normalizing this by $\xi(\sigma^k_H, \sigma^{k+1}_H)$,

$$UINPL(2, k) = DAF3(k) \left\{ \left( \frac{\sigma_k + \sigma^{k+1}_H}{2} \right)^2 - \sigma^{k+1}_H \right\}, \quad (4.b.16)$$

where

$$DAF3(k) = \frac{1}{\sigma^k_H - \sigma^{k+1}_H}. \quad (4.b.17)$$

Similar reasoning leads to the following expressions for the remaining fractional path lengths, for $\alpha_{21}$,

$$UINPL(3, k) = DAF3(k) \left\{ \left( \frac{\sigma_k + \sigma^k_H}{2} \right)^2 - \sigma^{k+1}_H \right\}, \quad (4.b.18)$$

for $\alpha_{11}$

$$UINPL(1, k) = DAF3(k) \left\{ \sigma^k_H - \left( \frac{\sigma_k + \sigma^k_H}{2} \right)^2 \right\}, \quad (4.b.19)$$

and for $\alpha_{12}$,

$$UINPL(4, k) = DAF3(k) \left\{ \sigma^k_H - \left( \frac{\sigma_k + \sigma^{k+1}_H}{2} \right)^2 \right\}. \quad (4.b.20)$$

The $UINPL$ are fractional layer amounts for path length that scale as $p^2$, i.e., $\bar{\xi}_{ij}$. 

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For variables that scale linearly in $p$, e.g., $\bar{u}_{ij}$, the following fractional layer amounts are used:

$$WINPL(1,k) = DAF4(k) \left\{ \frac{\sigma_H^k - \sigma^k}{2} \right\}, \quad (4.b.21)$$

$$WINPL(2,k) = DAF4(k) \left\{ \frac{\sigma^k - \sigma_H^{k+1}}{2} \right\}, \quad (4.b.22)$$

$$WINPL(3,k) = DAF4(k) \left\{ \left( \frac{\sigma_H^k + \sigma^k}{2} \right) - \sigma_H^{k+1} \right\}, \quad (4.b.23)$$

$$WINPL(4,k) = DAF4(k) \left\{ \sigma_H^k - \left( \frac{\sigma_H^{k+1} + \sigma^k}{2} \right) \right\}, \quad (4.b.24)$$

where

$$DAF4(k) = \frac{1}{\sigma_H^k - \sigma_H^{k+1}}. \quad (4.b.25)$$

These fractional layer amounts are directly analogous to the $UINPL$, but since $\bar{u}$ is linear in $p$, the squared terms are not present.

The variable $\beta_{ij}$ requires a mean pressure for the subdivided layer. These are

$$PINPL(1,k) = \frac{1}{2} \left\{ \frac{\sigma^k + \sigma_H^k}{2} + \sigma_H^k \right\}, \quad (4.b.26)$$

$$PINPL(2,k) = \frac{1}{2} \left\{ \frac{\sigma^k + \sigma_H^{k+1}}{2} + \sigma_H^{k+1} \right\}, \quad (4.b.27)$$

$$PINPL(3,k) = \frac{1}{2} \left\{ \frac{\sigma^k + \sigma_H^k}{2} + \sigma_H^{k+1} \right\}, \quad (4.b.28)$$

$$PINPL(4,k) = \frac{1}{2} \left\{ \frac{\sigma^k + \sigma_H^{k+1}}{2} + \sigma_H^k \right\}. \quad (4.b.29)$$

Finally, fractional layer amounts for ozone path lengths are needed, since ozone is interpolated linearly in physical thickness. These are given by

$$ZINPL(1,k) = \frac{1}{2} \ln \left( \frac{\sigma_H^k}{\sigma_H^{k+1}} \right), \quad (4.b.30)$$
c. Surface-Temperature Calculation

The surface temperature is specified as an external boundary condition over the oceans. Over land and sea ice, the surface temperature \( T_s^n \) is computed from the surface–energy–balance equation

\[
\sigma_B (T_s^n)^4 - F^S - F^1(p_s)^n + H_{K+1/2}^n + LR_{K+1/2}^n + Q_{ICE}^n = 0, \tag{4.c.1}
\]

where

\[
\sigma_B = \text{Stefan–Boltzmann constant}
\]

\[
F^S = \text{Net downward solar flux at surface}
\]

\[
F^1(p_s) = \text{Downward longwave flux at surface}
\]

\[
H_{K+1/2}^n = \text{Vertical flux of sensible heat at surface}
\]

\[
R_{K+1/2}^n = \text{Evaporation}
\]

\[
L = \text{Latent heat of condensation}
\]

\[
Q_{ICE}^n = \text{Heat conduction from sea ice (where present) to water below.}
\]

The terms \( F^S \) and \( F^1(p_s) \) are provided by the radiation parameterization described in section 4b.

The terms in (4.c.1) are given below. The latent heat flux (or evaporation) is

\[
R_{K+1/2}^n = \bar{\rho}^{-1}_K C_H D_w |\bar{V}_{K}^{n-1}| (q_s(T_s^n) - \bar{q}_K^{n-1}), \tag{4.c.2}
\]
where, as described in the vertical differencing, $K$ is the index of the first atmosphere level above the surface and $q_8$ is the saturation specific humidity for temperature $T_s^n$,

$$q_8 = \epsilon \frac{e^n_8}{p^n_{s-1} - (1 - \epsilon)e^n_8},$$

(4.c.3)

where $\epsilon = .622$ and $e^n_8$ is the saturation vapor pressure at temperature $T_s^n$ computed by linear interpolation within a table of values specified for $1^\circ$–temperature intervals. The drag coefficient $C_H$, evaporation factor $D_w$, and surface velocity $|V_K|$ are given following (4.c.9). The density $\bar{\rho}_K$ is computed from

$$\bar{\rho}_K = \frac{\sigma_K p^n_{s-1}}{RT^n_{v_K}},$$

(4.c.4)

using $p = \sigma p_s$, and the virtual temperature is

$$\bar{T}_{v_K} = \bar{T}_K \left[ 1 + \left( \frac{R_v}{R} - 1 \right) q^n_{K-1} \right].$$

(4.c.5)

The sensible heat flux is given by

$$H_{K+1/2} = \bar{\rho}_K C_{pK} C_H |V^n_{K-1}| \left( T^n_s - \frac{T^n_{K-1}}{\sigma_K} \right),$$

(4.c.6)

where

$$C_{pK} = C_p \left[ 1 + \left( \frac{C_{pK}}{C_p} - 1 \right) q^n_{K-1} \right].$$

(4.c.7)

The heat conduction out of the sea ice to the ocean below is

$$Q^n_{ICE} = K_I \frac{T^n_s - T_{SW}}{I},$$

(4.c.8)

where

$K_I =$ Thermal conductivity of ice = 2.092 W m$^{-1}$ K$^{-1}$

$I =$ Ice thickness = 2 m

(4.c.9)

$T_{SW} =$ Temperature of sea water below the pack ice = 271.2$^\circ$K

We now present the details of the drag coefficient computation. First, define a pseudo–velocity to provide a minimum exchange for the unstable case

$$|V_e| = \begin{cases} 
2(T^n_s - \bar{T}^{n-1}_K /\sigma_K)^{1/2} & T^n_s > \bar{T}^{n-1}_K /\sigma_K, \\
0 & T^n_s \leq \bar{T}^{n-1}_K /\sigma_K.
\end{cases}$$

(4.c.10)
Then

$$|\bar{V}_K^{n-1}| = \max \left( 1.0, \sqrt{(\bar{u}_K^{n-1})^2 + (\bar{v}_K^{n-1})^2 + |\bar{c}_V^2|} \right). \quad (4.c.11)$$

Note that, in any case, $|\bar{V}_K^{n-1}|$ has a minimum value of 1 m s$^{-1}$. The bulk Richardson number is given by

$$R_{IB} = \frac{gh}{T_K^{n-1}/\sigma_K} \left( \frac{T_{vK}^{n-1}}{\sigma_K} - T_s^{n-1} \right), \quad (4.c.12)$$

where the height of the boundary layer $h$ is taken to be the thickness of the first layer, with a minimum value of 500.0 m.

$$h = \max \left( 500.0, \frac{RT_{vK}^{n-1}}{g} \frac{\Delta \sigma_K}{\sigma_K} \right). \quad (4.c.13)$$

The virtual temperature $T_v$ is given by (4.c.5).

In order to provide a minimum exchange between the atmosphere and the surface in the stable case, the Richardson number is not allowed to exceed .5 times a critical value,

$$R_{IB} = \min \left( R_{IB}, 0.5R_{IC} \right), \quad (4.c.14)$$

where the critical value is

$$R_{IC} = 3.05. \quad (4.c.15)$$

Neutral drag coefficients are defined by

$$C_{UN} = \left\{ k^{-1} \ln \left( \frac{0.025h}{z_o} \right) + 8.4 \right\}^{-1}, \quad (4.c.16)$$

$$C_{\theta N} = \left\{ k^{-1} \ln \left( \frac{0.025h}{z_o} \right) + 7.3 \right\}^{-1}, \quad (4.c.17)$$

where the roughness length is

$$z_o = \begin{cases} 
0.25 \text{ m} & \text{over land, sea ice, and snow cover}, \\
0.001 \text{ m} & \text{over ocean},
\end{cases} \quad (4.c.18)$$

and

$$k = 0.4. \quad (4.c.19)$$
For the stable case, $R_{IB} \geq 0$,

$$C_U = C_{UN} (1 - R_{IB}/R_{IC}), \quad (4.c.20)$$

$$C_\theta = C_{\theta N} (1 - R_{IB}/R_{IC}), \quad (4.c.21)$$

and for the unstable case, $R_{IB} < 0$,

$$C_U = [C_{UN}^{-1} - 25.0 \exp (0.26\zeta - 0.030\zeta^2)]^{-1}, \quad (4.c.22)$$

$$C_\theta = [C_{\theta N}^{-1} + C_{UN}^{-1} - C_{\hat{U}N}^{-1}]^{-1}, \quad (4.c.23)$$

$$\zeta = \log_{10}(-R_{IB}) - 3.5. \quad (4.c.24)$$

Finally, the drag coefficients are

$$C_D = C_U^2, \quad (4.c.25)$$

$$C_H = C_U C_\theta. \quad (4.c.26)$$

The drag coefficient $C_H$ is used for sensible and latent heat fluxes [(4.c.2) and (4.c.6)]. The other drag coefficient $C_D$ will be used for momentum shortly.

The wetness factor $D_W$ in the latent heat flux (4.c.2), which sets the evaporation from the earth's surface to be a specified fraction of the evaporation from a saturated surface, is 1.0 over ocean, sea ice, and snow cover, 0.1 over grassland and scrub, 0.01 over deserts, and 0.25 over all other land types. These various land types are specified as external input parameters to the model.

The energy–balance equation (4.c.1) is implicit in $T^*_s$, since $T^*_s$ appears not only in the surface emission $\sigma_B(T^*_s)^4$ but also in the latent and sensible heat fluxes [(4.c.2) and (4.c.6)], the conduction from sea ice (4.c.9), and the bulk Richardson number [(4.c.10) and (4.c.12)]. The implicit equation is solved by the Newton–Raphson iterative procedure. The general form of the equation is

$$f(T_s) = 0. \quad (4.c.27)$$
The iterative solution is given by

\[ T_s^{(M+1)} = T_s^{(M)} - \frac{f(T_s^{(M)})}{f'(T_s^{(M)})}, \tag{4.c.28} \]

where \((M)\) indicates the iteration count.

The functional form involving \(T_s\) is

\[ f(T_s) = \sigma_B T_s^4 + b C_H(T_s) T_s + d C_H(T_s) q_s(T_s) + e T_s + c = 0. \tag{4.c.29} \]

The derivative of \(f\) with respect to \(T_s\) used in the denominator of (4.c.28) is

\[ f'(T_s) = 4\sigma_B T_s^3 + b \left[ C_H(T_s) + T_s \frac{\partial C_H(T_s)}{\partial T_s} \right] \]

\[ + d \left[ C_H(T_s) \frac{\partial q_s(T_s)}{\partial T_s} + q_s(T_s) \frac{\partial C_H(T_s)}{\partial T_s} \right] + e. \tag{4.c.30} \]

The variation of all terms in (4.c.1) with respect to \(T_s\) is included in \(f'\) except \(F^{Sn}\) and \(F^1(p_s)^n\). If the variation in \(C_H\) were not included but, rather, an earlier value such as \(T_s^{n-1}\) were used to calculate \(C_H\), which is then held fixed during the iteration, a \(2\Delta t\) oscillation could result as the state switched between stable and unstable on alternate time steps. To avoid such an oscillation, the current iterate of \(T_s^{(M)}\) is used to determine the stability for that iteration of (4.c.28).

For the stable case, \(RI_B \geq 0\),

\[ \frac{\partial C_U}{\partial T_s} = -\frac{C_{UN} \frac{\partial R_{IB}}{\partial T_s}}{R_{IC} \frac{\partial T_s}{\partial T_s}}, \tag{4.c.31} \]

\[ \frac{\partial C_{\theta}}{\partial T_s} = -\frac{C_{\theta N} \frac{\partial R_{IB}}{\partial T_s}}{R_{IC} \frac{\partial T_s}{\partial T_s}}. \tag{4.c.32} \]

For the unstable case, \(RI_B < 0\),

\[ \frac{\partial C_U}{\partial T_s} = C_U^2 25(0.26 - 0.060\zeta) \exp [0.26\zeta - 0.030\zeta^2] \frac{\partial \zeta}{\partial T_s}, \tag{4.c.33} \]

\[ \frac{\partial C_{\theta}}{\partial T_s} = \left( \frac{C_{\theta}}{C_U} \right)^2 \frac{\partial C_U}{\partial T_s}, \tag{4.c.34} \]
where

\[
\frac{\partial \zeta}{\partial T_*} = (\log e) \frac{1}{R_{IB}} \frac{\partial R_{IB}}{\partial T_*}, \tag{4.c.35}
\]

and

\[
\frac{\partial R_{IB}}{\partial T_*} = \begin{cases} 
-\frac{g^h}{T_k^{-1} - T_k^{-1}} & \text{for } R_{IB} \leq 0.5R_{IC}, \\
0 & \text{for } R_{IB} > 0.5R_{IC}.
\end{cases} \tag{4.c.36}
\]

Finally,

\[
\frac{\partial C_H}{\partial T_*} = C_U \frac{\partial C_\theta}{\partial T_*} + C_\theta \frac{\partial C_U}{\partial T_*}. \tag{4.c.37}
\]

There is the possibility of an oscillation between stable and unstable states on successive iterations and, therefore, of nonconvergence of (4.c.28). If the bulk Richardson number changes sign more than four times during the iterative process (4.c.28), the neutral values (4.c.16) and (4.c.17) with no dependence on $T_*$ are used for the remainder of the iterative process.

The derivative of $q_*$ with respect to $T_*$ is given by

\[
\frac{dq_*(T_*)}{dT_*} = \left[ \frac{T_0^{n-1}}{\left( T_0^{n-1} - (1 - \epsilon)e_* \right)} \right] \frac{L}{R_{H20}T_*^2 q_*(T_*)}, \tag{4.c.38}
\]

where $R_{H20}$ is the gas constant for water vapor and $L$ is the latent heat of evaporation.

The first guess for the iterative procedure is $T_*^{(0)} = T_*^{n-1}$, i.e., the surface temperature from the previous time step, and the procedure is repeated until

\[
|T_*^{(M+1)} - T_*^{(M)}| \leq \delta, \tag{4.c.39}
\]

at which point $T_*^n = T_*^{(M+1)}$. Initially, $\delta = 0.01$ in the convergence check (4.c.39). If convergence is not reached by the 40th iteration, the convergence criterion is doubled and the process is repeated. If $\delta$ exceeds 0.1 and convergence is still not reached, the model stops. Over snow-covered land or sea ice, the temperature $T_*^n$ is not allowed to exceed 273.16 K. If the iterative procedure gives a higher temperature at such a point, $T_*^n$ is reset to 273.16 K with the implicit assumption that the excess energy went to melt some snow.
d. Surface Fluxes

After the surface temperature is calculated over nonocean points, the sensible and latent heat fluxes and surface stresses are calculated at all points to serve as the lower boundary condition for the vertical diffusion. The surface fluxes of water vapor and sensible heat, $R^n_{K+1/2}$ and $H^n_{K+1/2}$, are given by (4.c.2) and (4.c.6), using the specified value of $T_s$ over ocean or new value of $T^n_s$ at all other points where it is calculated. The explicit surface stresses are given by

$$
\tau^n_{\lambda K+1/2} = -\bar{\rho}^{-1}_K C_D |\overline{V^n_K}| \overline{u^n_K}, \quad (4.d.1)
$$

$$
\tau^n_{\mu K+1/2} = -\bar{\rho}^{-1}_K C_D |\overline{V^n_K}| \overline{v^n_K}. \quad (4.d.2)
$$

The drag coefficient $C_D$ for momentum is given by (4.c.25).

In addition, the average evaporation rate from the surface from (4.c.2) is computed for diagnostic purposes.

$$
EVAP^n = EVAP^{n-1} + \frac{1}{\rho_{H_2O}} R^n_{K+1/2}. \quad (4.d.3)
$$

The $\rho_{H_2O}$ is included to convert the units from mass per unit area per second (kg m$^{-2}$ s$^{-1}$) to depth per second (m s$^{-1}$) of liquid water. These units are chosen for comparison with the precipitation. To convert to W m$^{-2}$ to compare with the sensible heat flux, one need only multiply by $L\rho_{H_2O}$. The average sensible heat flux from the surface from (4.c.6) is also computed for diagnostic purposes.

$$
HFLUX^n = HFLUX^{n-1} + \sigma^n_K H^n_{K+1/2} \quad (4.d.4)
$$
in W m$^{-2}$. The surface stresses $\tau_{\lambda}$ and $\tau_{\mu}$ (N m$^{-2}$) of (4.d.1) and (4.d.2) are written to the history tape for diagnostic purposes as is the average heat flux (W m$^{-2}$) associated with these stresses, i.e.,

$$
DUS^n = DUS^{n-1} - \overline{u^n_K} \tau^n_{\lambda K+1/2}, \quad (4.d.5)
$$

$$
DVS^n = DVS^{n-1} - \overline{v^n_K} \tau^n_{\mu K+1/2}. \quad (4.d.6)
$$
The evaporation and heat fluxes are initialized to zero after each history-tape write, then averaged over every time step between history-tape writes, and normalized or divided by the number of values entering the average just before the history-tape write.

**e. Nonlinear, Time-Split Vertical Diffusion**

As described in section 3a, the vertical diffusion is calculated first, before the main advection calculation. This time-split diffusion is

\[ u^{n+1}_k = \overline{u}^{n-1}_k + 2\Delta t \frac{g}{\overline{p}^{n-1}_s} \frac{\tau^{n+1}_{\lambda k+1/2} - \tau^{n+1}_{\lambda k-1/2}}{\Delta \sigma_k}, \tag{4.e.1} \]

\[ v^{n+1}_k = \overline{v}^{n-1}_k + 2\Delta t \frac{g}{\overline{p}^{n-1}_s} \frac{\tau^{n+1}_{\mu k+1/2} - \tau^{n+1}_{\mu k-1/2}}{\Delta \sigma_k}, \tag{4.e.2} \]

\[ T^{n+1}_k = \overline{T}^{n-1}_k + 2\Delta t \frac{g}{C_{p\lambda} \overline{p}^{n-1}_s} \frac{H^{n+1}_{k+1/2} - H^{n+1}_{k-1/2}}{\Delta \sigma_k}, \tag{4.e.3} \]

\[ q^{n+1}_k = \overline{q}^{n-1}_k + 2\Delta t \frac{g}{\overline{p}^{n-1}_s} \frac{R^{n+1}_{k+1/2} - R^{n+1}_{k-1/2}}{\Delta \sigma_k}, \tag{4.e.4} \]

where

\[ C_{p\lambda}^* = C_p \left[ 1 + \left( \frac{C_{p\lambda}}{C_p} - 1 \right) \overline{q}^{n-1}_k \right]. \tag{4.e.5} \]

The surface fluxes are given by the explicit forms (4.c.2), (4.c.6), (4.d.1), and (4.d.2):

\[ R^{n+1}_{k+1/2} = R^n_{K+1/2}, \tag{4.e.6} \]

\[ H^{n+1}_{K+1/2} = H^n_{K+1/2}, \tag{4.e.7} \]

\[ \tau^{n+1}_{\lambda K+1/2} = \tau^n_{\lambda K+1/2}, \tag{4.e.8} \]

\[ \tau^{n+1}_{\mu K+1/2} = \tau^n_{\mu K+1/2}. \tag{4.e.9} \]

Note that these surface fluxes are explicit and ensure conservation of energy and moisture in the surface exchange. The surface stresses \( \tau_\lambda \) and \( \tau_\mu \) could be made
implicit without affecting this conservation. In the free atmosphere, for \( k = 1 \) to \( K - 1 \),

\[
v^{n+1}_{\lambda k+1/2} = \left( \frac{\tilde{p}^{n-1}_{k+1/2}}{\tilde{P}^{n-1}_a} \right)^2 \frac{g}{\tilde{K}^{n-1}_{k+1/2}} \frac{u^{n+1}_{k+1} - u^{n+1}_k}{\sigma_{k+1} - \sigma_k}, \quad (4.e.10)
\]

\[
v^{n+1}_{\mu k+1/2} = \left( \frac{\tilde{p}^{n-1}_{k+1/2}}{\tilde{P}^{n-1}_a} \right)^2 \frac{g}{\tilde{K}^{n-1}_{k+1/2}} \frac{v^{n+1}_{k+1} - v^{n+1}_k}{\sigma_{k+1} - \sigma_k}, \quad (4.e.11)
\]

\[
R^{n+1}_{k+1/2} = \left( \frac{\tilde{p}^{n-1}_{k+1/2}}{\tilde{P}^{n-1}_a} \right)^2 \frac{g}{\tilde{K}^{n-1}_{k+1/2}} \frac{q^{n+1}_{k+1} - q^{n+1}_k}{\sigma_{k+1} - \sigma_k}, \quad (4.e.12)
\]

\[
H^{n+1}_{k+1/2} = C_{p_{k+1/2}}^* \left( \frac{\tilde{p}^{n-1}_{k+1/2}}{\tilde{P}^{n-1}_a} \right)^2 \frac{g}{\tilde{K}^{n-1}_{k+1/2}} \frac{T^{n+1}_{k+1}/\sigma_{k+1} - T^{n+1}_k/\sigma_k}{\sigma_{k+1} - \sigma_k}, \quad (4.e.13)
\]

where

\[
\tilde{\rho}^{n-1}_{k+1/2} = \frac{2\sigma_{k+1/2}}{R \left( \tilde{T}^{n-1}_{k+1} + \tilde{P}^{n-1}_k \right) \left[ 1 + (\frac{\tilde{R}^{n-1}}{\tilde{R}_k} - 1) \frac{1}{2} (\tilde{q}^{n-1}_{k+1} + \tilde{q}^{n-1}_k) \right]}, \quad (4.e.14)
\]

and

\[
C_{p_{k+1/2}}^* = C_p \left[ 1 + \left( \frac{C_{p_{k+1/2}}}{C_p} - 1 \right) \frac{1}{2} \left( \tilde{q}^{n-1}_{k+1} + \tilde{q}^{n-1}_k \right) \right]. \quad (4.e.15)
\]

\( \tilde{K}^{n-1}_{k+1/2} \) is then determined from the neutral \( K \) modulated by a function of Richardson number

\[
\tilde{K}^{n-1}_{k+1/2} = (\tilde{K}^{n-1}_N)_{k+1/2} \cdot f(R_{I_{k+1/2}}), \quad (4.e.16)
\]

with a minimum value for \( \tilde{K}^{n-1}_{k+1/2} \) of \( K_{\text{min}} = 0.1 \).

\[
f(R_I) = \begin{cases} 
\max \left( 0, 1 - R_I/R_{IC} \right) & \text{for } R_I \geq 0 \text{ (stable)} , \\
\sqrt{1 - 18R_I} & \text{for } R_I < 0 \text{ (unstable)} ,
\end{cases} \quad (4.e.17)
\]

with a critical Richardson number at which \( f \) goes to zero of

\[
R_{IC} = 0.2 . \quad (4.e.18)
\]

The neutral \( K_N \) is calculated by

\[
(\tilde{K}^{n-1}_N)_{k+1/2} = \ell^2_{k+1/2} \frac{g \sigma_{k+1/2}}{\Delta \sigma_{k+1/2}} \frac{\left[ (\tilde{u}^{n-1}_{k+1} - \tilde{u}^{n-1}_k)^2 + (\tilde{v}^{n-1}_{k+1} - \tilde{v}^{n-1}_k)^2 \right]}{R \tilde{T}^{n-1}_{v_{k+1/2}}}, \quad (4.e.19)
\]

with

\[
60
\]
\[ R \mathcal{T}_{v_{k+1/2}}^{n-1} = R \left[ 1 + \left( \frac{R_v}{R} - 1 \right) \frac{1}{2} (q_{k+1}^{n-1} + q_k^{n-1}) \right] \frac{1}{2} \left( \mathcal{T}_{k+1}^{n-1} + \mathcal{T}_k^{n-1} \right), \quad (4.e.20) \]

\[ \Delta \sigma_{k+1/2} = \sigma_{k+1} - \sigma_k, \quad (4.e.21) \]

and

\[ \ell_{k+1/2} = 30.0 \text{ m.} \quad (4.e.22) \]

The Richardson number in the free atmosphere is calculated from

\[ R_{I_{k+1/2}} = R \left[ 1 + \left( \frac{R_v}{R} - 1 \right) \frac{1}{2} (q_{k+1}^{n-1} + q_k^{n-1}) \right] \frac{\sigma_k^{k+1/2} \Delta \sigma_{k+1/2}}{\sigma_{k+1/2}} \]

\[ \times \left[ \frac{\mathcal{T}_k^{n-1}}{\sigma_k^\kappa - \mathcal{T}_{k+1}^{n-1}/\sigma_{k+1}^\kappa} \right]. \quad (4.e.23) \]

At the top of the model, the fluxes are set to 0,

\[ \mathcal{T}^{n+1} = R_{I_{1/2}}^{n+1} = R_{R_{1/2}}^{n+1} = H_{1/2}^{n+1} = 0. \quad (4.e.24) \]

The diffusion equations can be summarized as

\[ v_k^{n+1} - \{u^{n-1}\}_k = a_k \left( v_{k+1}^{n+1} - v_k^{n+1} \right) - c_k \left( v_{k+1}^{n+1} - v_{k-1}^{n+1} \right), \quad (4.e.25) \]

\[ v_k^{n+1} - \{v^{n-1}\}_k = a_k \left( v_{k+1}^{n+1} - v_k^{n+1} \right) - c_k \left( v_{k+1}^{n+1} - v_{k-1}^{n+1} \right), \quad (4.e.26) \]

\[ q_k^{n+1} - \{q^{n-1}\}_k = a_k \left( q_{k+1}^{n+1} - q_k^{n+1} \right) - c_k \left( q_{k+1}^{n+1} - q_{k-1}^{n+1} \right), \quad (4.e.27) \]

\[ T_k^{n+1} - \{T^{n-1}\}_k = a_k \left[ \left( \frac{\sigma_k}{\sigma_{k+1}} \right)^\kappa T_{k+1}^{n+1} - T_k^{n+1} \right] \]

\[ - c_k \left[ T_k^{n+1} - \left( \frac{\sigma_k}{\sigma_{k-1}} \right)^\kappa T_{k-1}^{n+1} \right], \quad (4.e.28) \]

for \( 1 \leq k \leq K - 1, \)

\[ \{u^{n-1}\}_k = u_k^{n-1}, \quad (4.e.29) \]

\[ \{v^{n-1}\}_k = v_k^{n-1}, \quad (4.e.30) \]

\[ \{q^{n-1}\}_k = q_k^{n-1}, \quad (4.e.31) \]

\[ \{T^{n-1}\}_k = T_k^{n-1}, \quad (4.e.32) \]
and for \( k = K \),
\[
\{ \bar{u}^{n-1} \}_K = \bar{u}^{n-1}_K + \frac{2\Delta t g}{\bar{p}^{n-1}_s \Delta \sigma_K} \sigma_{\lambda K+1/2} \tag{4.e.33}
\]
\[
\{ \bar{v}^{n-1} \}_K = \bar{v}^{n-1}_K + \frac{2\Delta t g}{\bar{p}^{n-1}_s \Delta \sigma_k} \sigma_{\mu K+1/2} \tag{4.e.34}
\]
\[
\{ \bar{q}^{n-1} \}_K = \bar{q}^{n-1}_K + \frac{2\Delta t g}{\bar{p}^{n-1}_s \Delta \sigma_K} R_{K+1/2} \tag{4.e.35}
\]
\[
\{ \bar{T}^{n-1} \}_K = \bar{T}^{n-1}_K + \frac{2\Delta t g \sigma_K}{C_p \bar{p}^{n-1}_s \Delta \sigma_K} H_{K+1/2} \tag{4.e.36}
\]

\[
a_k = \frac{2\Delta t}{\Delta \sigma_k (\sigma_{k+1} - \sigma_k)} \left( \frac{g \bar{p}^{n-1}_{k+1/2}}{\bar{p}^{n-1}_s} \right)^2 \bar{K}^{n-1}_{k+1/2} \tag{4.e.37}
\]
\[
a_K = 0, \tag{4.e.38}
\]
\[
c_1 = 0, \tag{4.e.39}
\]
\[
c_k = \frac{2\Delta t}{\Delta \sigma_k (\sigma_k - \sigma_{k-1})} \left( \frac{g \bar{p}^{n-1}_{k-1/2}}{\bar{p}^{n-1}_s} \right)^2 \bar{K}^{n-1}_{k-1/2} \tag{4.e.40}
\]

Following Richtmyer and Morton (1967, pp. 198–201), the solutions for \( u, v, \) and \( q \) have the form,
\[
u_{k+1}^{n+1} = F u_k + E_k u_{k+1}^{n+1}, \tag{4.e.41}
\]
where for \( k = 1 \),
\[
F_{u_1} = \frac{\{ \bar{u}^{n-1} \}_1}{b_1}, \tag{4.e.42}
\]
\[
E_1 = \frac{a_1}{b_1}, \tag{4.e.43}
\]
and for \( 2 \leq k \leq K - 1 \),
\[ F_{u_k} = \frac{\{\bar{w}^{n-1}\}_k + c_k F_{u_k-1}}{b_k - c_k E_{k-1}}, \quad (4.e.44) \]

\[ E_k = \frac{a_k}{b_k - c_k E_{k-1}}, \quad (4.e.45) \]

and for \( k = K \),

\[ E_K = 0, \quad (4.e.46) \]

\[ F_{u_K} = \frac{\{\bar{w}^{n-1}\}_K + c_K F_{u_K-1}}{b_K - c_K E_{K-1}}. \quad (4.e.47) \]

In the above, the diagonal coefficient is given by

\[ b_k = 1 + a_k + c_k. \quad (4.e.48) \]

The solution for \( T \) has the same form except that

\[ a_k \) is replaced by \( a_k \left( \frac{\sigma_k}{\sigma_{k+1}} \right)^{\kappa} \frac{C_{p_k+1/2}}{C_{p_k}}, \quad (4.e.49) \]

and

\[ c_k \) is replaced by \( c_k \left( \frac{\sigma_k}{\sigma_{k-1}} \right)^{\kappa} \frac{C_{p_k-1/2}}{C_{p_k}}. \quad (4.e.50) \]

The diagonal coefficient \( b_k \) is modified by the \( C_p^* \) factor but not by the \( \sigma \) factor, i.e.,

\[ b_k \) is replaced by \( 1 + a_k \frac{C_{p_k+1/2}}{C_{p_k}} + b_k \frac{C_{p_k-1/2}}{C_{p_k}}. \quad (4.e.51) \]

Note that, since the model actually forecasts vorticity and divergence, the new diffused velocity fields \( u_k^{*n+1} \) and \( v_k^{*n+1} \) are not explicitly carried by the model, and the corresponding diffused vorticity and divergence are not calculated. Rather, the net effects of the diffusion \( F_{uG}(u^{*n+1}) \) and \( F_{vG}(v^{*n+1}) \) are saved for addition to the nonlinear terms \( N_u^n \) and \( N_v^n \) in the vorticity and divergence equations (4.f.1) and (4.f.2).

\[ F_{uG}(u^{*n+1}) = \frac{u^{*n+1} - \bar{w}^{n-1}}{2\Delta t}, \quad (4.e.52) \]

\[ F_{vG}(v^{*n+1}) = \frac{v^{*n+1} - \bar{v}^{n-1}}{2\Delta t}. \quad (4.e.53) \]
The heating rate due to the diffusion is also calculated,

$$F_{TG}(T^{n+1}) = \frac{T^{n+1} - \bar{T}^{n-1}}{2\Delta t},$$  \hspace{1cm} (4.e.54)

and added to the nonlinear terms in (4.f.5). The frictional heating associated with (4.e.52) and (4.e.53) is also added to the nonlinear terms in (4.f.5). The individual heating rates $F_{TG}(T^{n+1})$, $-u^n F_{uG}(u^{n+1})/C_p^*$, and $-v^n F_{vG}(v^{n+1})/C_p^*$ with $C_p^*$ given by (4.e.5) are saved for diagnostic purposes. The new moisture $q^{n+1}$ is retained as the basis for the next time-split step in the moisture equation (4.f.12). The moisture tendency due to vertical diffusion is also saved for diagnostic purposes.

$$F_{qG}(q^{n+1}) = \frac{q^{n+1} - \bar{q}^{n-1}}{2\Delta t}. \hspace{1cm} (4.e.55)$$

f. Semi-Implicit Equations

The complete semi-implicit time differences in grid-point space are presented in this section. The absolute vorticity equation (2.b.4) in finite-difference form is purely explicit.

$$\eta^{*n+1} = \bar{\eta}^{n-1} + 2\Delta t \left[ \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} \left( N^n_u + \cos \phi F_{uG}(u^{n+1}) \right) 
- \frac{1}{a} \frac{\partial}{\partial \mu} \left( N^n_v + \cos \phi F_{uG}(u^{n+1}) \right) \right], \hspace{1cm} (4.f.1)$$

but the divergence equation (2.b.5), after substitution of the hydrostatic equation (2.f.16), contains implicit terms.

$$\delta^{*n+1} = \bar{\delta}^{n-1} + 2\Delta t \left[ \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} \left( N^n_u + \cos \phi F_{uG}(u^{n+1}) \right) 
+ \frac{1}{a} \frac{\partial}{\partial \mu} \left( N^n_v + \cos \phi F_{uG}(v^{n+1}) \right) \right] 
- 2\Delta t \nabla^2 \left[ E^n + \phi_\theta + RB(T^n_v - T^n) \right] \hspace{1cm} (4.f.2)$$

$$- 2\Delta t \frac{1}{2} \nabla^2 \left[ RB(T^{**n+1} + \bar{T}^{n-1}) 
+ RT_o(\bar{\ell}np_{**n+1} + \bar{\ell}np_{n-1}) \right].$$
The underbar denotes a vector over the vertical grid, and an undertilde denotes a matrix whose order is that of the vertical grid vectors. $\mathcal{B}$ is the matrix of the hydrostatic equation (3.b.11). In the divergence equation, the surface-pressure term and the component of the geopotential due to the vertical integral of temperature (not virtual temperature) are averaged in time for the semi-implicit algorithm. All other terms in the equation are treated explicitly. The centered terms $N_u^n$ and $N_v^n$ are

$$
N_u^n = \eta^n V^n - RT_v \frac{1}{a} \frac{\partial}{\partial \lambda} \ell n p_n - \sigma^n \frac{\partial U^n}{\partial \sigma},
$$

(4.f.3)

$$
N_v^n = -\eta^n U^n - RT_v (1 - \mu^2) \frac{\partial}{\partial \mu} \ell n p_n - \sigma^n \frac{\partial V^n}{\partial \sigma}.
$$

(4.f.4)

The discrete form of the vertical advection is given by (3.1). As mentioned at the beginning of section 4a, the derivatives of surface pressure, having been computed earlier from the spectral representation, are available at the grid points. The vertical diffusion terms $F_u G_k, F_v G_k$ were computed earlier (4.e.52) and (4.e.53).

The thermodynamic equation (2.c.1) is a little more complicated. Essentially, the terms in $\delta \frac{\partial T}{\partial \sigma}$ and $\kappa \frac{T_w}{p}$, involving $\delta$ multiplied by a mean temperature [from (2.d.7) and (2.d.5)], are treated implicitly and all other terms are treated explicitly.

$$
T'_k \, \text{n+1} = T'_k \, \text{n-1} + 2\Delta t \left\{ -\frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (U^n T'_k) 
$$

$$
- \frac{1}{a} \frac{\partial}{\partial \mu} (V^n T'_k) + \delta^n T'_k + Q^n_S + Q^n_{\ell w} + F_{TG_k} (T'^{n+1})
$$

$$
- \left[ \left[ u^n F_{uG_k} (u\, ^{n+1}) + v^n F_{vG_k} (v\, ^{n+1}) \right] \right]
$$

(4.f.5)

$$
+ 2\Delta t (T^2)_k - 2\Delta t \sum_{j=1}^{K} \tau_{kj} \frac{1}{2} (\delta_{j \, \text{n+1}} + \delta_{j \, \text{n-1}}),
$$

where $F_{TG_k}, F_{uG_k}$ and $F_{vG_k}$ are the vertical diffusions of potential temperature and momentum computed earlier in (4.e.54), (4.e.52), and (4.e.53), $C^*_{p,k}$ is given by
(4.e.5), and $Q_s$ and $Q_{lw}$ are the solar and longwave atmospheric heating rates from section 4b. The term $(T2)_k^n$ contains all the explicit components of $\sigma \frac{\partial T}{\partial \sigma}$ and $\kappa T_w/p$ that are evaluated at time $n$, while the last term contains the implicit components of these two terms.

\[
(T2)_k^n = -\sigma_{k+1/2} \frac{T_{k+1}^n - T_k^n}{2\Delta \sigma_k} - \sigma_{k-1/2} \frac{T_k^n - T_{k-1}^n}{2\Delta \sigma_k}
\]

\[
- \frac{T_{o_k+1} - T_{o_k}}{2\Delta \sigma_k} \left[ \sigma_{k+1/2} \sum_{j=1}^{K} V_j^n \cdot \nabla \ln p^s \Delta \sigma_j - \sum_{j=1}^{K} V_j^n \cdot \nabla \ln p^n \Delta \sigma_j \right]
\]

\[
- \frac{T_{o_k} - T_{o_k-1}}{2\Delta \sigma_k} \left[ \sigma_{k-1/2} \sum_{j=1}^{K} V_j^n \cdot \nabla \ln p^s \Delta \sigma_j - \sum_{j=1}^{K} V_j^n \cdot \nabla \ln p^n \Delta \sigma_j \right]
\]

\[
- \kappa T_o \sum_{j=1}^{k} C_{kj} V_j^n \cdot \nabla \ln p^s^n
\]

\[
- \kappa \left\{ \frac{T_{u_k}^n}{\left[ 1 + \left( \frac{C_{ex}}{C_p} - 1 \right) q_k^n \right]} - T_o \right\} \sum_{j=1}^{k} C_{kj} (\delta_j^n + V_j^n \cdot \nabla \ln p^s^n)
\]

\[
+ \kappa \frac{T_{u_k}^n}{\left[ 1 + \left( \frac{C_{ex}}{C_p} - 1 \right) q_k^n \right]} V_k^n \cdot \nabla \ln p^s^n,
\]

where $C_{kj}$ are the elements of the matrix used for the conversion term (3.b.20) and a sum from $j = 1$ to 0 is understood to be zero.

If we define

\[
\overline{\delta_j} = \frac{1}{2} \left( \delta_j^{*n+1} + \delta_j^{n-1} \right),
\]
the implicit term of (4.f.5) is
\[ \sum_{j=1}^{K} \tau_{kj} \delta_j^t = \kappa T_{ok} \sum_{j=1}^{k} C_{kj} \delta_j^t \]
\[ + \alpha_{k+} \left( \frac{\partial T_o}{\partial \sigma} \right)_{k+1/2} \left[ \sigma_{k+1/2} \sum_{j=1}^{K} \delta_j^t \Delta \sigma_j - \sum_{j=1}^{k} \delta_j^t \Delta \sigma_j \right] \]
\[ + \alpha_{k-} \left( \frac{\partial T_o}{\partial \sigma} \right)_{k-1/2} \left[ \sigma_{k-1/2} \sum_{j=1}^{K} \delta_j^t \Delta \sigma_j - \sum_{j=1}^{k-1} \delta_j^t \Delta \sigma_j \right], \tag{4.f.8} \]

where the \( \sigma \) temperature derivatives are given by (3.b.2) and a sum from \( j = 1 \) to \( 0 \) is understood to be zero. Since \( C_{kj} \) is lower triangular, the first summation on the right side of (4.f.8) can be extended to \( K \). Thus, the elements of the matrix \( \tau_{kj} \) are given by
\[ \tau_{kj} = \kappa T_{ok} C_{kj} \]
\[ + \frac{T_{ok+1} - T_{ok}}{2 \Delta \sigma_k} \left\{ \begin{array}{ll} \sigma_{k+1/2} \Delta \sigma_j & j > k, \\ \sigma_{k+1/2} \Delta \sigma_j - \Delta \sigma_j & j \leq k, \end{array} \right. \]
\[ + \frac{T_{ok} - T_{ok-1}}{2 \Delta \sigma_k} \left\{ \begin{array}{ll} \sigma_{k-1/2} \Delta \sigma_j & j > k - 1, \\ \sigma_{k-1/2} \Delta \sigma_j - \Delta \sigma_j & j \geq k - 1. \end{array} \right. \tag{4.f.9} \]

Note that, for \( k = K \), \( \sigma_{k+1/2} \Delta \sigma_j - \Delta \sigma_j = 0 \), and for \( k = 1 \), \( \sigma_{k-1/2} \Delta \sigma_j = 0 \), so that the apparently out-of-range \( T_{ok} \) and \( T_{oK+1} \) are not really used.

The thermodynamic equation can now be written as a vector equation, with the vectors extending over the vertical grid.
\[ T_{n+1} = T_n + 2 \Delta t \left[ (T1)_n + (T2)_n \right] - \Delta t \tau (\bar{\delta}^{*n+1} + \bar{\delta}^{n-1}), \tag{4.f.10} \]

where \( (T1)_n \) contains all the terms in brackets in (4.f.5).

The divergence term in the surface–pressure equation is also treated implic-
\[ \ell n p_s^{n+1} = \ell n p_s^{n-1} - 2\Delta t \sum_{j=1}^{K} V_j^n \cdot \nabla \ell n p_s^n \Delta \sigma_j \]  
(4.f.11)

- \[ - 2\Delta t \sum_{j=1}^{K} \frac{1}{2} (\delta_j^{n+1} + \delta_j^{n-1}) \Delta \sigma_j. \]

The final prognostic equation is that for specific humidity which is treated explicitly.

\[ q_k^{n+1} = q_k^{n+1} - 2\Delta t \left[ \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (U^n_k q_k^n) + \frac{1}{a^2 \mu} (V^n_k q_k^n) \right. \]

\[ - \delta_k^n q_k^n + \left( \dot{q} \frac{\partial q}{\partial \sigma} \right)_k - F^n_{qG_k} \]  
(4.f.12)

where the vertical advection differences are given in the usual way (3.b.1), and \( q_k^{n+1} \) is the moisture following the time-split vertical diffusion calculation [see discussion preceding (4.e.55)]. The actual solution of the semi-implicit equations for \( T, \delta, \) and \( \ell n p_s \) is done in spectral space where the inverses of the \( \nabla^2 \) and \( \nabla^4 \) operators have particularly simple forms.

**g. Combination of Terms**

In order to describe the transformation spectral space, for each equation we first group together all undifferentiated explicit terms, all explicit terms with longitudinal derivatives, and all explicit terms with meridional derivatives. Thus, the vorticity equation (4.f.1) is rewritten

\[ \eta^{n+1} = V + \frac{1}{a(1 - \mu^2)} \left[ \frac{\partial}{\partial \lambda} (V_{\lambda}) - (1 - \mu^2) \frac{\partial}{\partial \mu} (V_{\mu}) \right], \]  
(4.g.1)

where the explicit forms of the vectors \( V, V_{\lambda}, \) and \( V_{\mu} \) are given in Appendix 1 [(A1)-(A3)]. The divergence equation (4.f.2) is

\[ \delta^{n+1} = D + \frac{1}{a(1 - \mu^2)} \left[ \frac{\partial}{\partial \lambda} (D_{\lambda}) + (1 - \mu^2) \frac{\partial}{\partial \mu} (D_{\mu}) \right] - \nabla^2 D \nabla \]

\[ - \Delta t \nabla^2 (R \nabla T^{n+1} + RT_0 \ell n p_s^{n+1}). \]  
(4.g.2)
The mean component of the temperature is not included in the next-to-last term since the Laplacian of it is zero. The thermodynamic equation (4.f.5) is

\[ T'^{n+1} = T - \frac{1}{a(1-\mu^2)} \left[ \frac{\partial}{\partial \lambda} (T_\lambda) + (1 - \mu^2) \frac{\partial}{\partial \mu} (T_\mu) \right] - \Delta t \mathbf{\delta}^{n+1}, \tag{4.g.3} \]

The surface-pressure tendency (4.f.11) is

\[ \ell n p_s^{n+1} = PS - \Delta t \mathbf{\Pi}^T \mathbf{\delta}^{n+1}, \tag{4.g.4} \]

where \( \mathbf{\Pi}^T \) is the transpose of the vector with elements \( \Delta \sigma_k \). The specific humidity forecast equation is

\[ q'^{n+1} = Q - \frac{1}{a(1-\mu^2)} \left[ \frac{\partial}{\partial \lambda} (Q_\lambda) + (1 - \mu^2) \frac{\partial}{\partial \mu} (Q_\mu) \right]. \tag{4.g.5} \]

The grouped explicit terms in (4.g.2)-(4.g.5) are all given in Appendix 1 [(A4)-(A14)].

h. Transformation to Spectral Space

As mentioned in section 3c, the coefficients of the spectral representation are obtained by integration,

\[ \psi_n^m = \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_0^{2\pi} \psi(\lambda, \mu) e^{-im\lambda} d\lambda P_n^m(\mu) d\mu. \tag{4.h.1} \]

The inner integral is generally performed via an FFT subroutine. The result of this operation is denoted \( \psi^m(\mu) \),

\[ \psi^m(\mu) = \frac{1}{2\pi} \int_0^{2\pi} \psi(\lambda, \mu) e^{-im\lambda} d\lambda. \tag{4.h.2} \]

The outer integral is performed via Gaussian quadrature,

\[ \psi_n^m = \sum_{j=1}^J \psi^m(\mu_j) P_n^m(\mu_j) w_j, \tag{4.h.3} \]

where \( \mu_j \) denotes the Gaussian grid points in the meridional direction, \( w_j \) is the Gaussian weight at the \( j \)th grid point, and \( J \) the number of points from pole to pole.
Formally, Equations (4.g.1)-(4.g.5) are transformed to spectral space by performing the operations indicated in (4.h.1) to each term. We see that the equations basically contain three types of terms, for example, in the vorticity equation the undifferentiated term $V$, the longitudinally differentiated term $V_{\lambda}$, and the meridionally differentiated term $V_{\mu}$. All terms in the original equations were grouped into one of these terms on the Gaussian grid so that they could be transformed at once.

Transformation of the undifferentiated term is obtained by straightforward application of (4.h.1)-(4.h.3),

$$\{V\}_m = \sum_{j=1}^{J} V^m(\mu_j) P^m_n(\mu_j) w_j, \quad (4.h.4)$$

where $V^m(\mu_j)$ is the Fourier coefficient of $V$ with wavenumber $m$ at the Gaussian grid line $\mu_j$. The longitudinally differentiated term is handled by integration by parts using the cyclic boundary conditions,

$$\left\{ \frac{\partial}{\partial \lambda} (V_{\lambda}) \right\}^m = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{\partial V_{\lambda}}{\partial \lambda} e^{-im\lambda} d\lambda, \quad (4.h.5)$$

$$= im \frac{1}{2\pi} \int_{0}^{2\pi} V_{\lambda} e^{-im\lambda} d\lambda,$$

so that the Fourier transform is performed first, then the differentiation is carried out in spectral space. The transformation to spherical harmonic space then follows (4.h.4):

$$\left\{ \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (V_{\lambda}) \right\}_n^m = \sum_{j=1}^{J} im V^m_{\lambda}(\mu_j) \frac{P^m_n(\mu_j)}{a(1 - \mu_j^2)} w_j, \quad (4.h.6)$$

where $V^m_{\lambda}(\mu_j)$ is the Fourier coefficient of $V_{\lambda}$ with wavenumber $m$ at the Gaussian grid line $\mu_j$.

The latitudinally differentiated term is handled by integration by parts using
zero boundary conditions at the poles.

\[
\left\{ \frac{1}{a(1 - \mu^2)} (1 - \mu^2) \frac{\partial}{\partial \mu} (V_\mu) \right\}_n^m = \int_{-1}^{1} \frac{1}{a(1 - \mu^2)} (1 - \mu^2) \frac{\partial}{\partial \mu} (V_\mu)^m P_n^m d\mu,
\]

\[
= - \int_{-1}^{1} \frac{1}{a(1 - \mu^2)} (V_\mu)^m (1 - \mu^2) \frac{dP_n^m}{d\mu} d\mu.
\]

Defining the derivative of the associated Legendre polynomial by

\[
H_n^m = (1 - \mu^2) \frac{dP_n^m}{d\mu},
\]

(4.4.7) can be written

\[
\left\{ \frac{1}{a(1 - \mu^2)} (1 - \mu^2) \frac{\partial}{\partial \mu} (V_\mu) \right\}_n^m = - \sum_{j=1}^{J} (V_\mu)^m \frac{H_n^m(\mu_j)}{a(1 - \mu_j^2)} w_j.
\]

Similarly, the \(\nabla^2\) operator in the divergence equation can be converted to spectral space by sequential integration by parts and then applying the relationship,

\[
\nabla^2 P_n^m(\mu) e^{im\lambda} = \frac{-n(n+1)}{a^2} P_n^m(\mu) e^{im\lambda},
\]

(4.4.10) to each spherical harmonic function individually.

\[
\{\nabla^2 D_\nabla\}_n^m = \sum_{j=1}^{J} \frac{-n(n+1)}{a^2} D_\nabla^m(\mu_j) P_n^m(\mu_j) w_j,
\]

(4.4.11)

where \(D_\nabla^m(\mu)\) is the Fourier coefficient of the original grid variable \(D_\nabla\).

1. Solution of Semi-Implicit Equations

The prognostic equations can be converted to spectral form by summation over the Gaussian grid using (4.4.4), (4.4.6), and (4.4.9). The resulting equation for absolute vorticity is

\[
\eta_n^m = VS_n^m,
\]

(4.4.1)

where \(\eta_n^m\) denotes a spherical harmonic coefficient of \(\eta^{**n+1}\), and the form of \(VS_n^m\) as a summation over the Gaussian grid is given in Appendix 1 (A15).
The spectral form of the divergence equation (4.g.2) becomes

\[ \dot{\varepsilon}_n^m = D S_n^m + \Delta t \frac{n(n+1)}{a^2} \left[ R B T_n^m + R T_o (\ell n p_s)_n^m \right], \]  

(4.i.2)

where \( \dot{\varepsilon}_n^m \), \( T_n^m \), and \( (\ell n p_s)_n^m \) are spectral coefficients of \( \delta^{m+1} \), \( T^{m+1} \), and \( \ell n p_s^{m+1} \). The Laplacian of the total temperature in (4.g.2) is replaced by the equivalent Laplacian of the perturbation temperature in (4.i.2). \( D S_n^m \) is given in Appendix 1 (A16). The spectral thermodynamic equation is

\[ T_n^m = T S_n^m - \Delta t \tau \dot{\varepsilon}_n^m, \]  

(4.i.3)

with \( T S_n^m \) defined in Appendix 1 (A17), while the surface pressure equation is

\[ (\ell n p_s)_n^m = P S_n^m - \Delta t \Pi T \dot{\varepsilon}_n^m, \]  

(4.i.4)

and the specific humidity equation becomes

\[ q_n^m = Q S_n^m. \]  

(4.i.5)

\( P S_n^m \) and \( Q S_n^m \) are also given in Appendix 1 [(A18) and (A19)].

Equations (4.i.1) and (4.i.5) for vorticity and mixing ratio are explicit and complete at this point. However, the remaining equations (4.i.2)–(4.i.4) are coupled. They are solved by eliminating all variables except \( \dot{\varepsilon}_n^m \).

\[ A_n \delta_n^m = D S_n^m + \Delta t \frac{n(n+1)}{a^2} \left[ R B (T S)_n^m + R T_o (P S)_n^m \right], \]  

(4.i.6)

where

\[ A_n = I + \Delta t^{2n(n+1)} a^2 \left[ R B T + R T_o \Pi T \right], \]  

(4.i.7)

which is just a set of \( K \) simultaneous equations for the coefficients with given wavenumbers \((m, n)\) at each level and is solved by inverting \( A_n \). In order to prevent the accumulation of round-off error in the global mean divergence, which if exactly zero initially should remain exactly zero, \( (A_n)^{-1} \) is set to the null matrix rather than the identity, and the formal application of (4.i.6) then guarantees \( \delta_0^m = 0 \) always. Once \( \delta_n^m \) is known, \( T_n^m \) and \( (\ell n p_s)_n^m \) can be computed from (4.i.3) and
(4.i.4), respectively, and all prognostic variables are known at time \( n+1 \) as spherical harmonic coefficients. Note that the mean component \( T'_{n} \) is not necessarily zero since the perturbations are taken with respect to a specified \( T_{o} \).

**j. Horizontal Diffusion**

As mentioned earlier, the horizontal diffusion (3.a.7) is computed implicitly via time splitting after the forecast and transformations into spectral space (3.a.6). In the top three levels of the model, the \( \nabla^2 \) form is used:

\[
(\eta^{n+1})_n^m = (\eta^n)^m + 2\Delta t K_2 \left[ \nabla^2 (\eta^{n+1})_n^m + 2(\eta^{n+1})_n^m / a^2 \right], \quad (4.j.1)
\]

\[
(\delta^{n+1})_n^m = (\delta^n)^m + 2\Delta t K_2 \left[ \nabla^2 (\delta^{n+1})_n^m + 2(\delta^{n+1})_n^m / a^2 \right], \quad (4.j.2)
\]

\[
(T^{n+1})_n^m = (T^n)^m + 2\Delta t K_2 \left[ \nabla^2 (T^{n+1})_n^m \right], \quad (4.j.3)
\]

\[
(q^{n+1})_n^m = (q^n)^m + 2\Delta t K_2 \left[ \nabla^2 (q^{n+1})_n^m \right]. \quad (4.j.4)
\]

The solutions are just

\[
(\eta^{n+1})_n^m = K_2(\eta, \delta)(\eta^{n+1})_n^m, \quad (4.j.5)
\]

\[
(\delta^{n+1})_n^m = K_2(\eta, \delta)(\delta^{n+1})_n^m, \quad (4.j.6)
\]

\[
(T^{n+1})_n^m = K_2(T, q)(T^{n+1})_n^m, \quad (4.j.7)
\]

\[
(q^{n+1})_n^m = K_2(T, q)(q^{n+1})_n^m, \quad (4.j.8)
\]

where

\[
K_2(\eta, \delta) = \left\{ 1 + 2\Delta t K_2 \left[ \left( \frac{n(n+1)}{a^2} \right) - \frac{2}{a^2} \right] \right\}^{-1}, \quad (4.j.9)
\]

and

\[
K_2(T, q) = \left\{ 1 + 2\Delta t K_2 \left( \frac{n(n+1)}{a^2} \right) \right\}^{-1}. \quad (4.j.10)
\]

The extra term in (4.j.9) is added to prevent damping of uniform rotations. At all other levels, the \( \nabla^4 \) form is used.

\[
(\eta^{n+1})_n^m = (\eta^n)^m - 2\Delta t K_4 \left\{ \nabla^4 (\eta^{n+1})_n^m - [2(\eta^{n+1})_n^m / a^2]^2 \right\}. \quad (4.j.11)
\]
\[
(\delta^{n+1})^m_n = (\delta^{n+1})^m_n - 2\Delta t K_4 \left\{ \nabla^4 (\delta^{n+1})^m_n - \left[ \frac{2(\delta^{n+1})^m_n}{a^2} \right]^2 \right\} 
\]  
(4.j.12)

\[
(T^{n+1})^m_n = (T^{n+1})^m_n - 2\Delta t K_4 \nabla^4 (T^{n+1})^m_n, 
\]  
(4.j.13)

\[
(q^{n+1})^m_n = (q^{n+1})^m_n - 2\Delta t K_4 \nabla^4 (q^{n+1})^m_n, 
\]  
(4.j.14)

At these levels, the solutions are

\[
(\eta^{n+1})^m_n = K_4(\eta, \delta)(\eta^{n+1})^m_n, 
\]  
(4.j.15)

\[
(\delta^{n+1})^m_n = K_4(\eta, \delta)(\delta^{n+1})^m_n, 
\]  
(4.j.16)

\[
(T^{n+1})^m_n = K_4(T, q)(T^{n+1})^m_n, 
\]  
(4.j.17)

\[
(q^{n+1})^m_n = K_4(T, q)(q^{n+1})^m_n, 
\]  
(4.j.18)

where

\[
K_4(\eta, \delta) = \left\{ 1 + 2\Delta t K_4 \left[ \left( \frac{n(n+1)}{a^2} \right)^2 - \frac{4}{a^4} \right] \right\}^{-1}, 
\]  
(4.j.19)

and

\[
K_4(T, q) = \left\{ 1 + 2\Delta t K_4 \left( \frac{n(n+1)}{a^2} \right)^2 \right\}^{-1}. 
\]  
(4.j.20)

Again, the extra term is present in (4.j.19) to prevent damping of uniform rotations. The diffusion of \( T \) and \( q \) is actually not complete at this stage. In order to make the partial correction from \( \sigma \) to \( p \) in (2.i.7) and (2.i.8) local, it is not included until grid-point values are available. This requires that \( \nabla^4 \epsilon p_\sigma \) also be transformed from spectral to grid-point space. The values of the coefficients \( K_2 \) and \( K_4 \) are given in (2.i.9) and (2.i.10).

**k. Transformation from Spectral to Grid Space**

After the prognostic variables are completed at time \( n + 1 \) in spectral space

\[
(\eta^{n+1})^m_n, \ (\delta^{n+1})^m_n, \ (T^{n+1})^m_n, \ (\epsilon p_\sigma^{n+1})^m_n, \ (q^{n+1})^m_n, \]

they are transformed to grid space. For a variable \( \psi \), the transformation is given by

\[
\psi(\lambda, \mu) = \sum_{m=-M}^M \left[ \sum_{n=|m|}^N \psi_n^m P_n^m(\mu) \right] e^{im\lambda}. 
\]  
(4.k.1)
The inner sum is done essentially as a vector product over \( n \), and the outer is again performed by an FFT subroutine. The term needed for the remainder of the diffusion terms, \( \nabla^4 \ell n_{p*} \), is calculated from

\[
\nabla^4 \ell n^{**n+1}_{p*} = \sum_{m=-M}^{M} \left[ \sum_{n=|m|}^{N(m)} \left( \frac{n(n+1)}{a^2} \right)^2 \ell n_{p*}^{**n+1} P_n^m(\mu) \right] e^{i m \lambda}. \tag{4.k.2}
\]

In addition, the derivatives of \( \ell n_{p*} \) are needed on the grid for the terms involving \( \nabla \ell n_{p*} \) and \( \mathbf{V} \cdot \nabla \ell n_{p*} \),

\[
\mathbf{V} \cdot \nabla \ell n_{p*} = \frac{U}{a(1-\mu^2)} \frac{\partial \ell n_{p*}}{\partial \lambda} + \frac{V}{a(1-\mu^2)} (1-\mu^2) \frac{\partial \ell n_{p*}}{\partial \mu}. \tag{4.k.3}
\]

These required derivatives are given by

\[
\frac{\partial \ell n_{p*}}{\partial \lambda} = \sum_{m=-M}^{M} \frac{1}{i m} \left[ \sum_{n=|m|}^{N(m)} \ell n_{p*}^m P_n^m(\mu) \right] e^{i m \lambda}, \tag{4.k.4}
\]

and using (4.h.8),

\[
(1-\mu^2) \frac{\partial \ell n_{p*}}{\partial \mu} = \sum_{m=-M}^{M} \left[ \sum_{n=|m|}^{N(m)} \ell n_{p*}^m H_n^m(\mu) \right] e^{i m \lambda}, \tag{4.k.5}
\]

which involve basically the same operations as (4.k.2). The other variables needed on the grid are \( U \) and \( V \). These can be computed directly from the absolute vorticity and divergence coefficients using the relations

\[
\eta_n^m = -\frac{n(n+1)}{a^2} \psi_n^m + f_n^m, \tag{4.k.6}
\]

\[
\delta_n^m = -\frac{n(n+1)}{a^2} \chi_n^m, \tag{4.k.7}
\]

in which the only nonzero \( f_n^m \) is \( f_1^0 = \Omega/\sqrt{.375} \), and

\[
U = \frac{1}{a} \frac{\partial \chi}{\partial \lambda} - \frac{(1-\mu^2)}{a} \frac{\partial \psi}{\partial \mu}, \tag{4.k.8}
\]

\[
V = \frac{1}{a} \frac{\partial \psi}{\partial \lambda} + \frac{(1-\mu^2)}{a} \frac{\partial \chi}{\partial \mu}. \tag{4.k.9}
\]
Thus, the direct transformation is

\[
U = - \sum_{m=-M}^{M} a \sum_{n=|m|}^{N(m)} \left[ \frac{im}{n(n+1)} \epsilon_{m}^{n} P_{n}^{m}(\mu) - \frac{1}{n(n+1)} \eta_{n}^{m} H_{n}^{m}(\mu) \right] e^{im\lambda} - \frac{a}{2} \frac{\Omega}{\sqrt{0.375}} H_{1}^{P}, 
\]

(4.k.10)

\[
V = - \sum_{m=-M}^{M} a \sum_{n=|m|}^{N(m)} \left[ \frac{im}{n(n+1)} \eta_{n}^{m} P_{n}^{m}(\mu) \right. \\
+ \left. \frac{1}{n(n+1)} \epsilon_{m}^{n} H_{n}^{m}(\mu) \right] e^{im\lambda}. 
\]

(4.k.11)

The diffusion tendencies are also transformed back to grid space. The spectral coefficients for the tendencies follow from (4.j.1)-(4.j.4) and (4.j.11)-(4.j.14):

\[
F_{TS}(T^{**n+1})_{n} = (-1)^{i+1} K_{2i} \left[ \nabla^{2i}(T^{**n+1}) \right]_{n}, 
\]

(4.k.12)

\[
F_{qS}(q^{**n+1})_{n} = (-1)^{i+1} K_{2i} \left[ \nabla^{2i}(q^{**n+1}) \right]_{n}, 
\]

(4.k.13)

\[
F_{\eta S}(\eta^{**n+1})_{n} = (-1)^{i+1} K_{2i} \left\{ \nabla^{2i}(\eta^{**n+1}) + (-1)^{i+1} \left[ 2\eta^{**n+1}/a^{2} \right] \right\}, 
\]

(4.k.14)

\[
F_{\delta S}(\delta^{**n+1})_{n} = (-1)^{i} K_{2i} \left\{ \nabla^{2i}(\delta^{**n+1}) + (-1)^{i+1} \left[ 2\delta^{**n+1}/a^{2} \right] \right\}, 
\]

(4.k.15)

using \( i = 1 \) or \( 2 \) as appropriate for the \( \nabla^{2} \) or \( \nabla^{4} \) forms. These coefficients are transformed to grid space following (4.k.1) for the \( T \) and \( q \) terms and (4.k.10) and (4.k.11) for vorticity and divergence. Thus, the vorticity and divergence diffusion tendencies are converted to equivalent \( U \)- and \( V \)-diffusion tendencies.

1. Diffusion Correction

After grid-point values are calculated, the momentum diffusion tendencies are converted to heating rates and added to the temperature, and the partial correction of the \( \nabla^{4} \) diffusion from \( \sigma \) to \( p \) surfaces is applied to \( T \) and \( q \). The frictional heating rate is calculated from

\[
\hat{F}_{uS}(u^{**n+1}) = -u^{**n+1} F_{uS}(u^{**n+1})/C_{p}^{*}, 
\]

(4.l.1)

\[
\hat{F}_{vS}(v^{**n+1}) = -v^{**n+1} F_{vS}(v^{**n+1})/C_{p}^{*}, 
\]

(4.l.2)
with

\[ C_p^* = C_p \left[ 1 + \left( \frac{C_{px}}{C_p} - 1 \right) q^{**n+1} \right]. \tag{4.1.3} \]

These heating rates are then combined with the correction,

\[ \hat{T}_{k}^{n+1} = T_{k}^{***n+1} + 2\Delta t \left[ \hat{F}_{uS}(u^{***n+1})_k + \hat{F}_{vS}(v^{***n+1})_k \right] 
+ 2\Delta t \sigma_k \left( \frac{\partial T^{***n+1}}{\partial \sigma} \right)_k K_4 \nabla^4 \ell n p^{**n+1}, \tag{4.1.4} \]

\[ \hat{q}_{k}^{n+1} = q_{k}^{***n+1} + 2\Delta t \sigma_k \left( \frac{\partial q^{***n+1}}{\partial \sigma} \right)_k K_4 \nabla^4 \ell n p^{**n+1}. \tag{4.1.5} \]

The vertical derivatives of \( T \) and \( q \) are given by

\[
\left( \frac{\partial \psi}{\partial \sigma} \right)_1 = \frac{\psi_2 - \psi_1}{\sigma_2 - \sigma_1}, \tag{4.1.6}
\]

\[
\left( \frac{\partial \psi}{\partial \sigma} \right)_k = \frac{\psi_{k+1} - \psi_{k-1}}{\sigma_{k+1} - \sigma_{k-1}}, \quad 2 \leq k \leq K - 1; \tag{4.1.7}
\]

\[
\left( \frac{\partial \psi}{\partial \sigma} \right)_K = \frac{\psi_K - \psi_{K-1}}{\sigma_K - \sigma_{K-1}}. \tag{4.1.8}
\]

The corrections are added to the diffusion tendencies calculated earlier [(4.1.12) and (4.1.13)] to give the total tendencies for diagnostic purposes.

\[
\hat{F}_{TS}(T^{***n+1}) = F_{TS}(T^{***n+1}) 
+ 2\Delta t \sigma_k \left( \frac{\partial T^{***n+1}}{\partial \sigma} \right)_k K_4 \nabla^4 \ell n p^{**n+1}, \tag{4.1.9}
\]

\[
\hat{F}_{qS}(q^{***n+1}) = F_{qS}(q^{***n+1}) 
+ 2\Delta t \sigma_k \left( \frac{\partial q^{***n+1}}{\partial \sigma} \right)_k K_4 \nabla^4 \ell n p^{**n+1}. \tag{4.1.10}
\]

Note that the correction for \( T \) and \( q \) is applied only at the levels where the \( \nabla^4 \) form is used. At other levels, (4.1.4), (4.1.5), (4.1.9), and (4.1.10) would formally apply
with \( K_4 = 0 \). The remaining variables \( \eta, \delta, U, V \) require no further modification, so that formally
\[
\hat{\psi}^{n+1} = \psi^{n+1}
\]  
(4.1.11)
for them.

m. Convective Adjustment

At this point, the forecast to time \( n + 1 \) is complete except for the convective adjustment. The time index is first decremented to \( n \) and then the convective adjustment scheme of Manabe et al. (1965) is performed on the relabeled values now referred to as at time \( n \). The convective adjustment is done in several steps. First, any negative moisture values are modified to be zero by taking moisture from adjacent points in longitude and height if enough moisture is available at these points. If there is not enough moisture at these neighboring points for this local correction, then a second correction involving global redistribution is applied. At the same time, the global average mass of the model is set to a specified value to eliminate any very small drift that might occur in very long runs. Second, any layers that are unstable with respect to the dry adiabatic lapse rate are adjusted so that their lapse rates are dry adiabatic (dry convective adjustment). This dry adjustment is normally applied only in the stratosphere. Next, any layers that are supersaturated and are unstable with respect to the moist adiabatic lapse rate are adjusted so that they are just saturated and their lapse rate is moist adiabatic (moist convective adjustment). Finally, any points that are stable but supersaturated are modified to be just saturated (stable condensation). In each of the following four steps, the temperature before that step is indicated by \( \hat{T} \) and the adjusted value in that step by \( T \). Similarly for \( q \).

n. Elimination of Negative Specific Humidity

Because of the truncation in spectral space after the nonlinear advection and condensation processes of the previous time step, the grid-point values of moisture computed from (4.k.1) followed by (4.l.2) can be negative even though the Gaussian grid-point values which determined the coefficients originally are all nonnegative. This property is inherent in spectral-transform models and arises because the spec-
tral resolution is coarser than the grid resolution. It causes no problem with the basic advective fluid motion. However, such negative values can cause havoc in various computations associated with the physical parameterizations. To prevent such problems, we first attempt to make the specific humidity nonnegative by transporting moisture vertically and longitudinally from adjacent points in that plane. If enough moisture is available at these points, a local correction is made. If not, a global-conserving correction is made after the local correction is applied at all points.

If \( \hat{q}_{ki} < 0 \), the moisture deficit is given by

\[
D\hat{q}_{ki} = -\hat{q}_{ki} \Delta \sigma_k p_{si},
\]

(4.n.1)

where \( k \) denotes the vertical index and \( i \) denotes the longitudinal grid index. The amount of moisture available at the neighboring points is

\[
A\hat{q}_{ki} = \max [0, \hat{q}_{k+1,i} \Delta \sigma_k p_{si}] + \hat{q}_{k-1,i} \Delta \sigma_k p_{si}
\]

(4.n.2)

\[
+ \max [0, \hat{q}_{k,i+1} \Delta \sigma_k p_{si+1}] + \hat{q}_{k,i-1} \Delta \sigma_k p_{si-1}.
\]

If \( A\hat{q}_{ki} < D\hat{q}_{ki} \), nothing is done, but if \( A\hat{q}_{ki} \geq D\hat{q}_{ki} \), the following modification is made to the moisture field:

\[
q_{ki} = 0,
\]

(4.n.3)

\[
\beta = 1 - \frac{D\hat{q}_{ik}}{A\hat{q}_{ik}},
\]

(4.n.4)

\[
q(\text{ki})' = \begin{cases} 
\min (\hat{q}(\text{ki})', \beta \hat{q}(\text{ki})') & \text{for } (\text{ki})' = (k + 1, i) \text{ or } (k, i + 1), \\
\beta \hat{q}(\text{ki})' & \text{for } (\text{ki})' = (k - 1, i) \text{ or } (k, i - 1).
\end{cases}
\]

(4.n.5)

The min and max functions are used in (4.n.2) and (4.n.5) so that neighboring points which have not yet been adjusted and which have negative values do not enter into the calculation. Neighboring points that have already been considered for adjustment, but which were not adjusted because of insufficient moisture, enter into the available moisture calculation (4.n.2) as negative values. If the available
moisture (4.n.2) exceeds the deficit (4.n.1), these neighboring points are made less
negative (4.n.5), but are not brought to zero.

At the top and bottom points \( k = 1 \) or \( K \), the out-of-range values \( k - 1 \)
or \( k + 1 \), respectively, are not included in the available moisture calculation (4.n.2)
nor are modified by (4.n.5).

As noted above, this procedure does not always eliminate negative values. Any
remaining negative values are eliminated by a global borrowing scheme which
preserves the global average mass of moisture in layers (Royer, 1986). Let \([\ ]_k\)
denote the mass-weighted global average at each level,

\[
[\hat{q}]_k = \frac{1}{2L} \sum_{j=1}^{J} \sum_{i=1}^{I} \hat{q}_{ij} ps_{ij} \frac{\Delta \sigma_k}{g} w_j, \tag{4.n.6}
\]

where \( i \) is the longitudinal index, \( j \) is the latitudinal, and \( w_j \) is the Gaussian weight. The correction needed to make the moisture nonnegative at each point without
affecting positive values is

\[
\Delta \hat{q}_{ij} = \max (0, -\hat{q}_{ij}). \tag{4.n.7}
\]

To offset the increase in moisture by the process, all values are decreased propor-
tionally,

\[
q_{ij}^n = (1 - \alpha)(\hat{q}_{ij}^n + \Delta \hat{q}_{ij}^n), \tag{4.n.8}
\]

where \( \alpha \) is chosen to conserve globally the moisture in the layer,

\[
\alpha = \frac{[\Delta \hat{q}]^n}{[\hat{q}]^n + [\Delta \hat{q}]^n}. \tag{4.n.9}
\]

Typical values of \( \alpha \) in long runs vary from \( 1 \times 10^{-4} \) at the surface to \( 1 \times 10^{-3} \) at 300 mb decreasing to \( 1 \times 10^{-5} \) or less in the stratosphere.

This final corrected value of \( q \) is then used everywhere in the model except
possibly in the radiation. The radiation assumes an effective minimum value for \( q \)
of \( 1.5 \times 10^{-6} \ \text{kg kg}^{-1} \) in the bottom six levels and \( 0.1 \times 10^{-6} \ \text{kg kg}^{-1} \) in the other
levels; however, \( q \) is not reset to these values for the remaining calculations.
o. Mass Adjustment

To prevent the mass of the model from drifting slightly in very long runs, the global average dry mass is reset every time step to a specific value,

\[
p_{n_{s,ij}} = \hat{p}_{s,ij} + \bar{p} - \left\{ [\hat{p}_s]^n - g \sum_{k=1}^{K} [\hat{q}_k] \right\}, \tag{4.o.1}
\]

where

\[
[\hat{p}_s] = \frac{1}{2I} \sum_{j=1}^{J} \sum_{i=1}^{I} p_{s,ij} w_j, \tag{4.o.2}
\]

and \( \bar{p} \) is the specified value of the dry mass taken as 98222 Pa for average mountains, following Trenberth and Christy (1985). The correction in long runs is generally less than .02 Pa per time step and not of consistent sign, so that any accumulated drift without the correction would imply a much smaller average rate.

p. Dry Adiabatic Adjustment

If a layer is unstable with respect to the dry adiabatic lapse rate, dry adiabatic adjustment is performed. The layer is stable if

\[
\frac{\partial T}{\partial \sigma} < \frac{\kappa T}{\sigma}. \tag{4.p.1}
\]

In finite-difference form, this becomes

\[
\hat{T}_{k+1} - \hat{T}_k < C_{1k+1}(\hat{T}_{k+1} + \hat{T}_k) + \delta, \tag{4.p.2}
\]

where

\[
C_{1k+1} = \frac{\kappa (\sigma_{k+1} - \sigma_k)}{2\sigma_{k+1/2}}. \tag{4.p.3}
\]

If there are any unstable layers in a column, the temperature is adjusted so that (4.p.2) is satisfied everywhere in the column. The variable \( \delta \) represents a convergence criterion. The adjustment is done so that sensible heat is conserved.

\[
C_p(T_k \Delta \sigma_k + T_{k+1} \Delta \sigma_{k+1}) = C_p(\hat{T}_k \Delta \sigma_k + \hat{T}_{k+1} \Delta \sigma_{k+1}), \tag{4.p.4}
\]

and so that the layer has neutral stability.

\[
T_{k+1} - T_k = C_{1k+1}(T_{k+1} + T_k), \tag{4.p.5}
\]
where $\Delta \sigma_k$ is given by (3.b.5). As mentioned above, the hats denote the variables before adjustment.

Thus, the adjusted temperatures are given by

$$T_{k+1} = \frac{\Delta \sigma_k}{\Delta \sigma_{k+1} + \Delta \sigma_k C^{2k+1}} \hat{T}_k + \frac{\Delta \sigma_{k+1}}{\Delta \sigma_{k+1} + \Delta \sigma_k C^{2k+1}} \hat{T}_{k+1},$$

(4.p.6)

and

$$T_k = C^{2k+1} T_{k+1},$$

(4.p.7)

where

$$C^{2k+1} = \frac{1 - C^{1_{k+1}}}{1 + C^{1_{k+1}}}.$$  

(4.p.8)

Whenever the two layers undergo dry adjustment, the moisture is assumed to be completely mixed by the process as well. Thus, the specific humidity is changed in the two layers in a conserving manner to be the average value of the original values,

$$q_{k+1} = q_k = (\hat{q}_{k+1} \Delta \sigma_{k+1} + \hat{q}_k \Delta \sigma_k)/(\Delta \sigma_{k+1} + \Delta \sigma_k).$$

(4.p.9)

The layers are adjusted iteratively. Initially, $\delta = 0.01$ in the stability check (4.p.2). The column is passed through from $k = 1$ to $K - 1$ up to 15 times; each time unstable layers are adjusted until the entire column is stable. If convergence is not reached by the 15th pass, the convergence criterion is doubled, a message is printed, and the entire process is repeated. If $\delta$ exceeds 0.1 and the column is still not stable, the model stops.

In the frozen version of the model, the dry convective adjustment is only applied to the top four levels of the model. The vertical diffusion provides the stabilizing vertical mixing at other levels. Thus, in practice, momentum is mixed as well as moisture and potential temperature in the unstable case.

q. Moist Adiabatic Adjustment

If a layer is supersaturated and the lapse rate exceeds the moist–adiabatic lapse rate, the temperature and specific humidity are modified to make the layer stable and saturated. A point is defined to be supersaturated if

$$\frac{\hat{q}_k}{\hat{q}_{s_k}} > 0.9999.$$  

(4.q.1)
The saturation specific humidity \( q_{8k} \) is given by

\[
\hat{q}_{8k} = \varepsilon \frac{\hat{e}_{8k}}{p_k - (1 - \varepsilon)\hat{e}_{8k}}. \tag{4.q.2}
\]

The saturation vapor pressure \( \hat{e}_{8k} \) for temperature \( \hat{T}_k \) is determined by linear interpolation within a table of values specified for every degree of temperature, and \( \varepsilon = 0.622 \). The moist adiabatic lapse rate is given by

\[
\tilde{\gamma}_{w_{k+1/2}} = \frac{R}{C_p} \left( \frac{\hat{T}_k + \hat{T}_{k+1}}{2\sigma_{k+1/2}} \right) \left[ p_{k+1/2} + \frac{2L}{R} \frac{\hat{e}_{8k+1/2}}{(\hat{T}_k + \hat{T}_{k+1})} \right] \right] \right. \\
\left/ \left[ p_{k+1/2} + \alpha \varepsilon \frac{L}{C_p} \left( \frac{d\hat{e}_s}{dT} \right)_{k+1/2} \right] \right. 
\] \tag{4.q.3}

where \( \hat{e}_{8k+1/2} \) is the saturation vapor pressure for temperature \( \hat{T}_{k+1/2} = (\hat{T}_k + \hat{T}_{k+1})/2 \) determined by linear interpolation within a table of values specified for every degree of temperature and

\[
\left( \frac{d\hat{e}_s}{dT} \right)_{k+1/2} = \hat{e}_s \left( \hat{T}_{k+1/2} + .5 \right) - \hat{e}_s \left( \hat{T}_{k+1/2} - .5 \right). \tag{4.q.4}
\]

If

\[
\frac{\hat{T}_{k+1} - \hat{T}_k}{\sigma_{k+1} - \sigma_k} > \tilde{\gamma}_{w_{k+1/2}} + \delta, \tag{4.q.5}
\]

and levels \( k \) and \( k + 1 \) are supersaturated by (4.q.1), the layer is adjusted. If several contiguous layers are unstable, they are adjusted simultaneously. Let \( k_{BOT} \) be the index of the temperature at the bottom of a contiguous set of unstable layers and \( k_{TOP} \) be the index of the temperature at the top, so (4.q.5) holds for all \( k + 1 \leq k_{BOT} \) and for all \( k \geq k_{TOP} \). (Remember that the vertical index increases from top to bottom.) Then \( \hat{T}_k \) and \( \hat{q}_k \) are adjusted for \( k_{BOT} \geq k \geq k_{TOP} \) so that

\[
T_k = T_{k+1} - \tilde{\gamma}_{w_{k+1/2}}(\sigma_{k+1} - \sigma_k) \quad \text{for} \quad k = k_{BOT-1}, \cdots, k_{TOP}, \tag{4.q.6}
\]

and

\[
\sum_{k=k_{BOT}}^{k_{TOP}} (C_pT_k + Lq_k)\Delta \sigma_k = \sum_{k=k_{BOT}}^{k_{TOP}} (C_p\hat{T}_k + L\hat{q}_k)\Delta \sigma_k, \tag{4.q.7}
\]

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where, as before, the hats denote the variables before adjustment. Equation (4.q.6) can be rewritten

\[ T_k = T_{k_{BOT}} - \sum_{t=k_{BOT}-1}^{k} \hat{\gamma}_{t+1/2}(\sigma_{t+1} - \sigma_t), \]

(4.q.8)

for \( k = k_{BOT}-1, \ldots, k_{TOP} \). Since the layers are saturated after adjustment,

\[ q_k = \left[ \hat{q}_{s_k} + \frac{d\hat{q}_{s_k}}{dT}(T_k - \hat{T}_k) \right], \]

(4.q.9)

where

\[ \frac{d\hat{q}_{s_k}}{dT} = \left[ \frac{\sigma_k p_s}{(\sigma_k p_s - (1 - \epsilon)\hat{e}_s)} \right] \frac{L}{RH_{20}^2 T_k^2} \hat{q}_{s_k}, \]

(4.q.10)

and as before, \( \hat{q}_{s_k} \) is given by (4.q.2). If we define the difference between \( T_k \) and \( T_{k_{BOT}} \) by

\[ \Delta T_k = \sum_{t=k_{BOT}-1}^{k} \hat{\gamma}_{t+1/2}(\sigma_{t+1} - \sigma_t), \]

(4.q.11)

where \( \hat{\gamma}_{t+1/2} \) is given by (4.q.3) using the unadjusted temperatures, and (4.q.7)–(4.q.11) can be combined to give an equation for \( T_{k_{BOT}} \),

\[
\sum_{k=k_{BOT}}^{k_{TOP}} \left[ C_p(\hat{T}_k + \Delta T_k) + L \left( \hat{q}_k - \hat{q}_{s_k} + \frac{d\hat{q}_{s_k}}{dT}(\hat{T}_k + \Delta T_k) \right) \right] \Delta \sigma_k
\]

(4.q.12)

Once \( T_{k_{BOT}} \) is known, \( T \) at the other levels can be computed from (4.q.5) replacing \( \hat{T} \) by \( T \) on the left, without the \( \delta \) factor, and with the inequality sign replaced by equality, and \( q_k \) can be computed from (4.q.9). The iterative procedure represented by (4.q.6) and (4.q.7) is repeated until convergence (4.q.5) with a maximum of 20 iterations for \( k_{BOT} \geq k \geq k_{TOP} \), after which other sets of contiguous unstable layers in the column are adjusted by the same procedure. Once all subsets of the column have been adjusted, the column is searched again for unstable layers using the criteria (4.q.5) and (4.q.1). Initially, \( \delta = 0.01 \) and if convergence is not reached in 15 passes, it is modified in the same way as in the dry convective adjustment.
described following (4.p.9). During the iterative procedure, two indices are set for later use in the cloud parameterization associated with the radiation calculation. These are $K_{\text{CLOW}}$ and $K_{\text{CHI}}$ which specify the lowest level (largest $k$) and one plus the highest level (smallest $k$), respectively, affected by the moist convective adjustment. The change in the mixing ratio produced by this adjustment is assumed to produce precipitation. The condensation is assumed to take place where the temperature changed rather than by the simple change in $q$. Energetically, they are equivalent, but the levels at which they occur are different.

$$q_{\text{cu}_k} = p_s \frac{C_p}{\rho} \left( T_k - T_k^* \right) \frac{\Delta \sigma_k}{g}, \quad (4.q.13)$$

$$PREC_n = PREC_{n-1}^n + \frac{1}{2\Delta t} \sum_{k=1}^{K} q_{\text{cu}_k} / \rho_{H20}.$$

The $\rho_{H20}$ factor is included to convert the units from mass per unit area per second (kg m$^2$ s$^{-1}$) to depth per second (m s$^{-1}$) of liquid water. The rate of condensation at each level from the unstable adjustment $q_{\text{cu}_k} / (2\Delta t)$ is stored for use in the cloud calculation associated with the radiation. The average convective precipitation rate ($PREC_c$) is initialized to zero after each history-tape write, accumulated over every time step between history-tape writes, and normalized before each history-tape write.

r. Stable Condensation

If the lapse rate is stable, but the moisture is supersaturated at a point, i.e., if

$$\frac{\hat{q}_k}{\hat{q}_{sk}} > 1.0, \quad (4.r.1)$$

the temperature and moisture are adjusted simultaneously so that the point is just saturated. Saturation is defined as in the moist convective adjustment (4.q.1). The new specific humidity is given by

$$q_k = \left[ \hat{q}_{sk} + \frac{d\hat{q}_{sk}}{dT} \left( T_k - T_k^* \right) \right], \quad (4.r.2)$$
where \( q'_{sk} \) is given by (4.q.2) and \( dq_{sk}/dT \) by (4.q.10). The temperature change due to the release of latent heat during condensation is

\[
(T_k - \hat{T}_k) = \frac{L}{C_p} (\hat{q}_k - q_k).
\]  
(4.r.3)

Substitution of (4.r.3) into (4.r.2) gives the adjusted specific humidity,

\[
q_k = \hat{q}_k - (\hat{q}_k - \hat{q}_{sk}) \left/ \left(1 + \frac{L}{C_p} \frac{dq_{sk}}{dT}\right)\right..
\]  
(4.r.4)

Equations (4.r.4) and (4.r.3) are iterated twice. The corresponding stable precipitation at that level is given by

\[
q_{csk} = p_s (\hat{q}_k - q_k) \Delta \sigma_k / g.
\]  
(4.r.5)

The rate of stable precipitation \( q_{csk} / (2\Delta t) \) is added to that from the moist convective adjustment (4.q.13) to give the total rate of precipitation at each level for later use in the cloud parameterization.

\[
q_{ctk} = q_{csk} / (2\Delta t) + q_{cu_k} / (2\Delta t).
\]  
(4.r.6)

The stable precipitation over the entire column is given by

\[
PREC^n_s = PREC^{n-1}_s + \frac{1}{2\Delta t} \sum_{k=1}^{K} q_{csk} / \rho_{H2O}.
\]  
(4.r.7)

As with the convective precipitation (4.q.14), the average stable precipitation is initialized to zero after each history-tape write, accumulated over every time step between history-tape writes, and normalized before each history-tape write.

\section*{Cloud Parameterization}

Convective clouds form in the columns in which the moist convective adjustment described in section 4q takes place. The variables \( K_{LOW} \) and \( K_{HI} \), described preceding (4.q.13), specify the lowest level (largest \( k \)) and one plus the highest level (smallest \( k \)), respectively, affected by the moist convective adjustment. Convective clouds are thus centered at the temperature levels \( K_{LOW} \) at the bottom to \( K_{HI} - 1 \) at the top; the clouds are assumed to extend through the entire
layer from $k + 1/2$ to $k - 1/2$ for each $K_{CLOW} \geq k \geq K_{CHI} - 1$. Clouds are not allowed to form in the very thin first layer of the model or in the top two layers so that for the following $K_{CLOW}$ and $K_{CHI}$ are constrained by

$$K_{CLOW} \leq K - 1,$$

$$K_{CHI} - 1 > 2.$$  

The fractional cloud cover $A_k$ in each layer is given by

$$A_k = 0.3/(K_{CLOW} - K_{CHI} + 2), \quad K_{CLOW} \geq k \geq K_{CHI} - 1,$$

(4.s.2)

where $K_{CLOW} - K_{CHI} + 2$ is the number of layers having convective clouds. The total area of sky covered with these clouds $A_T$, assuming random overlap, is

$$A_T = 1 - \prod_{k=K_{CHI}-1}^{K_{CLOW}} (1 - A_k).$$

(4.s.3)

Thus, (4.s.2) and (4.s.3) imply that the total column cloud cover due to convective clouds is 30 percent. Note that clouds are formed at all levels between $K_{CLOW}$ and $K_{CHI} - 1$ even though moist convective adjustment may not have occurred at all these levels. The convective cloud scheme also forms a cloud layer of 95 percent just above the top convective region and just below the lowest convective layer. These cloud layers have emissivities of 1. If the moist adjustment occurred between the bottom two levels only, the cloud is assumed to form at the second and third levels from the bottom, i.e., if $A_{K-1} \neq 0$ and $A_{K-2} = 0$, $A_{K-2} = A_{K-1}$ after $A_{K-1}$ has been halved.

Nonconvective clouds form in columns where moist convective adjustment did not take place and wherever the stable condensation described in section 4r takes place. Thus, if $q_{ctk}$ given by (4.r.6) is not zero, nonconvective clouds are formed with 95 percent fractional cloud cover, i.e.,

$$\text{if } q_{ctk} > 0, \ A_k = 0.95, \ 3 \leq k \leq K - 1.$$  

(4.s.4)

Note again that no clouds form in the first model level $k = K$ or the top two levels. A further modification is made. If clouds are formed in either the second or third layer, they are assumed to form in both. Thus,
if \( q_{ctK-1} > 0 \) or \( q_{ctK-2} > 0 \),

\[
A_{K-1} = 0.95, \\
A_{K-2} = 0.95.
\] (4.5.5)

The nonconvective clouds are assigned emissivities which are a function of liquid water content given in (2.g.50).

In the description above, the index \( k \) corresponds to that of temperature levels (Figure 1), and the cloud is assumed to extend throughout the layer from \( k + 1/2 \) to \( k - 1/2 \). The radiation and cloud routines use a modified subscripting running from bottom to top and applied at half-sigma levels rather than that employed above to represent the same thing.

**t. Final/Initial Grid Values**

The modification of negative mixing ratio, the convective adjustments, and the stable condensation described above complete the computation procedure to give variables \( \psi^n \) as indicated by (3.a.9). The half-time smoothed values (4.a.11) at time \( n - 1 \) are accessed from their special buffer, and the second half of the time filter (3.a.10) is now applied to give the time-filtered values \( \overline{\psi}^{n-1} \) at time \( n - 1 \). Grid-point variables \( \psi^n \) and \( \overline{\psi}^{n-1} \) are now available for output to history files. The first half of the time filter (3.a.11) is applied to time level \( n \) and the result stored in a special buffer.

At this point, a full time cycle has been completed and we are back at the starting point with grid-point values at time \( n \) of the prognostic variables \( \eta^n, \delta^n, T^n, q^n, \ell np_s^n \), surface geopotential \( \Phi_s \), the wind components \( U^n \) and \( V^n \), and the derivatives of surface pressure, \( \partial \ell np_s^n / \partial \lambda \) and \( (1 - \mu^2) \partial \ell np_s^n / \partial \mu \). In addition, the time-smoothed values at time \( n - 1 \) of the prognostic variables are also available, \( \overline{\eta}^{n-1}, \overline{\delta}^{n-1}, \overline{T}^{n-1}, \overline{q}^{n-1}, \) and \( \overline{\ell np_s}^{n-1} \).

**u. Optional Interactive Surface Hydrology**

This section describes the modifications to the above algorithms that are implemented when the optional interactive surface hydrology is invoked.
First, the precipitation from the moist convective adjustment (4.q.13) and from the stable condensation (4.r.5) increases either the soil moisture or snow cover depending on the surface temperature and temperatures of the first two model levels above the surface.

If \( T_{n-1}^S \geq 273.16 \), or \( T_K^n \geq 273.16 \), or \( T_{K-1}^n \geq 273.16 \),

\[
W^{*n} = W^{n-1} + \frac{1}{2} \sum_{k=1}^{K} \frac{(q_{CU_k} + q_{CS_k})}{\rho H_2O},
\]

\[
S^{*n} = S^{n-1}.
\]

If \( T_{n-1}^S < 273.16 \), and \( T_K^n < 273.16 \), and \( T_{K-1}^n < 273.16 \),

\[
W^{*n} = W^{n-1},
\]

\[
S^{*n} = S^{n-1} + \frac{1}{2} \sum_{k=1}^{K} \frac{(q_{CU_k} + q_{CS_k})}{\rho H_2O}.
\]  \hspace{1cm} (4.u.1)

The \( \frac{1}{2} \) factor is included to account for the soil moisture and snow cover being forecast via forward differences over an interval \( \Delta t \), while the precipitation rates are effectively calculated via centered differences over an interval \( 2\Delta t \).

If the soil moisture then exceeds the field capacity, the runoff is accumulated and the soil moisture is set to the field capacity.

If \( W^{*n} > W_{FC} \),

\[
r_f^{*n} = r_f^{n-1} + \frac{(W^{*n} - W_{FC})}{\Delta t},
\]

\[
W^{*n} = W_{FC}.
\]

Otherwise

\[
r_f^{*n} = r_f^{n-1}.
\]  \hspace{1cm} (4.u.2)

As with the precipitation rates, the average runoff rate is initialized to zero after each history-tape write, accumulated over every time step between history-tape writes, and normalized by the number of time steps between history-tape writes before each history-tape write.
Prior to the surface-temperature calculation (4.c.1), the excess energy that would be available to melt snow if the surface temperature were 273.16 K is calculated over snow-covered areas. 

\[ E^n_z = -\left\{ \sigma_B T^4_s - F^s - F^l(p_e)^n + H^n_{K+1/2} + LR^n_{K+1/2} + Q^n_{ICE} \right\}, \]  

(4.u.3)

where \( T_s = 273.16 \) K and \( H \) and \( R \) are as (4.c.6) and (4.c.2) but with \( T^n_s \) replaced by \( T^* \). If energy is available to melt snow, the snow cover is decreased accordingly and the surface temperature \( T^n_s \) is set to 273.16 K rather than calculated by the iterative method (4.c.28). The amount of snow that melts is

\[ M^n_e = \begin{cases} \frac{\Delta t E^n_z}{L_{OH_2O}} & \text{if } E^n_z > 0, \\ 0 & \text{if } E^n_z \leq 0. \end{cases} \]  

(4.u.4)

If the value is nonzero, the snow cover and soil moisture are changed accordingly.

In the event that the energy available exceeds the amount required to melt all the snow, the excess goes into warming the surface temperature above 273.16.

\[ M^{*n}_e = \min (M^n_e, S^{*n}), \]  

(4.u.5)

\[ S^{**n} = S^{*n} - M^{*n}_e, \]  

(4.u.6)

\[ W^{**n} = W^{*n} + M^{*n}_e, \]  

(4.u.7)

Once again, if the soil moisture exceeds the field capacity \( W_{FC} \), the runoff is accumulated and the soil moisture is set to the field capacity.

If \( W^{**n} > W_{FC} \),

\[ r^{**n}_f = r^{*n}_f + (W^{**n} - W_{FC})/\Delta t, \]

\[ W^{**n} = W_{FC}. \]

Otherwise

\[ r^{**n}_f = r^{*n}_f. \]  

(4.u.8)
If there is no snowmelt or if there is excess energy after melting all the snow \( (M^n_c > S^*n) \), the surface-temperature calculation proceeds as described in section 4c. If there is excess energy, an extra term \( L \rho_{H_2O} S^*n/\Delta t \) is added to (4.c.1) and (4.c.29) to account for the energy used to melt the snow.

The only other modification in the surface-temperature calculation is that the wetness factor \( D_W \) for the evaporation (4.c.2) and described following (4.c.26) is calculated from the soil moisture

\[
D_W = \begin{cases} 
1.0 & \text{if } W^{**n} \geq W_C, \\
\frac{W^{**n}}{W_C} & \text{if } W < W_C, 
\end{cases}
\]  
(4.u.9)

with the critical value \( W_C \) determined from the field capacity \( W_{FC} \)

\[
W_C = 0.75 \ W_{FC},
\]  
(4.u.10)

and, as stated before, \( W_{FC} = 0.15 \) m.

After the new surface temperature is calculated, the snow cover and soil moisture are modified to account for sublimation or evaporation. Over snow-covered land,

\[
S^n = \max (0.0, S^{**n} - \Delta t R^n_{K+1/2}/\rho_{H_2O}).
\]  
(4.u.11)

The max function is included in case there is not enough snow to account for the sublimation in the surface-temperature calculation. In this case, the excess comes from the soil moisture after the snow is gone. Over bare land,

\[
W^n = W^{**n} - \Delta t R^n_{K+1/2}/\rho_{H_2O}.
\]  
(4.u.12)

Over land which lost its snow cover this time step,

\[
W^n = W^{**n} - (\Delta t R^n_{K+1/2}/\rho_{H_2O} - S^{**n}).
\]  
(4.u.13)

The soil moisture \( W^n \) is not allowed to become negative. If there is inadequate soil moisture to accommodate the evaporative flux, we accept the resulting small discrepancy in the energetics. Once again, the runoff is calculated in accordance with the possible negative fluxes.
If $W^n > W_{FC}$,

$$r^n_f = r^{**}_f + (W^n - W_{FC})/\Delta t,$$

$$W^n = W_{FC}.$$  

Otherwise

$$r^n_f = r^{**}_f.$$  \hspace{1cm} (4.u.14)
5. INITIAL AND BOUNDARY DATA

a. Initial Data

The previous section describes the central time loop of the model. In this section, we describe how the loop is started from data consistent with the spectral truncation. The basic initial data for the model consist of values of $u, v, T, q, p_s,$ and $\Phi_s$ on the Gaussian grid at time $t = 0$. From these, $U, V, T', \text{ and } \ell n p_s$ are computed on the grid using (2.b.2), (2.b.3), and (2.b.7). The Fourier coefficients of these variables $U^m, V^m, T'^m, q^m, \ell n p^m_s$, and $\Phi^m_s$ are determined via an FFT subroutine (4.h.2), and the spherical harmonic coefficients $T'^m_n, q^m_n, \ell n p^m_{sn}$, and $\Phi^m_{sn}$ are determined by Gaussian quadrature (4.h.3). The relative vorticity $\zeta$ and divergence $\delta$ spherical harmonic coefficients are determined directly from the Fourier coefficients $U^m$ and $V^m$ using the relations,

\begin{align}
\zeta &= \frac{1}{a(1 - \mu^2)} \frac{\partial V}{\partial \lambda} - \frac{1}{a} \frac{\partial U}{\partial \mu}, \\
\delta &= \frac{1}{a(1 - \mu^2)} \frac{\partial U}{\partial \lambda} + \frac{1}{a} \frac{\partial V}{\partial \mu}.
\end{align}

(5.a.1)

(5.a.2)

The relative vorticity and divergence coefficients are obtained by Gaussian quadrature directly, using (4.h.6) for the $\lambda$–derivative terms and (4.h.9) for the $\mu$–derivatives.

Once the spectral coefficients of the prognostic variables are available, the grid–point values of $\zeta, \delta, T', q, \ell n p_s,$ and $\Phi_s$ may be calculated from (4.k.1), the gradient $\nabla \ell n p_s$ from (4.k.4) and (4.k.5), and $U$ and $V$ from (4.k.10) and (4.k.11). The absolute vorticity $\eta$ is determined from the relative vorticity $\zeta$ by adding the appropriate associated Legendre function for $f$ (3.c.4). This process gives grid–point fields for all variables, including the surface geopotential, that are consistent with the spectral truncation even if the original grid–point data were not. These grid–point values are then convectively adjusted (including the mass and negative moisture corrections).

The first time step of the model is forward semi–implicit rather than centered.
semi-implicit, so only variables at \( t = 0 \) are needed. The model performs this forward step by setting the variables at time \( t = -\Delta t \) equal to those at \( t = 0 \) and by temporarily dividing \( 2\Delta t \) by 2 for this time step only. This is done so that formally the code and the centered prognostic equations of section 4 also describe this first forward step and no additional code is needed for this special step. The model loops through as indicated sequentially in section 4. The time step \( 2\Delta t \) is set to its original value before beginning the second time step.

b. Boundary Data

In addition to the initial grid-point values described in the previous section, the model also requires lower boundary conditions. The required data are surface temperature \( (T_s) \) at each ocean point, the surface geopotential at each point, a flag at each point to indicate whether the point is land, ocean, or sea ice, the soil moisture at each point, and the snow amount at each point. A surface temperature must also be provided at nonocean points, but it is only used as the first guess in the \( T_s \) calculation the first time and never thereafter. The soil moisture is used to determine the evaporation factor \( D_w \) in (4.c.2). Positive snow cover indicates that the grid point is covered with snow. The actual amount is not used in the frozen version. If the optional interactive soil hydrology is invoked, then soil moisture and snow cover are used as initial values for those fields which are then forecast as described in section 4u. These boundary data must be included in the data file containing the initial conditions.

When the model is run in the seasonal mode, the sea-surface temperatures, sea-ice locations, and snow cover are changed to reflect the seasonal changes. The sea-surface temperatures and sea-ice distributions are specified from the monthly mean analyses of Alexander and Mobley (1976). The mean monthly sea-surface temperature values are assigned the mid-month date and updated every 12 h at each grid point using linear interpolation.

The sea ice and snow cover are updated at the beginning of every month and are held fixed during the month. The snow cover is a function of latitude only. The minimum latitude for snow cover is estimated from climatic atlases for January
(42°, -55°) and July (68°, -51°). We calculated the minimum latitudes for other months from a cosine function, assuming January and July are the two extremes. All land points poleward of these snow lines are snow-covered.

The radiation parameterization requires monthly mean ozone mixing ratios to be specified as a function of the latitude grid, 23 vertical pressure levels, and time. The ozone path lengths are evaluated from the mixing-ratio data. The path lengths are interpolated to the model σ-layer interfaces for use in the radiation calculation. In the standard version, these path lengths are independent of longitude. The model includes an option to make them a function of longitude to account for the vertical displacement of the σ-levels over mountains. As with the sea-surface temperatures, the seasonal version assigns the monthly averages to the mid-month date and updates them every 12 h via linear interpolation. The actual mixing ratios used in the standard version were derived by Chervin (1986) from analyses of Dütsch (1978).

The radiation parameterization also requires that surface albedo be specified on the model grid for land points. The land albedos are constants (independent of time or moisture conditions); land albedos for snow-covered points are 50% weighted values of snow albedos and the constant land albedos. The surface albedo data for the CCM1 are composed of five quantities—the fraction of strong zenith-angle dependence and four surface albedos (two zenith angles and two spectral range groups). The original source of these data is the Matthews (1983) 1°–x–1° global 32-type vegetation data set, which was reduced to ten vegetation types. Narrow-band (0.2–0.5 μm, 0.5–0.7 μm, 0.7–0.85 μm, and 0.85–4.0 μm) spectral albedos (for diffuse incident radiation) were ascribed to each of these ten types (Briegleb et al., 1986). The ten surface types were segregated into two groups, based on solar zenith-angle dependence (strong or weak) and averaged to the CCM1 spectral intervals (0.2–0.9 μm, 0.9–4.0 μm). The 1°–x–1° data set was then averaged to the required horizontal resolution of the CCM1.
6. STATISTICS AND BUDGET CALCULATIONS

a. Statistics

At specified times during a model run, selected global average statistics are computed for diagnostic purposes. Let \( f_3 \) denote a global and vertical average and \( f_2 \) a horizontal global average. For an arbitrary variable \( \psi \), these are defined by

\[
\int_3 \psi dV = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} \psi_{ijk} w_j \Delta \sigma_k / 2I, \quad (6.a.1)
\]

and

\[
\int_2 \psi dA = \sum_{j=1}^{J} \sum_{i=1}^{I} \psi_{ijk} w_j / 2I, \quad (6.a.2)
\]

where recall that

\[
\sum_{j=1}^{J} w_j = 2. \quad (6.a.3)
\]

The quantities monitored are:

- global rms \( \eta(s^{-1}) = \left[ \int_3 (\eta^n)^2 dv \right]^{1/2} \),
- \( \delta(s^{-1}) = \left[ \int_3 (\delta^n)^2 dV \right]^{1/2} \),
- global rms \( T \) (K) = \( \left[ \int_3 (T_o + T''n)^2 dV \right]^{1/2} \),
- global average mass times g (Pa) = \( \int_2 p^n dA \),
- global average mass of moisture (kg m\(^{-2}\)) = \( \int_3 p^n q^n / g dV \),
- global average potential energy (J m\(^{-2}\)) = \( \int_3 C_p(T_o + T''n)p^n / g dV \)
  \[+ \int_2 p^n \Phi_s / g dA, \quad (6.a.9)\]
global average kinetic energy (J m$^{-2}$) = \( \int \left[ (u^n)^2 + (v^n)^2 \right] p^n/(2g) dV \), (6.a.10)

global average total energy = global average potential energy 
+ global average kinetic energy. (6.a.11)

The rms \( \eta, \delta, \) and \( T \) are also computed as a function of \( \sigma \),

\[
(rms \, \eta)_k = \left[ \int_2 (\eta_k^2) dA \right]^{1/2}, 
\]

\[
(rms \, \delta)_k = \left[ \int_2 (\delta_k^2) dA \right]^{1/2}, 
\]

\[
(rms \, T)_k = \left[ \int_2 (T_0^2 + T_k^2) dA \right]^{1/2}. 
\]

**b. Budget**

The model also includes a diagnostic energy-budget calculation. Various global-average source/sink and conversion terms are accumulated during the model run. At selected times, the initial global averages, current global averages, and accumulated global-average source/sink and conversion terms are printed out. In addition, the deficits, i.e., the sum of all source/sink or conversion terms not explicitly calculated, are determined as residuals. The initial values and accumulation quantities can be reinitialized during the run to monitor the budget over specified intervals.

The budget is calculated for one of the two interlaced time sequences associated with centered time differences. The basic time differences (3.a.5)–(3.a.7) can be written

\[
\psi^{n+2} = \psi^n + 2\Delta t (\Delta \psi)^{n+1}. 
\]

Thus, the value at time \( n_o + 2N \) is related to the value at some initial time \( n_o \) by

\[
\psi^{n_o + 2N} = \psi^{n_o} + 2\Delta t \sum_{i=1}^{N} (\Delta \psi)^{n_o + 2i - 1}. 
\]
The two interlaced sequences are then given by two sequential values for \( n_0 \). Of course, the two time sequences are not completely independent since the model includes the time filter (3.a.1) to couple them together. However, the change made to \( \psi^{n+2} \) by the time filter can be interpreted formally as part of (6.b.1), and its effect in the accumulation of one of the sequences (6.b.2) can be monitored either explicitly or by including its effect in the residual. In either case, if it becomes a major contributor to either sequence, it should be known.

Currently, the following global budget integrals are monitored. The integral notation is defined as before (6.a.1, 6.a.2). The global averages are computed for variables at time \( n - 1 \) after completion of the time smoothing described at the end of section 4. Therefore, the initial values are computed during iteration \( n = n_0 + 1 \). Later values for monitoring are computed during iteration \( n = n_0 + 2N + 1 \).

\[
\overline{Q}^{n-1} = \frac{L}{g} \int q^{n-1} p_s^n dV, \tag{6.b.3}
\]

\[
\overline{PE}^{n-1} = \frac{C_P}{g} \int T^{n-1} p_s^n dV, \tag{6.b.4}
\]

\[
\overline{SE}^{n-1} = \frac{1}{g} \int \Phi_s p_s^n dA, \tag{6.b.5}
\]

\[
\overline{KE}^{n-1} = \frac{1}{2g} \int [(\overline{u}^{n-1})^2 + (\overline{v}^{n-1})^2] p_s^n dV, \tag{6.b.6}
\]

\[
\overline{TE}^{n-1} = \overline{KE}^{n-1} + \overline{PE}^{n-1} + \overline{SE}^{n-1}. \tag{6.b.7}
\]

The units of all integrals have been normalized to be in energy per unit area (joules m\(^{-2}\)).

**Moisture.** The source/sink terms monitored for the moisture are:

- Stable condensation

\[
DWCDST = -\frac{L}{g} \int p_s^n (\overline{q}^n - q^n) dV, \tag{6.b.8}
\]

where \( \overline{q} \) now denotes the mixing ratio before the stable adjustment (4.r.4) and \( q \) the value after. The convective adjustment is calculated for variables at time \( n \).
Therefore, the accumulation corresponding to (6.b.2) is over

$$\sum_{i=1}^{N} (\tilde{q} - q)^{n_o+2i}.$$  \hfill (6.b.9)

- Unstable convective condensation

$$DWCDCU = -\frac{L}{g} \int_{\mathcal{V}} p_r^n (\tilde{q}^n - q^n) \, dV,$$  \hfill (6.b.10)

where $q$ now denotes the mixing ratio after the moist convective adjustment (4.q.9) and $\tilde{q}$ the value before. The time index for the accumulation is the same as for the stable condensation above (6.b.9).

- Evaporation from surface

$$DWFLUX = 2\Delta t L \int_{A} R_{K+1/2}^n \, dA,$$  \hfill (6.b.11)

where $R_{K+1/2}^n$ is given by (4.c.2). Equation (6.b.11) corresponds to time index $n$. Therefore, the accumulation for this term is done when $n = n_o + 2i - 1, i = 1, N,$ as indicated in (6.b.2).

The moisture deficit is calculated as a residual of the change in the global average minus the source/sink term.

$$DWDEF = \overline{Q}^{n_o+2N} - \overline{Q}^{n_o} - (DWCDST + DWDCCU + DWFLUX).$$  \hfill (6.b.12)

The vertical diffusion and negative specific–humidity remover both conserve moisture (except for round–off error). The horizontal advection is quadratic and therefore conserves with the usual choice of Gaussian grid. The vertical advection finite differences also conserve. Therefore, the deficit is attributable to the horizontal diffusion, time filter or nonconservation of the mass of the atmosphere ($p\sigma$).

Energy. The terms monitored for the energy budget are:

- Sensible heat flux from surface

$$DHFLUX = 2\Delta t \sigma K \int_{A} H_{K+1/2}^n \, dA,$$  \hfill (6.b.13)
where $H'_{K+1/2}$ is given by (4.c.6). The sensible heat flux is accumulated when $n = n_o + 2i - 1$, as with the evaporation (6.b.11).

- **Radiation**

$$DHRAD = 2\Delta t \frac{C_p}{g} \int_3 p^n_s (Q^n_{s_h} + Q^n_{lw_h}) dV,$$  \hspace{1cm} (6.b.14)

where $Q^n_{s_h}$ and $Q^n_{lw_h}$ are the heating rate returned by the radiation. The radiation is accumulated when $n = n_o + 2i - 1$.

- **Release of latent heat—stable condensation**

$$DHCDST = \frac{C_p}{g} \int_3 p^n_s (T^n - \hat{T}^n) dV,$$  \hspace{1cm} (6.b.15)

where $T$ denotes the temperature after the stable adjustment (4.r.3) and $\hat{T}$ the value before. As with the condensation, the latent heat is accumulated for $n = n_o + 2i, i = 1, N$ (6.b.9).

- **Release of latent heat—unstable convective adjustment**

$$DHCDCU = \frac{C_p}{g} \int_3 p^n_s (T^n - \hat{T}^n) dV,$$  \hspace{1cm} (6.b.16)

where $T$ now denotes the temperature after the moist convective adjustment (4.q.8) and (4.q.12) and $\hat{T}$ the value before. Note that from (4.r.3) $DHCDCU = -DWCDCU$. The accumulation is done as with the stable release of latent heat (6.b.15).

- **Energy conversion**

$$DHCON = 2\Delta t \frac{R}{g} \int_3 \left(1 + \frac{R_{H2O}}{R_{q_k}} q^n_k \right) T^n_k \omega^n_k dV,$$  \hspace{1cm} (6.b.17)

The conversion term is accumulated when $n = n_o + 2i - 1$. Notice that this explicit form of the conversion differs from the semi–implicit form actually used by the model. The difference is due to time truncation error.

- **Vertical temperature diffusion**

$$DHVDIF = 2\Delta t \frac{C_p}{g} \int_3 F_{TG_k}(T'^{n+1}) p^n_s dV,$$  \hspace{1cm} (6.b.18)
where $F_{TG}$ is given by (4.e.55). The vertical diffusion is accumulated when $n = n_o + 2i - 1$. $DHVDIF$ includes the sensible heat flux from the surface $DHFLUX$ and, therefore, $DHFLUX$ is not explicitly included in the energy budget. The two terms are not necessarily exactly equal because the vertical diffusion is not a priori conservative. The nonconservative aspect in practice is very small, however, as shown by comparing $DHVDIF$ and $DHFLUX$ from a model run.

- Vertical kinetic-energy dissipation and frictional heating

\[ DKEVDIF = -2\Delta t \frac{C_p}{g} \int_3 \left[ u_k^n F_{uG}(u^{n+1}) + v_k^n F_{vG}(v^{n+1}) \right] p_s^n dV, \quad (6.b.19) \]

where $F_{uG}$ and $F_{vG}$ are given by (4.e.52) and (4.e.53). The accumulation is over $n = n_o + 2i - 1$.

- Horizontal temperature diffusion

\[ DHHDIF = 2\Delta t \frac{C_p}{g} \int_3 \left[ F_{TS}(T^{n+1}) + \sigma_k \left( \frac{\partial T^{n+1}}{\partial \sigma} \right)_k K_4 \nabla^4 \ell n p_s^n \right] p_s^n dV, \quad (6.b.20) \]

where $F_{TS}$ is from the transformed values of (4.k.12) and the $\nabla^4$ component from (4.l.9) is only included at those levels where the $\nabla^4$ diffusion is applied. The accumulation is over $n = n_o + 2i$. Since the horizontal diffusion is applied to $T$, it does not a priori conserve energy.

- Horizontal kinetic-energy dissipation and frictional heating

\[ DKEHDIF = +2\Delta t \frac{C_p}{g} \int_3 \left[ \hat{F}_{uS}(u^{n+1}) + \hat{F}_{vS}(v^{n+1}) \right] p_s^n dV, \quad (6.b.21) \]

where $\hat{F}_{uS}$ and $\hat{F}_{vS}$ are from (4.l.1) and (4.l.2). The accumulation is over $n = n_o + 2i$.

The energy deficits are calculated by

\[ DPEDEF = \overline{PE}^{n_o+2N} - \overline{PE}^{n_o} + \overline{SE}^{n_o+2n} - \overline{SE}^{n_o} \]

\[ - (DHRAD + DHCDST + DHCDCU + DHCON) \]

\[ + DKEVDIF + DKEHDIF \]

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\[ + DHVDIF + DHHDIF), \quad (6.b.22) \]

\[ DKEDEF = KE^{n_0 + 2N} - KE^{n_0} \]

\[ + DHCON + DKEVDIF + DKEHDIF, \quad (6.b.23) \]

\[ DTEDEF = DPEDEF + DKEDEF. \quad (6.b.24) \]

The nonconservation arises from time truncation error, the time filter, horizontal temperature diffusion or from not properly including the variability of \( C_p \) with moisture in the energy definition, as it is in the model’s thermodynamic equation.

The mass correction applied every time step (4.o.1) is accumulated for diagnostic purposes,

\[ DMASS = DMASS + \bar{\rho}_s - \left\{ \bar{\rho}_s^n - g \sum_{k=1}^{k} [\bar{q}]_k \right\}. \quad (6.b.25) \]

The accumulation is over every time step rather than every other one, as in the budget quantities. The maximum value of the parameter \( \alpha \) associated with the global moisture borrowing (4.m.9) is also monitored,

\[ ALFMAX = \max(ALFMAX, \alpha). \quad (6.b.26) \]

This term is monitored every time step.

The maximum and minimum temperatures are also monitored every time step. These values and their locations are printed when the budget is calculated and printed.
APPENDIX 1—TERMS IN EQUATIONS

The terms of (4.g.1) are

\[ V = \bar{V}^{n-1}, \]  
(Al)

\[ V_\lambda = 2\Delta t \left[ N_v^n + \cos \phi F_v G (v^{n+1}) \right], \]  
(A2)

\[ V_\mu = 2\Delta t \left[ N_u^n + \cos \phi F_u G (u^{n+1}) \right]. \]  
(A3)

The terms of (4.g.2) are

\[ D = \bar{D}^{n-1}, \]  
(A4)

\[ D_\lambda = 2\Delta t \left[ N_u^n + \cos \phi F_u G (u^{n+1}) \right], \]  
(A5)

\[ D_\mu = 2\Delta t \left[ N_v^n + \cos \phi F_v G (v^{n+1}) \right], \]  
(A6)

\[ D_v = 2\Delta t \left[ E^n + \Phi_s + \frac{1}{2} \left( R B T_{n-1}^n + R T_o \xi n \eta_{n-1} \right) + R B (T_v^n - T^n) \right]. \]  
(A7)

The terms of (4.g.3) are

\[ T = \bar{T}^{n-1} + 2\Delta t \left[ Q_S^n + Q_{nw}^n + F_T G (T^{n+1}) + \delta^n T' + (T^2)^n \right] \]
\[ - \Delta t (\bar{\delta}^{n-1}), \]  
(A8)

\[ T_\lambda = 2\Delta t U^n T'^n, \]  
(A9)

\[ T_\mu = 2\Delta t V^n T'^n, \]  
(A10)

where a vector of the form \( \delta^n T'^n \) has elements \( \delta_k^n T'^n_k \). The term of (4.g.4) is

\[ PS = \xi n \eta_{n-1} - 2\Delta t \sum_{j=1}^{K} V_j^n \cdot \nabla \xi n \eta_{n} \Delta \sigma_j - \Delta t \sum_{j=1}^{K} \bar{\delta}_j^{-n-1} \Delta \sigma_j, \]  
(A11)

and the terms of (4.g.5) are

\[ Q = q^{n+1} + 2\Delta t \delta^n q^n - 2\Delta t \delta^n \frac{\partial q^n}{\partial \sigma}, \]  
(A12)
where the vertical advection is given the usual way (3.5.1) and

$$Q_\lambda = 2\Delta t U^n q^n,$$  \hspace{1cm} (A13)

$$Q_\mu = 2\Delta t V^n q^n.$$  \hspace{1cm} (A14)

The spectral transformation of the terms in the vorticity equation (4.i.1) is
given by

\[
V S^m_n = \sum_{j=1}^{J} \left[ V^m (\mu_j) P^m_n (\mu_j) + i m V^m (\mu_j) \frac{P^m_n (\mu_j)}{a(1 - \mu_j^2)} + V^m (\mu_j) \frac{H^m_n (\mu_j)}{a(1 - \mu_j^2)} \right] w_j.
\]  \hspace{1cm} (A15)

The spectral transformation of the explicit terms in the divergence equation (4.i.2) is

\[
D S^m_n = \sum_{j=1}^{J} \left\{ D^m (\mu_j) + \frac{n(n+1)}{a^2} D^o (\mu_j) \right\} P^m_n (\mu_j)
+ i m D^m (\mu_j) \frac{P^m_n (\mu_j)}{a(1 - \mu_j^2)} - D^m_\mu (\mu_j) \frac{H^m_n (\mu_j)}{a(1 - \mu_j^2)} \right\} w_j.
\]  \hspace{1cm} (A16)

The spectral transformation of the explicit term of the thermodynamic equation (4.i.3) is

\[
T S^m_n = \sum_{j=1}^{J} \left[ T^m (\mu_j) P^m_n (\mu_j) - i m T^m (\mu_j) \frac{P^m_n (\mu_j)}{a(1 - \mu_j^2)} + T^m (\mu_j) \frac{H^m_n (\mu_j)}{a(1 - \mu_j^2)} \right] w_j.
\]  \hspace{1cm} (A17)

The spectral transformation of the explicit terms of the surface pressure tendency equation (4.i.4) is

\[
PS^m_n = \sum_{j=1}^{J} P S^m (\mu_j) P^m_n (\mu_j) w_j.
\]  \hspace{1cm} (A18)

The spectral transformation of the terms in the mixing-ratio equation (4.i.5) is

\[
Q S^m_n = \sum_{j=1}^{J} \left[ Q^m (\mu_j) P^m_n (\mu_j) - i m Q^m (\mu_j) \frac{P^m_n (\mu_j)}{a(1 - \mu_j^2)} + Q^m (\mu_j) \frac{H^m_n (\mu_j)}{a(1 - \mu_j^2)} \right] w_j.
\]  \hspace{1cm} (A19)
## APPENDIX 2—PHYSICAL CONSTANTS

Following the American Meteorological Society convention, the model uses the International System of Units (SI) (see August 1974 *Bulletin of the American Meteorological Society*, Vol. 55, No. 8, pp. 926–930).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( 6.37122 \times 10^6 ) m</td>
<td>Radius of earth</td>
</tr>
<tr>
<td>( g )</td>
<td>( 9.80616 ) m s(^{-2} )</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>( 7.292 \times 10^{-5} ) s(^{-1} )</td>
<td>Angular velocity of rotation of sphere</td>
</tr>
<tr>
<td>( \sigma_B )</td>
<td>( 5.67 \times 10^{-8} ) W m(^{-2} ) K(^{-4} )</td>
<td>Stefan–Boltzmann constant</td>
</tr>
<tr>
<td>( R )</td>
<td>( 287.04 ) J kg(^{-1} ) K(^{-1} )</td>
<td>Gas constant for dry air</td>
</tr>
<tr>
<td>( C_p )</td>
<td>( 1.00464 \times 10^3 ) J kg(^{-1} ) K(^{-1} )</td>
<td>Specific heat capacity of dry air at constant pressure</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>( R/C_p )</td>
<td>Ratio of molecular weight of water vapor to that of dry air</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>( .622 )</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>( 2.5104 \times 10^6 ) J kg(^{-1} )</td>
<td>Latent heat of condensation</td>
</tr>
<tr>
<td>( R_v )</td>
<td>( 4.61 \times 10^2 ) J kg(^{-1} ) K(^{-1} )</td>
<td>Gas constant for water vapor</td>
</tr>
<tr>
<td>( \rho_{H2O} )</td>
<td>( 1.0 \times 10^3 ) kg m(^{-3} )</td>
<td>Density of liquid water</td>
</tr>
<tr>
<td>( C_{p_v} )</td>
<td>( 1.81 \times 10^3 ) J kg(^{-1} ) K(^{-1} )</td>
<td>Specific heat capacity of water vapor at constant pressure</td>
</tr>
</tbody>
</table>

The model code defines these constants to the stated accuracy. We do not mean to imply that these constants are known to this accuracy nor that the low-order digits are significant to the physical approximations employed.
ACKNOWLEDGMENTS

This model represents one stage in an evolutionary model-development process that was chosen to serve as a frozen basis of comparison for the next several years. Many investigators from both NCAR and the university community have contributed in various ways to this evolution.

The code itself has evolved from an adiabatic spectral model made available to us by the European Centre for Medium Range Weather Forecasts. Although the basic design of that code remains in CCM1, most details of the structure have been changed. L. Bath, M. Dias, and R. Wolski contributed substantially to this effort. B. Briegleb and P. Downey contributed to developing the radiation code. G. Williamson handled all operational running, processing, and data-archiving required during the development of CCM1.

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